Report for CSE 505 - Phase 3

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Code: https://github.com/Adoni/CSE505/tree/master/LTN

***Abstract*—We introcude a model named Logic Tensor Networks (LTN) in this report, which is invented by [1] in their paper "Logic Tensor Networks: Deep Learning and Logical Reasoning from Data and Knowledge". LTN constructs a multi-valued real logical system [2] with tensor networks, which map the components of a logical system, including constants, functions, predicates and clauses, into the tensor domain. Our contribution include three parts:**

1) **We implement their model;**
2) **We proposed a new structure named Convolutional Logic Tensor Networks (C-LTN), which gets better performance in the "smoker and friends" example;**
3) **We design several experiments to illustrate the performance of logical system implementation with tensor networks.**

Keywords*—*Multi-valued Real Logic, Tensor Network, Reasoning.

I. Paper Overview

A. Basic Information

Here is some basic information about the paper I selected:

- Title: Logic Tensor Networks: Deep Learning and Logical Reasoning from Data and Knowledge[1].
- Author: Luciano Serafini and Artur dAvila Garcez
- From: ArXiv.2016
- Abstract: This paper proposes Logic Tensor Networks (LTN) to integrate **learning** and **reasoning** together based on vector representation. The proposed model LTN represent each object with a vector and then converts each function on multiple objects into a manipulate on their vector representations. Then it also uses a s-norm operator to transform a predicate to a real number in $[0, 1]$, which means the confidence of this predicate. Finally, it defines a lose function for all predicates, which means it transfers the reasoning process into a learning and optimization problem.

B. Definitions

Recall that a first-order language L is composed by three parts:

- $\mathcal{C} = \{c_1, c_2, \dots\}$, the set of constant symbols;
- $\mathcal{F} = \{f_1, f_2, \dots\}$, the set of functional symbols;
- $\mathcal{P} = \{p_1, p_2, \dots\}$ the setof predicate symbols.

C. Groundings

LTN defines a term $\mathcal{G}$, called **grounding**, which contains $\mathcal{G}(c)$, $\mathcal{G}(f)$, $\mathcal{G}(P)$ respect to $c \in \mathcal{C}$, $f \in \mathcal{F}$, $P \in \mathcal{P}$:

Also, LTN define the manipulate of each clause ϕ.

1) Constant: For each constant, LTN allocate a n-dimension vector to it:

$$\mathcal{G}(c) \in \mathbb{R}^n \tag{1}$$

2) Function: For each function f/m on m parameters, LTN define it as a mapping on vector space, which means:

$$\mathcal{G}(f(t_1, t_2, \dots, t_m)) \in \mathbb{R}^{mn} \to \mathbb{R}^n \tag{2}$$

Note that here we specify the parameter of f is t_i instead of c_i because function f can be recursive like $f(f_1(c1, c2), f_2(c1, c3)$.

$$\mathcal{G}(f(t_1, t_2, \dots, t_m)) = \mathcal{G}(f)\mathcal{G}(t_1), \mathcal{G}(t_2), \dots, \mathcal{G}(t_m) \tag{3}$$

More specifically, $\mathcal{G}(f(t_1, t_2, \dots, t_m))$ is fitted by a linear function, which can be implemented with tensor network:

$$\begin{aligned}\mathcal{G}(f(t_1, t_2, \dots, t_m)) =& \mathcal{G}(f)(v_1, v_2, \dots, v_m)\\ =& \mathcal{G}(f)(v)\\ =& M_f v + N_f\end{aligned} \tag{4}$$

where $v = \langle v_1, v_2, \dots, v_m \rangle$, $M_f \in \mathbb{R}^{n \times mn}$, and $N_f \in \mathbb{R}^n$

3) Predicate: Like the grounding of a function,

$$\mathcal{G}(p(t_1, t_2, \dots, t_m)) \in \mathbb{R}^{mn} \to [0, 1] \tag{5}$$

Again, as t_i has been mapped to a vector, then:

$$\mathcal{G}(p(t_1, t_2, \dots, t_m)) = \mathcal{G}(P)\mathcal{G}(t_1), \mathcal{G}(t_2), \dots, \mathcal{G}(t_m) \tag{6}$$

