Universidad de Guadalajara, CUCEI

The Empire Strikes Back

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Contents

Think twice, code once

Template

tem.cpp

```cpp
#pragma GCC optimize("Ofast,unroll-loops,no-stack-protector")
#include <bits/stdc++.h>
using namespace std;

#ifdef LOCAL
#include "debug.h"
#else
#define debug(...)
#endif

#define df(b, e) ((b) > (e))
#define fore(i, b, e) for (auto i = (b) - df(b, e); i != e - df(b, e); i += 1 - 2 * df(b, e))
#define sz(x) int(x.size())
#define all(x) begin(x), end(x)
#define f first
#define s second
#define pb push_back

using lli = long long;
using ld = long double;
using ii = pair<int, int>;
using vi = vector<int>;

int main() {
  cin.tie(0)->sync_with_stdio(0), cout.tie(0);
  // solve the problem here D:
  return 0;
}
```

debug.h

```cpp
template <class A, class B>
ostream & operator << (ostream &os, const pair<A, B> & p) {
  return os << "(" << p.first << ", " << p.second << ")";
}

template <class A, class B, class C>
basic_ostream<A, B> & operator << (basic_ostream<A, B> &os, const C &c) {
  os << "[";
  for (const auto &x : c)
    os << ", " + 2 * (&x == &*begin(c)) << x;
  return os << "]";
}

void print(string s) { cout << endl; }

template <class H, class... T>
void print(string s, const H &h, const T&... t) {
  const static string reset = "\033[0m";
  bool ok = 1;
  do {
    if (s[0] == '\"') ok = 0;
    else cout << "\033[1;34m" << s[0] << reset;
    s = s.substr(1);
  } while (s.size() && s[0] != ',');
  if (ok) cout << ": " << "\033[1;95m" << h << reset;
  print(s, t...);
}
```

Randoms

```cpp
mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
uniform_int_distribution<>(l, r)(rng);
```

Fastio

```cpp
char gc() { return getchar_unlocked(); }

void readInt() {}
template <class H, class... T>
void readInt(H &h, T&... t) {
  char c, s = 1;
  while (isspace(c = gc()));
  if (c == '-') s = -1, c = gc();
  for (h = c - '0'; isdigit(c = gc()); h = h * 10 + c - '0');
  h *= s;
  readInt(t...);
}

void readFloat() {}
template <class H, class... T>
void readFloat(H &h, T&... t) {
  int c, s = 1, fp = 0, fpl = 1;
  while (isspace(c = gc()));
  if (c == '-') s = -1, c = gc();
  for (h = c - '0'; isdigit(c = gc()); h = h * 10 + c - '0');
  h *= s;
  if (h == '.')
    for (; isdigit(c = gc()); fp = fp * 10 + c - '0', fpl *= 10);
  h += (double)fp / fpl;
  readFloat(t...);
}
```

Compilation (gedit ~/.zshenv)

```bash
touch a_in{1..9} // make files a_in1, a_in2,..., a_in9
tee {a..m}.cpp < tem.cpp // "" with tem.cpp like base
cat > a_in1 // write on file a_in1
gedit a_in1 // open file a_in1
rm -r a.cpp // deletes file a.cpp :'(

red='\x1B[0;31m'
green='\x1B[0;32m'
noColor='\x1B[0m'
alias flags='-Wall -Wextra -Wshadow -D_GLIBCXX_ASSERTIONS -fmax-errors=3 -O2 -w'
go() { g++ --std=c++11 $2 ${flags} $1.cpp && ./a.out }
debug() { go $1 -DLOCAL < $2 }
run() { go $1 "" < $2 }

random() { // Make small test cases!!!
 g++ --std=c++11 $1.cpp -o prog
 g++ --std=c++11 gen.cpp -o gen
 g++ --std=c++11 brute.cpp -o brute
 for ((i = 1; i <= 200; i++)); do
  printf "Test case #$i"
  ./gen > in
  diff -uwi <(./prog < in) <(./brute < in) > $1_diff
  if [[ ! $? -eq 0 ]]; then
   printf "${red} Wrong answer ${noColor}\n"
   break
  else
   printf "${green} Accepted ${noColor}\n"
  fi
 done
}

test() {
 g++ --std=c++11 $1.cpp -o prog
 for ((i = 1; i <= 50; i++)); do
  [[ -f $1_in$i ]] || break
  printf "Test case #$i"
  diff -uwi <(./prog < $1_in$i) $1_out$i > $1_diff
```

```
  if [[ ! $? -eq 0 ]]; then
   printf "${red} Wrong answer ${noColor}\n"
  else
   printf "${green} Accepted ${noColor}\n"
  fi
 done
}
```

Bump allocator

```
static char buf[450 << 20];
void* operator new(size_t s) {
  static size_t i = sizeof buf; assert(s < i);
  return (void *) &buf[i -= s];
}
void operator delete(void *) {}
```

1 Data structures

1.1 Disjoint set with rollback

```
struct Dsu {
  vector<int> pr, tot;
  stack<ii> what;

  Dsu(int n = 0) : pr(n + 5), tot(n + 5, 1) {
    iota(all(pr), 0);
  }

  int find(int u) {
    return pr[u] == u ? u : find(pr[u]);
  }

  void unite(int u, int v) {
    u = find(u), v = find(v);
    if (u == v)
      what.emplace(-1, -1);
    else {
      if (tot[u] < tot[v])
        swap(u, v);
      what.emplace(u, v);
      tot[u] += tot[v];
      pr[v] = u;
    }
  }

  ii rollback() {
    ii last = what.top();
    what.pop();
    int u = last.f, v = last.s;
    if (u != -1) {
      tot[u] -= tot[v];
      pr[v] = v;
    }
    return last;
  }
};
```

1.2 Sparse table

```
int lg[N + 1], sp[1 + __lg(N)][N];

int query(int l, int r) {
  int sum = 0LL;
  fore (k, 1 + lg[N], 0)
    if (l + (1 << k) - 1 <= r) {
      sum += sp[k][l];
      l += (1 << k);
    }
  return sum;
}

int query(int l, int r) {
  int k = lg[r - l + 1];
  return min(sp[k][l], sp[k][r - (1 << k) + 1]);
}

void sparse() {
  fore (i, 2, N + 1)
    lg[i] = lg[i >> 1] + 1;
  fore (i, 0, n)
    sp[0][i] = a[i];
  for (int k = 1; (1 << k) <= n; k++)
    fore (l, 0, n - (1 << k) + 1) {
      int r = l + (1 << (k - 1));
      sp[k][l] = min(sp[k - 1][l], sp[k - 1][r]);
    }
}
```

1.3 Min-Max queue

```
template <class T>
struct MinQueue : deque< pair<T, int> > {
  // add a element to the right {val, pos}
  void add(T val, int pos) {
    while (!empty() && back().f >= val)
      pop_back();
    emplace_back(val, pos);
  }
  // remove all less than pos
  void rem(int pos) {
    while (front().s < pos)
      pop_front();
  }

  T qmin() { return front().f; }
};
```

1.4 Squirtle decomposition

The perfect block size is *squirtle* of N

```
int blo[N], cnt[N][B], a[N];

void update(int i, int x) {
  cnt[blo[i]][x]--;
  a[i] = x;
  cnt[blo[i]][x]++;
}

int query(int l, int r, int x) {
  int tot = 0;
  while (l <= r)
    if (l % B == 0 && l + B - 1 <= r) {
      tot += cnt[blo[l]][x];
      l += B;
    } else {
      tot += (a[l] == x);
      l++;
    }
  return tot;
}
```

1.5 In-Out trick

```
vector<int> in[N], out[N];
vector<Query> queries;

fore (x, 0, N) {
  for (int i : in[x])
    add(queries[i]);
  // solve
  for (int i : out[x])
    rem(queries[i]);
}
```

1.6 Parallel binary search

```
int lo[Q], hi[Q];
```

```
queue<int> solve[N];
vector<Query> queries;

fore (it, 0, 1 + __lg(N)) {
  fore (i, 0, sz(queries))
    if (lo[i] != hi[i]) {
      int mid = (lo[i] + hi[i]) / 2;
      solve[mid].emplace(i);
    }
  fore (x, 0, n) {
    // simulate
    while (!solve[x].empty()) {
      int i = solve[x].front();
      solve[x].pop();
      if (can(queries[i]))
        hi[i] = x;
      else
        lo[i] = x + 1;
    }
  }
}
```

1.7 Mo's algorithm

```
vector<Query> queries;
// N = 1e6, so aprox. sqrt(N) +/- C
uniform_int_distribution<int> dis(970, 1030);
const int blo = dis(rng);
sort(all(queries), [&](Query a, Query b) {
  const int ga = a.l / blo, gb = b.l / blo;
  if (ga == gb)
    return (ga & 1) ? a.r < b.r : a.r > b.r;
  return a.l < b.l;
});
int l = queries[0].l, r = l - 1;
for (Query &q : queries) {
  while (r < q.r)
    add(++r);
  while (r > q.r)
    rem(r--);
  while (l < q.l)
    rem(l++);
  while (l > q.l)
    add(--l);
  ans[q.i] = solve();
}
```

To make it faster, change the order to $hilbert(l, r)$

```
lli hilbert(int x, int y, int pw = 21, int rot = 0) {
  if (pw == 0)
    return 0;
  int hpw = 1 << (pw - 1);
  int k = ((x < hpw ? y < hpw ? 0 : 3 : y < hpw ? 1 :
      2) + rot) & 3;
  const int d[4] = {3, 0, 0, 1};
  lli a = 1LL << ((pw << 1) - 2);
  lli b = hilbert(x & (x ^ hpw), y & (y ^ hpw), pw - 1
      , (rot + d[k]) & 3);
  return k * a + (d[k] ? a - b - 1 : b);
}
```

Mo's algorithm with updates in $\mathcal{O}(n^{\frac{5}{3}})$

- Choose a *block* of size $n^{\frac{2}{3}}$
- Do a normal Mo's algorithm, in the *Query* definition add an extra variable for the *updatesSoFar*
- Sort the queries by the order ($l/block$, $r/block$, *updatesSoFar*)
- If the update lies inside the current query, update the data structure properly

```
struct Update {
  int pos, prv, nxt;
};

void undo(Update &u) {
  if (l <= u.pos && u.pos <= r) {
    rem(u.pos);
    a[u.pos] = u.prv;
    add(u.pos);
  } else {
    a[u.pos] = u.prv;
  }
}
```

