Exercise 1.

Problem of causal judgements.
Human behaviour can be modelled rather well with a simple learning algorithm.

The example is an adapted and simplified version presented in Spellman (1996):

- two fertilising liquids in two different colours: blue and red
- counts of growing vegetables: TOMATO and NO_TOMATO (i.e., no plant whatsoever) if they were given no liquid, one of the two liquids, or both liquids
- growing pot that is considered to be a constant background (i.e., an ambient or context)

- The participants' task was to draw causal inferences from what is, in essence, a 2×2 table of possibilities (red liquid: yes/no; blue liquid: yes/no; and two outcomes – TOMATO and NO_TOMATO):

TOMATO

		Blue Liquid	
		yes	**no**
Red Liquid	**yes**		
	no		

NO_TOMATO

		Blue Liquid	
		yes	**no**
Red Liquid	**yes**		
	no		

This information can be expanded with counts of how frequently each situation occurred:

Cues	Outcomes	Frequency
pot; red; blue	TOMATO	3
pot; red; blue	NO_TOMATO	0
pot; red	TOMATO	5
pot; red	NO_TOMATO	2
pot; blue	TOMATO	3
pot; blue	NO_TOMATO	5
pot	TOMATO	0
pot	NO_TOMATO	2

The most basic way of presenting this information is to present each learning trial separately (given the number of repetitions, as specified in the column Frequency):

Trial	Cues	Outcomes
1	pot; red; blue	TOMATO
2	pot; red; blue	TOMATO
3	pot; red; blue	TOMATO
4	pot; red	TOMATO
5	pot; red	TOMATO
6	pot; red	TOMATO
7	pot; red	TOMATO
8	pot; red	TOMATO
9	pot; red	NO_TOMATO
10	pot; red	NO_TOMATO
11	pot; blue	TOMATO
12	pot; blue	TOMATO
13	pot; blue	TOMATO
14	pot; blue	NO_TOMATO
15	pot; blue	NO_TOMATO
16	pot; blue	NO_TOMATO
17	pot; blue	NO_TOMATO
18	pot; blue	NO_TOMATO
19	pot	NO_TOMATO
20	pot	NO_TOMATO

Since identical trials are now grouped (i.e., repeated as many times as Frequency indicates), we may want to shuffle them to make each trial appear at random (i.e., in an unsystematic order):

Trial	Cues	Outcomes
1	pot	NO_TOMATO
2	pot; red; blue	TOMATO
3	pot; red	TOMATO
4	pot; red	NO_TOMATO
5	pot; blue	TOMATO
6	pot; blue	NO_TOMATO
7	pot; red	TOMATO
8	pot; blue	NO_TOMATO
9	pot; red	TOMATO
10	pot; red	TOMATO
11	pot; red; blue	TOMATO
12	pot; blue	TOMATO
13	pot; red; blue	TOMATO
14	pot; blue	NO_TOMATO
15	pot; red	TOMATO
16	pot; blue	NO_TOMATO
17	pot; red	NO_TOMATO
18	pot; blue	TOMATO
19	pot	NO_TOMATO
20	pot; blue	NO_TOMATO

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Learning rule that we will now use differentiates only three possible situations and, consequently, three possible changes. These are:

(1)	The cue is absent	**nothing** happens	$\Delta w_{ij}^{t} = 0$
(2)	The cue is present; The outcome is present	**positive evidence** that should strengthen the connection weight	$\Delta w_{ij}^{t} = \gamma\left(1 - \sum w_{.j}\right)$
(3)	The cue is present; The outcome is absent	**negative evidence** that should weaken the connection weight	$\Delta w_{ij}^{t} = \gamma\left(0 - \sum w_{.j}\right)$

At each time step, what the system learns about association strength between cues (pot and liquids) and outcomes (TOMATO or NO_TOMATO) is the sum of weights at the current step (w_{ij}^{t}) plus some change that is due to the current experience, i.e., learning (Δw_{ij}^{t}):

$$w_{ij}^{t+1} = w_{ij}^{t} + \Delta w_{ij}^{t}$$

There is only one *free parameter* γ – the learning rate (or speed of learning), which typically takes on a very small value (e.g., 0.01 or even 0.001).