Therefore we transfer a predicate into a series of manipulation on tensor space:

$$\begin{aligned}\mathcal{G}(p(t_1,t_2,\dots,t_m)) =&\mathcal{G}(f)(v_1,v_2,\dots,v_m)\\ =&\mathcal{G}(f)(v)\\ =&\sigma(u_p^T tanh(v^T W_P v + V_P v + B_P))\end{aligned} \tag{7}$$

where $v = \langle v_1, v_2, \dots, v_m\rangle \in \mathbb{R}^{mn}$, $W_P \in \mathbb{R}^{mn\times mn\times k}$, $V_P \in \mathbb{R}^{mn\times k}$, $B_P \in \mathbb{R}^k$, $u_P \in \mathbb{R}^k$, and σ is the sigmoid function.

4) Clause: In this project, we assume each clause is disjunctive normal form. So it's easy to get that

$$\mathcal{G}(\phi_1,\phi_2,\dots,\phi_k) = \mu(\mathcal{G}(\phi_1),\mathcal{G}(\phi_2),\dots,\mathcal{G}(\phi_k)) \tag{8}$$

where μ is the max function.

D. Optimization

So now we get the definition of all components of multi-valued first-order language in tensor networks. The next step is to define a proper lose function.

Saying we know the value of $\phi(x)$ in our dataset, where x is a constant. Intuitively, an optimal grouding should make $\mathcal{G}(\phi(t))$ as close as to $\phi(x)$.

II. Related Work

As described in the original paper [1], there are several models which are similar to LTN. All of them try to combine learning and logical reasoning together.

To avoid unnecessary details, here we try to compare LTN with some other models by listing the most important different features in Table I. It seems that LTN combines the advantages of those models, which gives some confidence about the power of this model.

III. Models

A. Logic Tensor Network (LTN)

As we said in Section 1, LTN maps the components of a logical system into the tensor domain. To be more precise,

- Constant c: $\mathcal{G}(c) \in \mathbb{R}^n$
- Function f: $\mathcal{G}(f(t_1,t_2,\dots,t_m)) = M_f v + N_f$
- Predicate p: $\mathcal{G}(p(t_1,t_2,\dots,t_m)) = \sigma(u_p^T tanh(v^T W_P v + V_P v + B_P))$
- Clause ϕ: $\mathcal{G}(\phi_1,\phi_2,\dots,\phi_k) = max(\mathcal{G}(\phi_1),\mathcal{G}(\phi_2),\dots,\mathcal{G}(\phi_k))$

It's clear to see that the model for a predicate in original LTN is implemented by multilayer perceptron (MLP). The key differencen with classical MLP is that the first layer is not linear layer but bilinear layer.

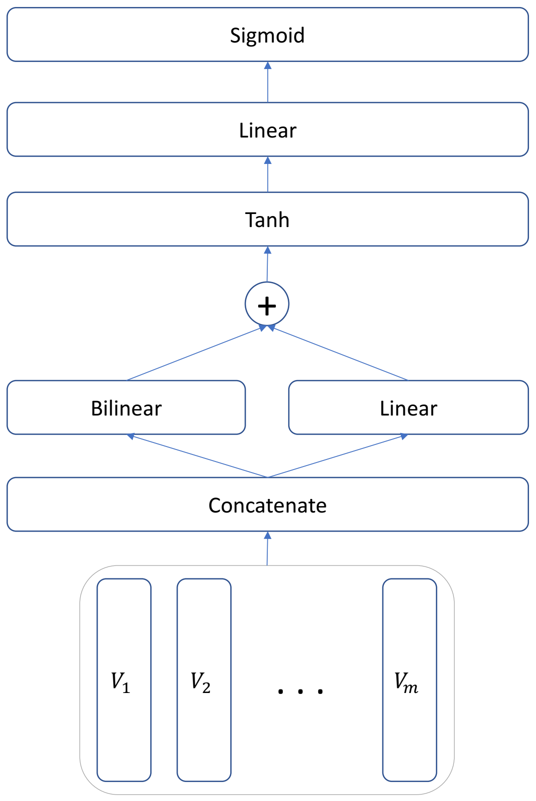


Figure 1: Model for Predicate in LTN

B. Convolutional Logic Tensor Network (C-LTN)

The predicate model in original LTN is based on MLP. However, recent development on deep learning shows that convolutional neural network (CNN) has more powerful ability. That makes us to extend LTN with CNN.

