

1 GENERAL INFORMATION

1.1 Objectives

This course provides a comprehensive and in-depth treatment of modern asset pricing theories. Extensive use is made of continuous time stochastic processes, stochastic calculus and optimal control. Particular emphasis will be placed on (i) consumption-portfolio choice problems and (ii) equilibrium asset pricing models. Advances involving nonseparable preferences, incomplete information, incomplete markets and agents diversity will be discussed.

1.2 Class material

- Problem Sets: on Questrom Tools.
- Textbooks:
 - Methods of Mathematical Finance, I. Karatzas and S. Shreve, Springer-Verlag, 1999.
 - Lectures on the Mathematics of Finance, I. Karatzas, CRM Monograph series, Montreal 1998.
 - Brownian Motion and Stochastic Calculus, I. Karatzas and S. Shreve, Springer-Verlag 1987.
 - Dynamic Asset Pricing Theory, D. Duffie, Princeton University Press, Third Edition, 2001.
- Other references.
 - Security Markets, D. Duffie, Academic Press, 1988.
 - Continuous Time Finance, R. Merton, Blackwell, 1992.
 - Stochastic Methods in Economics & Finance, A. Malliaris & W. Brock, North-Holland, 1982.
 - Stochastic Differential Equations: Theory and Applications, L. Arnold, Wiley and Sons, 1974.
 - Statistics of Random Processes I and II, R. Liptser and A. Shiriyayev, Springer-Verlag, 1977.
 - Deterministic & Stochastic Optimal Control, W. Fleming & R. Rishel, Springer-Verlag, 1975.
 - Diffusions, Markov Processes, Martingales: Ito Calculus, C. Rogers & D. Williams, Wiley, 1987.
 - Calcul Stochastique et Problemes de Martingales, J. Jacod, Springer-Verlag, 1979.
 - Stochastic Integration & Differential Eq.: a New Approach, P. Protter, Springer-Verlag, 1990.
 - Continuous Martingales and Brownian Motion, Revuz and M. Yor, Springer-Verlag, 1991.
 - The Malliavin Calculus and Related Topics, D. Nualart, Springer-Verlag, 1995.

2 STOCHASTIC PROCESSES & STOCHASTIC CALCULUS

- Duffie: Chapter 4, section 21.
 - Brock and Malliaris: Chapters 1 and 2.
 - Karatzas and Shreve:
 - Sections 1.1, 1.3, 1.5: stochastic processes, definitions.
 - Sections 2.1, 2.5: brownian motion.
 - Sections 3.1, 3.2 (A-C), 3.3 (A,B), 3.4 (D), 3.5 (A): stochastic integrals.
 - Sections 4.1, 4.4, 5.1, 5.2: partial and stochastic differential equations.
1. Clark, J.M.C. “The Representation of Functionals of Brownian Motion as Stochastic Integrals,” *Annals of Mathematical Statistics*, 41, 1970: 1282-1295.
 2. Girsanov, I.V. “On Transforming a Certain Class of Stochastic Processes by Absolutely Continuous Substitution of Measures,” *Theory of Probability and its Applications*, 5, 1960: 285-301.
 3. Merton, R.C. “On the Mathematics and Economic Assumptions of Continuous-Time Financial Models,” in *Financial Economics: Essays in Honor of Paul Cootner*, edited by W. F. Sharpe and C. M. Cootner. Englewood Cliffs, N.J.: Prentice Hall, 1982. (Chapter 3 in Continuous-Time Finance.)
 4. Merton, R. “Theory of Finance from the Perspective of Continuous Time,” *Journal of Financial and Quantitative Analysis*, 10, 1975: 659-674.

