Roll : 1715006 ; Name : Adowan Shahriar ; LAB - 2

Objectives:

- To be able to find the laplace transform by MATLAB function.
- To be able to find the inverse laplace transform.
- To use MATLAB to generate transfer function.
- To find the components of second order system. and step responses.

Introduction:

In mathematics, the LapLace Transform, named after its inventor Pierre - Simon LapLace, is an integral transform that converts a function of a real variable 's'. The transform has many applications in science and engineering because it is a tool for solving differential equations. In particular, it transforms differential equations into algebric equations and convolutions into multiplication.

The laplace transform of a function $f(t)$, defined for all real numbers $t \geq 0$, is the function $F(s)$, which is a unilateral transform defined by,

$$F(s) = \int_0^{\infty} f(t) e^{-st} \, dt$$

where s is a complex number frequency parameter.

$s = \sigma + j\omega$ with real numbers σ and ω.

In mathematics, the inverse Laplace transform of a function F(s) is the piecewise-continuous and exponentially-restricted real function f(t) is which has the property:

$$\mathcal{L}\{f\}(s) = \mathcal{L}\{f(t)\}(s) = F(s)$$

where, $\mathcal{L}$ denotes the Laplace transform.

A second order system exhibits a wide range of responses that must be analyzed and described. For example a second order system can display characterstics depending on component values, display damped or pure oscillations for its transient response.

Natural frequency: The natural frequency of a second order system is the frequency of oscillations of the system without damping. It is denoted as ω_n.

Damping Ratio: -

A viable definition for this quantity is that one compares the exponential decay frequency of the envelope to the natural frequency.

$$\zeta = \frac{\text{Exponential decay freq.}}{\text{Natural freq.}} = \frac{1}{2\pi} \frac{\text{Natural period (s)}}{\text{Exponential time Const.}}$$

Rise time, T_r: The time required for the waveform to go from 0.1 of the final value to 0.9 of the final value.

Peak time; T_p: The time required to reach the first or maximum, peak.

Percent overshoot, %S. The amount that the waveform overshoots the steady state, or final, value at the peak time, expressed as a percentage of the steady-state value.

settling time, T_s: The time required for the transient's damped oscillations to reach and stay within ±2% of the steady state value.

Task 1.a:

Editor - C:\Users\DELL\Untitled.m

Untitled.m

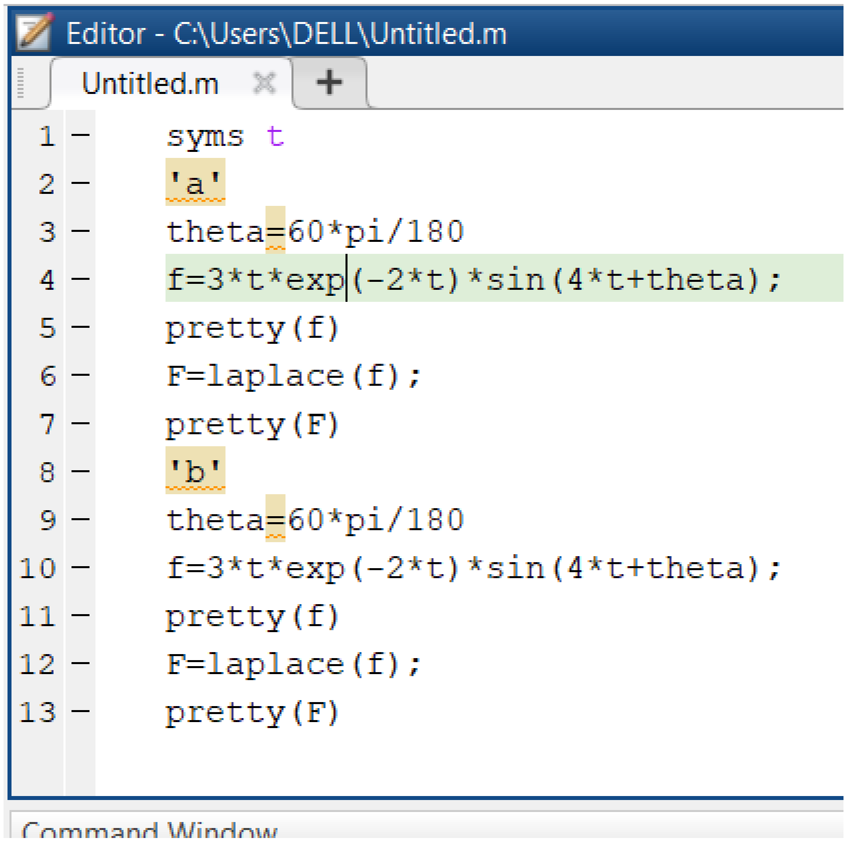
```
syms t
'a'
theta=45*pi/180
f=8*t^2*cos(3*t+theta);
pretty(f)
F=laplace(f);
pretty(F)
'b'
theta=60*pi/180
f=3*t*exp(-2*t)*sin(4*t+theta);
pretty(f)
F=laplace(f);
pretty(F)
```