Here we first follow up the implementation of $G(c)$, $G(f)$, and $G(\phi)$. However, we tend to use Convolutional Neural Network to implement $G(p)$.

First we need to construct the input matrix with the $v_1, v_2, \dots, v_m$. From LTN model, we learned that the bilinear manipulation is critical to get a good performance. Here we construct this bilinear function directly using input vectors of constants. That is, suppose we get m vectors called $x = [v_1, \dots, v_m]$, then $x \in \mathbb{R}^{k\times m}$, where k is the dimension of constant vectors. Therefore, $x^T x \in \mathbb{R}^{m\times m}$ a squre matrix, which can be treated as a 1-channel image.

The following step is trivial and the model is illustrated in Figure 2

C. Universal and Existential Quantifiers

One of the most difficult part of LTN is how to solve universal and existential quantifiers. In our implementation, we generated all the possible clause of a propositional by enumerating all variables.

For universal quantifier, the generated result is a set of clauses, while the result of existential quantifier is one

Model X	Feature of Model X	Feature of LTN
Markov Logic Network [3]	-The level of truth of a formula depends on the number of models that satisfy the formula -Works under the closed world assumption	-The level of truth of a complex formula is -determined by (fuzzy) logical reasoning -Works under open domain
Bayesian Logic [4]	Explicit probabilistic approach	Take the benefits of tensor networks for computational efficiency.
Knowledge Embedding [5]	-Function-free langauges -A special case of LTN -The semantics of the universal and existential quantifiers is based on the closed-world assumption (CWA)	-Provide groundings for functional symbols -A General model -Does not make the CWA -No specific t-norm

Table I: Comparison with Similar Model

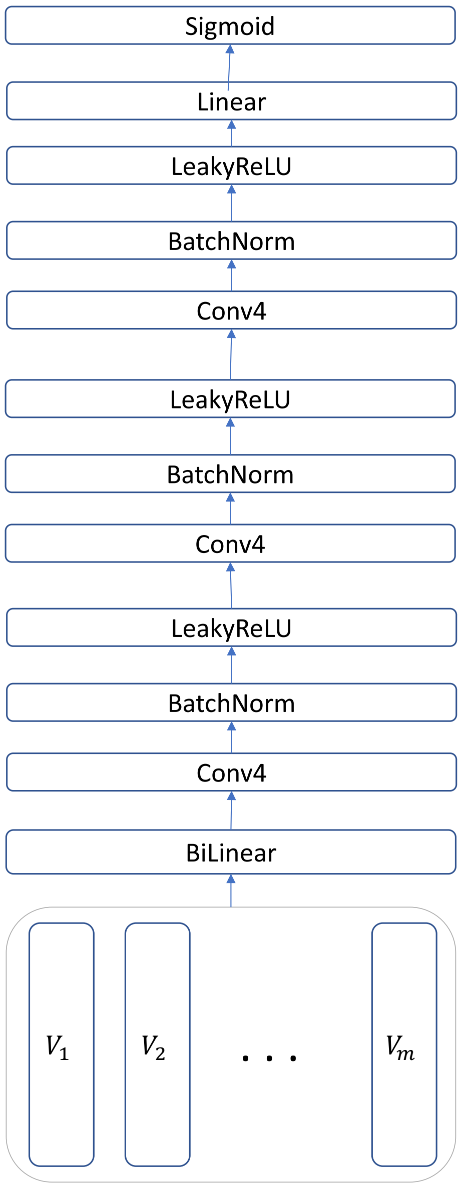


Figure 2: Model for Predicate in C-LTN

clause. For example, given two propositionals $\forall\neg F(x, x)$ and $\exists y F(a, y)$, if $x, y \in a, b, c$, then we can get the corresponding results as $\{F(a, a), F(b, b), F(c, c)\}$ and $F(a, a) \vee F(a, b) \vee F(a, c)$

D. *Weighted Knowledge Base*

As we said before, we generated all possible clauses. However, such generated clauses could be even more than the observed knowledge base, which would potentially caused the overfitting on generated knowledge base and underfitting on observed knowledge base.