- Solve the problem :D

```
l = queries[0].l, r = l - 1, upd = sz(updates) - 1;
for (Query &q : queries) {
  while (upd < q.upd)
    dodo(updates[++upd]);
  while (upd > q.upd)
    undo(updates[upd--]);
  // write down the normal Mo's algorithm
}
```

1.8 Ordered tree

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template <class K, class V = null_type>
using ordered_tree = tree<K, V, less<K>, rb_tree_tag,
     tree_order_statistics_node_update>;
// less_equal<K> for multiset, multimap (?
#define qrank order_of_key
#define qkth find_by_order
```

1.9 Unordered tree

```
struct chash {
  const uint64_t C = uint64_t(2e18 * 3) + 71;
  const int R = rng();
  uint64_t operator ()(uint64_t x) const {
    return __builtin_bswap64((x ^ R) * C); }
};

template <class K, class V = null_type>
using unordered_tree = gp_hash_table<K, V, chash>;
```

1.10 D-dimensional Fenwick tree

```
template <class T, int ...N>
struct Fenwick {
  T v = 0;
  void update(T v) { this->v += v; }
  T query() { return v; }
};

template <class T, int N, int ...M>
struct Fenwick<T, N, M...> {
  #define lsb(x) (x & -x)
  Fenwick<T, M...> fenw[N + 1];

  template <typename... Args>
  void update(int i, Args... args) {
    for (; i <= N; i += lsb(i))
      fenw[i].update(args...);
  }

  template <typename... Args>
  T query(int l, int r, Args... args) {
    T v = 0;
    for (; r > 0; r -= lsb(r))
      v += fenw[r].query(args...);
```

```
    for (--l; l > 0; l -= lsb(l))
      v -= fenw[l].query(args...);
    return v;
  }
};
```

1.11 Dynamic segment tree

```
struct Dyn {
  int l, r;
  lli sum = 0;
  Dyn *L, *R;

  Dyn(int l, int r) : l(l), r(r), L(0), R(0) {}

  void pull() {
    sum = (L ? L->sum : 0);
    sum += (R ? R->sum : 0);
  }

  void update(int p, lli v) {
    if (l == r) {
      sum += v;
      return;
    }
    int m = (l + r) / 2;
    if (p <= m) {
      if (!L)
        L = new Dyn(l, m);
      L->update(p, v);
    } else {
      if (!R)
        R = new Dyn(m + 1, r);
      R->update(p, v);
    }
    pull();
  }

  lli qsum(int ll, int rr) {
    if (rr < l || r < ll || r < l)
      return 0;
    if (ll <= l && r <= rr)
      return sum;
    int m = (l + r) / 2;
    return (L ? L->qsum(ll, rr) : 0) +
           (R ? R->qsum(ll, rr) : 0);
  }
};
```

1.12 Persistent segment tree

```
struct Per {
  int l, r;
  lli sum = 0;
  Per *L, *R;

  Per(int l, int r) : l(l), r(r), L(0), R(0) {}

  Per* pull() {
    sum = L->sum + R->sum;
    return this;
  }

  void build() {
    if (l == r)
      return;
    int m = (l + r) / 2;
    (L = new Per(l, m))->build();
    (R = new Per(m + 1, r))->build();
    pull();
  }

  Per* update(int p, lli v) {
    if (p < l || r < p)
      return this;
    Per* t = new Per(l, r);
    if (l == r) {
      t->sum = v;
      return t;
    }
    t->L = L->update(p, v);
    t->R = R->update(p, v);
    return t->pull();
  }

  lli qsum(int ll, int rr) {
    if (r < ll || rr < l)
      return 0;
    if (ll <= l && r <= rr)
      return sum;
    return L->qsum(ll, rr) + R->qsum(ll, rr);
  }
};

Per *tree[T];
```

1.13 Wavelet tree

```
struct Wav {
  #define iter int * // vector<int>::iterator
  int lo, hi;
  Wav *L, *R;
  vi amt;

  Wav(int lo, int hi) : lo(lo), hi(hi), L(0), R(0) {}

  void build(iter b, iter e) { // array 1-indexed
    if (lo == hi || b == e)
      return;
    amt.reserve(e - b + 1);
    amt.pb(0);
    int m = (lo + hi) / 2;
    for (auto it = b; it != e; it++)
      amt.pb(amt.back() + (*it <= m));
    auto p = stable_partition(b, e, [=](int x) {
      return x <= m;
    });
    (L = new Wav(lo, m))->build(b, p);
    (R = new Wav(m + 1, hi))->build(p, e);
  }

  int qkth(int l, int r, int k) {
    if (r < l)
      return 0;
    if (lo == hi)
      return lo;
    if (k <= amt[r] - amt[l - 1])
      return L->qkth(amt[l - 1] + 1, amt[r], k);
    return R->qkth(l - amt[l - 1], r - amt[r], k - amt
        [r] + amt[l - 1]);
  }

  int qleq(int l, int r, int mx) {
    if (r < l || mx < lo)
      return 0;
    if (hi <= mx)
      return r - l + 1;
    return L->qleq(amt[l - 1] + 1, amt[r], mx) +
           R->qleq(l - amt[l - 1], r - amt[r], mx);
  }
};
```

1.14 Li Chao tree

```
struct Fun {
  lli m = 0, c = inf;
```

```
  lli operator ()(lli x) const { return m * x + c; }
};

struct LiChao {
  Fun f;
  lli l, r;
  LiChao *L, *R;

  LiChao(int l, int r, Fun f = {}) :
    l(l), r(r), f(f), L(0), R(0) {}

  void add(Fun &g) {
    if (f(l) <= g(l) && f(r) <= g(r))
      return;
    if (g(l) < f(l) && g(r) < f(r)) {
      f = g;
      return;
    }
    lli m = (l + r) / 2;
    if (g(m) < f(m))
      swap(f, g);
    if (g(l) <= f(l))
      L = L ? (L->add(g), L) : new LiChao(l, m, g);
    else
     R = R ? (R->add(g), R) : new LiChao(m + 1, r, g);
  }

  lli query(lli x) {
    if (l == r)
      return f(x);
    lli m = (l + r) / 2;
    if (x <= m)
      return min(f(x), L ? L->query(x) : inf);
    return min(f(x), R ? R->query(x) : inf);
  }
};
```

1.15 Treap

```
typedef struct Node* Treap;
struct Node {
  uint32_t pri = rng();
  int val;
  Treap ch[2] = {0, 0};
  int sz = 1, flip = 0;
  Node(int val) : val(val) {}
};

void push(Treap t) {
  if (!t)
    return;
  if (t->flip) {
    swap(t->ch[0], t->ch[1]);
    for (Treap ch : t->ch) if (ch)
      ch->flip ^= 1;
    t->flip = 0;
  }
}

Treap pull(Treap t) {
  #define gsz(t) (t ? t->sz : 0)
  t->sz = 1;
  for (Treap ch : t->ch)
    push(ch), t->sz += gsz(ch);
  return t;
}

pair<Treap, Treap> split(Treap t, int val) {
  // <= val goes to the left, > val to the right
  if (!t)
    return {t, t};
  push(t);
  if (val < t->val) {
    auto p = split(t->ch[0], val);
    t->ch[0] = p.s;
    return {p.f, pull(t)};
  }
  auto p = split(t->ch[1], val);
  t->ch[1] = p.f;
  return {pull(t), p.s};
}

pair<Treap, Treap> splitsz(Treap t, int sz) {
  // <= sz goes to the left, > sz to the right
  if (!t)
    return {t, t};
  push(t);
  if (sz <= gsz(t->ch[0])) {
    auto p = splitsz(t->ch[0], sz);
    t->ch[0] = p.s;
    return {p.f, pull(t)};
  }
  auto p = splitsz(t->ch[1], sz - gsz(t->ch[0]) - 1);
  t->ch[1] = p.f;
  return {pull(t), p.s};
}

Treap merge(Treap l, Treap r) {
  if (!l || !r)
    return l ? l : r;
  push(l), push(r);
  if (l->pri > r->pri)
    return l->ch[1] = merge(l->ch[1], r), pull(l);
  else
    return r->ch[0] = merge(l, r->ch[0]), pull(r);
}

Treap qkth(Treap t, int k) { // 0-indexed
  if (!t)
    return t;
  push(t);
  int sz = gsz(t->ch[0]);
  if (sz == k)
    return t;
  return k < sz ? qkth(t->ch[0], k) : qkth(t->ch[1], k
      - sz - 1);
}

int qrank(Treap t, int val) { // 0-indexed
  if (!t)
    return -1;
  push(t);
  if (val < t->val)
    return qrank(t->ch[0], val);
  if (t->val == val)
    return gsz(t->ch[0]);
  return gsz(t->ch[0]) + qrank(t->ch[1], val) + 1;
}

Treap insert(Treap t, int val) {
  auto p1 = split(t, val);
  auto p2 = split(p1.f, val - 1);
  return merge(p2.f, merge(new Node(val), p1.s));
}

Treap erase(Treap t, int val) {
  auto p1 = split(t, val);
  auto p2 = split(p1.f, val - 1);
  return merge(p2.f, p1.s);
}
```

2 Graphs

2.1 Tarjan algorithm (SCC)

```
vector<vi> scc;
int tin[N], fup[N];
bitset<N> still;
stack<int> stk;
int timer = 0;

void tarjan(int u) {
  tin[u] = fup[u] = ++timer;
  still[u] = true;
  stk.push(u);
  for (int v : graph[u]) {
    if (!tin[v])
      tarjan(v);
    if (still[v])
      fup[u] = min(fup[u], fup[v]);
  }
  if (fup[u] == tin[u]) {
    scc.pb({});
    int v;
    do {
      v = stk.top();
      stk.pop();
      still[v] = false;
      scc.back().pb(v);
    } while (v != u);
  }
}
```

2.2 Kosaraju algorithm (SCC)

```
int scc[N], k = 0;
char vis[N];
vi order;

void dfs1(int u) {
  vis[u] = 1;
  for (int v : graph[u])
    if (vis[v] != 1)
      dfs1(v);
  order.pb(u);
}

void dfs2(int u, int k) {
  vis[u] = 2, scc[u] = k;
  for (int v : rgraph[u]) // reverse graph
    if (vis[v] != 2)
      dfs2(v, k);
}

void kosaraju() {
  fore (u, 1, n + 1)
    if (vis[u] != 1)
      dfs1(u);
  reverse(all(order));
  for (int u : order)
    if (vis[u] != 2)
      dfs2(u, ++k);
}
```

2.3 Two Sat

