



Universidad de Guadalajara,
CUCEI

The Empire Strikes Back

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Contents

1 Data structures	2	8 Graphs	12
1.1 DSU rollback	2	8.1 Cutpoints and bridges	12
1.2 Monotone queue	3	8.2 Topological sort	12
1.3 Stack queue	3	8.3 Kosaraju	12
1.4 In-Out trick	3	8.4 Tarjan	12
1.5 Parallel binary search $\mathcal{O}((n+q) \cdot \log n \cdot f)$	3	8.5 Isomorphism	12
1.6 Mos $\mathcal{O}((n+q) \cdot \sqrt{n} \cdot f)$	4	8.6 Two sat	12
1.7 Hilbert order	4	8.7 LCA	13
1.8 Sqrt decomposition	4	8.8 Virtual tree $\mathcal{O}(n \cdot \log n)$	13
1.9 Sparse table	4	8.9 Euler-tour + HLD + LCA $\mathcal{O}(n \cdot \log n \cdot f)$	13
1.10 Fenwick	4	8.10 Centroid $\mathcal{O}(n \cdot \log n \cdot f)$	14
1.11 Dynamic segtree	4	8.11 Guni $\mathcal{O}(n \cdot \log n \cdot f)$	14
1.12 Persistent segtree	5	8.12 Link-Cut tree $\mathcal{O}(n \cdot \log n \cdot f)$	14
1.13 Li Chao	5	9 Flows	16
1.14 Wavelet	5	9.1 Hopcroft Karp $\mathcal{O}(e\sqrt{v})$	16
1.15 Ordered tree	6	9.2 Hungarian $\mathcal{O}(n^2 \cdot m)$	16
1.16 Treap	6	9.3 Dinic $\mathcal{O}(\min(e \cdot \text{flow}, v^2 \cdot e))$	16
2 Dynamic programming	6	9.4 Min-Cost flow $\mathcal{O}(\min(e \cdot \text{flow}, v^2 \cdot e))$	17
2.1 All submasks of a mask	6	10 Game theory	17
2.2 Convex hull trick $\mathcal{O}(n^2) \Rightarrow \mathcal{O}(n)$	6	10.1 Grundy numbers	17
2.3 Digit dp	7	11 Math	18
2.4 Divide and conquer $\mathcal{O}(k \cdot n^2) \Rightarrow \mathcal{O}(k \cdot n \log n)$	7	11.1 Bits	18
2.5 Knapsack 01 $\mathcal{O}(n \cdot \text{Max}W)$	7	11.2 Bitset	18
2.6 Knuth $\mathcal{O}(n^3) \Rightarrow \mathcal{O}(n^2)$	7	11.3 Probability	18
2.7 Matrix exponentiation	7	11.4 Simplex	18
2.8 SOS dp	8	11.5 Xor basis	19
3 Geometry	8	12 Combinatorics	19
3.1 Geometry	8	12.1 Catalan	19
3.2 Radial order	8	12.2 Factorial	19
3.3 Sort along line	8	12.3 Factorial mod small prime	19
4 Point	8	12.4 Choose	19
4.1 Point	8	12.5 Pascal	19
4.2 Angle between vectors	9	12.6 Stars and bars	20
4.3 Closest pair of points $\mathcal{O}(n \log n)$	9	12.7 Lucas	20
4.4 KD Tree	9	12.8 Burnside lemma	20
5 Lines and segments	9	13 Number theory	20
5.1 Line	9	13.1 Amount of divisors $\mathcal{O}(n^{1/3})$	20
5.2 Segment	10	13.2 Chinese remainder theorem	20
5.3 Projection	10	13.3 Euclid $\mathcal{O}(\log(a \cdot b))$	20
5.4 Distance point line	10	13.4 Factorial factors	20
5.5 Distance point segment	10	13.5 Factorize sieve	20
5.6 Distance segment segment	10	13.6 Sieve	20
6 Circle	10	13.7 Phi $\mathcal{O}(\sqrt{n})$	20
6.1 Circle	10	13.8 Phi sieve	21
6.2 Distance point circle	10	13.9 Miller rabin $\mathcal{O}(\text{Witnesses} \cdot (\log n)^3)$	21
6.3 Common area circle circle	10	13.10 Pollard Rho $\mathcal{O}(n^{1/4})$	21
6.4 Minimum enclosing circle $\mathcal{O}(n)$ wow!!	11	14 Polynomials	21
7 Polygon	11	14.1 Berlekamp Massey	21
7.1 Area polygon	11	14.2 Lagrange $\mathcal{O}(n)$	21
7.2 Perimeter	11	14.3 FFT	22
7.3 Cut polygon line	11	14.4 Fast Walsh Hadamard Transform	22
7.4 Common area circle polygon $\mathcal{O}(n)$	11	14.5 Primitive root	22
7.5 Point in polygon	11	14.6 NTT	23
7.6 Convex hull $\mathcal{O}(n \log n)$	11		
7.7 Is convex	11		
7.8 Point in convex polygon $\mathcal{O}(\log n)$	12		

15 Strings	23
15.1 KMP	23
15.2 KMP automaton $\mathcal{O}(\text{Alphabet} * n)$	23
15.3 Z	23
15.4 Manacher	23
15.5 Hash	24
15.6 Min rotation	24
15.7 Suffix array $\mathcal{O}(n \log n)$	24
15.8 Aho Corasick $\mathcal{O}(\sum s_i)$	25
15.9 Eertree $\mathcal{O}(\sum s_i)$	25
15.10 Suffix automaton $\mathcal{O}(\sum s_i)$	26

Think twice, code once

Template.cpp

```
#pragma GCC optimize("Ofast,unroll-loops,no-stack-protector")
#include <bits/stdc++.h>
using namespace std;

#define fore(i, l, r) for (auto i = (l) - ((l) > (r)); i != (r) - ((l) > (r)); i += 1 - 2 * ((l) > (r)))
#define sz(x) int(x.size())
#define all(x) begin(x), end(x)
#define f first
#define s second
#define pb push_back

#ifdef LOCAL
#include "debug.h"
#else
#define debug(...)
#endif

using ld = long double;
using lli = long long;
using ii = pair<int, int>;
using vi = vector<int>;

int main() {
    cin.tie(0) -> sync_with_stdio(0), cout.tie(0);
    return 0;
}
```

Debug.h

```
#include <bits/stdc++.h>
using namespace std;

template <class A, class B>
ostream& operator<<(ostream& os, const pair<A, B>& p) {
    return os << "(" << p.first << ", " << p.second << ")";
}

template <class A, class B, class C>
basic_ostream<A, B>& operator<<(basic_ostream<A, B>& os, const C& c) {
    os << "[";
    for (const auto& x : c)
        os << ", " + 2 * (&x == &begin(c)) << x;
    return os << "]";
}

void print(string s) {
    cout << endl;
}

template <class H, class... T>
void print(string s, const H& h, const T&... t) {
```

```
const static string reset = "\033[0m", blue = "\033[1;34m",
    purple = "\033[3;95m";
bool ok = 1;
do {
    if (s[0] == '\n')
        ok = 0;
    else
        cout << blue << s[0] << reset;
    s = s.substr(1);
} while (s.size() && s[0] != ',');
if (ok)
    cout << ": " << purple << h << reset;
print(s, t...);
}
```

```
#define debug(...) print(__VA_ARGS__, __VA_ARGS__)
```

Randoms

```
mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
```

Compilation (gedit ~/.zshenv)

```
touch in{1..9} // make files in1, in2, ..., in9
tee {a..z}.cpp < tem.cpp // make files with tem.cpp
rm -r a.cpp // deletes file a.cpp :(

red = '\x1B[0;31m'
green = '\x1B[0;32m'
removeColor = '\x1B[0m'

compile() {
    alias flags='-Wall -Wextra -Wfatal-errors -Wshadow -w -mcmmodel=medium'
    g++-11 --std=c++17 $2 ${flags} $1.cpp -o $1
}

go() {
    file=$1
    name="${file%.*}"
    input=$2
    moreFlags=$3
    compile ${name} ${moreFlags}
    ./${name} < ${input}
}

run() { go $1 $2 "" }
debug() { go $1 $2 -DLOCAL }

random() { # Make small test cases!!!
    file=$1
    name="${file%.*}"
    compile ${name} ""
    compile gen ""
    compile brute ""

    for ((i = 1; i <= 300; i++)); do
        printf "Test case #${i}"
        ./gen > tmp
        diff -ywi <(. /name < tmp) <(. /brute < tmp) > $nameDiff
        if [[ $? -eq 0 ]]; then
            printf "${green} Accepted ${removeColor}\n"
        else
            printf "${red} Wrong answer ${removeColor}\n"
            break
        fi
    done
}
```

1 Data structures

1.1 DSU rollback

```

struct Dsu {
    vector<int> par, tot;
    stack<ii> mem;

    Dsu(int n = 1) : par(n + 1), tot(n + 1, 1) {
        iota(all(par), 0);
    }

    int find(int u) {
        return par[u] == u ? u : find(par[u]);
    }

    void unite(int u, int v) {
        u = find(u), v = find(v);
        if (u != v) {
            if (tot[u] < tot[v])
                swap(u, v);
            mem.emplace(u, v);
            tot[u] += tot[v];
            par[v] = u;
        } else {
            mem.emplace(-1, -1);
        }
    }

    void rollback() {
        auto [u, v] = mem.top();
        mem.pop();
        if (u != -1) {
            tot[u] -= tot[v];
            par[v] = v;
        }
    }
};

```

1.2 Monotone queue

```

template <class T, class F = less<T>>
struct MonotoneQueue {
    deque<pair<T, int>> pref;
    F f;

    void add(int pos, T val) {
        while (pref.size() && !f(pref.back().f, val))
            pref.pop_back();
        pref.emplace_back(val, pos);
    }

    void trim(int pos) { // >= pos
        while (pref.size() && pref.front().s < pos)
            pref.pop_front();
    }

    T query() {
        return pref.empty() ? T() : pref.front().f;
    }
};

```

1.3 Stack queue

```

template <class T, class F = function<T(const T&, const T&)>
>>
struct Stack : vector<T> {
    vector<T> s;
    F f;

    Stack(const F& f) : f(f) {}

    void push(T x) {
        this->pb(x);
        s.pb(s.empty() ? x : f(s.back(), x));
    }
};

```

```

T pop() {
    T x = this->back();
    this->pop_back();
    s.pop_back();
    return x;
}

T query() {
    return s.back();
}

};

template <class T, class F = function<T(const T&, const T&)>
>>
struct Queue {
    Stack<T> a, b;
    F f;

    Queue(const F& f) : a(f), b(f), f(f) {}

    void push(T x) {
        b.push(x);
    }

    T pop() {
        if (a.empty())
            while (!b.empty())
                a.push(b.pop());
        return a.pop();
    }

    T query() {
        if (a.empty())
            return b.query();
        if (b.empty())
            return a.query();
        return f(a.query(), b.query());
    }
};

```

1.4 In-Out trick

```

vector<int> in[N], out[N];
vector<Query> queries;

for (x, 0, N) {
    for (int i : in[x])
        add(queries[i]);
    // solve
    for (int i : out[x])
        rem(queries[i]);
}

```

1.5 Parallel binary search $\mathcal{O}((n + q) \cdot \log n \cdot f)$

```

int lo[Q], hi[Q];
queue<int> solve[N];
vector<Query> queries;

for (it, 0, 1 + __lg(N)) {
    for (i, 0, sz(queries))
        if (lo[i] != hi[i]) {
            int mid = (lo[i] + hi[i]) / 2;
            solve[mid].emplace(i);
        }
    for (x, 0, n) { // 0th-indexed
        // simulate
        while (!solve[x].empty()) {
            int i = solve[x].front();
            solve[x].pop();
            if (can(queries[i]))
                hi[i] = x;
            else