To solve this problem, we add a weight to each generated clause and confine that the sum of weights of clauses generated by the same propositional should equal to 1. In another word, if the generated clauses of ϕ is $\{\phi_1, \phi_2, \ldots, \phi_n\}$, then the weight for each ϕ_i should be $\frac{1}{n}$.

IV. Experiments

This section will explore the same example given by the original paper, which is called "friends and smokers" problem. We will firstly how the definition of this example respect to the symbols we use before. Then two experiments will be deducted on knowledge base with only observed facts and knowledge base with extra rules. Finally, we will show the effects of vector length.

The experiments details are as following: total training epoch is 1000 and choose the best one according to the accuracy; for the 3.2 and 3.3, the vector length is 50.

A. *Friends and Smokers*

- Constant $C = C1 \cup C2 = \{a, b, c, d, e, f, g, h\} \cup \{a, b, c, d, e, f, g, h\}$ is all people in two groups $C1$ and $C2$
- Functions: $S/1$, whether a person smokes; $F/2$, whether two persons are friends; $C/1$, whether a person have cancer.
- Predicate: all components of all clauses, including $F/2$, $S/1$ and $C/1$
- Clause: original clauses is defined as the yellow part of Figure.3. We need transfer them into disjunctive form.

The value of "observed facts" we know is shown in Figure.3.

$\mathcal{K}^{SFC}_{a\ldots h}$

$S(a),\ S(e),\ S(f),\ S(g),$
$\neg S(b),\ \neg S(c),\ \neg S(d),\ \neg S(g),\ \neg S(h),$
$F(a,b),\ F(a,e),\ F(a,f),\ F(a,g),\ F(b,c),$
$F(c,d),\ F(e,f),\ F(g,h),$
$\neg F(a,c),\ \neg F(a,d),\ \neg F(a,h),\ \neg F(b,d),\ \neg F(b,e),$
$\neg F(b,f),\ \neg F(b,g),\ \neg F(b,h),\ \neg F(c,e),\ \neg F(c,f),$
$\neg F(c,g),\ \neg F(c,h),\ \neg F(d,e),\ \neg F(d,f),\ \neg F(d,g),$
$\neg F(d,h),\ \neg F(e,g),\ \neg F(e,h),\ \neg F(f,g),\ \neg F(f,h),$
$C(a),\ C(e),$
$\neg C(b),\ \neg C(c),\ \neg C(d),\ \neg C(f),\ \neg C(g),\ \neg C(h)$

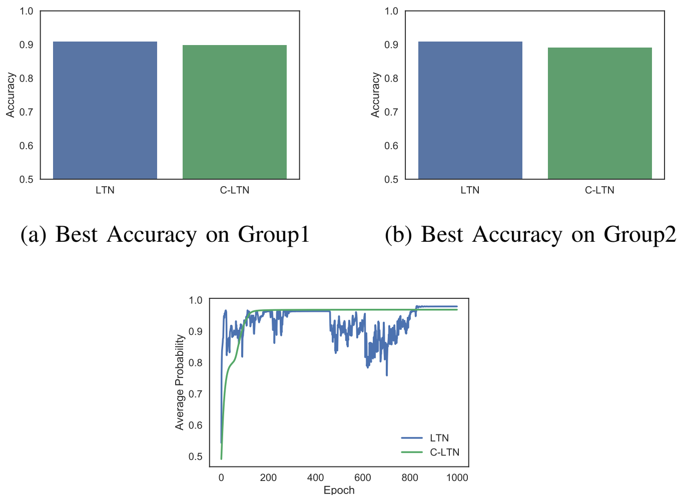
$\mathcal{K}^{SF}_{i\ldots n}$

$S(i),\ S(n),$
$\neg S(j),\ \neg S(k),$
$\neg S(l),\ \neg S(m),$
$F(i,j),\ F(i,m),$
$F(k,l),\ F(m,n),$
$\neg F(i,k),\ \neg F(i,l),$
$\neg F(i,n),\ \neg F(j,k),$
$\neg F(j,l),\ \neg F(j,m),$
$\neg F(j,n),\ \neg F(l,n),$
$\neg F(k,m),\ \neg F(l,m)$