```
void add(int u, int v) {
  graph[u].pb(v);
  rgraph[v].pb(u);
}

void implication(int u, int v) {
  #define neg(u) ((n) + (u))
  add(u, v);
  add(neg(v), neg(u));
}

pair<bool, vi> satisfy(int n) {
  kosaraju(2 * n); // size of the two-sat is 2 * n
  vi ans(n + 1, 0);
  fore (u, 1, n + 1) {
    if (scc[u] == scc[neg(u)])
      return {0, ans};
    ans[u] = scc[u] > scc[neg(u)];
  }
  return {1, ans};
}
```

2.4 Topological sort

2.5 Cutpoints and Bridges

```
int tin[N], fup[N], timer = 0;

void findWeakness(int u, int p = 0) {
  tin[u] = fup[u] = ++timer;
  int children = 0;
  for (int v : graph[u]) if (v != p) {
    if (!tin[v]) {
      ++children;
      findWeakness(v, u);
      fup[u] = min(fup[u], fup[v]);
      if (fup[v] >= tin[u] && p) // u is a cutpoint
      if (fup[v] > tin[u]) // bridge u -> v
    }
    fup[u] = min(fup[u], tin[v]);
  }
  if (!p && children > 1) // u is a cutpoint
}
```

2.6 Detect a cycle

```
bool cycle(int u) {
  vis[u] = 1;
  for (int v : graph[u]) {
    if (vis[v] == 1)
      return true;
    if (!vis[v] && cycle(v))
      return true;
  }
  vis[u] = 2;
  return false;
}
```

2.7 Euler tour for Mo's in a tree

Mo's in a tree, extended euler tour tin[u] = ++timer, tout[u] = ++timer

- u = lca(u, v), query(tin[u], tin[v])
- u $\neq$ lca(u, v), query(tout[u], tin[v]) + query(tin[lca], tin[lca])

2.8 Lowest common ancestor (LCA)

```
const int LogN = 1 + __lg(N);
int pr[LogN][N], dep[N];

void dfs(int u, int pr[]) {
  for (int v : graph[u])
    if (v != pr[u]) {
      pr[v] = u;
      dep[v] = dep[u] + 1;
      dfs(v, pr);
    }
}

int lca(int u, int v){
  if (dep[u] > dep[v])
    swap(u, v);
  fore (k, LogN, 0)
    if (dep[v] - dep[u] >= (1 << k))
```

```
    v = pr[k][v];
  if (u == v)
    return u;
  fore (k, LogN, 0)
    if (pr[k][v] != pr[k][u])
      u = pr[k][u], v = pr[k][v];
  return pr[0][u];
}

int dist(int u, int v) {
  return dep[u] + dep[v] - 2 * dep[lca(u, v)];
}

void init(int r) {
  dfs(r, pr[0]);
  fore (k, 1, LogN)
    fore (u, 1, n + 1)
      pr[k][u] = pr[k - 1][pr[k - 1][u]];
}
```

2.9 Guni

```
int tin[N], tout[N], who[N], sz[N], heavy[N], color[N
    ];
int timer = 0;

int dfs(int u, int pr = 0){
  sz[u] = 1, tin[u] = ++timer, who[timer] = u;
  for (int v : graph[u]) if (v != pr) {
    sz[u] += dfs(v, u);
    if (sz[v] > sz[heavy[u]])
      heavy[u] = v;
  }
  return tout[u] = timer, sz[u];
}

void guni(int u, int pr = 0, bool keep = 0) {
  for (int v : graph[u])
    if (v != pr && v != heavy[u])
      guni(v, u, 0);
  if (heavy[u])
    guni(heavy[u], u, 1);
  for (int v : graph[u])
    if (v != pr && v != heavy[u])
      fore (i, tin[v], tout[v] + 1)
        add(color[who[i]]);
  add(color[u]);
  // Solve the subtree queries here
  if (keep == 0)
    fore (i, tin[u], tout[u] + 1)
      rem(color[who[i]]);
}
```

2.10 Centroid decomposition

```
int cdp[N], sz[N];
bitset<N> rem;

int dfsz(int u, int p = 0) {
  sz[u] = 1;
  for (int v : graph[u])
    if (v != p && !rem[v])
      sz[u] += dfsz(v, u);
  return sz[u];
}

int centroid(int u, int n, int p = 0) {
  for (int v : graph[u])
    if (v != p && !rem[v] && 2 * sz[v] > n)
      return centroid(v, n, u);
  return u;
}

void solve(int u, int p = 0) {
  cdp[u = centroid(u, dfsz(u))] = p;
  rem[u] = true;
  for (int v : graph[u])
    if (!rem[v])
      solve(v, u);
}
```

2.11 Heavy-light decomposition

```
int pr[N], dep[N], sz[N], heavy[N], head[N], pos[N],
    who[N], timer = 0;
Lazy* tree; // generally a lazy segtree

int dfs(int u) {
  sz[u] = 1, heavy[u] = head[u] = 0;
  for (int v : graph[u]) if (v != pr[u]) {
    pr[v] = u;
    dep[v] = dep[u] + 1;
    sz[u] += dfs(v);
    if (sz[v] > sz[heavy[u]])
      heavy[u] = v;
  }
  return sz[u];
}

void hld(int u, int h) {
  head[u] = h, pos[u] = ++timer, who[timer] = u;
  if (heavy[u] != 0)
    hld(heavy[u], h);
  for (int v : graph[u])
    if (v != pr[u] && v != heavy[u])
      hld(v, v);
}

template <class F>
void processPath(int u, int v, F fun) {
  for (; head[u] != head[v]; v = pr[head[v]]) {
    if (dep[head[u]] > dep[head[v]])
      swap(u, v);
    fun(pos[head[v]], pos[v]);
  }
  if (dep[u] > dep[v])
    swap(u, v);
  if (u != v)
    fun(pos[heavy[u]], pos[v]);
  fun(pos[u], pos[u]); // process lca(u, v) too?
}

void updatePath(int u, int v, lli z) {
  processPath(u, v, [&](int l, int r) {
    tree->update(l, r, z);
  });
}

lli queryPath(int u, int v) {
  lli sum = 0;
  processPath(u, v, [&](int l, int r) {
    sum += tree->qsum(l, r);
  });
  return sum;
}
```

2.12 Link-Cut tree

```
typedef struct Node* Splay;
struct Node {
  int val, mx = 0;
  Splay ch[2] = {0, 0}, p = 0;
  int sz = 1, flip = 0;
  Node(int val) : val(val), mx(val) {}
};
```

```
void push(Splay u) {
  if (!u || !u->flip)
    return;
  swap(u->ch[0], u->ch[1]);
  for (Splay v : u->ch)
    if (v) v->flip ^= 1;
  u->flip = 0;
}

void pull(Splay u) {
  #define gsz(t) (t ? t->sz : 0)
  u->sz = 1, u->mx = u->val;
  for (Splay v : u->ch) if (v) {
    push(v);
    u->sz += gsz(v);
    u->mx = max(u->mx, v->mx);
  }
}

int dir(Splay u) {
  if (!u->p) return -2; // root of the LCT component
  if (u->p->ch[0] == u) return 0; // left child
  if (u->p->ch[1] == u) return 1; // right child
  return -1; // root of current splay tree
}

void add(Splay u, Splay v, int d) {
  if (v) v->p = u;
  if (d >= 0) u->ch[d] = v;
}

void rot(Splay u) { // assume p and p->p propagated
  int x = dir(u);
  Splay g = u->p;
  add(g->p, u, dir(g));
  add(g, u->ch[x ^ 1], x);
  add(u, g, x ^ 1);
  pull(g), pull(u);
}

void splay(Splay u) {
  #define isRoot(u) (dir(u) < 0)
  while (!isRoot(u) && !isRoot(u->p)) {
    push(u->p->p), push(u->p), push(u);
    rot(dir(u) == dir(u->p) ? u->p : u);
    rot(u);
  }
  if (!isRoot(u)) push(u->p), push(u), rot(u);
  push(u);
}

// puts u on the preferred path, splay (right subtree
    is empty)
void access(Splay u) {
  for (Splay v = u, last = 0; v; v = v->p) {
    splay(v); // now switch virtual children, i don't
        know what this means!!
    // if (last) v->vsub -= last->sub;
    // if (v->ch[1]) v->vsub += v->ch[1]->sub;
    v->ch[1] = last, pull(v), last = v;
  }
  splay(u);
}

void rootify(Splay u) {
  access(u), u->flip ^= 1, access(u);
}

Splay lca(Splay u, Splay v) {
  if (u == v) return u;
  access(u), access(v);
  if (!u->p) return 0;
  return splay(u), u->p ?: u;
}

bool connected(Splay u, Splay v) {
  return lca(u, v);
}

void link(Splay u, Splay v) { // make u parent of v,
    make sure they aren't connected
  if (!connected(u, v)) {
    rootify(v), access(u);
    add(v, u, 0), pull(v);
  }
}

void cut(Splay u) { // cut u from its parent
  access(u);
  u->ch[0] = u->ch[0]->p = 0;
  pull(u);
}

void cut(Splay u, Splay v) { // if u, v adj in tree
  rootify(u), access(v), cut(v);
}

Splay getRoot(Splay u) {
  access(u);
  while (u->ch[0])
    u = u->ch[0], push(u);
  return access(u), u;
}

Splay lift(Splay u, int k) {
  push(u);
  int sz = gsz(u->ch[0]);
  if (sz == k)
    return splay(u), u;
  return k < sz ? lift(u->ch[0], k) : lift(u->ch[1], k
      - sz - 1);
}

Splay ancestor(Splay u, int k) {
  return access(u), lift(u, gsz(u->ch[0]) - k);
}

Splay query(Splay u, Splay v) {
  return rootify(u), access(v), v;
}

Splay lct[N];
```

3 Flows

3.1 Dinic $\mathcal{O}(min(E \cdot flow, V^2E))$

If the network is massive, try to compress it by looking for patterns. Dinic with scaling works in $\mathcal{O}(EV \cdot log(maxCap))$.