```

```

        lo[i] = x + 1;
    }
}
}
}

1.6 Mos  $\mathcal{O}((n+q) \cdot \sqrt{n} \cdot f)$ 
struct Query {
    int l, r, i;
};

vector<Query> queries;

const int BLOCK = sqrt(N);
sort(all(queries), [&](Query& a, Query& b) {
    const int ga = a.l / BLOCK, gb = b.l / BLOCK;
    if (ga == gb)
        return a.r < b.r;
    return ga < gb;
});

int l = queries[0].l, r = l - 1;
for (auto& q : queries) {
    while (r < q.r)
        add(++r);
    while (r > q.r)
        rem(r--);
    while (l < q.l)
        rem(l++);
    while (l > q.l)
        add(--l);
    ans[q.i] = solve();
}

```

1.7 Hilbert order

```

lli hilbert(int x, int y, int pw = 21, int rot = 0) {
    if (pw == 0)
        return 0;
    int hpw = 1 << (pw - 1);
    int k = ((x < hpw ? y < hpw ? 0 : 3 : y < hpw ? 1 : 2) +
        rot) & 3;
    const int d[4] = {3, 0, 0, 1};
    lli a = 1LL << ((pw << 1) - 2);
    lli b = hilbert(x & (x ^ hpw), y & (y ^ hpw), pw - 1, (
        rot + d[k]) & 3);
    return k * a + (d[k] ? a - b - 1 : b);
}

```

1.8 Sqrt decomposition

```

const int BLOCK = sqrt(N);
int blo[N]; // blo[i] = i / BLOCK

void update(int i) {}

int query(int l, int r) {
    while (l <= r)
        if (l % BLOCK == 0 && l + BLOCK - 1 <= r) {
            // solve for block
            l += BLOCK;
        } else {
            // solve for individual element
            l++;
        }
}

```

1.9 Sparse table

```

template <class T, class F = function<T(const T&, const T&)
>>
struct Sparse {
    vector<T> sp[25];
    F f;
    int n;

```

```

    Sparse(T* begin, T* end, const F& f) : Sparse(vector<T>(
        begin, end), f) {}

```

```

    Sparse(const vector<T>& a, const F& f) : f(f), n(sz(a)) {
        sp[0] = a;
        for (int k = 1; (1 << k) <= n; k++) {
            sp[k].resize(n - (1 << k) + 1);
            fore (l, 0, sz(sp[k])) {
                int r = l + (1 << (k - 1));
                sp[k][l] = f(sp[k - 1][l], sp[k - 1][r]);
            }
        }
    }

    T query(int l, int r) {
        #warning Can give TLE D:, change it to a log table
        int k = __lg(r - l + 1);
        return f(sp[k][l], sp[k][r - (1 << k) + 1]);
    }
};

```

1.10 Fenwick

```

template <class T>
struct Fenwick {
    vector<T> fenw;

    Fenwick(int n) : fenw(n, T()) {} // 0-indexed

    void update(int i, T v) {
        for (; i < sz(fenw); i |= i + 1)
            fenw[i] += v;
    }

    T query(int i) {
        T v = T();
        for (; i >= 0; i &= i + 1, --i)
            v += fenw[i];
        return v;
    }

    int lower_bound(T v) {
        int pos = 0;
        for (int k = __lg(sz(fenw)); k >= 0; k--)
            if (pos + (1 << k) <= sz(fenw) && fenw[pos + (1 << k)
                - 1] < v) {
                pos += (1 << k);
                v -= fenw[pos - 1];
            }
        return pos + (v == 0);
    }
};

```

1.11 Dynamic segtree

```

template <class T>
struct Dyn {
    int l, r;
    Dyn *left, *right;
    T val;

    Dyn(int l, int r) : l(l), r(r), left(0), right(0) {}

    void pull() {
        val = (left ? left->val : T()) + (right ? right->val :
            T());
    }

    template <class... Args>
    void update(int p, const Args&... args) {
        if (l == r) {
            val = T(args...);
            return;

```

```

}
int m = (l + r) >> 1;
if (p <= m) {
    if (!left)
        left = new Dyn(l, m);
    left->update(p, args...);
} else {
    if (!right)
        right = new Dyn(m + 1, r);
    right->update(p, args...);
}
pull();
}

T query(int ll, int rr) {
    if (rr < l || r < ll || r < l)
        return T();
    if (ll <= l && r <= rr)
        return val;
    int m = (l + r) >> 1;
    return (left ? left->query(ll, rr) : T()) + (right ?
        right->query(ll, rr) : T());
}
};

```

1.12 Persistent segtree

```

template <class T>
struct Per {
    int l, r;
    Per *left, *right;
    T val;

    Per(int l, int r) : l(l), r(r), left(0), right(0) {}

    Per* pull() {
        val = left->val + right->val;
        return this;
    }

    void build() {
        if (l == r)
            return;
        int m = (l + r) >> 1;
        (left = new Per(l, m))->build();
        (right = new Per(m + 1, r))->build();
        pull();
    }

    template <class... Args>
    Per* update(int p, const Args&... args) {
        if (p < l || r < p)
            return this;
        Per* tmp = new Per(l, r);
        if (l == r) {
            tmp->val = T(args...);
            return tmp;
        }
        tmp->left = left->update(p, args...);
        tmp->right = right->update(p, args...);
        return tmp->pull();
    }

    T query(int ll, int rr) {
        if (r < ll || rr < l)
            return T();
        if (ll <= l && r <= rr)
            return val;
        return left->query(ll, rr) + right->query(ll, rr);
    }
};

```

1.13 Li Chao

```

struct LiChao {
    struct Fun {
        lli m = 0, c = -INF;
        lli operator()(lli x) const {
            return m * x + c;
        }
    } f;

    lli l, r;
    LiChao *left, *right;
    LiChao(lli l, lli r, Fun f) : l(l), r(r), f(f), left(0),
        right(0) {}

    void add(Fun& g) {
        lli m = (l + r) >> 1;
        bool bl = g(l) > f(l), bm = g(m) > f(m);
        if (bm)
            swap(f, g);
        if (l == r)
            return;
        if (bl != bm)
            left = left ? (left->add(g), left) : new LiChao(l, m,
                g);
        else
            right = right ? (right->add(g), right) : new LiChao(m
                + 1, r, g);
    }

    lli query(lli x) {
        if (l == r)
            return f(x);
        lli m = (l + r) >> 1;
        if (x <= m)
            return max(f(x), left ? left->query(x) : -INF);
        return max(f(x), right ? right->query(x) : -INF);
    }
};

```

1.14 Wavelet

```

struct Wav {
    int lo, hi;
    Wav *left, *right;
    vector<int> amt;

    template <class Iter>
    Wav(int lo, int hi, Iter b, Iter e) : lo(lo), hi(hi) { //
        array 1-indexed
        if (lo == hi || b == e)
            return;
        amt.reserve(e - b + 1);
        amt.pb(0);
        int mid = (lo + hi) >> 1;
        auto leq = [mid](auto x) {
            return x <= mid;
        };
        for (auto it = b; it != e; it++)
            amt.pb(amt.back() + leq(*it));
        auto p = stable_partition(b, e, leq);
        left = new Wav(lo, mid, b, p);
        right = new Wav(mid + 1, hi, p, e);
    }

    int kth(int l, int r, int k) {
        if (r < l)
            return 0;
        if (lo == hi)
            return lo;
        if (k <= amt[r] - amt[l - 1])
            return left->kth(amt[l - 1] + 1, amt[r], k);
    }
};

```

```

    return right->kth(l - amt[l - 1], r - amt[r], k - amt[r]
        + amt[l - 1]);
}

int count(int l, int r, int x, int y) {
    if (r < l || y < x || y < lo || hi < x)
        return 0;
    if (x <= lo && hi <= y)
        return r - l + 1;
    return left->count(amt[l - 1] + 1, amt[r], x, y) +
        right->count(l - amt[l - 1], r - amt[r], x, y);
}
};

```

1.15 Ordered tree

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template <class K, class V = null_type>
using OrderedTree = tree<K, V, less<K>, rb_tree_tag,
    tree_order_statistics_node_update>;
#define rank order_of_key
#define kth find_by_order

```

1.16 Treap

```

struct Treap {
    static Treap* null;
    Treap *left, *right;
    unsigned pri = rng(), sz = 0;
    int val = 0;

    void push() {
        // propagate like segtree, key-values aren't modified!!
    }

    Treap* pull() {
        sz = left->sz + right->sz + (this != null);
        // merge(left, this), merge(this, right)
        return this;
    }

    Treap() {
        left = right = null;
    }

    Treap(int val) : val(val) {
        left = right = null;
        pull();
    }

    template <class F>
    pair<Treap*, Treap*> split(const F& leq) { // {<= val, >
        val}
        if (this == null)
            return {null, null};
        push();
        if (leq(this)) {
            auto p = right->split(leq);
            right = p.f;
            return {pull(), p.s};
        } else {
            auto p = left->split(leq);
            left = p.s;
            return {p.f, pull()};
        }
    }

    Treap* merge(Treap* other) {
        if (this == null)
            return other;
    }
}

```

```

if (other == null)
    return this;
push(), other->push();
if (pri > other->pri) {
    return right = right->merge(other), pull();
} else {
    return other->left = merge(other->left), other->pull
        ();
}
}

pair<Treap*, Treap*> leftmost(int k) {
    return split([&](Treap* n) {
        int sz = n->left->sz + 1;
        if (k >= sz) {
            k -= sz;
            return true;
        }
        return false;
    });
}

auto split(int x) {
    return split([&](Treap* n) {
        return n->val <= x;
    });
}

Treap* insert(int x) {
    auto&& [leq, ge] = split(x);
    // auto && [le, eq] = split(x); // uncomment for set
    return leq->merge(new Treap(x))->merge(ge); // change
        leq for le for set
}

Treap* erase(int x) {
    auto&& [leq, ge] = split(x);
    auto&& [le, eq] = leq->split(x - 1);
    auto&& [kill, keep] = eq->leftmost(1); // comment for
        set
    return le->merge(keep)->merge(ge); // le->merge(ge) for
        set
}
}* Treap::null = new Treap;

```

2 Dynamic programming

2.1 All submasks of a mask

```
for (int B = A; B > 0; B = (B - 1) & A)
```

2.2 Convex hull trick $\mathcal{O}(n^2) \Rightarrow \mathcal{O}(n)$

```

dp[i] = min_{j < i} (dp[j] + b[j] * a[i])
dp[i][j] = min_{k < j} (dp[i - 1][k] + b[k] * a[j])
b[j] ≥ b[j + 1] optionally a[i] ≤ a[i + 1]

// for doubles, use INF = 1/.0, div(a,b) = a / b
struct Line {
    mutable lli m, c, p;
    bool operator<(const Line& l) const {
        return m < l.m;
    }
    bool operator<(lli x) const {
        return p < x;
    }
    lli operator()(lli x) const {
        return m * x + c;
    }
};

template <bool MAX>

```

```

struct DynamicHull : multiset<Line, less<>> {
    lli div(lli a, lli b) {
        return a / b - ((a ^ b) < 0 && a % b);
    }

    bool isect(iterator i, iterator j) {
        if (j == end())
            return i->p = INF, 0;
        if (i->m == j->m)
            i->p = i->c > j->c ? INF : -INF;
        else
            i->p = div(i->c - j->c, j->m - i->m);
        return i->p >= j->p;
    }

    void add(lli m, lli c) {
        if (!MAX)
            m = -m, c = -c;
        auto k = insert({m, c, 0}), j = k++, i = j;
        while (isect(j, k))
            k = erase(k);
        if (i != begin() && isect(--i, j))
            isect(i, j = erase(j));
        while ((j = i) != begin() && (--i)->p >= j->p)
            isect(i, erase(j));
    }

    lli query(lli x) {
        if (empty())
            return 0LL;
        auto f = *lower_bound(x);
        return MAX ? f(x) : -f(x);
    }
};

```

2.3 Digit dp

Counts the amount of numbers in $[l, r]$ such are divisible by k . (flag *nonzero* is for different lengths)
 It can be reduced to $dp(i, x, small)$, and has to be solved like $f(r) - f(l - 1)$