$\mathcal{K}^{SFC}$

$\forall x \neg F(x,x),\quad \forall xy(S(x) \wedge F(x,y) \to S(y)),$
$\forall xy(F(x,y) \to F(y,x)),\quad \forall x(S(x) \to C(x))$
$\forall x \exists y F(x,y),$

Figure 3: Friends and Smokers



(a) Best Accuracy on Group1 (b) Best Accuracy on Group2

(c) Probability w.r.t. Epoch

Figure 4: Fitting Observed Facts.

B. Fit Knowledge Base

In this experiment, we only use the oberserved facts, which is $K_{exp1} = K^{SFC}_{a...h} \cup K^{SF}_{i...n}$.

As there is no rules in this part, so we only compare the result of LTN and C-LTN. Figure 4 shows the result of this experiment, where Figure 4c is the probability change with respect to the training epoch, Figure 4a shows the accuracy on the first group and Figure 4b is that of the second group.

From the Figure 4, it's clear that both LTN and C-LTN fit the observed data very well. The reason why the accuracy on first group is not 100% is that there exists a contradictory pair: $S(g)$ and $\neg S(g)$. So it's not possible to make both of them 100% true.

Here we also shows the fitting result for each predicate on two groups in Table II and Table III. And it's clear that we have a good fitting to the obseverd data.

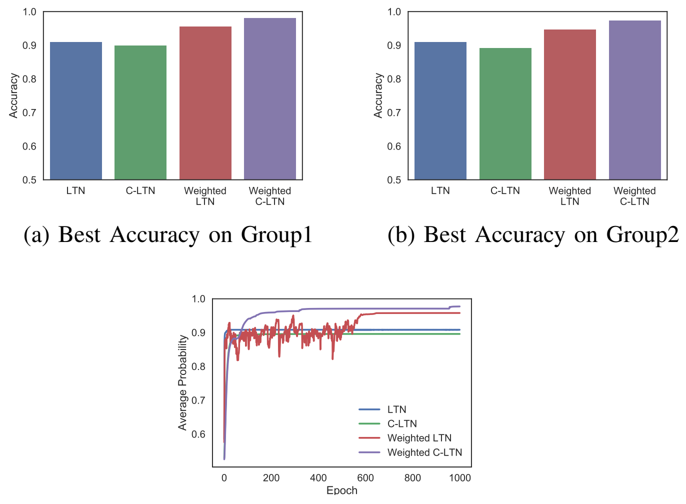
Finally, in Table IV, we shows the reliability of the rules on this situation. Even though that we didn't use the rules in this experiments, it still got a reasonable probability for each rule.

	S	C	F							
			a	b	c	d	e	f	g	h
a	1.00	1.00	1.00	1.00	0.00	0.00	1.00	1.00	1.00	0.00
b	0.00	0.00	1.00	0.11	1.00	0.00	0.00	0.00	0.00	0.00
c	0.00	0.00	0.00	1.00	0.01	1.00	0.00	0.00	0.00	0.00
d	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
e	1.00	1.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
f	1.00	0.00	1.00	0.00	0.00	0.00	1.00	0.01	0.00	0.00
g	0.14	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.08	1.00
h	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00

Table II: Fitting on Group1

	S	C	F					
			i	j	k	l	m	n
i	1.00	0.05	0.74	1.00	0.01	0.01	1.00	0.00
j	0.00	0.02	1.00	0.00	0.00	0.00	0.00	0.00
k	0.00	0.09	0.01	0.00	0.02	0.90	0.01	0.03
l	0.00	0.02	0.01	0.00	0.90	0.01	0.01	0.01
m	0.00	0.11	1.00	0.00	0.01	0.01	0.51	1.00
n	1.00	0.05	0.00	0.00	0.03	0.01	1.00	0.02

Table III: Fitting on Group2



(a) Best Accuracy on Group1 (b) Best Accuracy on Group2

(c) Probability w.r.t. Epoch

Figure 5: Learning from Observed Facts & Rules.