```
template <class F>
struct Dinic {
  struct Edge {
    int v, inv;
    F cap, flow;
    Edge(int v, F cap, int inv) :
      v(v), cap(cap), flow(0), inv(inv){}
  };

  F eps = (F) 1e-9, lim = (F) 1e-9;
  const bool scaling = 0;
  int s, t, n, m = 0;
  vector< vector<Edge> > g;
```

```
  vi dist, ptr;

  Dinic(int n, int ss = -1, int tt = -1) :
    n(n), g(n + 5), dist(n + 5), ptr(n + 5) {
    s = ss == -1 ? n + 1 : ss;
    t = tt == -1 ? n + 2 : tt;
  }

  void add(int u, int v, F cap) {
    g[u].pb(Edge(v, cap, sz(g[v])));
    g[v].pb(Edge(u, 0, sz(g[u]) - 1));
    lim = (scaling ? max(lim, cap) : lim);
    m += 2;
  }

  bool bfs() {
    fill(all(dist), -1);
    queue<int> qu({s});
    dist[s] = 0;
    while (sz(qu) && dist[t] == -1) {
      int u = qu.front(); qu.pop();
      for (Edge &e : g[u]) if (dist[e.v] == -1)
        if (scaling ? e.cap - e.flow >= lim : e.cap -
            e.flow > eps) {
          dist[e.v] = dist[u] + 1;
          qu.push(e.v);
        }
    }
    return dist[t] != -1;
  }

  F dfs(int u, F flow = numeric_limits<F>::max()) {
    if (flow <= eps || u == t)
      return max<F>(0, flow);
    for (int &i = ptr[u]; i < sz(g[u]); i++) {
      Edge &e = g[u][i];
      if (e.cap - e.flow > eps && dist[u] + 1 == dist[
          e.v]) {
       F pushed = dfs(e.v, min<F>(flow, e.cap - e.flow
           ));
        if (pushed > eps) {
          e.flow += pushed;
          g[e.v][e.inv].flow -= pushed;
          return pushed;
        }
      }
    }
    return 0;
  }

  F maxFlow() {
    F flow = 0;
    for (lim = scaling ? lim : 1; lim > eps; lim /= 2)
      while (bfs()) {
        fill(all(ptr), 0);
        while (F pushed = dfs(s))
          flow += pushed;
      }
    return flow;
  }
};
```

3.2 Min cost flow $\mathcal{O}(min(E \cdot flow, V^2 E))$

If the network is massive, try to compress it by looking for patterns.

```
template <class C, class F>
struct Mcmf {
  static constexpr F eps = (F) 1e-9;
  struct Edge {
    int u, v, inv;
    F cap, flow;
    C cost;
    Edge(int u, int v, C cost, F cap, int inv) :
      u(u), v(v), cost(cost), cap(cap), flow(0), inv(
          inv) {}
  };

  int s, t, n, m = 0;
  vector< vector<Edge> > g;
  vector<Edge*> prev;
  vector<C> cost;
  vi state;

  Mcmf(int n, int ss = -1, int tt = -1):
    n(n), g(n + 5), cost(n + 5), state(n + 5), prev(n
        + 5) {
    s = ss == -1 ? n + 1 : ss;
    t = tt == -1 ? n + 2 : tt;
  }

  void add(int u, int v, C cost, F cap) {
    g[u].pb(Edge(u, v, cost, cap, sz(g[v])));
    g[v].pb(Edge(v, u, -cost, 0, sz(g[u]) - 1));
    m += 2;
  }

  bool bfs() {
    fill(all(state), 0);
    fill(all(cost), numeric_limits<C>::max());
    deque<int> qu;
    qu.push_back(s);
    state[s] = 1, cost[s] = 0;
    while (sz(qu)) {
      int u = qu.front(); qu.pop_front();
      state[u] = 2;
      for (Edge &e : g[u]) if (e.cap - e.flow > eps)
        if (cost[u] + e.cost < cost[e.v]) {
          cost[e.v] = cost[u] + e.cost;
          prev[e.v] = &e;
          if (state[e.v] == 2 || (sz(qu) && cost[qu.
              front()] > cost[e.v]))
            qu.push_front(e.v);
          else if (state[e.v] == 0)
            qu.push_back(e.v);
          state[e.v] = 1;
        }
    }
    return cost[t] != numeric_limits<C>::max();
  }

  pair<C, F> minCostFlow() {
    C cost = 0; F flow = 0;
    while (bfs()) {
      F pushed = numeric_limits<F>::max();
      for (Edge* e = prev[t]; e != nullptr; e = prev[e
          ->u])
        pushed = min(pushed, e->cap - e->flow);
      for (Edge* e = prev[t]; e != nullptr; e = prev[e
          ->u]) {
        e->flow += pushed;
        g[e->v][e->inv].flow -= pushed;
        cost += e->cost * pushed;
      }
      flow += pushed;
    }
    return make_pair(cost, flow);
  }
};
```

3.3 Hopcroft-Karp $\mathcal{O}(E\sqrt{V})$

```
struct HopcroftKarp {
  int n, m = 0;
```

```
  vector<vi> g;
  vi dist, match;

  HopcroftKarp(int _n) : n(5 + _n), g(n + 5), dist(n +
      5), match(n + 5, 0) {}

  void add(int u, int v) {
    g[u].pb(v), g[v].pb(u);
    m += 2;
  }

  bool bfs() {
    queue<int> qu;
    fill(all(dist), -1);
    fore (u, 1, n + 1)
      if (!match[u])
        dist[u] = 0, qu.push(u);
    while (!qu.empty()) {
      int u = qu.front(); qu.pop();
      for (int v : g[u])
        if (dist[match[v]] == -1) {
          dist[match[v]] = dist[u] + 1;
          if (match[v])
            qu.push(match[v]);
        }
    }
    return dist[0] != -1;
  }

  bool dfs(int u) {
    for (int v : g[u])
      if (!match[v] || (dist[u] + 1 == dist[match[v]]
           && dfs(match[v]))) {
        match[u] = v, match[v] = u;
        return 1;
      }
    dist[u] = 1 << 30;
    return 0;
  }

  int maxMatching() {
    int tot = 0;
    while (bfs())
      fore (u, 1, n + 1)
        tot += match[u] ? 0 : dfs(u);
    return tot;
  }
};
```

3.4 Hungarian $\mathcal{O}(N^3)$

n jobs, m people

```
template <class C>
pair<C, vi> Hungarian(vector< vector<C> > &a) {
  int n = sz(a), m = sz(a[0]), p, q, j, k; // n <= m
 vector<C> fx(n, numeric_limits<C>::min()), fy(m, 0);
  vi x(n, -1), y(m, -1);
  fore (i, 0, n)
    fore (j, 0, m)
      fx[i] = max(fx[i], a[i][j]);
  fore (i, 0, n) {
    vi t(m, -1), s(n + 1, i);
    for (p = q = 0; p <= q && x[i] < 0; p++)
      for (k = s[p], j = 0; j < m && x[i] < 0; j++)
        if (abs(fx[k] + fy[j] - a[k][j]) < eps && t[j]
             < 0) {
          s[++q] = y[j], t[j] = k;
          if (s[q] < 0) for (p = j; p >= 0; j = p)
            y[j] = k = t[j], p = x[k], x[k] = j;
        }
    if (x[i] < 0) {
      C d = numeric_limits<C>::max();
      fore (k, 0, q + 1)
        fore (j, 0, m) if (t[j] < 0)
          d = min(d, fx[s[k]] + fy[j] - a[s[k]][j]);
      fore (j, 0, m)
        fy[j] += (t[j] < 0 ? 0 : d);
      fore (k, 0, q + 1)
        fx[s[k]] -= d;
      i--;
    }
  }
  C cost = 0;
  fore (i, 0, n) cost += a[i][x[i]];
  return make_pair(cost, x);
}
```

4 Strings

4.1 Hash

```
vi p = {10006793, 1777771, 10101283, 10101823, 1013635
    9, 10157387, 10166249};
vi mod = {999992867, 1070777777, 999727999, 1000008223
    , 1000009999, 1000003211, 1000027163, 1000002193,
     1000000123};
int pw[2][N];

struct Hash {
  vector<vi> h;

  Hash(string &s) : h(2, vi(sz(s) + 1, 0)) {
    fore (i, 0, 2)
      fore (j, 0, sz(s))
        h[i][j + 1] = (h[i][j] + lli(s[j] - 'a' + 1) *
             pw[i][j]) % mod[i];
  }

  lli query(int l, int r) {
    lli f[2] = {0, 0};
    fore (i, 0, 2)
      f[i] = 1LL * (h[i][r + 1] - h[i][l] + mod[i]) %
          mod[i] * pw[i][N - l - 1] % mod[i];
    return (f[0] << 32) | f[1];
  }
};

shuffle(all(p), rng), shuffle(all(mod), rng);
fore (i, 0, 2) {
  pw[i][0] = 1LL;
  fore (j, 1, N)
    pw[i][j] = 1LL * p[0] * pw[i][j - 1] % mod[i];
}
```

4.2 KMP

period $= n - lps[n-1]$, period$(abcabc) = 3$, $n \mod period \equiv 0$

```
int lps[N];

void pre(string &s) {
  int j = (lps[0] = 0);
  fore (i, 1, sz(s)) {
    while (j && s[i] != s[j])
      j = lps[j - 1];
    if (s[i] == s[j])
      j++;
    lps[i] = j;
  }
}
// how many times t occurs in s
int kmp(string &s, string &t) {
  pre(t);
  int j = 0, tot = 0;
  fore (i, 0, sz(s)) {
    while (j && s[i] != t[j])
```

```
        j = lps[j - 1];
      if (s[i] == t[j])
        j++;
      if (j == sz(t))
        tot++; // pos: i - sz(t) + 1;
    }
    return tot;
  }
```

4.3 KMP automaton

```
int go[N][A];

void kmpAutomaton(string &s) {
  s += "$";
  fore (i, 0, sz(s))
    fore (c, 0, A) {
      if (i && s[i] != 'a' + c)
        go[i][c] = go[lps[i - 1]][c];
      else
        go[i][c] = i + ('a' + c == s[i]);
    }
  s.pop_back();
}
```

4.4 Z algorithm

```
int z[N];

void zf(string &s) {
  for (int i = 1, l = 0, r = 0; i < sz(s); i++) {
    if (i <= r)
      z[i] = min(r - i + 1, z[i - l]);
    while (i + z[i] < sz(s) && s[i + z[i]] == s[z[i]])
      ++z[i];
    if (i + z[i] - 1 > r)
      l = i, r = i + z[i] - 1;
  }
}
```

4.5 Manacher algorithm

```
int aba[2][N];

void manacher(string &s) {
  fore (k, 0, 2) {
    int l = 0, r = 0;
    fore (i, 0, sz(s)) {
      if (i < r)
        aba[k][i] = min(r - i + !k, aba[k][l + r - i +
            !k]);
      int p = i - aba[k][i], q = i + aba[k][i] - !k;
      while (p >= 1 && q + 1 < sz(s) && s[p - 1] == s[
          q + 1])
        ++aba[k][i], --p, ++q;
      if (q > r)
        l = p, r = q;
    }
  }
}
```

4.6 Suffix array

- Duplicates $\sum_{i=1}^{n} lcp[i]$
- Longest Common Substring of various strings
  Add $notUsed$ characters between strings, i.e. $a+\$+b+\#+c$
  Use two-pointers to find a range $[l, r]$ such that all $notUsed$ characters are present, then $query(lcp[l+1], .., lcp[r])$ for that window is the common length.