```

#define state [i][x][small][big][nonzero]
int dp(int i, int x, bool small, bool big, bool nonzero) {
    if (i == sz(r))
        return x % k == 0 && nonzero;
    int& ans = mem state;
    if (done state != timer) {
        done state = timer;
        ans = 0;
        int lo = small ? 0 : l[i] - '0';
        int hi = big ? 9 : r[i] - '0';
        for (y, lo, max(lo, hi) + 1) {
            bool small2 = small | (y > lo);
            bool big2 = big | (y < hi);
            bool nonzero2 = nonzero | (x > 0);
            ans += dp(i + 1, (x * 10 + y) % k, small2, big2, nonzero2);
        }
    }
    return ans;
}

```

2.4 Divide and conquer $\mathcal{O}(k \cdot n^2) \Rightarrow \mathcal{O}(k \cdot n \log n)$

Split the array of size n into k continuous groups. $k \leq n$
 $cost(a, c) + cost(b, d) \leq cost(a, d) + cost(b, c)$ with $a \leq b \leq c \leq d$

```

void solve(int cut, int l, int r, int optl, int optr) {
    if (r < l)
        return;
    int mid = (l + r) / 2;
    pair<lli, int> best = {INF, -1};
    for (p, optl, min(mid, optr) + 1)
        best = min(best, {dp[~cut & 1][p - 1] + cost(p, mid), p});
    dp[cut & 1][mid] = best.f;
    solve(cut, l, mid - 1, optl, best.s);
    solve(cut, mid + 1, r, best.s, optr);
}

for (i, 1, n + 1)
    dp[1][i] = cost(1, i);
for (cut, 2, k + 1)
    solve(cut, cut, n, cut, n);

```

2.5 Knapsack 01 $\mathcal{O}(n \cdot MaxW)$

```

for (i, 0, n)
    for (int x = MaxW; x >= w[i]; x--)
        umax(dp[x], dp[x - w[i]] + cost[i]);

```

2.6 Knuth $\mathcal{O}(n^3) \Rightarrow \mathcal{O}(n^2)$

$$dp[l][r] = \min_{l \leq k \leq r} \{dp[l][k] + dp[k][r]\} + cost(l, r)$$

```

lli dp[N][N];
int opt[N][N];

for (len, 1, n + 1)
    for (l, 0, n) {
        int r = l + len - 1;
        if (r > n - 1)
            break;
        if (len <= 2) {
            dp[l][r] = 0;
            opt[l][r] = 1;
            continue;
        }
        dp[l][r] = INF;
        for (k, opt[l][r - 1], opt[l + 1][r] + 1) {
            lli cur = dp[l][k] + dp[k][r] + cost(l, r);
            if (cur < dp[l][r]) {
                dp[l][r] = cur;
                opt[l][r] = k;
            }
        }
    }
}

```

2.7 Matrix exponentiation

```

template <class T>
using Mat = vector<vector<T>>;

template <class T>
Mat<T> operator*(Mat<T>& a, Mat<T>& b) {
    Mat<T> c(sz(a), vector<T>(sz(b[0])));
    for (k, 0, sz(a[0]))
        for (i, 0, sz(a))
            for (j, 0, sz(b[0]))
                c[i][j] += a[i][k] * b[k][j];
    return c;
}

template <class T>
vector<T> operator*(Mat<T>& a, vector<T>& b) {
    assert(sz(a[0]) == sz(b));
    vector<T> c(sz(a), T());
    for (i, 0, sz(a))
        for (j, 0, sz(b))

```



```

    c[i] += a[i][j] * b[j];
    return c;
}

template <class T>
Mat<T> fpow(Mat<T>& a, lli n) {
    Mat<T> ans(sz(a), vector<T>(sz(a)));
    for (i, 0, sz(a))
        ans[i][i] = 1;
    for (; n > 0; n >>= 1) {
        if (n & 1)
            ans = ans * a;
        a = a * a;
    }
    return ans;
}

```

2.8 SOS dp

```

// N = amount of bits
// dp[mask] = Sum of all dp[x] such that 'x' is a submask
// of 'mask'
for (i, 0, N)
    for (mask, 0, 1 << N)
        if (mask >> i & 1) {
            dp[mask] += dp[mask ^ (1 << i)];
        }

```

3 Geometry

3.1 Geometry

```

const ld EPS = 1e-20;
const ld INF = 1e18;
const ld PI = acos(-1.0);
enum { ON = -1, OUT, IN, OVERLAP };

#define eq(a, b) (abs((a) - (b)) <= +EPS)
#define neq(a, b) (!eq(a, b))
#define geq(a, b) ((a) - (b) >= -EPS)
#define leq(a, b) ((a) - (b) <= +EPS)
#define ge(a, b) ((a) - (b) > +EPS)
#define le(a, b) ((a) - (b) < -EPS)

int sgn(ld a) {
    return (a > EPS) - (a < -EPS);
}

```

3.2 Radial order

```

struct Radial {
    Pt c;
    Radial(Pt c) : c(c) {}

    int cuad(Pt p) const {
        if (p.x > 0 && p.y >= 0)
            return 0;
        if (p.x <= 0 && p.y > 0)
            return 1;
        if (p.x < 0 && p.y <= 0)
            return 2;
        if (p.x >= 0 && p.y < 0)
            return 3;
        return -1;
    }

    bool operator()(Pt a, Pt b) const {
        Pt p = a - c, q = b - c;
        if (cuad(p) == cuad(q))
            return p.y * q.x < p.x * q.y;
        return cuad(p) < cuad(q);
    }
};

```

3.3 Sort along line

```

void sortAlongLine(vector<Pt>& pts, Line l) {
    sort(all(pts), [&](Pt a, Pt b) {
        return a.dot(l.v) < b.dot(l.v);
    });
}

```

4 Point

4.1 Point

```

struct Pt {
    ld x, y;
    explicit Pt(ld x = 0, ld y = 0) : x(x), y(y) {}

    Pt operator+(Pt p) const {
        return Pt(x + p.x, y + p.y);
    }

    Pt operator-(Pt p) const {
        return Pt(x - p.x, y - p.y);
    }

    Pt operator*(ld k) const {
        return Pt(x * k, y * k);
    }

    Pt operator/(ld k) const {
        return Pt(x / k, y / k);
    }

    ld dot(Pt p) const {
        // 0 if vectors are orthogonal
        // - if vectors are pointing in opposite directions
        // + if vectors are pointing in the same direction
        return x * p.x + y * p.y;
    }

    ld cross(Pt p) const {
        // 0 if collinear
        // - if b is to the right of a
        // + if b is to the left of a
        // gives you 2 * area
        return x * p.y - y * p.x;
    }

    ld norm() const {
        return x * x + y * y;
    }

    ld length() const {
        return sqrt(norm());
    }

    Pt unit() const {
        return (*this) / length();
    }

    ld angle() const {
        ld ang = atan2(y, x);
        return ang + (ang < 0 ? 2 * acos(-1) : 0);
    }

    Pt perp() const {
        return Pt(-y, x);
    }

    Pt rotate(ld angle) const {
        // counter-clockwise rotation in radians
        // degree = radian * 180 / pi
        return Pt(x * cos(angle) - y * sin(angle), x * sin(
            angle) + y * cos(angle));
    }
}

```

```

}

int dir(Pt a, Pt b) const {
    // where am I on the directed line ab
    return sgn((a - *this).cross(b - *this));
}

bool operator<(Pt p) const {
    return eq(x, p.x) ? le(y, p.y) : le(x, p.x);
}

bool operator==(Pt p) const {
    return eq(x, p.x) && eq(y, p.y);
}

bool operator!=(Pt p) const {
    return !(*this == p);
}

friend ostream& operator<<(ostream& os, const Pt& p) {
    return os << "(" << p.x << ", " << p.y << ")";
}

friend istream& operator>>(istream& is, Pt& p) {
    return is >> p.x >> p.y;
}
};

```

4.2 Angle between vectors

```

double angleBetween(Pt a, Pt b) {
    double x = a.dot(b) / a.length() / b.length();
    return acosl(max(-1.0, min(1.0, x)));
}

```

4.3 Closest pair of points $\mathcal{O}(n \log n)$

```

pair<Pt, Pt> closestPairOfPoints(vector<Pt>& pts) {
    sort(all(pts), [&](Pt a, Pt b) {
        return le(a.y, b.y);
    });
    set<Pt> st;
    ld ans = INF;
    Pt p, q;
    int pos = 0;
    for (i, 0, sz(pts)) {
        while (pos < i && geq(pts[i].y - pts[pos].y, ans))
            st.erase(pts[pos++]);
        auto lo = st.lower_bound(Pt(pts[i].x - ans - eps, -INF));
        auto hi = st.upper_bound(Pt(pts[i].x + ans + eps, -INF));
        for (auto it = lo; it != hi; ++it) {
            ld d = (pts[i] - *it).length();
            if (le(d, ans))
                ans = d, p = pts[i], q = *it;
        }
        st.insert(pts[i]);
    }
    return {p, q};
}

```

4.4 KD Tree

```

struct Pt {
    // Geometry point mostly
    ld operator[](int i) const {
        return i == 0 ? x : y;
    }
};

struct KDTree {
    Pt p;
    int k;

```

```

    KDTree *left, *right;

    template <class Iter>
    KDTree(Iter l, Iter r, int k = 0) : k(k), left(0), right(
        0) {
        int n = r - l;
        if (n == 1) {
            p = *l;
            return;
        }
        nth_element(l, l + n / 2, r, [&](Pt a, Pt b) {
            return a[k] < b[k];
        });
        p = *(l + n / 2);
        left = new KDTree(l, l + n / 2, k ^ 1);
        right = new KDTree(l + n / 2, r, k ^ 1);
    }

    pair<ld, Pt> nearest(Pt x) {
        if (!left && !right)
            return {(p - x).norm(), p};
        vector<KDTree*> go = {left, right};
        auto delta = x[k] - p[k];
        if (delta > 0)
            swap(go[0], go[1]);
        auto best = go[0]->nearest(x);
        if (best.f > delta * delta)
            best = min(best, go[1]->nearest(x));
        return best;
    }
};

```

5 Lines and segments

5.1 Line

```

struct Line {
    Pt a, b, v;

    Line() {}
    Line(Pt a, Pt b) : a(a), b(b), v((b - a).unit()) {}

    bool contains(Pt p) {
        return eq((p - a).cross(b - a), 0);
    }

    int intersects(Line l) {
        if (eq(v.cross(l.v), 0))
            return eq((l.a - a).cross(v), 0) ? INF : 0;
        return 1;
    }

    int intersects(Seg s) {
        if (eq(v.cross(s.v), 0))
            return eq((s.a - a).cross(v), 0) ? INF : 0;
        return a.dir(b, s.a) != a.dir(b, s.b);
    }

    template <class Line>
    Pt intersection(Line l) { // can be a segment too
        return a + v * ((l.a - a).cross(l.v) / v.cross(l.v));
    }

    Pt projection(Pt p) {
        return a + v * proj(p - a, v);
    }

    Pt reflection(Pt p) {
        return a * 2 - p + v * 2 * proj(p - a, v);
    }
};

```

5.2 Segment

```
struct Seg {
    Pt a, b, v;

    Seg() {}
    Seg(Pt a, Pt b) : a(a), b(b), v(b - a) {}

    bool contains(Pt p) {
        return eq(v.cross(p - a), 0) && leq((a - p).dot(b - p),
            0);
    }

    int intersects(Seg s) {
        int d1 = a.dir(b, s.a), d2 = a.dir(b, s.b);
        if (d1 != d2)
            return s.a.dir(s.b, a) != s.a.dir(s.b, b);
        return d1 == 0 && (contains(s.a) || contains(s.b) || s.
            contains(a) || s.contains(b)) ? INF : 0;
    }

    template <class Seg>
    Pt intersection(Seg s) { // can be a line too
        return a + v * ((s.a - a).cross(s.v) / v.cross(s.v));
    }
};
```

5.3 Projection

