Lecture 21: Dynamics

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November 3, 2021

Ch.19: Impulse and Momentum:

Previously (linear (straight) move):

$$m\mathbf{v}_1 + \int \mathbf{F} dt = m\mathbf{v}_2$$

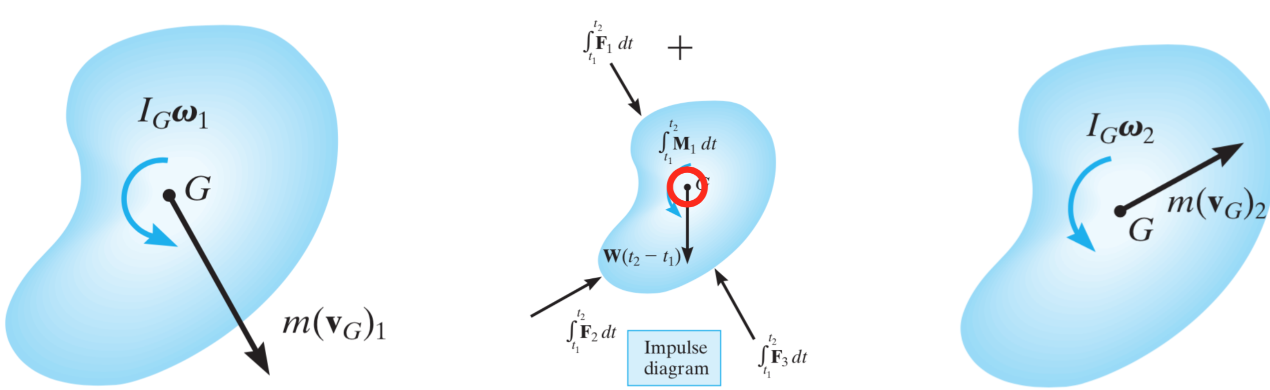
For rotation:

- Around the centroid G:

$$I_G\omega_1 + \sum \int M_G dt = I_G\omega_2$$

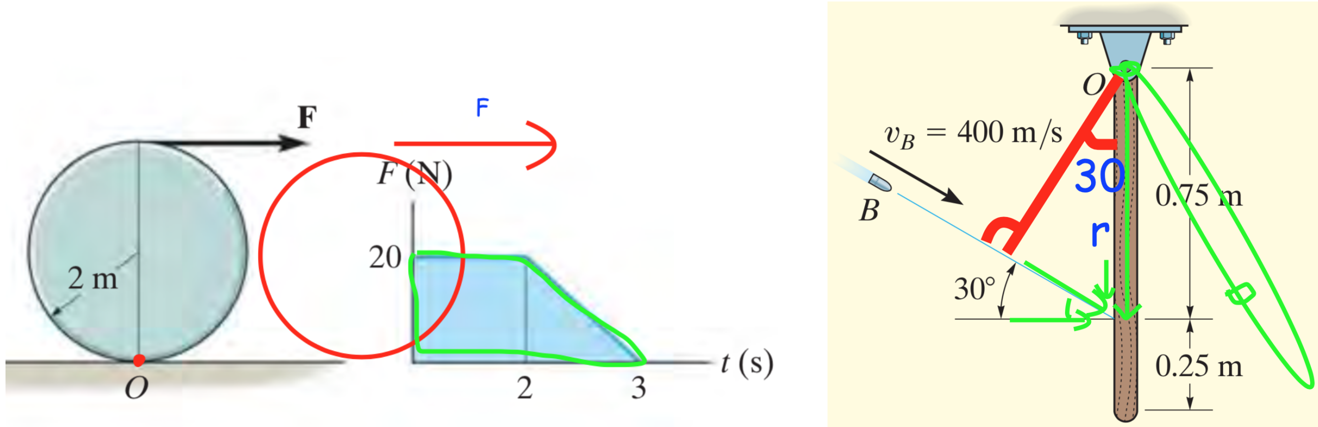
- Around the instantaneous center O:

$$I_O\omega_1 + \sum \int M_O dt = I_O\omega_2$$



(a) Initial Momentum (b) Applied Impulse (c) Resulted Momentum

Figure 1: Change in the momentum due to Impulse. (Linear + angular)



(a) P. 19-2

(b) Example 19-7

Figure 2: Solved problems.