```
struct SuffixArray {
  int n;
  string s;
  vi sa, lcp;
  SuffixArray(string &s) : n(sz(s) + 1), s(s), sa(n),
      lcp(n) {
    vi top(max(256, n)), rk(n);
    fore (i, 0, n)
      top[rk[i] = s[i] & 255]++;
    partial_sum(all(top), top.begin());
    fore (i, 0, n)
      sa[--top[rk[i]]] = i;
    vi sb(n);
    for (int len = 1, j; len < n; len <<= 1) {
      fore (i, 0, n) {
        j = (sa[i] - len + n) % n;
        sb[top[rk[j]]++] = j;
      }
      sa[sb[top[0] = 0]] = j = 0;
      fore (i, 1, n) {
        if (rk[sb[i]] != rk[sb[i - 1]] || rk[sb[i] +
            len] != rk[sb[i - 1] + len])
          top[++j] = i;
        sa[sb[i]] = j;
      }
      copy(all(sa), rk.begin());
      copy(all(sb), sa.begin());
      if (j >= n - 1)
        break;
    }
    for (int i = 0, j = rk[lcp[0] = 0], k = 0; i < n -
        1; i++, k++)
      while (k >= 0 && s[i] != s[sa[j - 1] + k])
        lcp[j] = k--, j = rk[sa[j] + 1];
  }

  char at(int i, int j) {
    int k = sa[i] + j;
    return k < n ? s[k] : 'z' + 1;
  }

  bool count(string &t) {
    ii lo(0, n - 1), hi(0, n - 1);
    fore (i, 0, sz(t)) {
      while (lo.f + 1 < lo.s) {
        int mid = (lo.f + lo.s) / 2;
        if (at(mid, i) < t[i])
          lo.f = mid;
        else
          lo.s = mid;
      }
      while (hi.f + 1 < hi.s) {
        int mid = (hi.f + hi.s) / 2;
        if (t[i] < at(mid, i))
          hi.s = mid;
        else
          hi.f = mid;
      }
      int p1 = (at(lo.f, i) == t[i] ? lo.f : lo.s);
      int p2 = (at(hi.s, i) == t[i] ? hi.s : hi.f);
      if (at(p1, i) != t[i] || at(p2, i) != t[i] || p1
           > p2)
        return 0;
      lo = hi = ii(p1, p2);
    }
    return lo.s - lo.f + 1;
  }
};
```

4.7 Suffix automaton

- $len[u] - len[link[u]] =$ distinct strings
- Number of different substrings (dp)

$$diff(u) = 1 + \sum_{v \in trie[u]} diff(v)$$

- Total length of all different substrings (2 x dp)

$$totLen(u) = \sum_{v \in trie[u]} diff(v) + totLen(v)$$

- Leftmost occurrence $pos[u] = len[u] - 1$
  if is **clone** then $pos[clone] = pos[q]$
- All occurrence positions
- Smallest cyclic shift
  Construct sam of $s + s$, find the lexicographically smallest path of $sz(s)$
- Shortest non-appearing string

$$nonAppearing(u) = \min_{v \in trie[u]} nonAppearing(v) + 1$$

```
struct SuffixAutomaton {
  vector< map<char, int> > trie;
  vi link, len;
  int last;

  SuffixAutomaton() { last = newNode(); }

  int newNode() {
    trie.pb({}), link.pb(-1), len.pb(0);
    return sz(trie) - 1;
  }

  void extend(char c) {
    int u = newNode();
    len[u] = len[last] + 1;
    int p = last;
    while (p != -1 && !trie[p].count(c)) {
      trie[p][c] = u;
      p = link[p];
    }
    if (p == -1)
      link[u] = 0;
    else {
      int q = trie[p][c];
      if (len[p] + 1 == len[q])
        link[u] = q;
      else {
        int clone = newNode();
        len[clone] = len[p] + 1;
        trie[clone] = trie[q];
        link[clone] = link[q];
        while (p != -1 && trie[p][c] == q) {
          trie[p][c] = clone;
          p = link[p];
        }
        link[q] = link[u] = clone;
      }
    }
    last = u;
  }

  string qkthSubstring(lli kth, int u = 0) {
    // number of different substrings (dp)
    string s = "";
    while (kth > 0)
      for (auto &[c, v] : trie[u]) {
        if (kth <= diff(v)) {
          s.pb(c), kth--, u = v;
          break;
        }
        kth -= diff(v);
      }
    return s;
  }

  void occurs() {
    // occ[u] = 1, occ[clone] = 0
    vi who;
    fore (u, 1, sz(trie))
      who.pb(u);
    sort(all(who), [&](int u, int v) {
      return len[u] > len[v];
    });
    for (int u : who)
      occ[link[u]] += occ[u];
  }

  int qocc(string &s, int u = 0) {
    for (char c : s) {
      if (!trie[u].count(c))
        return 0;
      u = trie[u][c];
    }
    return occ[u];
  }

  int longestCommonSubstring(string &s, int u = 0) {
    int mx = 0, clen = 0;
    for (char c : s) {
      while (u && !trie[u].count(c)) {
        u = link[u];
        clen = len[u];
      }
      if (trie[u].count(c))
        u = trie[u][c], clen++;
      mx = max(mx, clen);
    }
    return mx;
  }

  string smallestCyclicShift(int n, int u = 0) {
    string s = "";
    fore (i, 0, n) {
      char c = trie[u].begin()->f;
      s += c;
      u = trie[u][c];
    }
    return s;
  }

  int leftmost(string &s, int u = 0) {
    for (char c : s) {
      if (!trie[u].count(c))
        return -1;
      u = trie[u][c];
    }
    return pos[u] - sz(s) + 1;
  }
} sam;
```

4.8 Aho corasick

```
struct AhoCorasick {
  vector< map<char, int> > trie;
  vi link, cnt;

  AhoCorasick() { newNode(); }

  int newNode() {
    trie.pb({}), link.pb(0), cnt.pb(0);
    return sz(trie) - 1;
  }

  void insert(string &s, int u = 0) {
    for (char c : s) {
      if (!trie[u][c])
        trie[u][c] = newNode();
      u = trie[u][c];
```

```
    }
    cnt[u]++;
  }

  int go(int u, char c) {
    while (u && !trie[u].count(c))
      u = link[u];
    return trie[u][c];
  }

  void pushLinks() {
    queue<int> qu;
    qu.push(0);
    while (!qu.empty()) {
      int u = qu.front();
      qu.pop();
      for (auto &[c, v] : trie[u]) {
        if (v == 0) {
          v = trie[link[u]][c];
          continue;
        }
        link[v] = u ? go(link[u], c) : 0;
        cnt[v] += cnt[link[v]];
        qu.push(v);
      }
    }
  }

  int match(string &s, int u = 0) {
    int ans = 0;
    for (char c : s)
      u = go(u, c), ans += cnt[u];
    return ans;
  }
} aho;
```

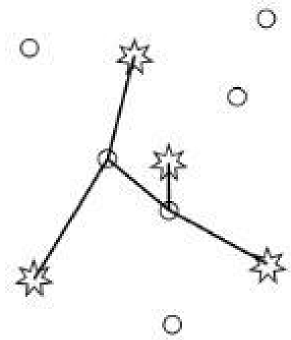
4.9 Eertree

```
struct Eertree {
  vector< map<char, int> > trie;
  vi link, len;
  string s = "$";
  int last;

  Eertree() {
    last = newNode(), newNode();
    link[0] = 1, len[1] = -1;
  }

  int newNode() {
    trie.pb({}), link.pb(0), len.pb(0);
    return sz(trie) - 1;
  }

  int go(int u) {
    while (s[sz(s) - len[u] - 2] != s.back())
      u = link[u];
    return u;
  }

  void extend(char c) {
    s += c;
    int u = go(last);
    if (!trie[u][c]) {
      int v = newNode();
      len[v] = len[u] + 2;
      link[v] = trie[go(link[u])][c];
      trie[u][c] = v;
    }
    last = trie[u][c];
  }
} eert;
```

5 Dynamic Programming

5.1 Steiner-tree DP

n nodes, k terminal nodes, unite all terminal nodes doing a Steiner tree



```
fore (u, k, n)
  fore (a, 0, k)
    umin(dp[u][1 << a], dist[u][a]);
fore (A, 0, (1 << k))
  fore (u, k, n) {
    for (int B = A; B > 0; B = (B - 1) & A)
      umin(dp[u][A], dp[u][B] + dp[u][A ^ B]);
    fore (v, k, n)
      umin(dp[v][A], dp[u][A] + dist[u][v]);
  }
```

5.2 Matrix Chain Multiplication

```
int dp(int l, int r) {
  if (l > r)
    return 0LL;
  int &ans = mem[l][r];
  if (!done[l][r]) {
    done[l][r] = true, ans = inf;
    fore (k, l, r + 1) // split in [l, k] [k + 1, r]
      ans = min(ans, dp(l, k) + dp(k + 1, r) + add);
  }
  return ans;
}
```

5.3 Digit DP

Counts the amount of numbers in $[l, r]$ such are divisible by k. (flag *nonzero* is for different lengths)
It can be reduced to $dp(i, x, small)$, and has to be solve like $f(r) - f(l - 1)$