3 OPTIMAL CONSUMPTION-PORTFOLIO POLICIES

3.1 The Standard Model

1. Cox, J.C. and C. Huang. “A Variational Problem arising in Financial Economics,” *Journal of Mathematical Economics*, 20, 1991: 465-487.
2. * Cox, J.C. and C. Huang. “Optimal Consumption and Portfolio Policies when Asset Prices follow a Diffusion Process,” *Journal of Economic Theory*, 49, 1989: 33-83.
3. Fischer, S. “The Demand for Index Bonds,” *Journal of Political Economy*, 83, 1975: 509-534.
4. * Karatzas, I., J.P. Lehoczky and S.E. Shreve. “Optimal Portfolio and Consumption Decisions for a ‘Small Investor’ on a Finite Horizon,” *SIAM Journal of Control and Optimization*, 25, 1987: 1557-1586.
5. * Merton, R.C. “Lifetime Portfolio Selection under Uncertainty: the Continuous Time Case,” *Review of Economic Studies*, 51, 1969: 247-257.
6. * Merton, R.C. “Optimum Consumption and Portfolio Rules in a Continuous Time Model,” *Journal of Economic Theory*, 3, 1971: 373-413.

3.2 Optimal Portfolios

1. * Detemple, J.B., R. Garcia and M. Rindisbacher, “A Monte-Carlo Method for Optimal Portfolios,” *Journal of Finance*, 58, 2003: 401-446.
2. Detemple, J., R. Garcia and M. Rindisbacher, “Intertemporal Asset Allocation: a Comparison of Methods,” *Journal of Banking and Finance*, 29, 2005: 2821-2848.

3. Detemple, J., R. Garcia and M. Rindisbacher, "Representation Formulas for Malliavin Derivatives of Diffusion Processes," *Finance and Stochastics*, 9, 2005: 349-367.
4. Detemple, J. and M. Rindisbacher, "Dynamic Asset Allocation: Portfolio Decomposition Formula and Applications," *Review of Financial Studies*, 23, 2010: 25-100.
5. Karatzas, I., Ocone, D. and J. Li, "An Extension of Clark's Formula," *Stochastics and Stochastics Reports*, 37, 1991: 127-131.
6. Nualart, D. and E. Pardoux, "Stochastic Calculus with Anticipating Integrands," *Probability Theory and Related Fields*, 78, 1988: 535-581.
7. Ocone, D., "Malliavin's Calculus and Stochastic Integral representation of Functionals of Diffusion Processes," *Stochastic*, 12, 1984: 161-185.
8. Ocone, D., "A Guide to the Stochastic Calculus of Variations," *Lecture Notes in Mathematics*, 1316, 1988: 1-79.
9. * Ocone, D. and I. Karatzas. "Representation of Optimal Portfolios via Clark's Formula," *Stochastics and Stochastics Reports*, 34, 1991: 187-220.
10. Pliska, S. "A Stochastic Calculus Model of Continuous Trading: Optimal Portfolios," *Mathematics of Operations Research*, 11, 1986: 371-382.

3.3 Nonseparable Preferences

1. Bodie, Z., J.B. Detemple, S. Otruba and S. Walter, "Optimal Consumption-Portfolio Choices and Retirement Planning," *Journal of Economic Dynamics and Control*, 28, 2004: 1115-1148.
2. * Detemple, J.B. and I. Karatzas, "Non-Addictive Habits: Optimal Consumption-Portfolio Policies," *Journal of Economic Theory*, 113, 2003: 265-285.
3. Detemple, J.B. and F. Zapatero. "Optimal Consumption-Portfolio Policies with Habit Formation," *Mathematical Finance*, 2, 1992: 35-58.
4. * Hindy A. and C. Huang, "Optimal Consumption and Portfolio Rules with Durability and Local Substitution," *Econometrica*, 61, 1993: 85-121.
5. Kreps, D. and E. Porteus. "Temporal von Neumann-Morgenstern and Induced Preferences" *Journal of Economic Theory*, 20, 1979: 81-109.
6. Kreps, D. and E. Porteus. "Dynamic Choice Theory and Dynamic Programming," *Econometrica*, 47, 1979: 91-100.
7. Pollak, R.A. "Habit Formation and Dynamic Demand Functions," *Journal of Political Economy*, 78, 1970: 745-763.
8. Ryder, H.E. and G.M. Heal. "Optimal Growth with Intertemporally Dependent Preferences," *Review of Economic Studies*, 40, 1973: 1-33.
9. Selden, L. "A New Representation of Preferences over 'Certain x Uncertain' Consumption Pairs: The 'Ordinal Certainty Equivalence Hypothesis,'" *Econometrica*, 46, 1978: 1045-60.
10. Sundaresan, S.M. "Portfolio Selection with Intertemporally Dependent Preferences," First Boston Working Paper #87-02, Columbia University, Graduate School of Business, 1984.