```
ld proj(Pt a, Pt b) {
    return a.dot(b) / b.length();
}
```

5.4 Distance point line

```
ld distance(Pt p, Line l) {
    Pt q = l.projection(p);
    return (p - q).length();
}
```

5.5 Distance point segment

```
ld distance(Pt p, Seg s) {
    if (le((p - s.a).dot(s.b - s.a), 0))
        return (p - s.a).length();
    if (le((p - s.b).dot(s.a - s.b), 0))
        return (p - s.b).length();
    return abs((s.a - p).cross(s.b - p) / (s.b - s.a).length
        ());
}
```

5.6 Distance segment segment

```
ld distance(Seg a, Seg b) {
    if (a.intersects(b))
        return 0.L;
    return min({distance(a.a, b), distance(a.b, b), distance(
        b.a, a), distance(b.b, a)});
}
```

6 Circle

6.1 Circle

```
struct Cir : Pt {
    ld r;
    Cir() {}
    Cir(ld x, ld y, ld r) : Pt(x, y), r(r) {}
    Cir(Pt p, ld r) : Pt(p), r(r) {}

    int inside(Cir c) {
        ld l = c.r - r - (*this - c).length();
        return ge(l, 0) ? IN : eq(l, 0) ? ON : OVERLAP;
    }

    int outside(Cir c) {
        ld l = (*this - c).length() - r - c.r;
        return ge(l, 0) ? OUT : eq(l, 0) ? ON : OVERLAP;
    }
};
```

```
int contains(Pt p) {
    ld l = (p - *this).length() - r;
    return le(l, 0) ? IN : eq(l, 0) ? ON : OUT;
}

Pt projection(Pt p) {
    return *this + (p - *this).unit() * r;
}

vector<Pt> tangency(Pt p) {
    // point outside the circle
    Pt v = (p - *this).unit() * r;
    ld d2 = (p - *this).norm(), d = sqrt(d2);
    if (leq(d, 0))
        return {}; // on circle, no tangent
    Pt v1 = v * (r / d), v2 = v.perp() * (sqrt(d2 - r * r)
        / d);
    return {*this + v1 - v2, *this + v1 + v2};
}

vector<Pt> intersection(Cir c) {
    ld d = (c - *this).length();
    if (eq(d, 0) || ge(d, r + c.r) || le(d, abs(r - c.r)))
        return {}; // circles don't intersect
    Pt v = (c - *this).unit();
    ld a = (r * r + d * d - c.r * c.r) / (2 * d);
    Pt p = *this + v * a;
    if (eq(d, r + c.r) || eq(d, abs(r - c.r)))
        return {p}; // circles touch at one point
    ld h = sqrt(r * r - a * a);
    Pt q = v.perp() * h;
    return {p - q, p + q}; // circles intersects twice
}

template <class Line>
vector<Pt> intersection(Line l) {
    // for a segment you need to check that the point lies
    // on the segment
    ld h2 = r * r - l.v.cross(*this - l.a) * l.v.cross(*
        this - l.a) / l.v.norm();
    Pt p = l.a + l.v * l.v.dot(*this - l.a) / l.v.norm();
    if (eq(h2, 0))
        return {p}; // line tangent to circle
    if (le(h2, 0))
        return {}; // no intersection
    Pt q = l.v.unit() * sqrt(h2);
    return {p - q, p + q}; // two points of intersection (
        chord)
}

Cir(Pt a, Pt b, Pt c) {
    // find circle that passes through points a, b, c
    Pt mab = (a + b) / 2, mcb = (b + c) / 2;
    Seg ab(mab, mab + (b - a).perp());
    Seg cb(mcb, mcb + (b - c).perp());
    Pt o = ab.intersection(cb);
    *this = Cir(o, (o - a).length());
}
};
```

6.2 Distance point circle

```
ld distance(Pt p, Cir c) {
    return max(0.L, (p - c).length() - c.r);
}
```

6.3 Common area circle circle

```
ld commonArea(Cir a, Cir b) {
    if (le(a.r, b.r))
        swap(a, b);
}
```

```

ld d = (a - b).length();
if (leq(d + b.r, a.r))
    return b.r * b.r * PI;
if (geq(d, a.r + b.r))
    return 0.0;
auto angle = [&](ld x, ld y, ld z) {
    return acos((x * x + y * y - z * z) / (2 * x * y));
};
auto cut = [&](ld x, ld r) {
    return (x - sin(x)) * r * r / 2;
};
ld a1 = angle(d, a.r, b.r), a2 = angle(d, b.r, a.r);
return cut(a1 * 2, a.r) + cut(a2 * 2, b.r);
}

```

6.4 Minimum enclosing circle $\mathcal{O}(n)$ wow!!

```

Cir minEnclosing(vector<Pt>& pts) { // a bunch of points
    shuffle(all(pts), rng);
    Cir c(0, 0, 0);
    fore (i, 0, sz(pts))
        if (!c.contains(pts[i])) {
            c = Cir(pts[i], 0);
            fore (j, 0, i)
                if (!c.contains(pts[j])) {
                    c = Cir((pts[i] + pts[j]) / 2, (pts[i] - pts[j]).
                        length() / 2);
                    fore (k, 0, j)
                        if (!c.contains(pts[k]))
                            c = Cir(pts[i], pts[j], pts[k]);
                }
        }
    return c;
}

```

7 Polygon

7.1 Area polygon

```

ld area(const vector<Pt>& pts) {
    ld sum = 0;
    fore (i, 0, sz(pts))
        sum += pts[i].cross(pts[(i + 1) % sz(pts)]);
    return abs(sum / 2);
}

```

7.2 Perimeter

```

ld perimeter(const vector<Pt>& pts) {
    ld sum = 0;
    fore (i, 0, sz(pts))
        sum += (pts[(i + 1) % sz(pts)] - pts[i]).length();
    return sum;
}

```

7.3 Cut polygon line

```

vector<Pt> cut(const vector<Pt>& pts, Line l) {
    vector<Pt> ans;
    int n = sz(pts);
    fore (i, 0, n) {
        int j = (i + 1) % n;
        if (geq(l.v.cross(pts[i] - l.a), 0)) // left
            ans.pb(pts[i]);
        Seg s(pts[i], pts[j]);
        if (l.intersects(s) == 1) {
            Pt p = l.intersection(s);
            if (p != pts[i] && p != pts[j])
                ans.pb(p);
        }
    }
    return ans;
}

```

7.4 Common area circle polygon $\mathcal{O}(n)$

```

ld commonArea(Cir c, const vector<Pt>& poly) {

```

```

    auto arg = [&](Pt p, Pt q) {
        return atan2(p.cross(q), p.dot(q));
    };
    auto tri = [&](Pt p, Pt q) {
        Pt d = q - p;
        ld a = d.dot(p) / d.norm(), b = (p.norm() - c.r * c.r)
            / d.norm();
        ld det = a * a - b;
        if (leq(det, 0))
            return arg(p, q) * c.r * c.r;
        ld s = max(0.L, -a - sqrt(det)), t = min(1.L, -a + sqrt
            (det));
        if (t < 0 || 1 <= s)
            return arg(p, q) * c.r * c.r;
        Pt u = p + d * s, v = p + d * t;
        return u.cross(v) + (arg(p, u) + arg(v, q)) * c.r * c.r
            ;
    };
    ld sum = 0;
    fore (i, 0, sz(poly))
        sum += tri(poly[i] - c, poly[(i + 1) % sz(poly)] - c);
    return abs(sum / 2);
}

```

7.5 Point in polygon

```

int contains(const vector<Pt>& pts, Pt p) {
    int rays = 0, n = sz(pts);
    fore (i, 0, n) {
        Pt a = pts[i], b = pts[(i + 1) % n];
        if (ge(a.y, b.y))
            swap(a, b);
        if (Seg(a, b).contains(p))
            return ON;
        rays ^= (leq(a.y, p.y) && le(p.y, b.y) && p.dir(a, b) >
            0);
    }
    return rays & 1 ? IN : OUT;
}

```

7.6 Convex hull $\mathcal{O}(n \log n)$

```

vector<Pt> convexHull(vector<Pt> pts) {
    vector<Pt> hull;
    sort(all(pts), [&](Pt a, Pt b) {
        return a.x == b.x ? a.y < b.y : a.x < b.x;
    });
    pts.erase(unique(all(pts)), pts.end());
    fore (i, 0, sz(pts)) {
        while (sz(hull) >= 2 && hull.back().dir(pts[i], hull[sz
            (hull) - 2]) < 0)
            hull.pop_back();
        hull.pb(pts[i]);
    }
    hull.pop_back();
    int k = sz(hull);
    fore (i, sz(pts), 0) {
        while (sz(hull) >= k + 2 && hull.back().dir(pts[i],
            hull[sz(hull) - 2]) < 0)
            hull.pop_back();
        hull.pb(pts[i]);
    }
    hull.pop_back();
    return hull;
}

```

7.7 Is convex

```

bool isConvex(const vector<Pt>& pts) {
    int n = sz(pts);
    bool pos = 0, neg = 0;
    fore (i, 0, n) {
        Pt a = pts[(i + 1) % n] - pts[i];
        Pt b = pts[(i + 2) % n] - pts[(i + 1) % n];

```

```

int dir = sgn(a.cross(b));
if (dir > 0)
    pos = 1;
if (dir < 0)
    neg = 1;
}
return !(pos && neg);
}

```

7.8 Point in convex polygon $\mathcal{O}(\log n)$

```

bool contains(const vector<Pt>& a, Pt p) {
    int lo = 1, hi = sz(a) - 1;
    if (a[0].dir(a[lo], a[hi]) > 0)
        swap(lo, hi);
    if (p.dir(a[0], a[lo]) >= 0 || p.dir(a[0], a[hi]) <= 0)
        return false;
    while (abs(lo - hi) > 1) {
        int mid = (lo + hi) >> 1;
        (p.dir(a[0], a[mid]) > 0 ? hi : lo) = mid;
    }
    return p.dir(a[lo], a[hi]) < 0;
}

```

8 Graphs

8.1 Cutpoints and bridges

```

int tin[N], fup[N], timer = 0;

void weakness(int u, int p = -1) {
    tin[u] = fup[u] = ++timer;
    int children = 0;
    for (int v : graph[u])
        if (v != p) {
            if (!tin[v]) {
                ++children;
                weakness(v, u);
                fup[u] = min(fup[u], fup[v]);
                if (fup[v] >= tin[u] && !(p == -1 && children < 2))
                    // u is a cutpoint
                    if (fup[v] > tin[u]) // bridge u -> v

```

8.2 Topological sort

```

vector<int> order;
int indeg[N];

void topologicalSort() { // first fill the indeg[]
    queue<int> qu;
    for (u, 1, n + 1)
        if (indeg[u] == 0)
            qu.push(u);
    while (!qu.empty()) {
        int u = qu.front();
        qu.pop();
        order.pb(u);
        for (auto& v : graph[u])
            if (--indeg[v] == 0)
                qu.push(v);
    }
}

```

8.3 Kosaraju

```

int scc[N], k = 0;
char vis[N];
vector<int> order;

void dfs1(int u) {
    vis[u] = 1;
    for (int v : graph[u])

```

```

    if (vis[v] != 1)
        dfs1(v);
    order.pb(u);
}

void dfs2(int u, int k) {
    vis[u] = 2, scc[u] = k;
    for (int v : rgraph[u]) // reverse graph
        if (vis[v] != 2)
            dfs2(v, k);
}

```

```

void kosaraju() {
    for (u, 1, n + 1)
        if (vis[u] != 1)
            dfs1(u);
    reverse(all(order));
    for (int u : order)
        if (vis[u] != 2)
            dfs2(u, ++k);
}

```

8.4 Tarjan