```
#define state [i][x][small][big][nonzero]
int dp(int i, int x, bool small, bool big, bool
     nonzero) {
  if (i == sz(r))
    return x % k == 0 && nonzero;
  int &ans = mem state;
  if (done state != timer) {
    done state = timer;
    ans = 0;
    int lo = small ? 0 : l[i] - '0';
    int hi = big ? 9 : r[i] - '0';
    fore (y, lo, max(lo, hi) + 1) {
      bool small2 = small | (y > lo);
      bool big2 = big | (y < hi);
      bool nonzero2 = nonzero | (x > 0);
      ans += dp(i + 1, (x * 10 + y) % k, small2, big2,
           nonzero2);
    }
  }
  return ans;
}
```

5.4 Knapsack 0/1

```
for (auto &cur : items)
  fore (w, W + 1, cur.w) // [cur.w, W]
    umax(dp[w], dp[w - cur.w] + cur.cost);
```

5.5 Convex Hull Trick $\mathcal{O}(n^2) \Rightarrow \mathcal{O}(n)$

$dp[i] = \min_{j<i}(dp[j] + b[j] * a[i])$
$dp[i][j] = \min_{k<j}(dp[i-1][k] + b[k] * a[j])$
$b[j] \geq b[j+1]$ optionally $a[i] \leq a[i+1]$

```
// for doubles, use inf = 1/.0, div(a,b) = a / b
struct Line {
  mutable lli m, c, p;
  bool operator < (const Line &l) const { return m < l
      .m; }
  bool operator < (lli x) const { return p < x; }
  lli operator ()(lli x) const { return m * x + c; }
};

lli bet(const Line &a, const Line &b) {
  if (a.m == b.m)
    return a.c > b.c ? inf : -inf;
  // can just be a / b?
  return (b.c - a.c) / (a.m - b.m);
  // lli divi(lli a, lli b) { return a / b - ((a ^ b)
      < 0 && a % b); }
  // return divi(b.c - a.c, a.m - b.m);
}

struct DynamicHull : multiset<Line, less<>> {
  bool isect(iterator x, iterator y) {
    if (y == end()) {
      x->p = inf;
      return 0;
    }
    x->p = bet(*x, *y);
    return x->p >= y->p;
  }

  void add(lli m, lli c) {
    auto z = insert({m, c, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y))
      isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }

  lli query(lli x) {
    if (empty()) return 0LL;
    auto f = *lower_bound(x);
    return f(x);
  }
```

5.6 Divide and conquer $\mathcal{O}(kn^2) \Rightarrow \mathcal{O}(k \cdot nlogn)$

Split the array of size n into k continuous groups. $k \leq n$
$cost(a,c) + cost(b,d) \leq cost(a,d) + cost(b,c)$ with $a \leq b \leq c \leq d$

```
void dc(int cut, int l, int r, int optl, int optr) {
  if (r < l)
    return;
  int mid = (l + r) / 2;
  pair<lli, int> best = {inf, -1};
  fore (p, optl, min(mid, optr) + 1) {
    lli nxtGroup = dp[~cut & 1][p - 1] + cost(p, mid);
    if (nxtGroup < best.f)
      best = {nxtGroup, p};
  }
  dp[cut & 1][mid] = best.f;
  int opt = best.s;
  dc(cut, l, mid - 1, optl, opt);
  dc(cut, mid + 1, r, opt, optr);
}

fore (i, 1, n + 1)
  dp[1][i] = cost(1, i);
fore (cut, 2, k + 1)
  dc(cut, cut, n, cut, n);
```

5.7 Knuth optimization $\mathcal{O}(n^3) \Rightarrow \mathcal{O}(n^2)$

$dp[l][r] = \min_{l \leq k \leq r}\{dp[l][k] + dp[k][r]\} + cost(l,r)$

```
fore (len, 1, n + 1)
  fore (l, 0, n) {
    int r = l + len - 1;
    if (r > n - 1)
      break;
    if (len <= 2) {
      dp[l][r] = 0;
      opt[l][r] = l;
      continue;
    }
    dp[l][r] = inf;
    fore (k, opt[l][r - 1], opt[l + 1][r] + 1) {
      lli cur = dp[l][k] + dp[k][r] + cost(l, r);
      if (cur < dp[l][r]) {
        dp[l][r] = cur;
        opt[l][r] = k;
      }
    }
  }
```

5.8 Do all submasks of a mask

```
for (int B = A; B > 0; B = (B - 1) & A)
```

6 Game Theory

6.1 Grundy Numbers

If the moves are consecutive $S = \{1, 2, 3, ..., x\}$ the game can be solved like $stackSize \pmod{x+1} \neq 0$

```
int mem[N];

int mex(set<int> &st) {
  int x = 0;
  while (st.count(x))
    x++;
  return x;
}

int grundy(int n) {
  if (n < 0)
    return inf;
  if (n == 0)
    return 0;
  int &g = mem[n];
  if (g == -1) {
    set<int> st;
    for (int x : {a, b})
      st.insert(grundy(n - x));
    g = mex(st);
  }
  return g;
}
```

7 Combinatorics

7.1 Factorial

```
fac[0] = 1LL;
fore (i, 1, N)
  fac[i] = lli(i) * fac[i - 1] % mod;
ifac[N - 1] = fpow(fac[N - 1], mod - 2);
```

```
fore (i, N - 1, 0)
  ifac[i] = lli(i + 1) * ifac[i + 1] % mod;
```

7.2 Factorial mod *smallPrime*

```
lli facMod(lli n, int p) {
  lli r = 1LL;
  for (; n > 1; n /= p) {
    r = (r * ((n / p) % 2 ? p - 1: 1)) % p;
    fore (i, 2, n % p + 1)
      r = r * i % p;
  }
  return r % p;
}
```

7.3 Lucas theorem

Convierte $\binom{n}{k}$ mod p, con $n, k \geq 2e6$ y $p \leq 1e7$

$$\binom{n}{k} \equiv \prod_{i=0}^{n} \binom{n_i}{k_i} \mod p$$

```
lli lucas(lli n, lli k) {
  if (k == 0)
    return 1LL;
  return lucas(n / mod, k / mod) * choose(n % mod, k %
       mod) % mod;
}
```

7.4 N choose K

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\binom{n}{k_1, k_2, ..., k_m} = \frac{n!}{k_1! * k_2! * ... * k_m!}$$

```
lli choose(int n, int k) {
  if (n < 0 || k < 0 || n < k)
    return 0LL;
  return fac[n] * ifac[k] % mod * ifac[n - k] % mod;
}

lli choose(int n, int k) {
  double r = 1;
  fore (i, 1, k + 1)
    r = r * (n - k + i) / i;
  return lli(r + 0.01);
}
```

7.5 Catalan

```
catalan[0] = 1LL;
fore (i, 0, N) {
  catalan[i + 1] = catalan[i] * lli(4 * i + 2) % mod *
       fpow(i + 2, mod - 2) % mod;
}
```

7.6 Prime factors of N!

```
vector< pair<lli, int> > factorialFactors(int n) {
  vector< pair<lli, int> > fac;
  for (lli p : primes) {
    if (n < p)
      break;
    lli mul = 1LL, k = 0;
    while (mul <= n / p) {
      mul *= p;
      k += n / mul;
    }
    fac.emplace_back(p, k);
  }
  return fac;
}
```

8 Number Theory

8.1 Goldbach conjecture

- All number ≥ 6 can be written as sum of 3 *primes*
- All even number > 2 can be written as sum of 2 *primes*

8.2 Sieve of Eratosthenes

Numbers up to $2e8$

```
int erat[N >> 6];
#define bit(i) ((i >> 1) & 31)
#define prime(i) !(erat[i >> 6] >> bit(i) & 1)

void bitSieve() {
  for (int i = 3; i * i < N; i += 2) if (prime(i))
    for (int j = i * i; j < N; j += (i << 1))
      erat[j >> 6] |= 1 << bit(j);
}
```

To factorize divide x by $factor[x]$ until is equal to 1

```
void factorizeSieve() {
  iota(factor, factor + N, 0);
  for (int i = 2; i * i < N; i++) if (factor[i] == i)
    for (int j = i * i; j < N; j += i)
      factor[j] = i;
}
```

Use it if you need a huge amount of $phi[x]$ up to some N

```
void phiSieve() {
  isp.set(); // bitset<N> is faster
  iota(phi, phi + N, 0);
  fore (i, 2, N) if (isp[i])
    for (int j = i; j < N; j += i) {
      isp[j] = (i == j);
      phi[j] /= i;
      phi[j] *= i - 1;
    }
 }
```

8.3 Phi of euler

```
lli phi(lli n) {
  if (n == 1)
    return 0;
  lli r = n;
  for (lli i = 2; i * i <= n; i++)
    if (n % i == 0) {
      while (n % i == 0)
        n /= i;
      r -= r / i;
    }
  if (n > 1)
    r -= r / n;
  return r;
}
```

8.4 Miller-Rabin

```
bool compo(lli p, lli d, lli n, lli k) {
  lli x = fpow(p % n, d, n), i = k;
  while (x != 1 && x != n - 1 && p % n && i--)
    x = mul(x, x, n);
  return x != n - 1 && i != k;
}

bool miller(lli n) {
  if (n < 2 || n % 6 % 4 != 1)
    return (n | 1) == 3;
  int k = __builtin_ctzll(n - 1);
  lli d = n >> k;
  for (lli p : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31
       , 37}) {
    if (compo(p, d, n, k))
      return 0;
    if (compo(2 + rng() % (n - 3), d, n, k))
      return 0;
  }
  return 1;
}
```

8.5 Pollard-Rho

```
lli f(lli x, lli c, lli mod) {
  return (mul(x, x, mod) + c) % mod;
}

lli rho(lli n) {
  while (1) {
    lli x = 2 + rng() % (n - 3), c = 1 + rng() % 20, y
        = f(x, c, n), g;
    while ((g = __gcd(n + y - x, n)) == 1)
      x = f(x, c, n), y = f(f(y, c, n), c, n);
    if (g != n) return g;
  }
  return -1;
}

void pollard(lli n, map<lli, int> &fac) {
  if (n == 1) return;
  if (n % 2 == 0) {
    fac[2]++;
    pollard(n / 2, fac);
    return;
  }
  if (miller(n)) {
    fac[n]++;
    return;
  }
  lli x = rho(n);
  pollard(x, fac);
  pollard(n / x, fac);
}
```

8.6 Amount of divisors