3.4 Incomplete Markets, Portfolio and Liquidity Constraints

1. * Cuoco, D. "Optimal consumption and Equilibrium Prices with Portfolio Constraints and Stochastic Income," *Journal of Economic Theory*, 72, 1997: 33-73.
2. Cuoco D. and J. Cvitanic, "Optimal Consumption Choices for a 'Large' Investor," *Journal of Economic Dynamics and Control*, 22, 1998: 401-436.
3. * Cvitanic, J. and I. Karatzas. "Convex Duality in Constrained Portfolio Optimization," *Annals of Applied Probability*, 2, 1992: 767-818.
4. Detemple, J. and A. Serrat, "Dynamic Equilibrium with Liquidity Constraints," *Review of Financial Studies*, 16, 2003: 597-629.
5. Detemple, J. and M. Rindisbacher, "Closed Form Solutions for Optimal Portfolio Selection with Stochastic Interest Rate and Investment Constraints," *Mathematical Finance*, 15, 2005: 539-568.
6. He, H. and H.F. Pages. "Labor Income, Borrowing Constraints and Equilibrium Asset Prices: a Duality Approach," *Economic Theory*, 3:663-696.
7. * He, H. and N.D. Pearson. "Consumption and Portfolio Policies with Incomplete Markets and Short-Sale Constraints: The Infinite-Dimensional Case," *Journal of Economic Theory*, 54, 1991: 259-304.
8. Jouini E. and H. Kallal, "Portfolio Choice and Market Frictions," Working Paper, Stern School of Business, New York University, 1993.
9. * Karatzas, I., J.P. Lehoczky, S.E. Shreve and G-L. Xu. "Martingale and Duality Methods for Utility Maximization in an Incomplete market," *SIAM Journal of Control and Optimization*, 29, 1991: 702-730.

3.5 Transaction Costs

1. Davis, M. and A. Norman. "Portfolio Selection with Transaction Costs," *Mathematics of Operations Research*, 15, 1990: 676-713.
2. Magill, M. and G. Constantinides. "Portfolio Selection with Transaction Costs," *Journal of Economic Theory*, 1976.

3.6 Incomplete Information - Anticipative Information

1. Detemple, J.B. "Asset Pricing in a Production Economy with Incomplete Information," *Journal of Finance*, 41, No. 2, 1986: 383-391.
2. Detemple, J.B. "Further Results on Asset Pricing with Incomplete Information," *Journal of Economic Dynamics and Control*, 15, No. 3, 1991: 425-453.
3. Detemple, J.B. "Demande de Portefeuille et Politique de Couverture de Risque sous Information Incomplete," *Actualite Economique*, 69, 1993: 45-70.
4. * Dothan, M.U. and D. Feldman. "Equilibrium Interest Rates and Multiperiod Bonds in a Partially Observable Economy," *Journal of Finance*, 41, 1986: 369-382.
5. * Gennotte, G. "Optimal Portfolio Choice under Incomplete Information," *Journal of Finance*, 41, 1986: 733-746.
6. * Detemple, J. and M Rindisbacher. "A Structural Model of Dynamic Market Timing," *Review of Financial Studies*, 26, 2013, 2492-2547

