

$$\eta = (Q, \Sigma, \Gamma, \delta, q_0, 1, F)$$
$$\Sigma = \{0, 1\}$$
$$\Gamma = \{B, I\} \cup \Sigma$$
$$Q = \{q_0, q_1, q_2, q_3, q_4, q_\infty, q_{-\infty}, q_{\rightarrow}, q_{\leftarrow}\}$$
$$F := \{q, r\}$$

- i.  $(x, \alpha) = (0, \varepsilon)$
- ii.  $(x, \alpha) = (100, @)$
- iii.  $(x, \alpha) = (3, @@\%)$
- iv.  $(x, \alpha) = (100, @\%)$

Hand-drawn state transition diagram for a Turing machine. States are  $q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7$ .  $q_0$  is the start state, and  $q_7$  is the final state. Transitions are labeled with input/output/move. The diagram shows a sequence of states with various transitions, including self-loops and jumps.

i).  $(0, \varepsilon) =$

$$d_0 = q_0 p_0 \vdash q_1 p_1 \vdash q_2 p_2 \vdash q_3 p_3 \vdash q_4 p_4 \vdash q_5 p_5 \vdash q_6 p_6 \vdash q_7 p_7 \vdash q_8 p_8 \vdash q_9 p_9 \vdash q_{10} p_{10} \vdash q_{11} p_{11} \vdash q_{12} p_{12} \vdash q_{13} p_{13} \vdash q_{14} p_{14} \vdash q_{15} p_{15} \vdash q_{16} p_{16} \vdash q_{17} p_{17} \vdash q_{18} p_{18} \vdash q_{19} p_{19} \vdash q_{20} p_{20} \vdash q_{21} p_{21} \vdash q_{22} p_{22} \vdash q_{23} p_{23} \vdash q_{24} p_{24} \vdash q_{25} p_{25} \vdash q_{26} p_{26} \vdash q_{27} p_{27} \vdash q_{28} p_{28} \vdash q_{29} p_{29} \vdash q_{30} p_{30} \vdash q_{31} p_{31} \vdash q_{32} p_{32} \vdash q_{33} p_{33} \vdash q_{34} p_{34} \vdash q_{35} p_{35} \vdash q_{36} p_{36} \vdash q_{37} p_{37} \vdash q_{38} p_{38} \vdash q_{39} p_{39} \vdash q_{40} p_{40} \vdash q_{41} p_{41} \vdash q_{42} p_{42} \vdash q_{43} p_{43} \vdash q_{44} p_{44} \vdash q_{45} p_{45} \vdash q_{46} p_{46} \vdash q_{47} p_{47} \vdash q_{48} p_{48} \vdash q_{49} p_{49} \vdash 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$$j|) (100, @) =$$
$$d_0 = q_0 B^{100} B @ \vdash q_1 |^{19} B @ \vdash | q_1 |^{18} \vdash B |^{100} q_1 B @ \vdash B |^{100} B q_2 @ \vdash B |^{100} B @ q_3 \\ \vdash B |^{100} B q @ \vdash B |^{100} q B @ \vdash B |^{100} | q @ \vdash B |^{100} | | q \vdash B |^{100} | q_1 \\ \vdash B |^{100} q_1 \vdash \dots \vdash q B |^{100} |$$

iii)  $(3, @ @ \%) =$

$q_0 B_{111} B @ @ \% \vdash B q_1, ||| B @ @ \% \vdash B | q_1, || B @ @ \% \vdash \dots \vdash B ||| q_1, 0 @ @ \% \vdash$   
 $B ||| B q_2 @ @ \% \vdash B ||| B @ q_1 @ \% \vdash B ||| B @ @ q_2 \% \vdash B ||| B @ @ \% q_3 \vdash B ||| B @ @ q_2 \%$   
 $\vdash B ||| B @ q_1 @ \% \vdash B ||| B q_1 @ @ \% \vdash B ||| q_1 0 @ @ \% \vdash B ||| q_1 @ @ \% \vdash B ||| q_1 @ \% \vdash B ||| q_1 q_2$   
 $\vdash B ||| q_1 q_1 \vdash B ||| q_1 q_1 \vdash B ||| q_1 q_1 \vdash B ||| q_1 q_1 \vdash \dots \vdash B q_1 ||| q_1 \vdash q_1 B ||| q_1$   
 $\vdash q_1 B ||| q_1$