```

int tin[N], fup[N];
bitset<N> still;
stack<int> stk;
int timer = 0;

void tarjan(int u) {
    tin[u] = fup[u] = ++timer;
    still[u] = true;
    stk.push(u);
    for (auto& v : graph[u]) {
        if (!tin[v])
            tarjan(v);
        if (still[v])
            fup[u] = min(fup[u], fup[v]);
    }
    if (fup[u] == tin[u]) {
        int v;
        do {
            v = stk.top();
            stk.pop();
            still[v] = false;
            // u and v are in the same scc
        } while (v != u);
    }
}

```

8.5 Isomorphism

```

lli dp[N], h[N];

lli f(lli x) {
    // K * n <= 9e18
    static uniform_int_distribution<lli> uid(1, K);
    if (!mp.count(x))
        mp[x] = uid(rng);
    return mp[x];
}

lli hsh(int u, int p = -1) {
    dp[u] = h[u] = 0;
    for (auto& v : graph[u]) {
        if (v == p)
            continue;
        dp[u] += hsh(v, u);
    }
    return h[u] = f(dp[u]);
}

```

8.6 Two sat

```

// 1-indexed
struct TwoSat {
    int n;
    vector<vector<int>> imp;

    TwoSat(int k) : n(k + 1), imp(2 * n) {}

    // a || b
    void either(int a, int b) {
        a = max(2 * a, -1 - 2 * a);
        b = max(2 * b, -1 - 2 * b);
        imp[a ^ 1].pb(b);
        imp[b ^ 1].pb(a);
    }

    // if a then b
    // a b   a => b
    // F F   T
    // T T   T
    // F T   T
    // T F   F
    void implies(int a, int b) {
        either(~a, b);
    }

    // setVal(a): set a = true
    // setVal(~a): set a = false
    void setVal(int a) {
        either(a, a);
    }

    optional<vector<int>> solve() {
        int k = sz(imp);
        vector<int> s, b, id(sz(imp));
        function<void(int)> dfs = [&](int u) {
            b.pb(id[u] = sz(s)), s.pb(u);
            for (int v : imp[u]) {
                if (!id[v])
                    dfs(v);
                else
                    while (id[v] < b.back())
                        b.pop_back();
            }
            if (id[u] == b.back())
                for (b.pop_back(), ++k; id[u] < sz(s); s.pop_back())
                    id[s.back()] = k;
        };
        vector<int> val(n);
        for (u, 0, sz(imp))
            if (!id[u])
                dfs(u);
        for (u, 0, n) {
            int x = 2 * u;
            if (id[x] == id[x ^ 1])
                return nullopt;
            val[u] = id[x] < id[x ^ 1];
        }
        return optional(val);
    }
};

```

8.7 LCA

```

const int LogN = 1 + __lg(N);
int par[LogN][N], depth[N];

void dfs(int u, int par[]) {
    for (auto& v : graph[u])
        if (v != par[u]) {
            par[v] = u;

```

```

            depth[v] = depth[u] + 1;
            dfs(v, par);
        }
    }

    int lca(int u, int v) {
        if (depth[u] > depth[v])
            swap(u, v);
        for (k, LogN, 0)
            if (dep[v] - dep[u] >= (1 << k))
                v = par[k][v];
        if (u == v)
            return u;
        for (k, LogN, 0)
            if (par[k][v] != par[k][u])
                u = par[k][u], v = par[k][v];
        return par[0][u];
    }

    int dist(int u, int v) {
        return depth[u] + depth[v] - 2 * depth[lca(u, v)];
    }

    void init(int r) {
        dfs(r, par[0]);
        for (k, 1, LogN)
            for (u, 1, n + 1)
                par[k][u] = par[k - 1][par[k - 1][u]];
    }
}

```

8.8 Virtual tree $\mathcal{O}(n \cdot \log n)$

```

vector<int> virt[N];

int virtualTree(vector<int>& ver) {
    auto byDfs = [&](int u, int v) {
        return tin[u] < tin[v];
    };
    sort(all(ver), byDfs);
    for (i, sz(ver), 1)
        ver.pb(lca(ver[i - 1], ver[i]));
    sort(all(ver), byDfs);
    ver.erase(unique(all(ver)), ver.end());
    for (int u : ver)
        virt[u].clear();
    for (i, 1, sz(ver))
        virt[lca(ver[i - 1], ver[i])].pb(ver[i]);
    return ver[0];
}

```

8.9 Euler-tour + HLD + LCA $\mathcal{O}(n \cdot \log n \cdot f)$

Solves subtrees and paths problems.

```

int par[N], nxt[N], depth[N], sz[N];
int tin[N], tout[N], who[N], timer = 0;

int dfs(int u) {
    sz[u] = 1;
    for (auto& v : graph[u])
        if (v != par[u]) {
            par[v] = u;
            depth[v] = depth[u] + 1;
            sz[u] += dfs(v);
            if (graph[u][0] == par[u] || sz[v] > sz[graph[u][0]])
                swap(v, graph[u][0]);
        }
    return sz[u];
}

void hld(int u) {
    tin[u] = ++timer, who[timer] = u;
    for (auto& v : graph[u])
        if (v != par[u]) {

```

```

    nxt[v] = (v == graph[u][0] ? nxt[u] : v);
    hld(v);
}
tout[u] = timer;
}

template <bool OverEdges = 0, class F>
void processPath(int u, int v, F f) {
    for (; nxt[u] != nxt[v]; u = par[nxt[u]]) {
        if (depth[nxt[u]] < depth[nxt[v]])
            swap(u, v);
        f(tin[nxt[u]], tin[u]);
    }
    if (depth[u] < depth[v])
        swap(u, v);
    f(tin[v] + OverEdges, tin[u]);
}

void updatePath(int u, int v, lli z) {
    processPath(u, v, [&](int l, int r) {
        tree->update(l, r, z);
    });
}

void updateSubtree(int u, lli z) {
    tree->update(tin[u], tout[u], z);
}

lli queryPath(int u, int v) {
    lli sum = 0;
    processPath(u, v, [&](int l, int r) {
        sum += tree->query(l, r);
    });
    return sum;
}

lli querySubtree(int u) {
    return tree->query(tin[u], tout[u]);
}

int lca(int u, int v) {
    int last = -1;
    processPath(u, v, [&](int l, int r) {
        last = who[l];
    });
    return last;
}

```

8.10 Centroid $\mathcal{O}(n \cdot \log n \cdot f)$

Solves "all pairs of nodes" problems

```

int cdp[N], sz[N];
bitset<N> rem;

int dfsz(int u, int p = -1) {
    sz[u] = 1;
    for (int v : graph[u])
        if (v != p && !rem[v])
            sz[u] += dfsz(v, u);
    return sz[u];
}

int centroid(int u, int size, int p = -1) {
    for (int v : graph[u])
        if (v != p && !rem[v] && 2 * sz[v] > size)
            return centroid(v, size, u);
    return u;
}

void solve(int u, int p = -1) {

```

```

    cdp[u = centroid(u, dfsz(u))] = p;
    rem[u] = true;
    for (int v : graph[u])
        if (!rem[v])
            solve(v, u);
}

```

8.11 Guni $\mathcal{O}(n \cdot \log n \cdot f)$

Solve subtrees problems

```

int cnt[C], color[N];
int sz[N];

int guni(int u, int p = -1) {
    sz[u] = 1;
    for (auto& v : graph[u])
        if (v != p) {
            sz[u] += guni(v, u);
            if (sz[v] > sz[graph[u][0]] || p == graph[u][0])
                swap(v, graph[u][0]);
        }
    return sz[u];
}

void update(int u, int p, int add, bool skip) {
    cnt[color[u]] += add;
    fore (i, skip, sz[graph[u]])
        if (graph[u][i] != p)
            update(graph[u][i], u, add, 0);
}

void solve(int u, int p = -1, bool keep = 0) {
    fore (i, sz[graph[u]], 0)
        if (graph[u][i] != p)
            solve(graph[u][i], u, !i);
    update(u, p, +1, 1); // add
    // now cnt[i] has how many times the color i appears in
    // the subtree of u
    if (!keep)
        update(u, p, -1, 0); // remove
}

```

8.12 Link-Cut tree $\mathcal{O}(n \cdot \log n \cdot f)$

Solves dynamic trees problems, can handle subtrees and paths
maybe with a high constant

```

struct LinkCut {
    struct Node {
        Node *left{0}, *right{0}, *par{0};
        bool rev = 0;
        int sz = 1;
        int sub = 0, vsub = 0; // subtree
        lli path = 0; // path
        lli self = 0; // node info
    }

    void push() {
        if (rev) {
            swap(left, right);
            if (left)
                left->rev ^= 1;
            if (right)
                right->rev ^= 1;
            rev = 0;
        }
    }

    void pull() {
        sz = 1;
        sub = vsub + self;
        path = self;
    }
}

```

```

    if (left) {
        sz += left->sz;
        sub += left->sub;
        path += left->path;
    }
    if (right) {
        sz += right->sz;
        sub += right->sub;
        path += right->path;
    }
}

void addVsub(Node* v, lli add) {
    if (v)
        vsub += 1LL * add * v->sub;
}

};

vector<Node> a;

LinkCut(int n = 1) : a(n) {}

void splay(Node* u) {
    auto assign = [&](Node* u, Node* v, int d) {
        if (v)
            v->par = u;
        if (d >= 0)
            (d == 0 ? u->left : u->right) = v;
    };
    auto dir = [&](Node* u) {
        if (!u->par)
            return -1;
        return u->par->left == u ? 0 : (u->par->right == u ?
            1 : -1);
    };
    auto rotate = [&](Node* u) {
        Node *p = u->par, *g = p->par;
        int d = dir(u);
        assign(p, d ? u->left : u->right, d);
        assign(g, u, dir(p));
        assign(u, p, !d);
        p->pull(), u->pull();
    };
    while (~dir(u)) {
        Node *p = u->par, *g = p->par;
        if (~dir(p))
            g->push();
        p->push(), u->push();
        if (~dir(p))
            rotate(dir(p) == dir(u) ? p : u);
        rotate(u);
    }
    u->push(), u->pull();
}

void access(int u) {
    Node* last = NULL;
    for (Node* x = &a[u]; x; last = x, x = x->par) {
        splay(x);
        x->addVsub(x->right, +1);
        x->right = last;
        x->addVsub(x->right, -1);
        x->pull();
    }
    splay(&a[u]);
}

void reroot(int u) {
    access(u);
    a[u].rev ^= 1;
}

```

```

}

void link(int u, int v) {
    reroot(v), access(u);
    a[u].addVsub(v, +1);
    a[v].par = &a[u];
    a[u].pull();
}

void cut(int u, int v) {
    reroot(v), access(u);
    a[u].left = a[v].par = NULL;
    a[u].pull();
}

int lca(int u, int v) {
    if (u == v)
        return u;
    access(u), access(v);
    if (!a[u].par)
        return -1;
    return splay(&a[u]), a[u].par ? -1 : u;
}

int depth(int u) {
    access(u);
    return a[u].left ? a[u].left->sz : 0;
}

// get k-th parent on path to root
int ancestor(int u, int k) {
    k = depth(u) - k;
    assert(k >= 0);
    for (;;) a[u].push() {
        int sz = a[u].left->sz;
        if (sz == k)
            return access(u), u;
        if (sz < k)
            k -= sz + 1, u = u->ch[1];
        else
            u = u->ch[0];
    }
    assert(0);
}

lli queryPath(int u, int v) {
    reroot(u), access(v);
    return a[v].path;
}

lli querySubtree(int u, int x) {
    // query subtree of u, x is outside
    reroot(x), access(u);
    return a[u].vsub + a[u].self;
}

void update(int u, lli val) {
    access(u);
    a[u].self = val;
    a[u].pull();
}

Node& operator[](int u) {
    return a[u];
}

};

```


9 Flows

9.1 Hopcroft Karp $\mathcal{O}(e\sqrt{v})$

```
struct HopcroftKarp {
    int n, m;
    vector<vector<int>> graph;
    vector<int> dist, match;

