

Midterm

Multiple Choice [2 points each]

- 1) a -> De Morgan's Law
- 2) d -> Exclusive Disjunction
- 3) e -> Transposition Logic (Contrapositive)
- 4) a -> Union (Venn Diagram) Everything that is in either of the sets
- 5) c -> Intersection (Venn Diagram) Only the things that are in both of the sets
- 6) e -> The function is neither one-to-one or onto because values less than -1 on the y-axis are never used. It passes the horizontal line test, but since possible y-values belong to the set of \mathbf{R} and $f(x) \in \mathbf{R}$, not all possible values are used
- 7) a, c, d -> $20 \bmod 60 = .333$; all $80 \bmod 60$ has remainder of .333 as well as 380 and 800 mod 60.

Using the Grey Matter a Little Bit [4 point each]

- 8) Nothing much can be said about Steve getting a scholarship. Although the argument is valid, it has no bearing on whether any of the statements in the argument are true. It is a Modus Ponens argument ($P \rightarrow Q \wedge Q \rightarrow S$). Steve could have gotten a scholarship by different means (i.e. he got a scholarship because he scored high on a written test) other than by training hard (or not) and by winning the game (or not). It is a one direction argument; therefore Steve could have gotten a scholarship by training hard and winning the game, but also by other means..
For example, if the argument were to be an "If and only if" (Biconditional proposition \leftrightarrow), then you could say that Steve got a scholarship only because he trained hard and won the game, but it's not "if and only if"; therefore, nothing much can be said about Steve.

- 9) a) $365(\text{days}) \times 10(\text{years}) = 3650 \text{ days}$;
 $3650 \bmod 7(\text{days of week}) = 3$;
 Therefore, 3 days after Thursday would be on a **Sunday**.

- b) $365 * 10 = 3650$;
 $3650 + 3 = 3653$; // Plus 3 leap years
 $3653 \bmod 7 = 6$;
 Therefore, 6 days after Thursday would be on a **Wednesday**.

- c) $3000 - 2015 = 985 \text{ years}$;
 $985 / 4 = 246 \text{ possible leap years}$;
 $985 * 365 = 359525 \text{ days}$;
 $246 - 8 = 238 \text{ days extra}$; // -8 because there are 8 years that are NOT leap years
 (2100, 2200, 2300, 2500, 2600, 2700, 2900, 3000)
 $359525 + 238 = 359763 \text{ total days}$;
 $359763 \bmod 7 = 5$;
 Therefore, 5 days after Thursday would be on a **Tuesday**.

$$10) \frac{f(n)}{g(n)} = \frac{n^4 + 3n^3 + 6n^2 + 3n + 1}{n^4} < \frac{n^4 + 3n^4 + 6n^4 + 3n^4 + n^4}{n^4} = 14$$

Choose $C = 14$, $k = 1$

Thus, $n^4 + 3n^3 + 6n^2 + 3n + 1$ is $O(n^4)$
because $n^4 + 3n^3 + 6n^2 + 3n + 1 \leq 14n^4$ whenever $n > 1$ ($n > k$)

```
11) float StandardDeviation(int numbers[], int size) {
    float sqr_sum = 0; float mean = 0;
    for (i=0; i<size; i++)
        sqr_sum += numbers[i]*numbers[i];
    for (i=0; i<size; i++)
        mean += numbers[i];
    mean /= size;
    return sqrt(sqr_sum/size - pow(mean, 2)); }
```

The Big-Theta runtime in this case is n^2 and the Space usage is n because of the pow and square root functions.

if $n = 100$ # of Iterations = 1,000,000
 Runtime = n^2
 Space Usage = n

Short Answer (Maybe not so much) [4 point each]

12)	A	B	I	$A \wedge B$	$\neg(A \wedge B)$	$\neg A \vee \neg B$
	T	T		T	F	F
	T	F		F	T	T
	F	T		F	T	T
	F	F		F	T	T

- 13) I would use the Induction Proof by Base Case = 1 to prove the statement, and the Induction proof by $n = k + 1$ to disprove the statement.

Let $n =$ "I just ate", $k =$ "Therefore there is no world hunger"

Proof by Base Case: 1 (Prove)

Prove $n = k$;

If $n = 1$, then $k = 1$; Therefore $n = k$ is true by Base Case: 1

Proof by Induction (Disprove)

Assume true for $n = k$;

Let $n = k + 1$;

If $n = 1$, then $k = 2$; Therefore $n \neq k$. Disproving it by Induction.

- 14) Proof by

Base Case: 1

Let $n = 1 \Rightarrow 1 = [n(n+1)]/2 \Rightarrow 1 = 2/2 \Rightarrow 1 = 1 \checkmark$

Induction Step

Assume true for $n = k$

Assume true for $n = k + 1 \Rightarrow n = [k(k+1)]/2$ is true

$1 + 2 + \dots + k + (k+1) = [(k+1)(k+2)]/2$

$[k(k+1)]/2 + (k+1) = [k^2 + 3k + 2]/2$

$(k/2)^2 + (3/2)k + 1 = (k/2)^2 + (3/2)k + 1 \checkmark$

Alternative, one can use the Direct Proof to prove that

$$1 + 2 + 3 + \dots + (n-2) + (n-1) + n = [n(n+1)] / 2$$

15)

3	1	0	3	0
625	125	25	5	1
1875	125	0	15	0

Multiply by the number above
 $\Rightarrow 1875 + 125 + 0 + 15 + 0$
 $= 2015$

Therefore, **31030₅** is **2015₁₀**

16) $0 \equiv (3 \cdot a_1 + a_2 + 3 \cdot a_3 + a_4 + 3 \cdot a_5 + \dots + a_{10} + 3 \cdot a_{11} + a_{12}) \bmod 10$

```
bool ValidateCode(int upc_code[12]);
{
    int sum = 0;
    int checkdigit = 0;
    for (int i=0; i < 12; i++)
    {
        if ( i % 2 == 0 )
            sum += ( 3 * upc_code[i]);
        else
            sum += upc_code[i];
    }
    checkdigit = sum % 10;
    if (checkdigit == 0)
        return true;
    else
        return false;
}
```

17)

18)

```
#include <iostream>
#include <string>

using namespace std;

int main()
{
    string msg;
    int length, before = 0;

    cout << "Enter your phrase: \n";
    getline(cin, msg);

    length = (int)msg.length();

    // TO DECRYPT IT

    for (int element = 0; element < length; element++)
    {
```

```
        if(isalpha(msg[element]))
        {
            msg[element] = toupper(msg[element]);
            cout << (char)((((int)msg[element] - 65 - before) + 26) % 26) + 65);
            before = (int)msg[element] - 65;
        }
    }

    cout << endl;

    return 0;
}
```