```
lli divs(lli n) {
  lli cnt = 1LL;
  for (lli p : primes) {
    if (p * p * p > n)
      break;
    if (n % p == 0) {
      lli k = 0;
      while (n > 1 && n % p == 0)
        n /= p, ++k;
      cnt *= (k + 1);
    }
  }
  lli sq = mysqrt(n); // A binary search, the last x *
      x <= n
  if (miller(n))
    cnt *= 2;
  else if (sq * sq == n && miller(sq))
    cnt *= 3;
  else if (n > 1)
    cnt *= 4;
  return cnt;
}
```

8.7 Bézout's identity

$a_1 * x_1 + a_2 * x_2 + ... + a_n * x_n = g$
$g = \gcd(a_1, a_2, ..., a_n)$

8.8 GCD

$a \leq b; \gcd(a + k, b + k) = \gcd(b - a, a + k)$

8.9 LCM

$x = p * lcm(a_1, a_2, ..., a_k) + q, 0 \leq q \leq lcm(a_1, a_2, ..., a_k)$
$x \pmod{a_i} \equiv q \pmod{a_i}$ as $a_i \mid lcm(a_1, a_2, ..., a_k)$

8.10 Euclid

```
pair<lli, lli> euclid(lli a, lli b) {
  if (b == 0)
    return {1, 0};
  auto p = euclid(b, a % b);
  return {p.s, p.f - a / b * p.s};
}
```

8.11 Chinese remainder theorem

```
pair<lli, lli> crt(pair<lli, lli> a, pair<lli, lli> b)
     {
  if (a.s < b.s)
    swap(a, b);
  auto p = euclid(a.s, b.s);
  lli g = a.s * p.f + b.s * p.s, l = a.s / g * b.s;
  if ((b.f - a.f) % g != 0)
    return {-1, -1}; // no solution
 p.f = a.f + (b.f - a.f) % b.s * p.f % b.s / g * a.s;
  return {p.f + (p.f < 0) * l, l};
}
```

9 Math

9.1 Progressions

Arithmetic progressions

$$a_n = a_1 + (n - 1) * diff$$

$$\sum_{i=1}^{n} a_i = n * \frac{a_1 + a_n}{2}$$

Geometric progressions

$$a_n = a_1 * r^{n-1}$$

$$\sum_{k=1}^{n} a_1 * r^k = a_1 * \left(\frac{r^{n+1} - 1}{r - 1}\right) : r \neq 1$$

9.2 Mod multiplication

```
lli mul(lli x, lli y, lli mod) {
  lli r = 0LL;
  for (x %= mod; y > 0; y >>= 1) {
    if (y & 1) r = (r + x) % mod;
    x = (x + x) % mod;
  }
  return r;
}
```

9.3 Fpow

```
lli fpow(lli x, lli y, lli mod) {
  lli r = 1;
  for (; y > 0; y >>= 1) {
    if (y & 1) r = mul(r, x, mod);
    x = mul(x, x, mod);
  }
  return r;
}
```

9.4 Fibonacci

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n = \begin{bmatrix} fib_{n+1} & fib_n \\ fib_n & fib_{n-1} \end{bmatrix}$$

10 Geometry

10.1 Point

```
const ld eps = 1e-9;
#define eq(a, b) fabs((a) - (b)) <= eps
#define neq(a, b) fabs((a) - (b)) > eps
#define geq(a, b) (a) - (b) >= -eps
#define ge(a, b) (a) - (b) > eps
#define le(a, b) (b) - (a) > eps
#define leq(a, b) (b) - (a) >= -eps
#define sq(x) (x) * (x)

int sgn(ld x) {
  return x > 0 ? 1 : (x < 0 ? -1 : 0);
}

template <class T>
```

```cpp
struct Point {
  typedef Point<T> P;
  T x, y;
  explicit Point(T x = 0, T y = 0) : x(x), y(y) {}
  P operator + (const P &p) const {
    return P(x + p.x, y + p.y); }
  P operator - (const P &p) const {
    return P(x - p.x, y - p.y); }
  P operator * (T k) const {
    return P(x * k, y * k); }
  P operator / (T k) const {
    return P(x / k, y / k); }

  T dot(const P &p) { return x * p.x + y * p.y; }
  T cross(const P &p) { return x * p.y - y * p.x; }
  double length() const { return sqrtl(norm()); }
  T norm() const { return sq(x) + sq(y); } // double ?
  double angle() { return atan2(y, x); }

  P perp() const { return P(-y, x); }
  P unit() const { return (*this) / length(); }
  P rotate (double angle) const {
    return P(x * cos(angle) - y * sin(angle),
             x * sin(angle) + y * cos(angle)); }

  bool operator == (const P &p) const {
    return eq(x, p.x) && eq(y, p.y); }
  bool operator != (const P &p) const {
    return neq(x, p.x) || neq(y, p.y); }

  friend ostream & operator << (ostream &os, P &p) {
    return os << "(" << p.x << ", " << p.y << ")";
  }
};
typedef Point<double> P;

double ccw(P a, P b, P c) {
  return (b - a).cross(c - a);
}
```

10.2 Angle Between Vectors

```cpp
double angleBetween(P a, P b) {
  double x = a.dot(b) / a.length() / b.length();
  return acos(max(ld(-1.0), min(ld(1.0), ld(x))));
}
```

10.3 Area Poligon

```cpp
double area(vector<P> &p) {
  double sum = 0;
  fore (i, 0, n)
    sum += p[i].cross(p[(i + 1) % sz(p)]);
  return abs(sum / 2);
}
```

10.4 Area Poligon In Circle

```cpp
vector<P> intersectLineCircle(const P &a, const P &v,
    const P &c, ld r) {
  ld h2 = r * r - v.cross(c - a) * v.cross(c - a) / v.
      norm();
  P p = a + v * v.dot(c - a) / v.norm();
  if (eq(h2, 0))
    return {p};  // line tangent to circle
  else if (le(h2, 0))
    return {};  // no intersection
  else {
    point u = v.unit() * sqrt(h2);
    return {p - u, p + u};  // two points of
        intersection (chord)
  }
}
bool pointInLine(const P &a, const P &v, const P &p) {
  return eq((p - a).cross(v), 0);
}
bool pointInSegment(const P &a, const P &b, const P &p
    ) {
  return pointInLine(a, b - a, p) && leq((a - p).dot(b
       - p), 0);
}
int pointInCircle(const P &c, ld r, const P &p) {
  ld l = (p - c).length() - r;
  return (le(l, 0) ? 1 : (eq(l, 0) ? -1 : 0));
}
vector<P> intersectSegmentCircle(const P &a, const P &
    b, const point &c, ld r) {
  vector<P> points = intersectLineCircle(a, b - a, c,
      r), ans;
  for (const P &p : points) {
    if (pointInSegment(a, b, p)) ans.pb(p);
  }
  return ans;
}
ld signed_angle(const P &a, const P &b) {
  return sgn(a.cross(b)) * acosl(a.dot(b) / (a.length
      () * b.length()));
}
ld intersectPolygonCircle(const vector<P> &points,
    const P &c, ld r) {
  int n = points.size();
  ld ans = 0;
  for (int i = 0; i < n; ++i) {
    P p = points[i], q = points[(i + 1) % n];
    bool p_inside = (pointInCircle(c, r, p) != 0);
    bool q_inside = (pointInCircle(c, r, q) != 0);
    if (p_inside && q_inside) {
      ans += (p - c).cross(q - c);
    } else if (p_inside && !q_inside) {
      P s1 = intersectSegmentCircle(p, q, c, r)[0];
      P s2 = intersectSegmentCircle(c, q, c, r)[0];
      ans += (p - c).cross(s1 - c) + r * r *
          signed_angle(s1 - c, s2 - c);
    } else if (!p_inside && q_inside) {
      P s1 = intersectSegmentCircle(c, p, c, r)[0];
      P s2 = intersectSegmentCircle(p, q, c, r)[0];
      ans += (s2 - c).cross(q - c) + r * r *
          signed_angle(s1 - c, s2 - c);
    } else {
      auto info = intersectSegmentCircle(p, q, c, r);
      if (info.size() <= 1) {
        ans += r * r * signed_angle(p - c, q - c);
      } else {
        P s2 = info[0], s3 = info[1];
        P s1 = intersectSegmentCircle(c, p, c, r)[0];
        P s4 = intersectSegmentCircle(c, q, c, r)[0];
        ans += (s2 - c).cross(s3 - c) + r * r * (
            signed_angle(s1 - c, s2 - c) +
            signed_angle(s3 - c, s4 - c));
      }
    }
  }
  return abs(ans) / 2;
}
```

10.5 Closest Pair Of Points

```cpp
pair<P, P> cpp(vector<P> points) {
  sort(all(points), [&](P a, P b) {
    return le(a.y, b.y);
  });
  set<P> st;
  ld ans = inf;
  P p, q;
  int pos = 0, n = sz(points);
```

```
  fore (i, 0, n) {
    while (pos < i && geq(points[i].y - points[pos].y,
         ans))
      st.erase(points[pos++]);
    auto lo = st.lower_bound({points[i].x - ans - eps,
         -inf});
    auto hi = st.upper_bound({points[i].x + ans + eps,
         -inf});
    for (auto it = lo; it != hi; ++it) {
      ld d = (points[i] - *it).length();
      if (le(d, ans))
        ans = d, p = points[i], q = *it;
    }
    st.insert(points[i]);
  }
  return {p, q};
}
```

10.6 Convex Hull

```
vector<P> convexHull(vector<P> &pts) {
  int n = sz(pts);
  vector<P> low, up;
  sort(all(pts), [&](P a, P b) {
    return a.x == b.x ? a.y < b.y : a.x < b.x;
  });
  pts.erase(unique(all(pts)), pts.end());
  if (n <= 2)
    return pts;
  fore (i, 0, n) {
    while(sz(low) >= 2 && (low.end()[-1] - low.end()[-
        2]).cross(pts[i] - low.end()[-1]) <= 0)
      low.pop_back();
    low.pb(pts[i]);
  }
  fore (i, n, 0) {
    while(sz(up) >= 2 && (up.end()[-1] - up.end()[-2])
        .cross(pts[i] - up.end()[-1]) <= 0)
      up.pop_back();
    up.pb(pts[i]);
  }
  low.pop_back(), up.pop_back();
  low.insert(low.end(), all(up));
  return low;
}
```

10.7 Distance Point Line