- **P19-2:**

Angular impulse around point O: (from $t = [0, 3]$ the disk generates a line):

$$\begin{aligned}\int M_O dt &= \int_0^2 4 \times 20 dt + \int_2^3 4 \times (60 - 20t) dt \\ &= \int_0^3 4F dt = 4 \times (40 + 10) = 100 N.m.s\end{aligned}$$

- **Example 19-7**

Conservation of angular momentum H:

$$H_1 = H_2$$

$$H_{1,b} + H_{1,B} = H_2$$

$$\begin{aligned}H_{1,b,O} &= \mathbf{r} \times m\mathbf{v} \\ &= (-0.75\mathbf{j}) \times 0.004 \times (400 \cos 30\mathbf{i} - 400 \sin 30\mathbf{j}) \\ &= 1.03\mathbf{k}\end{aligned}$$

Note: for a beam rotating around the corner O:

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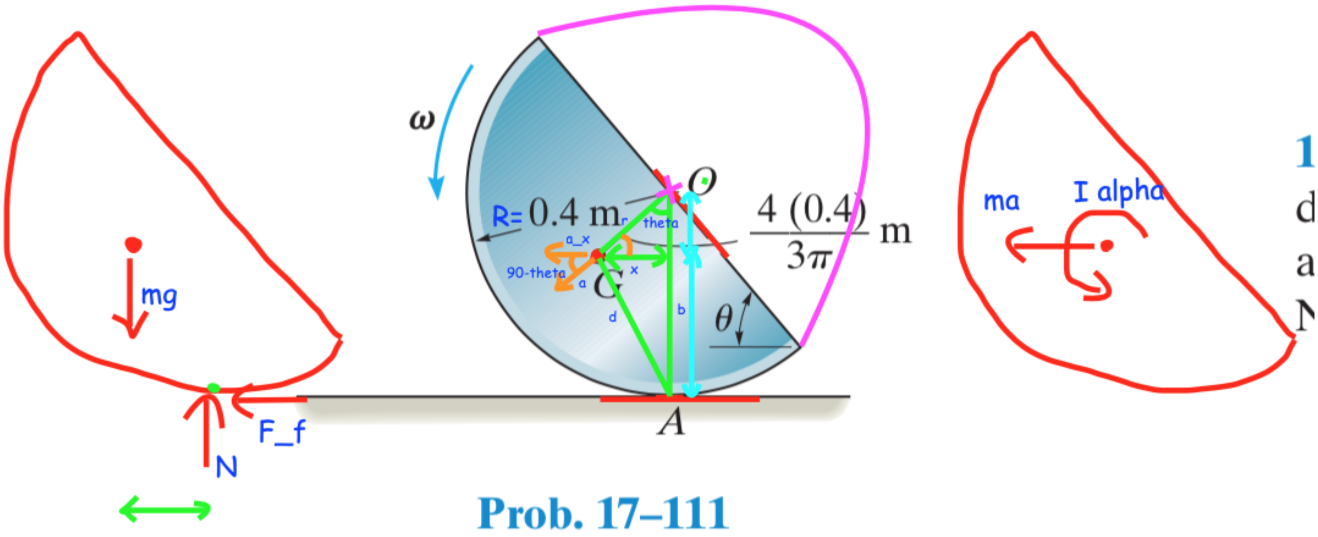

Figure 3: 17-111

$$
\begin{aligned}
I &= I_G + md^2 \\
&= \frac{1}{12}ml^2 + m\left(\frac{l}{2}\right)^2 \\
&= \frac{1}{3}ml^2
\end{aligned}
$$

$$
\begin{aligned}
H_2 = I_2\omega &= \left(\frac{1}{3}m_Bl^2 + m_bd^2\right)\omega \\
&= \left(\frac{1}{3} \times 5 \times 1^2 + 0.004 \times 0.75^2\right)\omega
\end{aligned}
$$

$$1.03 + 0 = 1.67\omega$$

$$\rightarrow \omega = 0.6\frac{rad}{s}$$

19-55: Exercise.

17-111:
$m = 10kg$
$\omega_1 = 4\frac{rad}{s}$
$\theta_1 = 60^\circ$
$\mu_s = 0.5$

Slips?
Maximum frictional force:

$$\begin{aligned} F_{f,max} &= \mu_s N = \mu_s mg \\ &= 0.5 \times 10 \times 10 = 50N \end{aligned}$$

For linear move:

$$\sum F_x = ma_x$$

$$F_f = 10a_x$$

For rotation:

$$\sum M_A = I_A \alpha$$

$$\begin{aligned} M_A &= mgx = 10 \times 10 \times \frac{4 \times 0.4}{3 \times 3.14} \times \sin 30 \\ &\approx 8.5N.m \end{aligned}$$

$$\begin{aligned} I_A &= I_G + md^2 \\ &= (I_O - mr^2) + md^2 \\ &= \frac{1}{4}mR^2 - mr^2 + md^2 \\ &= \frac{1}{4} \times 10 \times 0.4^2 - 10 \times \left(\frac{4 \times 0.4}{3 \times 3.14}\right)^2 + 10 \times \left(0.4 - \frac{4 \times 0.4}{3\pi} \cos 60^\circ\right)^2 \\ &= 0.4 - 0.29 + 0.99 = 1.1kg.m^2 \end{aligned}$$

$$\alpha = \frac{M_A}{I_A} = \frac{8.5}{1.1} = 7.72\frac{rad}{s^2}$$

$$\begin{aligned} F_{req} &= ma = mR\alpha \\ &= 10 \times 0.4 \times 7.72 \\ &= 30.9N \end{aligned}$$

$$F_{req} < F_{slip}$$

$\rightarrow$ It will hold it and will not slip.