Output:

```
>> Untitled

ans =

    'a'


theta =

    0.7854

 2   /      pi \
t  cos| 3 t + -- | 8
     \      4 /
```

```
       /                  2  \           /                  3  \
       |     6        24 s    |           |    6 s        8 s    |
sqrt(2) | --------- - --------- | 4 - sqrt(2) | --------- - --------- | 4
       |   2     2     2     3 |           |   2     2     2     3 |
       \ (s  + 9)    (s  + 9)  /           \ (s  + 9)    (s  + 9)  /



ans =


   'b'



theta =


   1.0472


   /      pi \
t sin| 4 t + -- | exp(-2 t) 3
   \       3 /


                  /      1        (2 s + 4) (s + 2) \
           sqrt(3) | ------------- - ----------------- | 3
                  |       2              2      2 |
  (2 s + 4) 6           \ (s + 2)  + 16    ((s + 2)  + 16)  /
---------------- - -----------------------------------------------
      2      2                       2
((s + 2)  + 16)
```

Task 1.b:




```
syms t
'a'
theta=60*pi/180
f=3*t*exp(-2*t)*sin(4*t+theta);
pretty(f)
F=laplace(f);
pretty(F)
'b'
theta=60*pi/180
f=3*t*exp(-2*t)*sin(4*t+theta);
pretty(f)
F=laplace(f);
pretty(F)
```

Output:

```
>> Untitled

ans =

   'a'

theta =

   0.7854

 2   /      pi \
```

```
t  cos| 3 t + -- | 8
      \        4 /

        /                 2  \           /                  3  \
        |     6         24 s     |            |     6 s        8 s     |
sqrt(2) | --------- - --------- | 4 - sqrt(2) | --------- - --------- | 4
        |   2     2     2     3 |            |   2     2     2     3 |
        \ (s  + 9)    (s  + 9)  /            \ (s  + 9)    (s  + 9)  /


ans =

    'b'


theta =

    1.0472

     /       pi \
t sin| 4 t + -- | exp(-2 t) 3
     \        3 /

                      /       1         (2 s + 4) (s + 2) \
               sqrt(3) | ------------- - ----------------- | 3
                      |        2               2      2 |
  (2 s + 4) 6           \ (s + 2)  + 16    ((s + 2)  + 16)  /
---------------- - -----------------------------------------------
       2      2                        2
```

```
((s + 2)  + 16)

>> Untitled

ans =

    'a'


theta =

    1.0472

Undefined function or variable 'e'.

Error in Untitled (line 4)
f=3*t*e^(-2*t)*sin(4*t+theta);

>> Untitled

ans =

    'a'


theta =

    1.0472
```

```
    /      pi \
t sin| 4 t + -- | exp(-2 t) 3
    \       3 /

                       /      1        (2 s + 4) (s + 2) \
                sqrt(3) | ------------- - ----------------- | 3
                       |       2               2     2 |
  (2 s + 4) 6          \ (s + 2)  + 16    ((s + 2)  + 16)  /
---------------- - ---------------------------------------------
       2     2                       2
((s + 2)  + 16)


ans =

    'b'


theta =

    1.0472

    /      pi \
t sin| 4 t + -- | exp(-2 t) 3
    \       3 /

                       /      1        (2 s + 4) (s + 2) \
                sqrt(3) | ------------- - ----------------- | 3
                       |       2               2     2 |
```

```
 (2 s + 4) 6          \ (s + 2)  + 16    ((s + 2)  + 16)  /
---------------- - -----------------------------------------------
      2     2                      2
((s + 2)  + 16)
```

Task 2.a:

Editor - C:\Users\DELL\Untitled.m

Untitled.m

```
clc
clear all
close all
syms s

g=((s^2+3*s+10)*(s+5))/((s+3)*(s+4)*(s^2+2*s+100))
pretty(g)
F=ilaplace(g)
pretty(F)
```

Output:

Command Window

```
g =

((s + 5)*(s^2 + 3*s + 10))/((s + 3)*(s + 4)*(s^2 + 2*s + 100))

                  2
     (s + 5) (s  + 3 s + 10)
--------------------------------
                     2
(s + 3) (s + 4) (s  + 2 s + 100)


F =

(20*exp(-3*t))/103 - (7*exp(-4*t))/54 + (5203*exp(-t)*(cos(3*11^(1/2)*t) - (11^(1/2)*sin(3*11^(1/2)*t))/57233))/5562

                                        /                      sqrt(11) sin(3 sqrt(11) t) \
                              exp(-t) | cos(3 sqrt(11) t) - -------------------------- | 5203
exp(-3 t) 20   exp(-4 t) 7            \                                57233           /
------------ - ----------- + ---------------------------------------------------------------
     103            54                                    5562
```