iv)  $(100, @ \%) =$

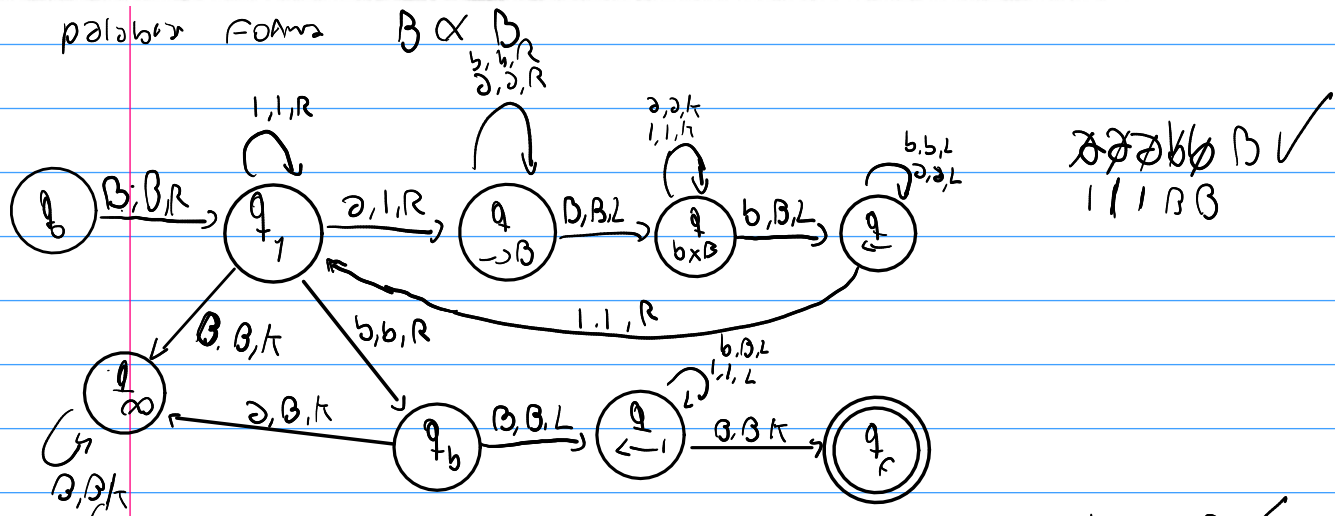
$g_0 \in I^{100} B \mathbb{Q} / \Gamma \vdash B g_1 \in I^{100} B \mathbb{Q} / \Gamma \vdash B g_2 \in I^{99} B \mathbb{Q} / \Gamma \vdash \dots \vdash B g_{i-1} \in I^{100} B \mathbb{Q} / \Gamma \vdash B g_i \in I^{100} B \mathbb{Q} / \Gamma$   
 $\vdash B g_{i+1} \in I^{100} B \mathbb{Q} / \Gamma \vdash B g_{i+2} \in I^{100} B \mathbb{Q} / \Gamma \vdash B g_{i+3} \in I^{100} B \mathbb{Q} / \Gamma$   
 $\vdash B g_{i+4} \in I^{100} B \mathbb{Q} / \Gamma \vdash \dots \vdash B g_{i+j} \in I^{100} B \mathbb{Q} / \Gamma \dots$

2. Sea  $f : \{a^n b^{n+1} : n \geq 0\} \rightarrow \omega$  dada por  $f(\alpha) = |\alpha|_a$ .

(a) Dar una máquina de Turing  $M$  con alfabeto de terminales  $\{a, b\}$  y de a lo sumo 12 estados que compute a  $f$ .

(b) Exhiba sucesiones de descripciones instantáneas que muestren que  $M$  funciona correctamente para los inputs en  $\{ba, aabb, aabbb\}$ .

Importante: Si  $M$  no funciona correctamente para cada uno de los inputs del punto (c), recibirá 0 puntos por este problema.