    HopcroftKarp(int k) : n(k + 1), graph(n), dist(n), match(
        n, 0) {} // 1-indexed!!

    void add(int u, int v) {
        graph[u].pb(v), graph[v].pb(u);
    }

    bool bfs() {
        queue<int> qu;
        fill(all(dist), -1);
        for (u, 1, n)
            if (!match[u])
                dist[u] = 0, qu.push(u);
        while (!qu.empty()) {
            int u = qu.front();
            qu.pop();
            for (int v : graph[u])
                if (dist[match[v]] == -1) {
                    dist[match[v]] = dist[u] + 1;
                    if (match[v])
                        qu.push(match[v]);
                }
        }
        return dist[0] != -1;
    }

    bool dfs(int u) {
        for (int v : graph[u])
            if (!match[v] || (dist[u] + 1 == dist[match[v]] &&
                dfs(match[v]))) {
                match[u] = v, match[v] = u;
                return 1;
            }
        dist[u] = 1 << 30;
        return 0;
    }

    int maxMatching() {
        int tot = 0;
        while (bfs())
            for (u, 1, n)
                tot += match[u] ? 0 : dfs(u);
        return tot;
    }
};
```

9.2 Hungarian $\mathcal{O}(n^2 \cdot m)$

n jobs, m people

```
template <class C>
pair<C, vector<int>> Hungarian(vector<vector<C>>& a) { //
    max assignment
    int n = sz(a), m = sz(a[0]), p, q, j, k; // n <= m
    vector<C> fx(n, numeric_limits<C>::min()), fy(m, 0);
    vector<int> x(n, -1), y(m, -1);
    for (i, 0, n)
        for (j, 0, m)
            fx[i] = max(fx[i], a[i][j]);
    for (i, 0, n) {
        vector<int> t(m, -1), s(n + 1, i);
        for (p = q = 0; p <= q && x[i] < 0; p++)
            for (k = s[p], j = 0; j < m && x[i] < 0; j++)
                if (abs(fx[k] + fy[j] - a[k][j]) < EPS && t[j] < 0)
```

```
                {
                    s[++q] = y[j], t[j] = k;
                    if (s[q] < 0)
                        for (p = j; p >= 0; j = p)
                            y[j] = k = t[j], p = x[k], x[k] = j;
                }
            if (x[i] < 0) {
                C d = numeric_limits<C>::max();
                for (k, 0, q + 1)
                    for (j, 0, m)
                        if (t[j] < 0)
                            d = min(d, fx[s[k]] + fy[j] - a[s[k]][j]);
                for (j, 0, m)
                    fy[j] += (t[j] < 0 ? 0 : d);
                for (k, 0, q + 1)
                    fx[s[k]] -= d;
                i--;
            }
        }
        C cost = 0;
        for (i, 0, n)
            cost += a[i][x[i]];
        return make_pair(cost, x);
    }
}
```

9.3 Dinic $\mathcal{O}(\min(e \cdot \text{flow}, v^2 \cdot e))$

```
template <class F>
struct Dinic {
    struct Edge {
        int v, inv;
        F cap, flow;
        Edge(int v, F cap, int inv) : v(v), cap(cap), flow(0),
            inv(inv) {}
    };

    F EPS = (F)1e-9;
    int s, t, n;
    vector<vector<Edge>> graph;
    vector<int> dist, ptr;

    Dinic(int n) : n(n), graph(n), dist(n), ptr(n), s(n - 2),
        t(n - 1) {}

    void add(int u, int v, F cap) {
        graph[u].pb(Edge(v, cap, sz(graph[v])));
        graph[v].pb(Edge(u, 0, sz(graph[u]) - 1));
    }

    bool bfs() {
        fill(all(dist), -1);
        queue<int> qu({s});
        dist[s] = 0;
        while (sz(qu) && dist[t] == -1) {
            int u = qu.front();
            qu.pop();
            for (Edge& e : graph[u])
                if (dist[e.v] == -1)
                    if (e.cap - e.flow > EPS) {
                        dist[e.v] = dist[u] + 1;
                        qu.push(e.v);
                    }
        }
        return dist[t] != -1;
    }

    F dfs(int u, F flow = numeric_limits<F>::max()) {
        if (flow <= EPS || u == t)
            return max<F>(0, flow);
        for (int& i = ptr[u]; i < sz(graph[u]); i++) {
```

```

Edge& e = graph[u][i];
if (e.cap - e.flow > EPS && dist[u] + 1 == dist[e.v])
{
    F pushed = dfs(e.v, min<F>(flow, e.cap - e.flow));
    if (pushed > EPS) {
        e.flow += pushed;
        graph[e.v][e.inv].flow -= pushed;
        return pushed;
    }
}
}
return 0;
}

F maxFlow() {
    F flow = 0;
    while (bfs()) {
        fill(all(ptr), 0);
        while (F pushed = dfs(s))
            flow += pushed;
    }
    return flow;
}

bool leftSide(int u) {
    // left side comes from sink
    return dist[u] != -1;
}
};

```

9.4 Min-Cost flow $\mathcal{O}(\min(e \cdot \text{flow}, v^2 \cdot e))$

```

template <class C, class F>
struct Mcmf {
    struct Edge {
        int u, v, inv;
        F cap, flow;
        C cost;
        Edge(int u, int v, C cost, F cap, int inv) : u(u), v(v),
            cost(cost), cap(cap), flow(0), inv(inv) {}
    };

    F EPS = (F)1e-9;
    int s, t, n;
    vector<vector<Edge>> graph;
    vector<Edge*> prev;
    vector<C> cost;
    vector<int> state;

    Mcmf(int n) : n(n), graph(n), cost(n), state(n), prev(n),
        s(n - 2), t(n - 1) {}

    void add(int u, int v, C cost, F cap) {
        graph[u].pb(Edge(u, v, cost, cap, graph[v]));
        graph[v].pb(Edge(v, u, -cost, 0, sz(graph[u]) - 1));
    }

    bool bfs() {
        fill(all(state), 0);
        fill(all(cost), numeric_limits<C>::max());
        deque<int> qu;
        qu.push_back(s);
        state[s] = 1, cost[s] = 0;
        while (sz(qu)) {
            int u = qu.front();
            qu.pop_front();
            state[u] = 2;
            for (Edge& e : graph[u])
                if (e.cap - e.flow > EPS)
                    if (cost[u] + e.cost < cost[e.v]) {
                        cost[e.v] = cost[u] + e.cost;

```

```

prev[e.v] = &e;
if (state[e.v] == 2 || (sz(qu) && cost[qu.front()] > cost[e.v]))
    qu.push_front(e.v);
else if (state[e.v] == 0)
    qu.push_back(e.v);
state[e.v] = 1;
}
}
return cost[t] != numeric_limits<C>::max();
}

pair<C, F> minCostFlow() {
    C cost = 0;
    F flow = 0;
    while (bfs()) {
        F pushed = numeric_limits<F>::max();
        for (Edge* e = prev[t]; e != nullptr; e = prev[e->u])
            pushed = min(pushed, e->cap - e->flow);
        for (Edge* e = prev[t]; e != nullptr; e = prev[e->u])
        {
            e->flow += pushed;
            graph[e->v][e->inv].flow -= pushed;
            cost += e->cost * pushed;
        }
        flow += pushed;
    }
    return make_pair(cost, flow);
}
};

```

10 Game theory

10.1 Grundy numbers

If the moves are consecutive $S = \{1, 2, 3, \dots, x\}$ the game can be solved like $\text{stackSize} \pmod{x+1} \neq 0$

```

int mem[N];

int mex(set<int>& st) {
    int x = 0;
    while (st.count(x))
        x++;
    return x;
}

int grundy(int n) {
    if (n < 0)
        return INF;
    if (n == 0)
        return 0;
    int& g = mem[n];
    if (g == -1) {
        set<int> st;
        for (int x : {a, b})
            st.insert(grundy(n - x));
        g = mex(st);
    }
    return g;
}

```

11 Math

11.1 Bits

Bits++	
Operations on <i>int</i>	Function
<code>x & ~x</code>	Least significant bit in <i>x</i>
<code>__lg(x)</code>	Most significant bit in <i>x</i>
<code>c = x&x, r = x+c; (((r^x) >> 2)/c) r</code>	Next number after <i>x</i> with same number of bits set
<code>__builtin_</code>	Function
<code>popcount(x)</code>	Amount of 1's in <i>x</i>
<code>clz(x)</code>	0's to the left of biggest bit
<code>ctz(x)</code>	0's to the right of smallest bit

11.2 Bitset

Bitset<Size>	
Operation	Function
<code>_Find_first()</code>	Least significant bit
<code>_Find_next(idx)</code>	First set bit after index <i>idx</i>
<code>any(), none(), all()</code>	Just what the expression says
<code>set(), reset(), flip()</code>	Just what the expression says x2
<code>to_string('.', 'A')</code>	Print 011010 like .AA.A.

11.3 Probability

Conditional

The event A happens and the event B has already happened

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

If **independent** events

$$P(A|B) = P(A), P(B|A) = P(B)$$

Bayes theorem

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Binomial

$$B = \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x}$$

n = number of trials

x = number of **success** from *n* trials

p = probability of **success** on a single trial

Geometric

Probability of success at the *n*th-event after failing the others

$$G = (1-p)^{n-1} \cdot p$$

n = number of trials

p = probability of *success* on a single trial

Poisson

$$Po = \frac{\lambda^k \cdot e^{-\lambda}}{k!}$$

λ = number of times an event is expected (occurs / time)

k = number of occurring events in the limited period of time

Example: The event happens 4 times per minute and we want *k* events to happen in 10 minutes, then $\lambda = 4 \cdot 10 = 40$

Expected value

$$E_x = \sum_{\forall x} x \cdot p(x)$$

11.4 Simplex

// maximize `c^t x` s.t. `ax <= b, x >= 0`

```
template <class T>
pair<T, vector<T>> simplex(vector<vector<T>> a, vector<T> b
, vector<T> c) {
    const T EPS = 1e-9;
    T sum = 0;
    int n = b.size(), m = c.size();
    vector<int> p(m), q(n);
    iota(all(p), 0), iota(all(q), m);
```

```
    auto pivot = [&](int x, int y) {
        swap(p[y], q[x]);
        b[x] /= a[x][y];
        for (i, 0, m)
            if (i != y)
                a[x][i] /= a[x][y];
        a[x][y] = 1 / a[x][y];
        for (i, 0, n)
            if (i != x && abs(a[i][y]) > EPS) {
                b[i] -= a[i][y] * b[x];
                for (j, 0, m)
                    if (j != y)
                        a[i][j] -= a[i][y] * a[x][j];
                a[i][y] = -a[i][y] * a[x][y];
            }
        sum += c[y] * b[x];
        for (i, 0, m)
            if (i != y)
                c[i] -= c[y] * a[x][i];
        c[y] = -c[y] * a[x][y];
    };
```

```
    while (1) {
        int x = -1, y = -1;
        ld mn = -EPS;
        for (i, 0, n)
            if (b[i] < mn)
                mn = b[i], x = i;
        if (x < 0)
            break;
        for (i, 0, m)
            if (a[x][i] < -EPS) {
                y = i;
                break;
            }
        assert(y >= 0); // no solution to Ax <= b
        pivot(x, y);
    }
```

```
    while (1) {
        int x = -1, y = -1;
        ld mx = EPS;
        for (i, 0, m)
            if (c[i] > mx)
                mx = c[i], y = i;
        if (y < 0)
            break;
        ld mn = 1e200;
        for (i, 0, n)
            if (a[i][y] > EPS && b[i] / a[i][y] < mn) {
                mn = b[i] / a[i][y], x = i;
            }
        assert(x >= 0); // c^t x is unbounded
        pivot(x, y);
    }
```

```

vector<T> ans(m);
for (i, 0, n)
    if (q[i] < m)
        ans[q[i]] = b[i];
return {sum, ans};
}

```

11.5 Xor basis

