CISC-820 Project 4: Dimensionality Reduction & Classification

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1 Objective

This report explores the use of Principal Component Analysis (PCA) for reducing the dimensions of facial images. The dataset consists of 400 images from 40 people with 10 images per person. PCA using Singular Value Decomposition (SVD) is used to reduce the dimensionality by identifying eigenfaces, which represent key facial features. The report focuses on how PCA can compress data while preserving important details and how it can be used to reconstruct images using different numbers of eigenfaces. It examines the visual appearance of the eigenfaces and discusses their decreasing contribution as the number of eigenfaces increases. Additionally, the report investigates the reconstruction quality of facial images with PCA and the minimum number of eigenfaces required to achieve a reasonable reconstruction error.

2 Using PCA to Extract Eigenfaces

2.1 How Do the Leading Eigenfaces Look as an Image?

The first few eigenfaces extracted using PCA show increasingly recognizable facial features. For example, the first eigenface appears as a blurry image resembling the outline of a face with general features like the chin, eyes, and hair (Figure 1). As we move to higher eigenfaces, details like cheekbones, eyebrows, and ear shapes become clearer. These eigenfaces represent important facial structures and can be combined to reconstruct the original images.

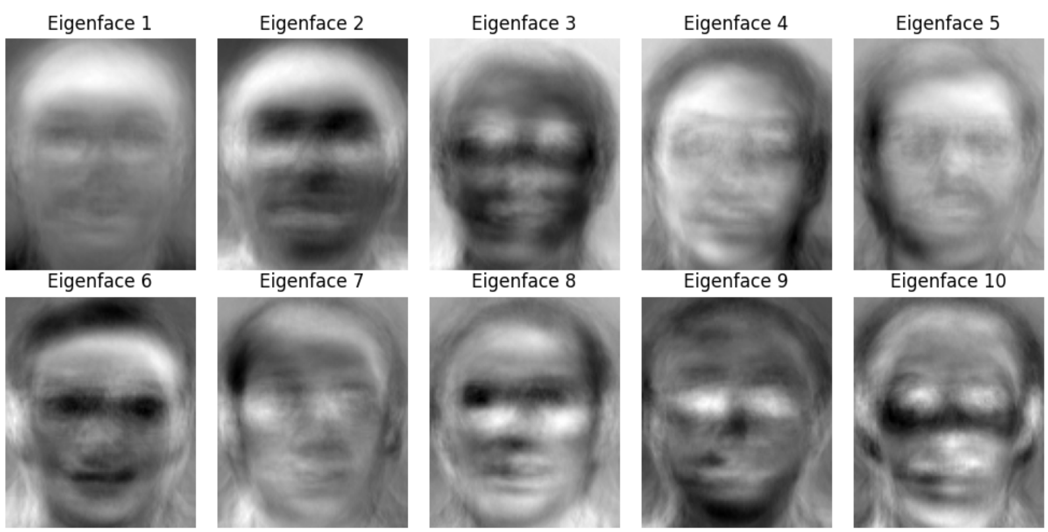
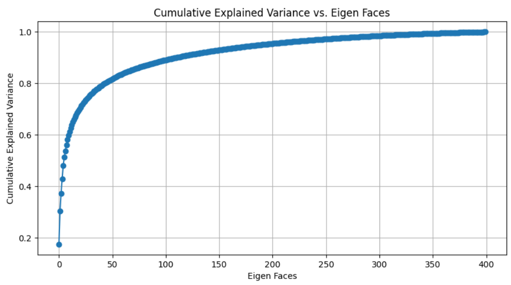