C. Learn From Rule

Firgure 5 shows the result of two models on datasets. Like the result in last subsection, this figure containts three subfigures, including probability changes and best accuracy on two groups.

Propositional	Group1	Group2
$\forall \neg F(x,x)$	0.75	0.666667
$\forall xy F(x,y) \to F(y,x)$	0.984375	0.944444
$\forall x \exists y F(x,y)$	1	1
$\forall xy S(x)F(x,y) \to S(y)$	0.953125	0.916667
$\forall x S(x) \to C(x)$	0.75	0.6666

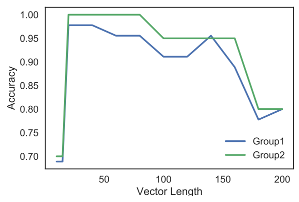
Table IV: Learned Rules

	S	C	F							
			a	b	c	d	e	f	g	h
a	1.00	1.00	1.00	1.00	0.00	0.00	1.00	1.00	1.00	0.00
b	0.00	0.00	1.00	0.05	1.00	0.00	0.00	0.00	0.00	0.00
c	0.00	0.00	0.00	1.00	0.93	1.00	0.00	0.00	0.00	0.00
d	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
e	1.00	1.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
f	1.00	0.00	1.00	0.00	0.00	0.00	1.00	0.01	0.00	0.00
g	0.49	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.10	1.00
h	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00

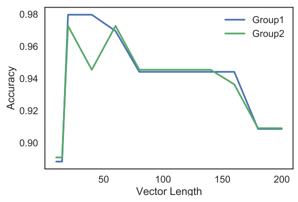
Table V: Fitting and Learning on Group1

	S	C	F					
			i	j	k	l	m	n
i	1.00	0.84	0.13	1.00	0.00	0.00	1.00	0.00
j	0.00	0.25	1.00	0.00	0.00	0.00	0.00	0.00
k	0.00	0.76	0.00	0.00	0.13	1.00	0.00	0.03
l	0.00	0.08	0.00	0.00	1.00	0.05	0.00	0.00
m	0.00	0.54	1.00	0.00	0.00	0.00	0.32	1.00
n	1.00	0.04	0.00	0.00	0.03	0.00	1.00	0.05

Table VI: Fitting and Learning on Group2



(a) Fitting Observed Facts



(b) Learning from Rules

Figure 6: Best Accuracy w.r.t. Vector Length.

Some interesting phenomenon can be concluded from the results.

First, by comparing results on weighted and unweighted dataset, we can see that training on weighted dataset gets a better performance. As we said before, this is because the weighted method treat each proposal as one clause, which could avoid the unbalanced training.

Second, C-LTN on weighted datasets shows the best performance. That's because CNN has better fitting ability and more stable in training.

D. Parameter Sensitive

In this part, we want to test the effect of vector length. We enumerate the vector length from 1 to 100 and shows the best accuracy on two groups. We did our experiments on both observed data and weighted dataset with rule. As the performance of C-LNT is better than LNT, we only shows the result of C-LNT, which is shown in Figure 6.

Figure 4a shows the result in observed dataset. From this figure, we can conclude that the performance is not always getting better with the increase of vector length. That's may because the under training problem. That is, as the

Actually, we want to find the appropriate vector lenght that is enough for fitting the data

Propositional	Group1	Group2
$\forall \neg F(x,x)$	0.625	0.5
$\forall xy F(x,y) \to F(y,x)$	0.984375	0.916667
$\forall x \exists y F(x,y)$	1	1
$\forall xy S(x) F(x,y) \to S(y)$	0.9375	0.916667
$\forall x S(x) \to C(x)$	0.75	0.666667

Table VII: Learned Rules

V. Conclusion

In this report, we gives a overview of paper "Logic Tensor Networks: Deep Learning and Logical Reasoning from Data and Knowledge" [1]. Besides implementing the original LTN model, we also proposed a kind of variation for LTN, called Convolutional LTN (C-LTN). Experiments "smoker and friends" example shows that C-LTN is not only easy to train, but also preforms better when fitting the data and learning from rules.

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