2. Sea  $\Sigma = \{ @, \% \}$  y sea  $f : D_f \subseteq \Sigma^* \rightarrow \omega$  una función  $\Sigma$ -efectivamente computable. Supongamos que  $@ \in D_f$ . Sea  $P$  un procedimiento efectivo que compute a  $f$ .

(a) Utilizando  $P$  diseñe un procedimiento efectivo  $Q$  que enumere a  $L = \{ (\alpha, \beta) \in D_f \times D_f : f(\alpha) = f(\beta) \}$ . (note que  $L \neq \emptyset$  ya que  $(@, @) \in L$ ).

(b) Justifique por qué  $Q$  que enumera a  $L$

Como  $f$  es  $\Sigma$ -e.c., entonces por lema sabemos que  $D_f$  es enumerable. Entonces sea  $P$  el proc que compute a  $f$ ,  $P_{D_f}$  el proc que enumere a  $D_f$  entonces definiremos  $Q$  como el proc que enumere a  $S \subseteq \omega \times \omega$  donde  $P_{D_f}$

E1: tomamos  $x \in \omega$  como dato de entrada, si es 0 devuelve  $(@, @)$  sino es 2.

E2: caso -  $P_{D_f}$  con entrada  $(x)_1$  y guarda a  $A$   $A \leftarrow * \prec (x)_1$   
 $P_{D_f}$  con entrada  $(x)_2$  y guarda en  $B$   $B \leftarrow * \prec ((x)_2)$

$K \leftarrow P$  de  $A$

$Z \leftarrow P$  de  $B$

E3: si  $K = Z$  devuelve  $(A, B)$ , sino  $(@, @)$ . termina.

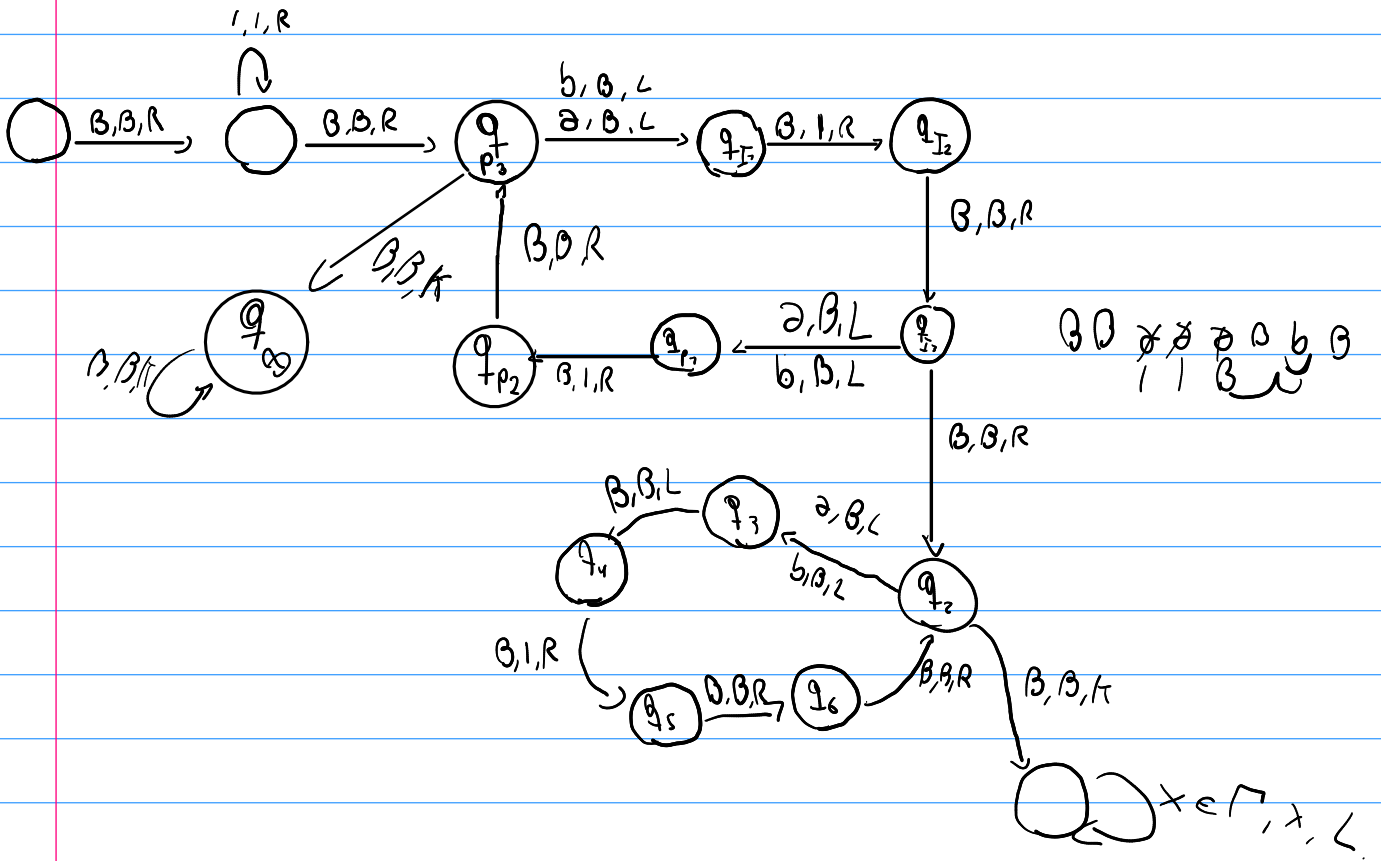
Para cada  $(\alpha, \beta) \in L$  hay un  $x \in \omega$  tal que  $Q$  de  $x$ , devuelve a  $(\alpha, \beta)$   $x = 2^{\# \prec (\alpha)} \cdot 3^{\# \prec (\beta)}$  (justificación de •)