4 ASSET PRICING MODELS

4.1 Separable vNM Preferences

1. * Breeden, D.T. "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," *Journal of Financial Economics*, 7, 1979: 265-296 .
2. Brock, W.A. "Asset Pricing in a Production Economy," in *Economics of Information*, J. McCall editor, University of Chicago Press, 1982: 1-43.
3. Cornell, B. "The Consumption Based Asset Pricing Model: a Note on Potential Tests and Applications," *Journal of Financial Economics*, 9, 1981.
4. * Cox, J.C., J.E. Ingersoll and S.A. Ross. "An Intertemporal General Equilibrium Model of Asset Prices," *Econometrica*, 53, 1985: 363-384.
5. Duffie, D. "Stochastic Equilibria: Existence, Spanning Number and the "No Expected Gains from Trade" Hypothesis," *Econometrica*, 54, 1986: 1161-1183.
6. Duffie, D. and C. Huang. "Implementing Arrow-Debreu Equilibria by Continuous Trading of Few Long-Lived Securities," *Econometrica*, 53, 1985: 1337-1356.
7. * Duffie, D. and W. Zame. "The Consumption-Based Capital Asset Pricing Model," *Econometrica*, 57, 1989: 1279-1297.
8. Grossman, S.J. and R.J. Shiller. "The Determinants of the Variability of Stock Market Prices," *American Economic Review*, 71, 1981: 222-227.
9. Hall, R.E. "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy*, 86, 1978: 971-987.
10. * Harrison, J.M. and D. Kreps. "Martingales and Arbitrage in Multiperiod Security Markets," *Journal of Economic Theory*, 20, 1979: 381-408.
11. Harrison, J.M. and S. Pliska. "Martingales and Stochastic Integrals in the Theory of Continuous Trading," *Stochastic Processes and their Applications*, 11, 1981: 215-260.
12. He, H. and H. Leland. "On Equilibrium Asset Price Processes," *Review of Financial Studies*, 6, 1993: 593-617.
13. Hellwig, M.F. "On the Validity of the Intertemporal Capital Asset Pricing Model," Core Discussion Paper #774, 1977.
14. Huang, C. "Information Structures and Viable Price Systems," *Journal of Mathematical Economics*, 14, 1985: 215-240.
15. * Huang, C. "An Intertemporal General Equilibrium Asset Pricing Model: the Case of Diffusion Information," *Econometrica*, 55, 1987: 117-142.
16. Karatzas, I., J.P. Lehoczky and S.E. Shreve. "Existence, Uniqueness of Multi-Agent Equilibrium in a Stochastic, Dynamic Consumption/Investment Model," *Math. of Operations Research*, 15, 1990: 80-128.
17. Kreps, D. "Arbitrage and Equilibrium in Economies with Infinitely many Commodities," *Journal of Mathematical Economics*, 8, 1981: 15-35.
18. Lucas, R.E. "Asset Prices in an Exchange Economy," *Econometrica*, 46, 1978: 1429-45.
19. * Merton, R.C. "An Intertemporal Capital Asset Pricing Model," *Econometrica*, 41, 1973: 867-87.

20. Mehra, R. and E.C. Prescott. "The Equity Premium: a Puzzle," *Journal of Monetary Economics*, 15, 1985, 145-161.
21. Wang, J. "The Term Structure of Interest Rates in a Pure Exchange Economy with Heterogeneous Investors," *Journal of Financial Economics*, 41, 1996: 75-110.

4.2 Nonseparable Preferences

1. Abel, A. "Asset Prices under Habit Formation and Catching up with the Jones," *American Economic Review*, 80, 1990: 38-42.
2. Bergman, Y.Z. "Time Preference and Capital Asset Pricing Models," *Journal of Financial Economics*, 14, 1985: 145-159.
3. * Constantinides, G.M. "Habit Formation: a Resolution of the Equity Premium Puzzle," *Journal of Political Economy*, 98, 1990: 519-543.
4. * Detemple, J.B. and F. Zapatero. "Asset Prices in an Exchange Economy with Habit Formation", *Econometrica*, 59, 1991: 1633-1657.
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6. Duffie, D. and C. Skiadas. "Continuous Time Security Pricing: A Utility Gradient Approach," *Journal of Mathematical Economics*, 1993.
7. Epstein, L.G. and S. Zin. "Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: a Theoretical Framework," *Econometrica*, 57, 1989: 937-969.
8. Heaton, J. "The Interaction between Time-nonseparable Preferences and Time Aggregation," *Econometrica*, 61, 1993: 353-385.
9. Hindy, A. and C. Huang. "On Intertemporal Preferences with a Continuous Time Dimension II: The Case of Uncertainty," *Econometrica*, 60: 1992: 781-801.
10. * Sundaresan, S.M. "Intertemporally Dependent Preferences and the Volatility of Consumption and Wealth," *Review of Financial Studies*, 2, 1989: 73-89.

