

Graphs & Equations Worked Solutions

① Consider $(1, 5)$ and $(2, 8)$ at $x=0, y=2$

$$y = mx + c \quad m = \frac{\Delta y}{\Delta x} = \frac{(8-5)}{(2-1)} = \frac{3}{1}$$

$$\therefore Y = 3x + 2 \quad \checkmark$$

② Consider $(0, -2.5), (1, -3)$

$$y = mx + c \quad m = \frac{\Delta y}{\Delta x} = \frac{-3 - (-2.5)}{1 - 0} = \frac{-0.5}{1} = -\frac{1}{2}$$

$$\therefore Y = -\frac{1}{2}x - 2.5 \quad \checkmark$$

③ Consider $(2, 1), (4, 7), (0, -1)$

$$y = ax^2 + bx + c$$

$$(2, 1) : 1 = a(2)^2 + b(2) - 1$$

$$1 = 4a + 2b - 1$$

$$b = 1 - 2a \quad \curvearrowleft$$

$$(4, 7) : 7 = a(4)^2 + b(4) - 1$$

$$4 = 8a$$

$$a = \frac{1}{2} \quad \therefore b = 1 - 2(\frac{1}{2}) = 0$$

$$\therefore Y = \frac{1}{2}x^2 + 1 \quad \checkmark$$

④ Consider $(-3, 41), (3, 7), (2, -1), (1, -1), (0, 1)$

$$y = ax^3 + bx^2 + cx + d$$

$$\begin{aligned} ① (-3, 41) : \quad -41 &= a(-3)^3 + b(-3)^2 + c(-3) - 1 \\ -42 &= -27a + 9b - 3c \end{aligned}$$

$$\begin{aligned} ② (3, 7) : \quad 6 &= 27a + 9b + 3c \end{aligned}$$

$$\begin{aligned} ③ (2, -1) : \quad -2 &= 8a + 4b + 2c \end{aligned}$$

$$\begin{aligned} ④ (1, -1) : \quad -2 &= a + b + c \end{aligned}$$

$$\begin{aligned} ③ - 4 \times ④ : \quad -2 &= 8a + 4b + 2c \\ -8 &= 4a + 4b + 4c - \\ 6 &= 4a - 2c \\ \therefore c &= 2a - 3 \end{aligned}$$

$$\begin{aligned} ② - ①: \quad 6 &= 27a + 9b + 3c \\ -42 &= -27a + 9b + 3c - \\ 48 &= 54a + 6c \end{aligned}$$

$$48 = 54a + 6(2a - 3)$$

$$66 = 66a$$

$$\therefore a = \underline{\underline{1}}$$

if $a = 1$,

$$c = 2(1) - 3 = \underline{-1}$$

if $a = 1$ and $c = -1$

$$③ -2 = 8(1) + 4b + 2(-1)$$

$$-8 = 4b$$

$$b = -2$$

$$\therefore \boxed{y = x^3 - 2x^2 - x + 1} \checkmark$$