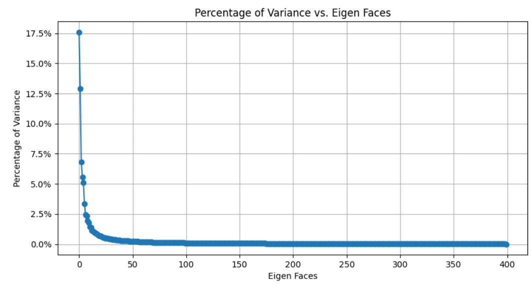
Figure 1: Example Image of Eigenfaces

2.2 How Does the Importance of Eigenfaces Decrease?

We computed the first 50 eigenfaces from the 400 facial images, capturing 81.61% of the total variance. As more eigenfaces were included, the amount of variance they explained decreased. For example, the first 100 eigenfaces explained 89.06% of the variance, meaning the second set of 50 eigenfaces only added 7.4% to the total. The first 400 eigenfaces captured 99.42% of the variance. Beyond this, adding more eigenfaces contributed little to the total variance. This indicates that the most important features are captured by the first few eigenfaces, with diminishing returns as we add more (Figure 2).



(a) Cumulative Explained Variance



(b) Percentage of Variance

Figure 2: Variance Captured by Eigenfaces

2.3 Explanation of Variance

The variance captured by the eigenfaces is an important aspect of PCA. The following graphs illustrate how the variance is distributed among the eigenfaces:

- **Cumulative Explained Variance:** This graph (Figure 2a) shows the cumulative variance explained by the eigenfaces as more eigenfaces are included. It helps us understand how many eigenfaces are needed to capture a significant portion of the total variance. For example, the first 50 eigenfaces capture 81.61% of the total variance, while the first 100 eigenfaces capture 89.06%. The first 400 eigenfaces capture 99.42% of the total variance.
- **Percentage of Variance:** This graph (Figure 2b) shows the percentage of variance explained by each eigenface. It helps us identify the most important eigenfaces that contribute the most to the total variance. The first few eigenfaces capture a large portion of the variance, while the contribution of each subsequent eigenface decreases.

3 Facial Image Reconstruction with PCA

3.1 Difference Between Reconstructed and Original Images as the Number of Eigenfaces Increases

As we use more eigenfaces for reconstruction, the quality of the reconstructed images improves. With fewer eigenfaces (e.g., 10, 50), the reconstructed images are blurry and lack detail. As the number of eigenfaces increases (e.g., 150, 250, 350, 400), the reconstructed images closely resemble the original images. Using 400 eigenfaces captures nearly all the variance, resulting in a reconstruction that closely matches the original image. This demonstrates that higher-dimensional representations are necessary to preserve fine details in facial images (Figure 3).

3.2 Number of Eigenfaces Required to Recover an Original Face with Reasonable Errors

We assessed the reconstruction quality using the Mean Squared Error (MSE) between the original and reconstructed images. We found that using 350 eigenfaces gave us a reconstruction with a low MSE of 8.9894, meaning the reconstruction was very close to the original. Therefore, using around 350 eigenfaces is sufficient to achieve an accurate reconstruction with minimal error.

(a) (b)

Figure 3: Reconstructed facial images using different numbers of eigenfaces

4 Classification Results

In classifying the images, we divided the images into a subset of 35 classes and then 8 images per class. We kept the remaining 2 images as our test images. We used KNN, Logistic Regression, and Custom Linear Regression as our classification methods. The K-Nearest Neighbors (KNN) method achieved an accuracy of 99.25%. Logistic Regression performed exceptionally well with an accuracy of 100.00%. However, the Custom Linear Regression method showed significantly lower accuracy at 7.00%. This could be due to the continuous nature of the predictions made by the linear regression model, which may not align well with the discrete class labels required for classification tasks.

5 Conclusion

In this report, we demonstrated how PCA using SVD can be used to reduce the dimensionality of facial images from 10,304 features down to around 350, while still maintaining a low reconstruction error. We found that 400 eigenfaces provide near-perfect reconstruction, capturing 99.99% of the total variance. We also discussed how eigenfaces represent key facial features, such as the eyes, mouth, and chin, and how they can be combined to reconstruct images with varying levels of accuracy. This study highlights the effectiveness of PCA for dimensionality reduction in face recognition and reconstruction tasks.

The classification results showed that KNN and Logistic Regression performed exceptionally well, with accuracies of 99.25% and 100.00%, respectively. However, the Custom Linear Regression method had a significantly lower accuracy of 7.00%, indicating that it may not be suitable for this classification task.