```

template <int D>
struct XorBasis {
    using Num = bitset<D>;
    array<Num, D> basis, keep;
    vector<int> from;
    int n = 0, id = -1;

    XorBasis() : from(D, -1) {
        basis.fill(0);
    }

    bool insert(Num x) {
        ++id;
        Num k;
        for (i, D, 0)
            if (x[i]) {
                if (!basis[i].any()) {
                    k[i] = 1, from[i] = id, keep[i] = k;
                    basis[i] = x, n++;
                    return 1;
                }
                x ^= basis[i], k ^= keep[i];
            }
        return 0;
    }

    optional<Num> find(Num x) {
        // is x in xor-basis set?
        // v ^ (v ^ x) = x
        Num v;
        for (i, D, 0)
            if (x[i]) {
                if (!basis[i].any())
                    return nullopt;
                x ^= basis[i];
                v[i] = 1;
            }
        return optional(v);
    }

    optional<vector<int>> recover(Num x) {
        auto v = find(x);
        if (!v)
            return nullopt;
        Num tmp;
        for (i, D, 0)
            if (v.value()[i])
                tmp ^= keep[i];
        vector<int> ans;
        for (int i = tmp._Find_first(); i < D; i = tmp._Find_next(i))
            ans.pb(from[i]);
        return ans;
    }

    optional<Num> operator[](lli k) {
        lli tot = (1LL << n);
        if (k > tot)
            return nullopt;
        Num v = 0;
        for (i, D, 0)

```

```

        if (basis[i]) {
            lli low = tot / 2;
            if ((low < k && v[i] == 0) || (low >= k && v[i]))
                v ^= basis[i];
            if (low < k)
                k -= low;
            tot /= 2;
        }
        return optional(v);
    }
};

```

12 Combinatorics

12.1 Catalan

```

catalan[0] = 1LL;
for (i, 0, N) {
    catalan[i + 1] = catalan[i] * lli(4 * i + 2) % mod * fpow
        (i + 2, mod - 2) % mod;
}

```

12.2 Factorial

```

fac[0] = 1LL;
for (i, 1, N)
    fac[i] = lli(i) * fac[i - 1] % mod;
ifac[n - 1] = fpow(fac[n - 1], mod - 2, mod);
for (int i = n - 1; i >= 0; i--)
    ifac[i] = lli(i + 1) * ifac[i + 1] % mod;

```

12.3 Factorial mod small prime

```

lli facMod(lli n, int p) {
    lli r = 1LL;
    for (; n > 1; n /= p) {
        r = (r * ((n / p) % 2 ? p - 1 : 1)) % p;
        for (i, 2, n % p + 1)
            r = r * i % p;
    }
    return r % p;
}

```

12.4 Choose

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\binom{n}{k_1, k_2, \dots, k_m} = \frac{n!}{k_1! * k_2! * \dots * k_m!}$$

```

lli choose(int n, int k) {
    if (n < 0 || k < 0 || n < k)
        return 0LL;
    return fac[n] * ifac[k] % mod * ifac[n - k] % mod;
}

```

```

lli choose(int n, int k) {
    lli r = 1;
    int to = min(k, n - k);
    if (to < 0)
        return 0;
    for (i, 0, to)
        r = r * (n - i) / (i + 1);
    return r;
}

```

12.5 Pascal

```

for (i, 0, N) {
    choose[i][0] = choose[i][i] = 1;
    for (int j = 1; j <= i; j++)
        choose[i][j] = choose[i - 1][j - 1] + choose[i - 1][j];
}

```

12.6 Stars and bars

Enclosing n objects in k boxes

$$\binom{n+k-1}{k-1} = \binom{n+k-1}{n}$$

12.7 Lucas

Changes $\binom{n}{k} \bmod p$, with $n \geq 2e6, k \geq 2e6$ and $p \leq 1e7$

$$\binom{n}{k} \equiv \prod_{i=0}^n \binom{n_i}{k_i} \bmod p$$

```

lli lucas(lli n, lli k) {
    if (k == 0)
        return 1LL;
    return lucas(n / mod, k / mod) * choose(n % mod, k % mod)
        % mod;
}

```

12.8 Burnside lemma

$$|classes| = \frac{1}{|G|} \cdot \sum_{x \in G} f(x)$$

13 Number theory

13.1 Amount of divisors $\mathcal{O}(n^{1/3})$

```

ull amountOfDivisors(ull n) {
    ull cnt = 1;
    for (auto p : primes) {
        if (1LL * p * p * p > n)
            break;
        if (n % p == 0) {
            ull k = 0;
            while (n > 1 && n % p == 0)
                n /= p, ++k;
            cnt *= (k + 1);
        }
    }
    ull sq = mysqrt(n); // the last x * x <= n
    if (miller(n))
        cnt *= 2;
    else if (sq * sq == n && miller(sq))
        cnt *= 3;
    else if (n > 1)
        cnt *= 4;
    return cnt;
}

```

13.2 Chinese remainder theorem

```

pair<lli, lli> crt(pair<lli, lli> a, pair<lli, lli> b) {
    if (a.s < b.s)
        swap(a, b);
    auto p = euclid(a.s, b.s);
    lli g = a.s * p.f + b.s * p.s, l = a.s / g * b.s;
    if ((b.f - a.f) % g != 0)
        return {-1, -1}; // no solution
    p.f = a.f + (b.f - a.f) % b.s * p.f % b.s / g * a.s;
    return {p.f + (p.f < 0) * l, l};
}

```

13.3 Euclid $\mathcal{O}(\log(a \cdot b))$

```

pair<lli, lli> euclid(lli a, lli b) {
    if (b == 0)
        return {1, 0};
}

```

```

auto p = euclid(b, a % b);
return {p.s, p.f - a / b * p.s};
}

```

13.4 Factorial factors

```

vector<ii> factorialFactors(lli n) {
    vector<ii> fac;
    for (auto p : primes) {
        if (n < p)
            break;
        lli mul = 1LL, k = 0;
        while (mul <= n / p) {
            mul *= p;
            k += n / mul;
        }
        fac.emplace_back(p, k);
    }
    return fac;
}

```

13.5 Factorize sieve

```

int factor[N];

void factorizeSieve() {
    iota(factor, factor + N, 0);
    for (int i = 2; i * i < N; i++)
        if (factor[i] == i)
            for (int j = i * i; j < N; j += i)
                factor[j] = i;
}

```

```

map<int, int> factorize(int n) {
    map<int, int> cnt;
    while (n > 1) {
        cnt[factor[n]]++;
        n /= factor[n];
    }
    return cnt;
}

```

13.6 Sieve

```

bitset<N> isPrime;
vector<int> primes;

void sieve() {
    isPrime.set();
    isPrime[0] = isPrime[1] = 0;
    for (int i = 2; i * i < N; ++i)
        if (isPrime[i])
            for (int j = i * i; j < N; j += i)
                isPrime[j] = 0;
    for (i = 2, N)
        if (isPrime[i])
            primes.pb(i);
}

```

13.7 Phi $\mathcal{O}(\sqrt{n})$

```

lli phi(lli n) {
    if (n == 1)
        return 0;
    lli r = n;
    for (lli i = 2; i * i <= n; i++)
        if (n % i == 0) {
            while (n % i == 0)
                n /= i;
            r -= r / i;
        }
    if (n > 1)
        r -= r / n;
    return r;
}

```

13.8 Phi sieve

```
bitset<N> isPrime;
int phi[N];

void phiSieve() {
    isPrime.set();
    iota(phi, phi + N, 0);
    for (i, 2, N)
        if (isPrime[i])
            for (int j = i; j < N; j += i) {
                isPrime[j] = (i == j);
                phi[j] = phi[j] / i * (i - 1);
            }
}
```

13.9 Miller rabin $\mathcal{O}(\text{Witnesses} \cdot (\log n)^3)$

```
ull mul(ull x, ull y, ull mod) {
    lli ans = x * y - mod * ull(1.L / mod * x * y);
    return ans + mod * (ans < 0) - mod * (ans >= lli(mod));
}

// use mul(x, y, mod) inside fpow
bool miller(ull n) {
    if (n < 2 || n % 6 % 4 != 1)
        return (n | 1) == 3;
    ull k = __builtin_ctzll(n - 1), d = n >> k;
    for (ull p : {2, 325, 9375, 28178, 450775, 9780504, 17952
        65022}) {
        ull x = fpow(p % n, d, n), i = k;
        while (x != 1 && x != n - 1 && p % n && i--)
            x = mul(x, x, n);
        if (x != n - 1 && i != k)
            return 0;
    }
    return 1;
}
```

13.10 Pollard Rho $\mathcal{O}(n^{1/4})$

```
ull rho(ull n) {
    auto f = [n](ull x) {
        return mul(x, x, n) + 1;
    };
    ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
    while (t++ % 40 || __gcd(prd, n) == 1) {
        if (x == y)
            x = ++i, y = f(x);
        if (q = mul(prd, max(x, y) - min(x, y), n))
            prd = q;
        x = f(x), y = f(f(y));
    }
    return __gcd(prd, n);
}

// if used multiple times, try memorization!!
// try factoring small numbers with sieve
void pollard(ull n, map<ull, int>& fac) {
    if (n == 1)
        return;
    if (miller(n)) {
        fac[n]++;
    } else {
        ull x = rho(n);
        pollard(x, fac);
        pollard(n / x, fac);
    }
}
```

14 Polynomials

14.1 Berlekamp Massey

```
template <class T>
```

```
struct BerlekampMassey {
    int n;
    vector<T> s, t, pw[20];

    vector<T> combine(vector<T> a, vector<T> b) {
        vector<T> ans(sz(t) * 2 + 1);
        for (int i = 0; i <= sz(t); i++)
            for (int j = 0; j <= sz(t); j++)
                ans[i + j] += a[i] * b[j];
        for (int i = 2 * sz(t); i > sz(t); --i)
            for (int j = 0; j < sz(t); j++)
                ans[i - 1 - j] += ans[i] * t[j];
        ans.resize(sz(t) + 1);
        return ans;
    }

    BerlekampMassey(const vector<T>& s) : n(sz(s)), t(n), s(s)
    ) {
        vector<T> x(n), tmp;
        t[0] = x[0] = 1;
        T b = 1;
        int len = 0, m = 0;
        for (i, 0, n) {
            ++m;
            T d = s[i];
            for (int j = 1; j <= len; j++)
                d += t[j] * s[i - j];
            if (d == 0)
                continue;
            tmp = t;
            T coef = d / b;
            for (int j = m; j < n; j++)
                t[j] -= coef * x[j - m];
            if (2 * len > i)
                continue;
            len = i + 1 - len;
            x = tmp;
            b = d;
            m = 0;
        }
        t.resize(len + 1);
        t.erase(t.begin());
        for (auto& x : t)
            x = -x;
        pw[0] = vector<T>(sz(t) + 1), pw[0][1] = 1;
        for (i, 1, 20)
            pw[i] = combine(pw[i - 1], pw[i - 1]);
    }

    T operator[](lli k) {
        vector<T> ans(sz(t) + 1);
        ans[0] = 1;
        for (i, 0, 20)
            if (k & (1LL << i))
                ans = combine(ans, pw[i]);
        T val = 0;
        for (i, 0, sz(t))
            val += ans[i + 1] * s[i];
        return val;
    }
};
```

14.2 Lagrange $\mathcal{O}(n)$

```
template <class T>
struct Lagrange {
    int n;
    vector<T> y, suf, fac;

    Lagrange(vector<T>& y) : n(sz(y)), y(y), suf(n + 1, 1),
        fac(n, 1) {
```

```

    fore (i, 1, n)
        fac[i] = fac[i - 1] * i;
}

T operator[](lli k) {
    for (int i = n - 1; i >= 0; i--)
        suf[i] = suf[i + 1] * (k - i);

    T pref = 1, val = 0;
    fore (i, 0, n) {
        T num = pref * suf[i + 1];
        T den = fac[i] * fac[n - 1 - i];
        if ((n - 1 - i) % 2)
            den *= -1;
        val += y[i] * num / den;
        pref *= (k - i);
    }
    return val;
}
};

```

14.3 FFT