Task 2.b:

```
clc
clear all
close all
syms s

g=(s^3+4*s^2+2*s+6)/((s+8)*(s^2+8*s+3)*(s*s+5*s+7))
pretty(g)
F=ilaplace(g)
pretty(F)
```

Output:

g =

(s^3 + 4*s^2 + 2*s + 6)/((s + 8)*(s^2 + 8*s + 3)*(s^2 + 5*s + 7))

```
       3     2
      s  + 4 s  + 2 s + 6
-------------------------------------
       2            2
(s + 8) (s  + 8 s + 3) (s  + 5 s + 7)
```

F =

(1199*exp(-4*t)*(cosh(13^(1/2)*t) - (4262*13^(1/2)*sinh(13^(1/2)*t))/15587))/417 - (65*exp(-(5*t)/2)*(cos((3^(1/2)*t)/2) + (131*3^(1/2)*sin((3^(1/2)*t)/2))/15))/4309 - (266*exp(-8*t))/93

```
       /              4262 sqrt(13) sinh(sqrt(13) t) \
exp(-4 t) | cosh(sqrt(13) t) - ------------------------------ | 1199
       \                        15587          /
------------------------------------------------------------------
                      417

            /                        / sqrt(3) t \    \
            |                 sqrt(3) sin| --------- | 131 |
      /  5 t \ |    / sqrt(3) t \           \    2    /    |
   exp| - --- | | cos| --------- | + ---------------------------- | 65
      \   2  / \   \    2    /              15           /     exp(-8 t) 266
  - ----------------------------------------------------------------- - -------------
                         4309                                  93
```

Task 3.a:

```
Editor - C:\Users\DELL\Untitled.m
Untitled.m
clc
clear all
close all
'a'
'i'
%Rational expression method
s=zpk('s')
F=[(5*(s+15)*(s+26)*(s+72))]/[(s*(s+55)*(s*s+5*s+30)*(s+56)*(s*s+27*s+52))]
clear
'ii'
%Rational expression method
s=tf('s')
F=[(5*(s+15)*(s+26)*(s+72))]/[(s*(s+55)*(s*s+5*s+30)*(s+56)*(s*s+27*s+52))]
pause
```

Output:

Command Window

```
ans =

    'a'


ans =

    'i'


s =

  s

Continuous-time zero/pole/gain model.


F =

                5 (s+15) (s+26) (s+72)
  ---------------------------------------------------
  s (s+55) (s+56) (s+24.91) (s+2.087) (s^2 + 5s + 30)

Continuous-time zero/pole/gain model.
```

```
ans =

    'ii'


s =

  s

Continuous-time transfer function.


F =

                        5 s^3 + 565 s^2 + 16710 s + 140400
  -----------------------------------------------------------------------------
  s^7 + 143 s^6 + 6849 s^5 + 123717 s^4 + 788690 s^3 + 3.469e06 s^2 + 4.805e06 s

Continuous-time transfer function.
```

Task 3.b:

Editor - C:\Users\DELL\Untitled.m

Untitled.m

```
clc
clear all
close all
'a'
'i'
%Rational expression method
s=zpk('s')
F=[(s^4+25*s^3+20*s*s+15*s+42)]/[(s^5+13*s^4+9*s^3+37*s*s+35*s+50)]
clear
'ii'
%Rational expression method
s=tf('s')
F=[(s^4+25*s^3+20*s*s+15*s+42)]/[(s^5+13*s^4+9*s^3+37*s*s+35*s+50)]
pause
```

Output:

Command Window

```

ans =

    'a'


ans =

    'i'


s =

  s

Continuous-time zero/pole/gain model.


F =

        (s+24.2) (s+1.35) (s^2 - 0.5462s + 1.286)
  ------------------------------------------------------
  (s+12.5) (s^2 + 1.463s + 1.493) (s^2 - 0.964s + 2.679)

Continuous-time zero/pole/gain model.
```

```
ans =

    'ii'


s =

  s

Continuous-time transfer function.


F =

        s^4 + 25 s^3 + 20 s^2 + 15 s + 42
  -----------------------------------------
  s^5 + 13 s^4 + 9 s^3 + 37 s^2 + 35 s + 50

Continuous-time transfer function.
```

Task 4:

Editor - C:\Users\DELL\Untitled.m

Untitled.m

```
clc
clear all
close all
numa=100;
dena=[1 15 6];
Ta=tf(numa,dena)
omegana=sqrt(dena(3))
zetaa=dena(2)/(2*omegana)
Ts=4/(zetaa*omegana)
Tp=pi/(omegana*sqrt(1-zetaa^2))
Tr=(1.76*zetaa^3 - 0.417*zetaa^2 + 1.039*zetaa + 1)/omegana
percenta=exp(-zetaa*pi/sqrt(1-zetaa^2))*100
step(Ta)
title('(a)')
```

Output:

Ta =

```
      100
  --------------
  s^2 + 15 s + 6
```

Continuous-time transfer function.

omegana =

    2.4495

zetaa =

    3.0619

Ts =

    0.5333

Tp =

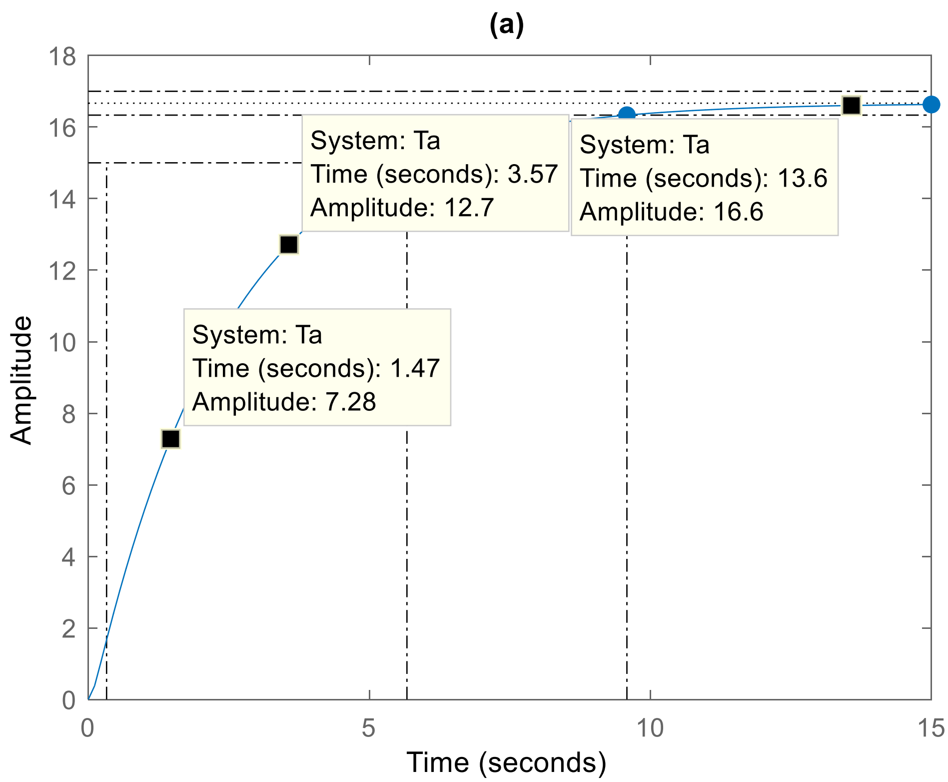
   0.0000 - 0.4432i

Tr =

20.7360

percenta =

-98.3435 -18.1263i



Task 5.a:

Editor - C:\Users\DELL\Untitled.m

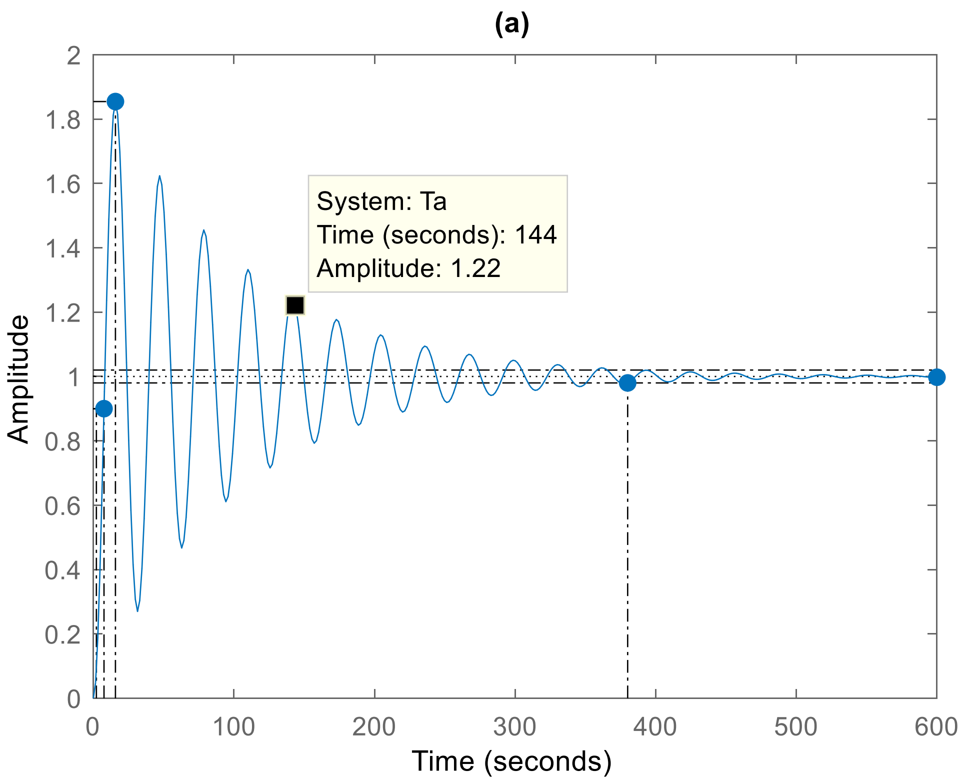
Untitled.m

```
clc
clear all
close all
numa=.04
dena=[1 .02 .04];
Ta=tf(numa,dena)
omegana=sqrt(dena(3))
zetaa=dena(2)/(2*omegana)
Ts=4/(zetaa*omegana)
Tp=pi/(omegana*sqrt(1-zetaa^2))
Tr=(1.76*zetaa^3 - 0.417*zetaa^2 + 1.039*zetaa + 1)/omegana
percenta=exp(-zetaa*pi/sqrt(1-zetaa^2))*100
step(Ta)
title('(a)')
```

