# FE 918
# Advanced Topics in Stochastic Calculus
Fall Semester 2023

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Meets on Mondays from 12:30 to 15:15 in room HAR 603.

This is a standalone PhD-level course in Stochastic Calculus. It covers the most advanced version of this subject and follows very closely the landmark books "*Probablités et Potentiel, Chapitres V à VII: Theorie des Martingales*" by Dellacherie et Meyer, "*Limit Theorems for Stochastic Processes*" by Jacod and Shiryaev, and "*Calcul Stochastique et Problèmes de Martingales*" by J. Jacod, in addition to the landmark papers on the semimartingale topology by M. Emery. The main objective is to provide an in-depth knowledge of the theory of semimartingales. One benefit from this course is that students will acquire the necessary prerequisites to follow up with the Delbaen-Schachermayer results on the *Fundamental Theorem of Asset Pricing*, in addition to other closely related recent papers by Yu. Kabanov and K. Kardaras. Students will have access to the preliminary (draft) version of a graduate textbook on the subject that contains a large number of exercises. Detailed slides will be provided.

**Prerequisites:** Real Analysis and Measure-Theoretic Probability Theory. Some knowledge of Functional Analysis will be helpful. Motivated students who have not taken courses on these subjects at a sufficiently high level may still be able to follow, provided they can quickly learn the needed material on the go.

**Tentative list of topics** (some may have to be skipped in the interest of time):

1. Optional Times, Debuts, and Filtrations
2. Optional and Predictable Projections
3. Supermartingales and Local Martingales
4. Processes of Finite Variation and Dual Projections
5. Doob-Meyer Decomposition
6. Quasimartingales and Rao's Decomposition Theorem
7. Semimartingales and their Invariance Properties
8. Bichteler-Dellacherie-Mokobodzky Theorem
9. Stochastic Integrals with Predictable Integrands
10. The Square Bracket
11. Young Duality, Garsia-Neveu's Theorem, and Other Inequalities
12. Itô-Tanaka-Meyer Formulas and Girsanov's Theorem
13. Emery's Topology
14. Stochastic Integration in Higher Dimensions
15. Compensated Stochastic Integrals
16. Extremal Laws and Predictable Representation
17. Random Measures
18. Semimartingale Characteristics
19. Martingale Problems
20. Stability of Solutions to SDEs

**Grading**: Midterm (Monday, October 30) 40% and Final exam 50%.