```
double distance_P_line(P a, P v, P p){
  return (proj(p - a, v) - (p - a)).length();
}
```

10.8 Get Circle

```
pair<P, double> getCircle(P m, P n, P p){
  P c = intersectLines((n + m) / 2, (n - m).perp(), (p
      + m) / 2, (p - m).perp());
  double r = (c - m).length();
  return {c, r};
}
```

10.9 Intersects Line

```
int intersectLinesInfo (P a1, P v1, P a2, P v2) { // v
    1 = b - a, v2 = d - c
  if (v1.cross(v2) == 0)
    return (a2 - a1).cross(v1) == 0 ? -1 : 0; // -1:
        infinity Ps, 0: no Ps
  else
    return 1; // single P
}

P intersectLines (P a1, P v1, P a2, P v2) {
  return a1 + v1 * ((a2 - a1).cross(v2) / v1.cross(v2)
      );
}
```

10.10 Intersects Line Segment

```
int intersectLineSegmentInfo(P a, P v, P c, P d) {
  P v2 = d - c;
  ld det = v.cross(v2);
  if (det == 0) {
    if ((c - a).cross(v) == 0)
     return -1; // infinity points
   else
     return 0; //no points
  } else
     return sgn(v.cross(c - a)) != sgn(v.cross(d - a))
         ;
}
```

10.11 Intersects Segment

```
int intersectSegmentsInfo(const P &a, const P &b,
     const P &c, const P &d) {
  P v1 = b - a, v2 = d - c;
  int t = sgn(v1.cross(c - a)), u = sgn(v1.cross(d - a
      ));
  if (t == u) {
    if (t == 0) {
      if (PInSegment(a, b, c) || PInSegment(a, b, d)
          || PInSegment(c, d, a) || PInSegment(c, d,
          b))
        return -1; // infinity Ps
      else
        return 0; // no P
    } else
      return 0; // no P
  } else
      return sgn(v2.cross(a - c)) != sgn(v2.cross(b -
          c)); // 1: single P 0: no P
}
```

10.12 Is Convex

```
bool isConvex(vector<P> points) {
  int n = sz(points);
  bool hasPos = false, hasNeg = false;
  fore (i, 0, n) {
    P first = points[(i+1)%n] - points[i];
    P second = points[(i+2)%n] - points[(i+1)%n];
    double sign = first.cross(second);
    if (sign > 0) hasPos = true;
    if (sign < 0) hasNeg = true;
  }
  return !(hasPos && hasNeg);
}
```

10.13 Perimeter

```
double perimeter(vector<P> points){
  int n = sz(points);
  double sum = 0;
  fore (i, 0, n)
    sum += (points[(i + 1) % n] - points[i]).length();
  return sum;
}
```

10.14 Point In Convex Polygon logN

```
// log(n)
// first preprocess: seg = process(points)
// for each query: PInConvexPolygon(seg, p - Ps[0])
vector<P> process(const vector<P> &Ps) {
  int n = sz(Ps);
  rotate(Ps.begin(), min_element(all(Ps), [&](P a, P b
      ) {
    return a.x == b.x ? a.y < b.y : a.x < b.x;
  }), Ps.end());
  vector<P> seg(n - 1);
  fore (i, 0, n - 1)
```

```
    seg[i] = Ps[i + 1] - Ps[0];
  return seg;
}

bool PInConvexPolygon(const vector<P> &seg, const P &p
    ) {
  int n = sz(seg);
  if (neq(seg[0].cross(p), 0) && sgn(seg[0].cross(p))
      != sgn(seg[0].cross(seg[n - 1])))
    return false;
  if (neq(seg[n - 1].cross(p), 0) && sgn(seg[n - 1].
      cross(p)) != sgn(seg[n - 1].cross(seg[0])))
    return false;
  if (eq(seg[0].cross(p), 0))
    return geq(seg[0].length(), p.length());
  int l = 0, r = n - 1;
  while (r - l > 1) {
    int m = l + ((r - l) >> 1);
    if (geq(seg[m].cross(p), 0))
      l = m;
    else
      r = m;
  }
  return eq(fabs(seg[l].cross(seg[l + 1])), fabs((p -
      seg[l]).cross(p - seg[l + 1])) +
          fabs(p.cross(seg[l])) + fabs(p.cross(seg[l +
               1])));
}
```

10.15 Point In Polygon

```
int pointInPolygon(const vector<P> &points, P p) { //
    O(N)
 int n = sz(points), ans = 0;
 fore (i, 0, n) {
  P a = points[i], b = points[(i + 1) % n];
  if (pointInSegment(a, b, p))
      return -1; // on perimeter
  if (a.y > b.y)
      swap(a,b);
  if (a.y <= p.y && b.y > p.y && (a - p).cross(b - p)
      > 0)
      ans ^= 1;
 }
 return ans ? 1 : 0; // inside, outside
}
```

10.16 Point In Segment

```
bool pointInSegment(P a, P b, P p){
  return (b - a).cross(p - a) == 0 && (a - p).dot(b -
      p) <= 0;
}
```

10.17 Points Of Tangency

```
pair<P, P> PsOfTangency(P c, double r, P p){
  P v = (p - c).unit() * r;
  double cos_theta = r / (p - c).length();
  double theta = acos(max(-1.0, min(1.0, cos_theta)));
  return {c + v.rotate(-theta), c + v.rotate(theta)};
}
```

10.18 Projection

```
P proj(P a, P v){
    v = v / v.unit();
    return v * a.dot(v);
}
```

10.19 Projection Line

```
P proj_line(P a, P v, P p){
  return a + proj(p - a, v);
}
```

10.20 Reflection Line

```
P reflection_line(P a, P v, P p){
  return a*2 - p + proj(p - a, v)*2;
}
```

10.21 Signed Distance Point Line

```
double signed_distance_P_line(P a, P v, P p){
  return v.cross(p - a) / v.length();
}
```

10.22 Sort Along Line

```
void sort_along_line(P a, P v, vector<P> & Ps){
  sort(Ps.begin(), Ps.end(), [](P u, P w){
    return u.dot(v) < w.dot(v);
  });
}
```

10.23 Intersects Line Circle

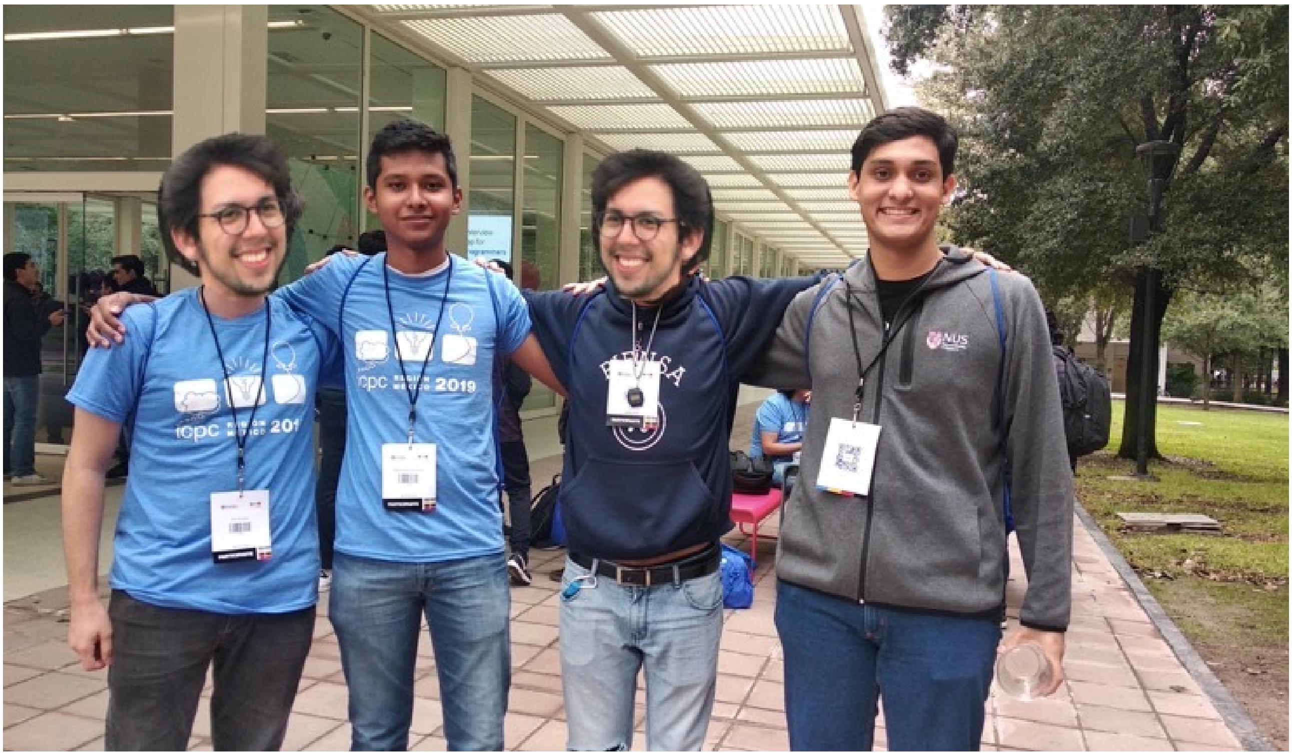
```
vector<P> intersectLineCircle(P a, P v, P c, double r)
    {
  P p = proj_line(a, v, c);
  double d = (p - c).length();
  double h = sq(r) - sq(d);
  if(h == 0)
  return {p}; //line tangent to circle
  else if(h < 0)
  return {}; //no intersection
  else {
    P u = v.unit() * sqrt(h);
    return {p - u, p + u}; //two Ps of intersection (
        chord)
  }
}
```

11 Bit tricks

Bits++	
Operations on *int*	Function
`x & -x`	Least significant bit in x
`__lg(x)`	Most significant bit in x
`c = x&-x, r = x+c; (((r^x) » 2)/c) | r`	Next number after x with same number of bits set
__builtin_	Function
popcount(x)	Amount of 1's in x
clz(x)	0's to the **left** of biggest bit
ctz(x)	0's to the **right** of smallest bit

11.1 Bitset

Bitset<Size>	
Operation	Function
_Find_first()	Least significant bit
_Find_next(idx)	First set bit after index *idx*
any(), none(), all()	Just what the expression says
set(), reset(), flip()	Just what the expression says x2
to_string('.', 'A')	Print 011010 like .AA.A.



The end...