Output:

```
numa =

   0.0400



Ta =

      0.04
  ------------------
  s^2 + 0.02 s + 0.04

Continuous-time transfer function.
```

```
omegana =

    0.2000


zetaa =

    0.0500


Ts =

   400


Tp =

   15.7276


Tr =

    5.2556


percenta =

   85.4468
```



Task 5.b:

Editor - C:\Users\DELL\Untitled.m

Untitled.m

```
clc
clear all
close all
numa=1.05*10^7
dena=[1 1.06*10^3 1.05*10^7];
Ta=tf(numa,dena)
omegana=sqrt(dena(3))
zetaa=dena(2)/(2*omegana)
Ts=4/(zetaa*omegana)
Tp=pi/(omegana*sqrt(1-zetaa^2))
Tr=(1.76*zetaa^3 - 0.417*zetaa^2 + 1.039*zetaa + 1)/omegana
percenta=exp(-zetaa*pi/sqrt(1-zetaa^2))*100
step(Ta)
title('(a)')
```

Output:

```
numa =

   10500000


Ta =

        1.05e07
  ----------------------
  s^2 + 1060 s + 1.05e07

Continuous-time transfer function.


omegana =

   3.2404e+03


zetaa =

   0.1636


Ts =

   0.0075
```

Tp =

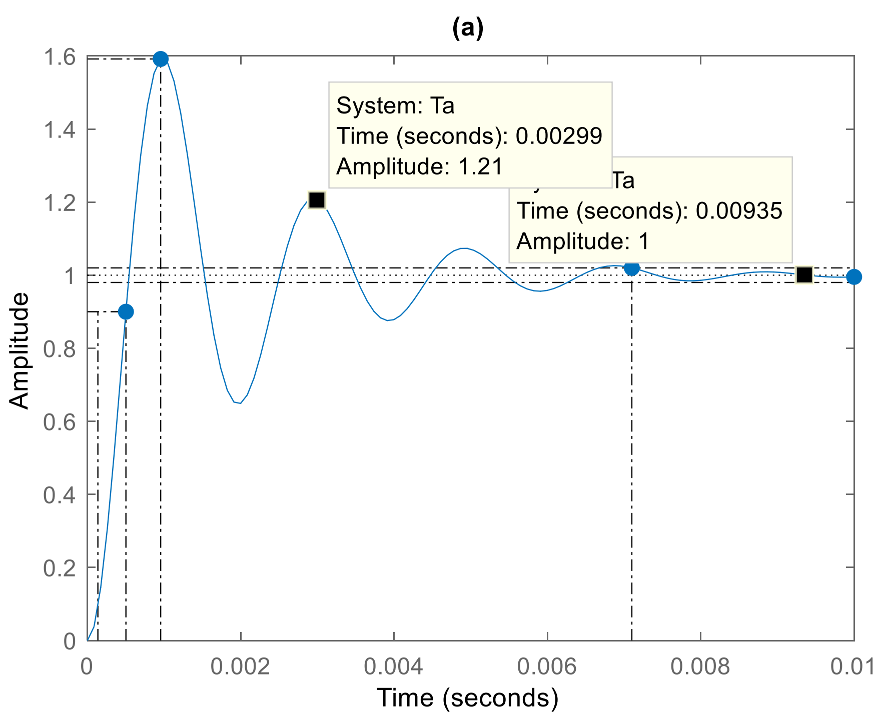
9.8275e-04

Tr =

3.5999e-04

percenta =

59.4011



Task 5.c:

