1. **By equation manipulation** to get the variable on one side of the equation and numbers on the other. For example:
1.1

$\mathbf{0 = 3x - 12}$	*// add 12 to both sides, to isolate the* ***x****'s*
$\mathbf{12 = 3x}$	*// divide both sides by 3, to reduce to single* ***x***
$\mathbf{12/3 = x}$	*// then simplify*
$\mathbf{x = 4}$	

Again, using the same principle but for a more complicated equation
1.2

$\mathbf{5x = 10x/3 + 2}$	*// get all the* ***x****'s on one side*
$\mathbf{5x - 10x/3 = 2}$	*// multiply both sides by 3*
$\mathbf{15x - 10x = 6}$	*// simplify*
$\mathbf{5x = 6}$	*// divide both sides by 5*
$\mathbf{x = 6/5}$	

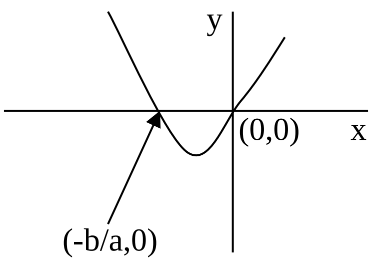
NB
This is fine when the ***only*** *power of* ***x*** *is one (linear) - there is only one answer.*
If the equation is a ***quadratic*** *(has a term* $\mathbf{x^2}$*), then there are two values for* ***x*** *that satisfy the equation. Modifying equation 1.1 to* $\mathbf{0 = 3x^2 - 12}$ *then after the same steps …*
$\mathbf{x^2 = 4}$ *so there are two solutions,* ***x = +2 or x = -2.***

2. **Iterative approximations[1].** Put a value for **x** into the equation- record the error.
Try another value- if the error increases, try one the other side of the first. If the error reduces try another- in the same direction. Keep going until you find two values that straddle the required value of the equation, then close-in to find a value of **x** that **satisfies** the equation[2]. This provides **one** solution and will work for any equation, but remember that there is only a single solution when the only **power** of **x** in the equation is one. If the degree is 2 (highest power of **x** is 2) then there are 2 solutions, degree three permits 3 solutions and so on.

3. **Factorisation.**
3.1 If there are **two terms**, write the equation with zero on one side:

$\mathbf{ax^2 + bx = 0}$	*// take the* ***common factor*** *of each term*
$\mathbf{x(ax + b) = 0}$	*// if the product =0, then one factor must =0*
x = 0, and x = -b/a	*// two solutions would satisfy this equation*

Graphic illustration:
(of $f(x)= ax^2 + bx$)



Conversely, given the above plot, it can be seen that:
$y = (x + (-b/a))*(x - 0)$

[1] This method may seem crude, but it is the method that was traditionally used to accurately calculate bond yields in the stock market (£billions of turnover per day), since they cannot be solved by any direct method.
[2] It is amenable to solving with a nested loop program (try it in 'c').

3.2 Special case, difference of squares

When an equation is the difference of two squared values (**terms**) i.e:

$$\mathbf{4x^2 - 9b^2 = 0} \; . . . . \mathbf{(1)}$$

it can be written as a product of, the difference, and the sum, of the square roots of the two terms:

(2x - 3b)(2x + 3b) = 0

so **<u>x = +3b/2 or x = -3b/2</u>** // *both **satisfy** this equation*

Another way of deriving this[3] would be to say that from (1) we have:

$4x^2 = 9b^2$
+/-2x = +/-3b *// taking square roots of both sides*
x = +/- 3b/2

3.3 Three terms:

for ax^2 + bx + c . . . .(2)

3.31 If a = 1

let **c = mn**, such that **b = (m+n)**
then
x^2 +(m+n)x + mn = 0 //substituting in (2)
(x + m)(x + n) = 0
<u>x = -m, x = -n, both satisfy the equation (2)</u>

3.32 If a != 1

let **a = pq,** and **c = rs,** such that **b = (pr + qs)**
then
pqx^2 + (ps + qr)x + rs = 0
(px + r)(qx + s) = 0
<u>x = -r/p, x = -s/q, both satisfy the equation (2)</u>

3.4 Complete the Square:

Put **'x'** terms to LHS and number value to RHS.

ax^2 + bx = -c // *divide by* **'a'**
x^2 + (b/a)x = -c/a // *add (1/2 the **x** coeff)^2 to both sides*
$x^2 + (b/a)x + (b/2a)^2 = -c/a + (b/2a)^2$ . . . . (3)
$(x + b/2a)^2$ = RHS
$x + b/2a = +/-(RHS)^{1/2}$
$x = -b/2a +/-(RHS)^{1/2}$

3.5 Formula method:

From (3) above:

$x^2 + (b/a)x + (b/2a)^2 = (-4ac + b^2)/4a^2$
$(x + b/2a)^2 = (-4ac + b^2)/ (2a)^2$
$x + b/2a = +/- (-4ac + b^2)^{1/2} / 2a$
$x = -b/2a +/- (-4ac + b^2)^{1/2} / 2a$
<u>$x = \{-b +/-(b^2 - 4ac)^{1/2}\} / 2a$</u> // *general formula*

[3] In this case it is obvious, but some times it can be harder to see.

Questions

Solve the following equations:

linear:

1. $3x - 7 = 32$
2. $(x + 2)/3 + (2x + 1)/5 = 6$
3. $2/x = 2 + 5/2x$

factorisation:

4. $8x^2 - x = 0$
5. $9x^2 - 49 = 0$
6. $x^2 -9x + 20 = 0$
7. $8x^2 + 15 = 22x$

complete the square:

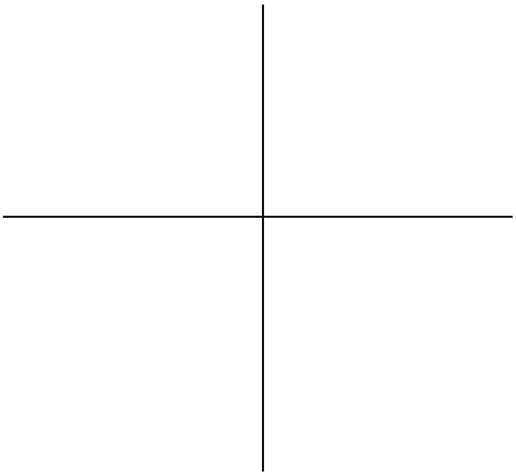
8. $x^2 - 8x = 2$
9. $4x^2 - 3x -2 = 0$

formula:

10. $x^2 + 8x + 6 = 0$

Sketch the Graphs - showing the co-ordinates of two points on each graph.

(5) $y = 9x^2 - 49$



(9) $y = 4x^2 - 3x -2$