Sample Problems

Prove each of the following identities.

1. $\tan x \sin x + \cos x = \sec x$
2. $\dfrac{1}{\tan x} + \tan x = \dfrac{1}{\sin x \cos x}$
3. $\sin x - \sin x \cos^2 x = \sin^3 x$
4. $\dfrac{\cos \alpha}{1 + \sin \alpha} + \dfrac{1 + \sin \alpha}{\cos \alpha} = 2 \sec \alpha$
5. $\dfrac{\cos x}{1 - \sin x} - \dfrac{\cos x}{1 + \sin x} = 2 \tan x$
6. $\cos^2 x = \dfrac{\csc x \cos x}{\tan x + \cot x}$
7. $\dfrac{\sin^4 x - \cos^4 x}{\sin^2 x - \cos^2 x} = 1$
8. $\dfrac{\tan^2 x}{\tan^2 x + 1} = \sin^2 x$
9. $\dfrac{1 - \sin x}{\cos x} = \dfrac{\cos x}{1 + \sin x}$
10. $1 - 2\cos^2 x = \dfrac{\tan^2 x - 1}{\tan^2 x + 1}$
11. $\tan^2 \theta = \csc^2 \theta \tan^2 \theta - 1$
12. $\sec x + \tan x = \dfrac{\cos x}{1 - \sin x}$
13. $\dfrac{\csc \beta}{\sin \beta} - \dfrac{\cot \beta}{\tan \beta} = 1$
14. $\sin^4 x - \cos^4 x = 1 - 2\cos^2 x$
15. $(\sin x - \cos x)^2 + (\sin x + \cos x)^2 = 2$
16. $\dfrac{\sin^2 x + 4 \sin x + 3}{\cos^2 x} = \dfrac{3 + \sin x}{1 - \sin x}$
17. $\dfrac{\cos x}{1 - \sin x} - \tan x = \sec x$
18. $\tan^2 x + 1 + \tan x \sec x = \dfrac{1 + \sin x}{\cos^2 x}$

Practice Problems

Prove each of the following identities.

1. $\tan x + \dfrac{\cos x}{1+\sin x} = \dfrac{1}{\cos x}$
2. $\tan^2 x + 1 = \sec^2 x$
3. $\dfrac{1}{1-\sin x} - \dfrac{1}{1+\sin x} = 2\tan x \sec x$
4. $\tan x + \cot x = \sec x \csc x$
5. $\dfrac{1+\tan^2 x}{1-\tan^2 x} = \dfrac{1}{\cos^2 x - \sin^2 x}$
6. $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$
7. $\dfrac{1-\cos x}{\sin x} + \dfrac{\sin x}{1-\cos x} = 2\csc x$
8. $\dfrac{\sec x - 1}{\sec x + 1} = \dfrac{1-\cos x}{1+\cos x}$
9. $1 + \cot^2 x = \csc^2 x$
10. $\dfrac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$
11. $\dfrac{\cot x - 1}{\cot x + 1} = \dfrac{1-\tan x}{1+\tan x}$
12. $(\sin x + \cos x)(\tan x + \cot x) = \sec x + \csc x$
13. $\dfrac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \sin x \cos x$
14. $\dfrac{\cos x + 1}{\sin^3 x} = \dfrac{\csc x}{1-\cos x}$
15. $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} = 4\tan x \sec x$
16. $\csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$
17. $\dfrac{\sin^2 x}{\cos^2 x + 3\cos x + 2} = \dfrac{1-\cos x}{2+\cos x}$
18. $\dfrac{\tan x + \tan y}{\cot x + \cot y} = \tan x \tan y$
19. $\dfrac{1+\tan x}{1-\tan x} = \dfrac{\cos x + \sin x}{\cos x - \sin x}$
20. $(\sin x - \tan x)(\cos x - \cot x) = (\sin x - 1)(\cos x - 1)$