Editor - C:\Users\DELL\Untitled.m

Untitled.m

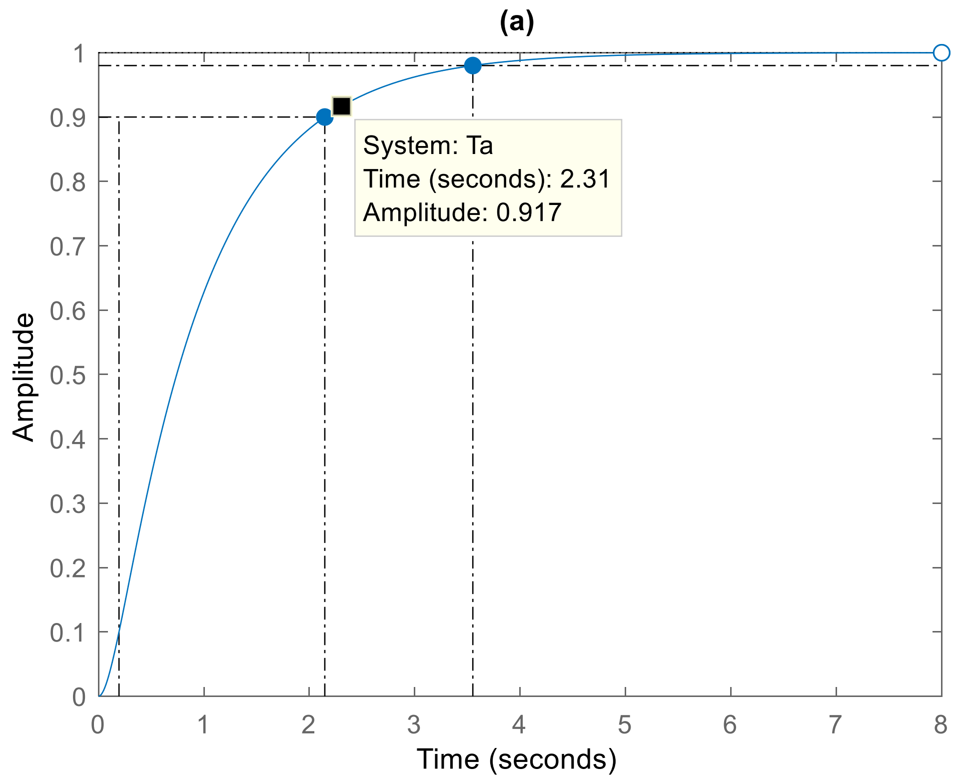
```
clc
clear all
close all
numa=9
dena=[1 9 9];
Ta=tf(numa,dena)
omegana=sqrt(dena(3))
zetaa=dena(2)/(2*omegana)
Ts=4/(zetaa*omegana)
Tp=pi/(omegana*sqrt(1-zetaa^2))
Tr=(1.76*zetaa^3 - 0.417*zetaa^2 + 1.039*zetaa + 1)/omegana
percenta=exp(-zetaa*pi/sqrt(1-zetaa^2))*100
step(Ta)
title('(a)')
```

Output:

```
numa =

     9


Ta =

       9
  -------------
  s^2 + 9 s + 9
```

Continuous-time transfer function.

```
omegana =

     3


zetaa =

    1.5000


Ts =

    0.8889


Tp =

   0.0000 - 0.9366i


Tr =

    2.5201


percenta =

 -47.7230 -87.8778i
```



>>

Task 5.d:

Editor - C:\Users\DELL\Untitled.m

Untitled.m

```
clc
clear all
close all
numa=9
dena=[1 2 9];
Ta=tf(numa,dena)
omegana=sqrt(dena(3))
zetaa=dena(2)/(2*omegana)
Ts=4/(zetaa*omegana)
Tp=pi/(omegana*sqrt(1-zetaa^2))
Tr=(1.76*zetaa^3 - 0.417*zetaa^2 + 1.039*zetaa + 1)/omegana
percenta=exp(-zetaa*pi/sqrt(1-zetaa^2))*100
step(Ta)
title('(a)')
```

\

Output:

```
numa =

     9


Ta =

       9
  -------------
  s^2 + 2 s + 9

Continuous-time transfer function.


omegana =

     3


zetaa =

    0.3333


Ts =

     4
```