4.3 Heterogeneity, information and beliefs

1. Abel, A. B. "Asset Pricing under Heterogeneous Beliefs: Implications for the Equity Premium," Working Paper, University of Pennsylvania, 1989.
2. Basak, S. "A Model of Dynamic Equilibrium Asset Pricing with Heterogeneous Beliefs and Extraneous Risk," *Journal of Economic Dynamics and Control*, 24, 2000: 63-95.
3. Detemple J.B., "Asset Pricing with Asymmetric Information: the Case of Conditionally Gaussian Information Structures," *Finance*, 24, 2003: 15-44.
4. Detemple, J.B., "Asset Pricing in an Intertemporal Partially-Revealing Rational Expectations Equilibrium," *Journal of Mathematical Economics*, 38, 2002: 219-248.
5. * Detemple, J. and S. Murthy, "Intertemporal Asset Pricing with Heterogeneous Beliefs," *Journal of Economic Theory*, 62, 1994: 294-320.
6. Detemple, J., M. Rindisbacher and S. Robertson., "Dynamic Noisy Rational Expectations Equilibrium with Insider Information," *Econometrica* 88, 2020: 2697-2737.

7. Detemple, J., M. Rindisbacher and S. Robertson, "Dynamic Noisy Rational Expectations Equilibrium with Insider Information: Welfare and Regulation," forthcoming *Journal of Economic Dynamics and Control*.
8. Wang, J. "A Model of Intertemporal Asset Pricing under Asymmetric Information," *Review of Economic Studies*, 60, 1993: 249-282.
9. Williams, J. "Capital Asset Prices with Heterogeneous Beliefs," *Journal of Financial Economics*, 5, 1977: 219-239.

4.4 Heterogeneous Agents and Market Frictions

1. Back, K., "Asset Pricing for General Processes," *Journal of Math. Economics*, 20, 1991: 371-395.
2. Constantinides, G.M. "Intertemporal Asset Pricing with Heterogeneous Consumers and without Demand Aggregation," *Journal of Business*, 55, 1982: 253-267.
3. Basak, S. and B. Croitoru. "Equilibrium Mispricing in a Capital Market with Portfolio Constraints," *Review of Financial Studies*, 13, 2000: 715-748.
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5. * Basak, S. and D. Cuoco. "An Equilibrium Model with Restricted Stock Market Participation," *Review of Financial Studies*, 11, 1998: 309-341.
6. * Constantinides, G.M. and D. Duffie. "Asset Pricing with Heterogeneous Consumers," *Journal of Political Economy*, 1992.
7. Cuoco, D. and H. He, "Dynamic Equilibrium in Infinite-Dimensional Economies with Incomplete Financial Markets," Working Paper, University of Pennsylvania, 1994.
8. * Detemple, J. and S. Murthy. "Equilibrium Asset Prices and No-Arbitrage with Portfolio Constraints," *Review of Financial Studies*, 10, 1997: 1133-1174.
9. Detemple, J. and A. Serrat, "Dynamic Equilibrium with Liquidity Constraints," *Review of Financial Studies*, 16, 2003: 597-629.
10. Dumas, B. "Two-Person Dynamic Equilibrium in the Capital Market," *Review of Financial Studies*, 3, 1989: 157-188.
11. Dutta J. and H. Polemarchakis, "Asset Markets and Equilibrium Processes," *Review of Economic Studies*, 57, 1990: 229-254.
12. Heaton, J. and D.J. Lucas, "Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing," *Journal of Political Economy*, 104, 1996: 443-487.
13. Jouini, E. and H. Kallal, "Martingales and Arbitrage in Securities Markets with Transaction Costs," *Journal of Economic Theory*, 66, 1995: 178-197.