Sea  $(\alpha, \beta) \in L$  y  $p, q \in \omega$  tal que  $P_{D_f}$  devuelve  $\alpha$  de  $p$  y  $P_{D_f}$  "  $\beta$  "  $q$  entonces sea  $x = 2^p \cdot 3^q$  entonces  $Q$  devuelve  $(\alpha, \beta)$  partiendo de  $x$ .

ex 13d) guida 4.

$$F: D_F \rightarrow W$$
$$D_F := \{ (x, \alpha, \beta) \in \omega \times \Sigma^{k,2} : |\alpha| \in \text{image} \}$$

$$(x, \alpha, \beta) = x + |\alpha| + |\beta|$$



1. Sean  $S \subseteq \omega$  y  $L \subseteq \{ @, \uparrow \}^*$  conjuntos  $\{ @, \uparrow \}$ -efectivamente enumerables tales que  $(0, \varepsilon) \in S \times L$ . Dar un procedimiento efectivo que enumere a  $S \times L$ . Explique por qué su procedimiento efectivo funciona correctamente.

Seon  $P_S$  y  $P_L$  los proc que enumeran a  $S$  y  $L$ , entonces defino  $P$  proc que enumera a  $S \times L$ .

E1: sea  $x \in W$  dato de entrada, si  $x=0$ , devuelvo  $(0, \varepsilon)$  y termino.

E2: corto  $-P_S$  desde  $(X)_1$  y lo guardo en A  
 $-P_L$  desde  $(X)_2$  y lo guardo en B

E3: devuelvo  $(A, B)$ .

enumera a 5xL, porq.

- Cont de fator de entrada e  $w$
- termina para todo  $x \in W$  e dá como resultado  $(x, \alpha) \in S \times L$
- se  $(x, \alpha) \in S \times L$ , existe um  $z = 2^p \cdot 3^k$  tq  $IP$  partindo de  $z$  devolve  $x, \alpha$ , donde.
  - $IP_p$  partindo de  $p$  devolve  $x$ .
  - $IP_k$  " " " "  $\alpha$ .

- $$\mathcal{L} \coloneqq \{(x, \alpha) \in S : f(x, \alpha) = @!!\}$$

Como  $f$  es computable  $\Rightarrow S$  es enumerable.  $P_S$  enumera  $S$ ,  $P_f$  computa  $f$ .  
Sea  $h$  la proc que enumera a  $L$ :  $Seq \leq 0$  en  $\mathbb{Z}$

e2: carro  $|P_r$  deve  $x$  e guarnição em A.

e3) corrige IP de drc de A y si el resultado es @!! devuelve A. sino (100)

- பொலிவாக்கம்