Tp =

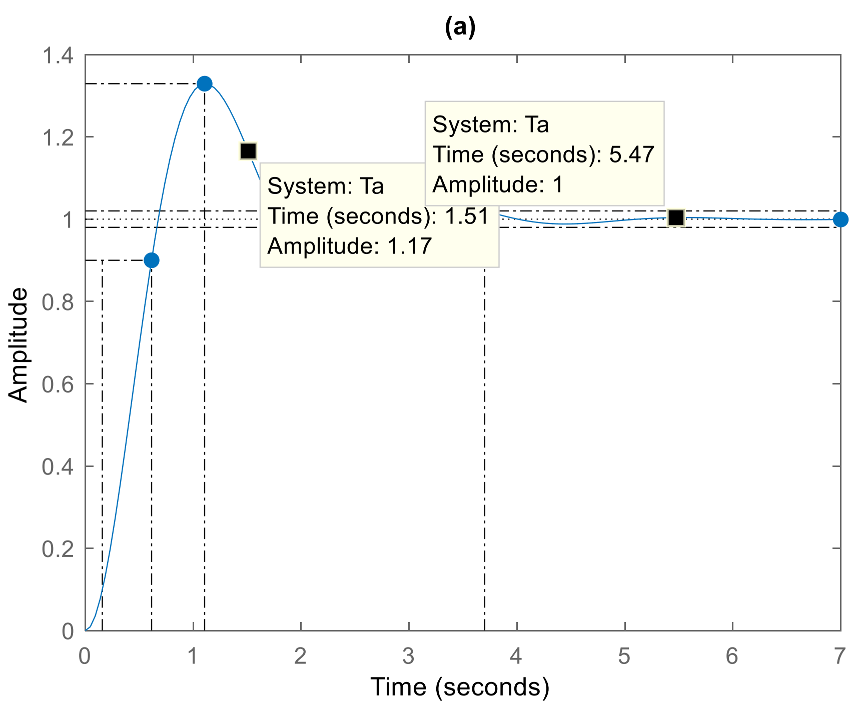
1.1107

Tr =

0.4551

percenta =

32.9322



>>

Task:5.e:

Editor - C:\Users\DELL\Untitled.m

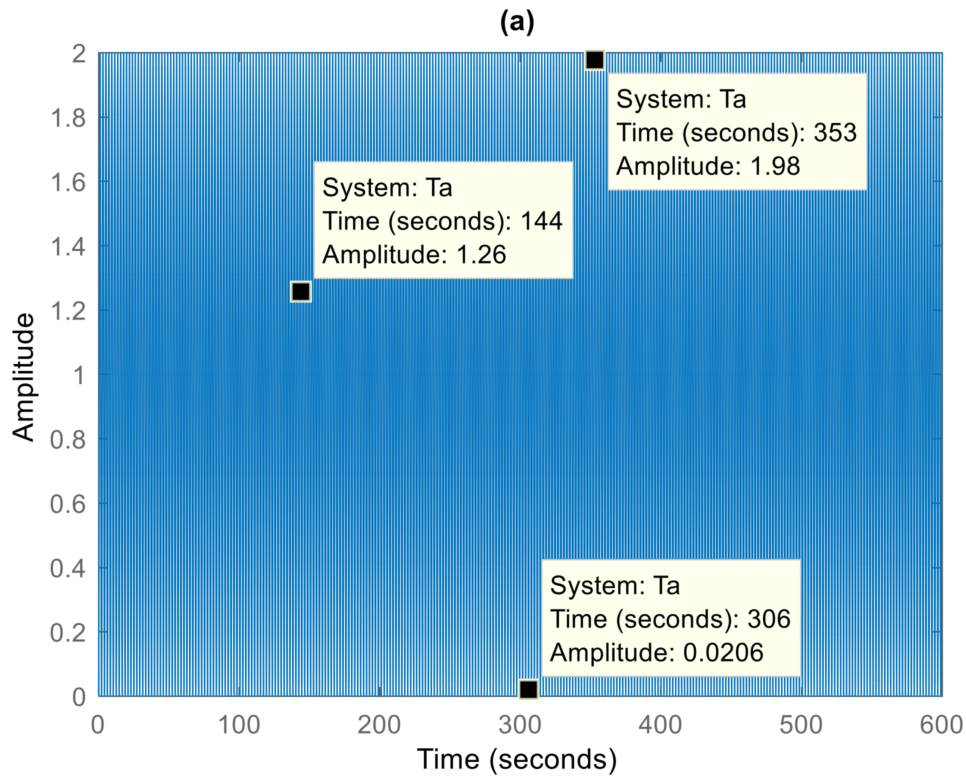
Untitled.m

```
clc
clear all
close all
numa=9
dena=[1 0 9];
Ta=tf(numa,dena)
omegana=sqrt(dena(3))
zetaa=dena(2)/(2*omegana)
Ts=4/(zetaa*omegana)
Tp=pi/(omegana*sqrt(1-zetaa^2))
Tr=(1.76*zetaa^3 - 0.417*zetaa^2 + 1.039*zetaa + 1)/omegana
percenta=exp(-zetaa*pi/sqrt(1-zetaa^2))*100
step(Ta)
title('(a)')
```