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$\gamma = 0.01$

1st TRIAL	**pot NO_TOMATO** $[\Delta w_{ij}^{t} = \gamma(1 - \sum w_{.j})]$ $\sum w_{.j} = 0$ $\left(1 - \sum w_{.j}\right) = (1 - 0) = 1$ $\gamma \times 1 = 0.01 \times 1 = 0.01$
	pot TOMATO?

	pot		**red**		**blue**	
Trial	**Outcome**		**Outcome**		**Outcome**	
	TOMATO	**NO_TOMATO**	**TOMATO**	**NO_TOMATO**	**TOMATO**	**NO_TOMATO**
1	0.0000	0.0100	0.0000	0.0000	0.0000	0.0000

2nd TRIAL	**pot**, **red**, **blue TOMATO** $[\Delta w_{ij}^{t} = \gamma(1 - \sum w_{.j})]$ $\sum w_{.j} = 0$ $\left(1 - \sum w_{.j}\right) = (1 - 0) = 1$ $\gamma \times 1 = 0.01 \times 1 = 0.01$
	pot NO_TOMATO $[\Delta w_{ij}^{t} = \gamma(0 - \sum w_{.j})]$ $\sum w_{.j} = 0.01$ $\gamma\left(0 - \sum w_{.j}\right) = 0.01(0 - 0.01) = -0.0001$ $w_{ij}^{t+1} = w_{ij}^{t} + \Delta w_{ij}^{t} = 0.01 + (-0.0001) = 0.0099$
	red, blue TOMATO? **red, blue NO_TOMATO**?

	pot		**red**		**blue**	
Trial	**Outcome**		**Outcome**		**Outcome**	
	TOMATO	**NO_TOMATO**	**TOMATO**	**NO_TOMATO**	**TOMATO**	**NO_TOMATO**
1	0.0000	0.0100	0.0000	0.0000	0.0000	0.0000
2	0.0100	0.0099	0.0100	-0.0001	0.0100	-0.0001

3rd TRIAL	**pot**, **red TOMATO** $[\Delta w_{ij}^{t} = \gamma(1 - \sum w_{.j})]$ $\sum w_{.j} = ?$ $\gamma\left(1 - \sum w_{.j}\right) = ?$
	pot, red NO_TOMATO $[\Delta w_{ij}^{t} = \gamma(0 - \sum w_{.j})]$

Cue and Outcome	**Change** ($\Delta w_{ij}^{t} = \gamma(1 - \sum w_{kj})$)	**Update** ($w_{ij}^{t+1} = w_{ij}^{t} + \Delta w_{ij}^{t}$)
pot → TOMATO	$0.01(1 - 0.02) = 0.0098$	$0.01 + 0.0098 = 0.0198$
red → TOMATO	$0.01(1 - 0.02) = 0.0098$	$0.01 + 0.0098 = 0.0198$
pot → NO_TOMATO	$0.01(0 - 0.0098) = -0.000098$	$0.0099 - 0.000098 = 0.009802$
red → NO_TOMATO	$0.01(0 - 0.0098) = -0.000098$	$-0.0001 - 0.000098 = -0.000198$

Trial	**pot**		**red**		**blue**	
	Outcome		**Outcome**		**Outcome**	
	TOMATO	**NO_TOMATO**	**TOMATO**	**NO_TOMATO**	**TOMATO**	**NO_TOMATO**
1	0.0000	0.0100	0.0000	0.0000	0.0000	0.0000
2	0.0100	0.0099	0.0100	-0.0001	0.0100	-0.0001
3	0.0198	0.0098	0.0198	-0.0002	0.0100	-0.0001