Output:

```
numa =

     9


Ta =

     9
  -------
  s^2 + 9

Continuous-time transfer function.
```

```
omegana =

     3


zetaa =

     0


Ts =

   Inf


Tp =

    1.0472


Tr =

    0.3333


percenta =

   100
```



>>

Task 5.f:

Editor - C:\Users\DELL\Untitled.m

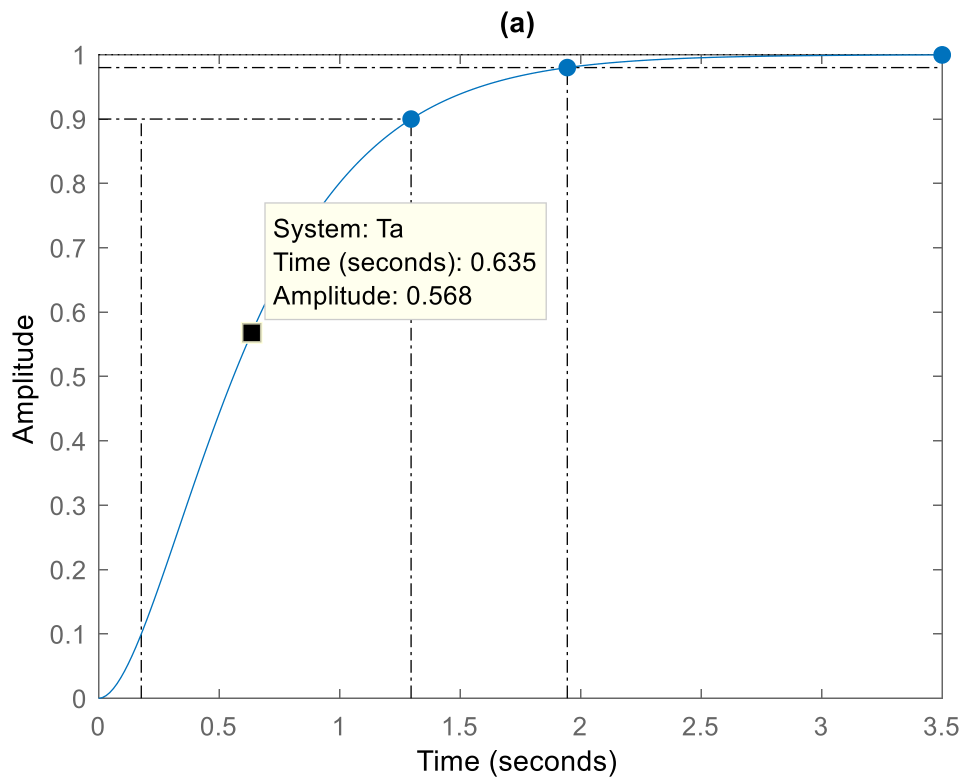
Untitled.m

```
clc
clear all
close all
numa=9
dena=[1 6 9];
Ta=tf(numa,dena)
omegana=sqrt(dena(3))
zetaa=dena(2)/(2*omegana)
Ts=4/(zetaa*omegana)
Tp=pi/(omegana*sqrt(1-zetaa^2))
Tr=(1.76*zetaa^3 - 0.417*zetaa^2 + 1.039*zetaa + 1)/omegana
percenta=exp(-zetaa*pi/sqrt(1-zetaa^2))*100
step(Ta)
title('(a)')
```

Output:

```
numa =

     9


Ta =

       9
  -------------
  s^2 + 6 s + 9

Continuous-time transfer function.


omegana =

     3


zetaa =

     1


Ts =

    1.3333
```

```
Tp =

   Inf


Tr =

    1.1273


percenta =

     0
```


(a)
System: Ta
Time (seconds): 0.635
Amplitude: 0.568
Amplitude
Time (seconds)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.5
1
1.5
2
2.5
3
3.5

>>

Discussion:

We learned and implemented some constructive MATLAB functions in this lab. Such as laplace (); ilaplace () function. These functions can readily calculate the defined form of equations. And we've learned how to define system under a certain variable system. Likewise we converted inverse laplace from 's' system into 't' system. We've done some algebric calculations to calculate second order system parameters. Although online classes are very struggling for proper learning process but ~~our~~ we tried our best in corporating with the class teacher.

Conclusion:

We were going through some pathetic circumstance due to covid 19. But through this lab we were lucky to conduct our regular lab classes. In this lab, we practiced doing lablace and inverse laplace transform through MATLAB software. We also learned how to calculate second order system parameter for time response prospect.