```

template <class Complex>
void FFT(vector<Complex>& a, bool inv = false) {
    const static double PI = acos(-1.0);
    static vector<Complex> root = {0, 1};
    int n = sz(a);
    for (int i = 1, j = 0; i < n - 1; i++) {
        for (int k = n >> 1; (j ^ k) < k; k >>= 1)
            ;
        if (i < j)
            swap(a[i], a[j]);
    }
    int k = sz(root);
    if (k < n)
        for (root.resize(n); k < n; k <= 1) {
            Complex z(cos(PI / k), sin(PI / k));
            fore (i, k >> 1, k) {
                root[i << 1] = root[i];
                root[i << 1 | 1] = root[i] * z;
            }
        }
    for (int k = 1; k < n; k <= 1)
        for (int i = 0; i < n; i += k << 1)
            fore (j, 0, k) {
                Complex t = a[i + j + k] * root[j + k];
                a[i + j + k] = a[i + j] - t;
                a[i + j] = a[i + j] + t;
            }
    if (inv) {
        reverse(1 + all(a));
        for (auto& x : a)
            x /= n;
    }
}

template <class T>
vector<T> convolution(const vector<T>& a, const vector<T>&
    b) {
    if (a.empty() || b.empty())
        return {};

    int n = sz(a) + sz(b) - 1, m = n;
    while (n != (n & -n))
        ++n;

    vector<complex<double>> fa(all(a)), fb(all(b));
    fa.resize(n), fb.resize(n);
    FFT(fa, false), FFT(fb, false);
    fore (i, 0, n)

```

```

        fa[i] *= fb[i];
    FFT(fa, true);

    vector<T> ans(m);
    fore (i, 0, m)
        ans[i] = round(real(fa[i]));
    return ans;
}

template <class T>
vector<T> convolutionTrick(const vector<T>& a,
    const vector<T>& b) { // 2 FFT's
    instead of 3!!

    if (a.empty() || b.empty())
        return {};

    int n = sz(a) + sz(b) - 1, m = n;
    while (n != (n & -n))
        ++n;

    vector<complex<double>> in(n), out(n);
    fore (i, 0, sz(a))
        in[i].real(a[i]);
    fore (i, 0, sz(b))
        in[i].imag(b[i]);

    FFT(in, false);
    for (auto& x : in)
        x *= x;
    fore (i, 0, n)
        out[i] = in[-i & (n - 1)] - conj(in[i]);
    FFT(out, false);

    vector<T> ans(m);
    fore (i, 0, m)
        ans[i] = round(imag(out[i]) / (4 * n));
    return ans;
}

```

14.4 Fast Walsh Hadamard Transform

```

template <char op, bool inv = false, class T>
vector<T> FWHT(vector<T> f) {
    int n = f.size();
    for (int k = 0; (n - 1) >> k; k++)
        for (int i = 0; i < n; i++)
            if (i >> k & 1) {
                int j = i ^ (1 << k);
                if (op == '^')
                    f[j] += f[i], f[i] = f[j] - 2 * f[i];
                if (op == '|')
                    f[i] += (inv ? -1 : 1) * f[j];
                if (op == '&')
                    f[j] += (inv ? -1 : 1) * f[i];
            }
    if (op == '^' && inv)
        for (auto& i : f)
            i /= n;
    return f;
}

```

14.5 Primitive root

```

int primitive(int p) {
    auto fpow = [&](lli x, int n) {
        lli r = 1;
        for (; n > 0; n >>= 1) {
            if (n & 1)
                r = r * x % p;
            x = x * x % p;
        }
        return r;
    };
}

```

```

for (int g = 2; g < p; g++) {
    bool can = true;
    for (int i = 2; i * i < p; i++)
        if ((p - 1) % i == 0) {
            if (fpow(g, i) == 1)
                can = false;
            if (fpow(g, (p - 1) / i) == 1)
                can = false;
        }
    if (can)
        return g;
}
return -1;
}

```

14.6 NTT

```

template <const int G, const int M>
void NTT(vector<Modular<M>>& a, bool inv = false) {
    static vector<Modular<M>> root = {0, 1};
    static Modular<M> primitive(G);
    int n = sz(a);
    for (int i = 1, j = 0; i < n - 1; i++) {
        for (int k = n >> 1; (j ^= k) < k; k >>= 1)
            ;
        if (i < j)
            swap(a[i], a[j]);
    }
    int k = sz(root);
    if (k < n)
        for (root.resize(n); k < n; k <= 1) {
            auto z = primitive.pow((M - 1) / (k <= 1));
            fore (i, k >> 1, k) {
                root[i <= 1] = root[i];
                root[i <= 1 | 1] = root[i] * z;
            }
        }
    for (int k = 1; k < n; k <= 1)
        for (int i = 0; i < n; i += k <= 1)
            fore (j, 0, k) {
                auto t = a[i + j + k] * root[j + k];
                a[i + j + k] = a[i + j] - t;
                a[i + j] = a[i + j] + t;
            }
    if (inv) {
        reverse(1 + all(a));
        auto invN = Modular<M>(1) / n;
        for (auto& x : a)
            x = x * invN;
    }
}

template <int G = 3, const int M = 998244353>
vector<Modular<M>> convolution(vector<Modular<M>> a, vector<Modular<M>> b) {
    // find G using primitive(M)
    // Common NTT couple (3, 998244353)
    if (a.empty() || b.empty())
        return {};

    int n = sz(a) + sz(b) - 1, m = n;
    while (n != (n & -n))
        ++n;
    a.resize(n, 0), b.resize(n, 0);

    NTT<G, M>(a), NTT<G, M>(b);
    fore (i, 0, n)
        a[i] = a[i] * b[i];
    NTT<G, M>(a, true);
}

```

```

return a;
}

```

15 Strings

15.1 KMP

```

template <class T>
vector<int> lps(T s) {
    vector<int> p(sz(s), 0);
    for (int j = 0, i = 1; i < sz(s); i++) {
        while (j && s[i] != s[j])
            j = p[j - 1];
        if (s[i] == s[j])
            j++;
        p[i] = j;
    }
    return p;
}

```

```

// positions where t is on s
template <class T>
vector<int> kmp(T& s, T& t) {
    vector<int> p = lps(t), pos;
    for (int j = 0, i = 0; i < sz(s); i++) {
        while (j && s[i] != t[j])
            j = p[j - 1];
        if (s[i] == t[j])
            j++;
        if (j == sz(t))
            pos.pb(i - sz(t) + 1);
    }
    return pos;
}

```

15.2 KMP automaton $\mathcal{O}(\text{Alphabet} * n)$

```

template <class T, int ALPHA = 26>
struct KmpAutomaton : vector<vector<int>> {
    KmpAutomaton() {}
    KmpAutomaton(T s) : vector<vector<int>>(sz(s) + 1, vector<int>(ALPHA)) {
        s.pb(0);
        vector<int> p = lps(s);
        auto& nxt = *this;
        nxt[0][s[0] - 'a'] = 1;
        fore (i, 1, sz(s))
            fore (c, 0, ALPHA)
                nxt[i][c] = (s[i] - 'a' == c ? i + 1 : nxt[p[i] - 1][c]);
    }
};

```

15.3 Z

```

template <class T>
vector<int> getZ(T& s) {
    vector<int> z(sz(s), 0);
    for (int i = 1, l = 0, r = 0; i < sz(s); i++) {
        if (i <= r)
            z[i] = min(r - i + 1, z[i - l]);
        while (i + z[i] < sz(s) && s[i + z[i]] == s[z[i]])
            ++z[i];
        if (i + z[i] - 1 > r)
            l = i, r = i + z[i] - 1;
    }
    return z;
}

```

15.4 Manacher

```

template <class T>
vector<vector<int>> manacher(T& s) {
    vector<vector<int>> pal(2, vector<int>(sz(s), 0));
    fore (k, 0, 2) {
        int l = 0, r = 0;

```



```

    fore (i, 0, sz(s)) {
        int t = r - i + !k;
        if (i < r)
            pal[k][i] = min(t, pal[k][l + t]);
        int p = i - pal[k][i], q = i + pal[k][i] - !k;
        while (p >= 1 && q + 1 < sz(s) && s[p - 1] == s[q + 1])
            ++pal[k][i], --p, ++q;
        if (q > r)
            l = p, r = q;
    }
}
return pal;
}

```

15.5 Hash

```

using Hash = int; // maybe an array<int, 2>
Hash pw[N], ipw[N];

struct Hashing {
    static constexpr int P = 10166249, M = 1070777777;
    vector<Hash> h;

    static void init() {
        const int Q = inv(P, M);
        pw[0] = ipw[0] = 1;
        fore (i, 1, N) {
            pw[i] = 1LL * pw[i - 1] * P % M;
            ipw[i] = 1LL * ipw[i - 1] * Q % M;
        }
    }

    Hashing(string& s) : h(sz(s) + 1, 0) {
        fore (i, 0, sz(s)) {
            lli x = s[i] - 'a' + 1;
            h[i + 1] = (h[i] + x * pw[i]) % M;
        }
    }

    Hash query(int l, int r) {
        return 1LL * (h[r + 1] - h[l] + M) * ipw[l] % M;
    }

    friend pair<Hash, int> merge(vector<pair<Hash, int>>&
        cuts) {
        pair<Hash, int> ans = {0, 0};
        fore (i, sz(cuts), 0) {
            ans.f = (cuts[i].f + 1LL * ans.f * pw[cuts[i].s] % M)
                % M;
            ans.s += cuts[i].s;
        }
        return ans;
    }
};

```

15.6 Min rotation

```

template <class T>
int minRotation(T& s) {
    int n = sz(s), i = 0, j = 1;
    while (i < n && j < n) {
        int k = 0;
        while (k < n && s[(i + k) % n] == s[(j + k) % n])
            k++;
        (s[(i + k) % n] <= s[(j + k) % n] ? j : i) += k + 1;
        j += i == j;
    }
    return i < n ? i : j;
}

```

15.7 Suffix array $\mathcal{O}(n \log n)$

- Duplicates $\sum_{i=1}^n lcp[i]$

- Longest Common Substring of various strings
Add *notUsed* characters between strings, i.e. $a + \$ + b + \# + c$
Use two-pointers to find a range $[l, r]$ such that all *notUsed* characters are present, then *query*($lcp[l + 1], \dots, lcp[r]$) for that window is the common length.

```

template <class T>
struct SuffixArray {
    int n;
    T s;
    vector<int> sa, pos, dp[25];

    SuffixArray(const T& x) : n(sz(x) + 1), s(x), sa(n), pos(
        n) {
        s.pb(0);
        fore (i, 0, n)
            sa[i] = i, pos[i] = s[i];
        vector<int> nsa(sa), npos(n), cnt(max(260, n), 0);
        for (int k = 0; k < n; k ? k *= 2 : k++) {
            fill(all(cnt), 0);
            fore (i, 0, n)
                nsa[i] = (sa[i] - k + n) % n, cnt[pos[i]]++;
            partial_sum(all(cnt), cnt.begin());
            for (int i = n - 1; i >= 0; i--)
                sa[--cnt[pos[nsa[i]]]] = nsa[i];
            for (int i = 1, cur = 0; i < n; i++) {
                cur += (pos[sa[i]] != pos[sa[i - 1]] || pos[sa[i]
                    + k] % n != pos[sa[i - 1] + k] % n);
                npos[sa[i]] = cur;
            }
            pos = npos;
            if (pos[sa[n - 1]] >= n - 1)
                break;
        }
        dp[0].assign(n, 0);
        for (int i = 0, j = pos[0], k = 0; i < n - 1; ++i, ++k)
            while (k >= 0 && s[i] != s[sa[j - 1] + k])
                dp[0][j] = k--, j = pos[sa[j] + 1];
        for (int k = 1, pw = 1; pw < n; k++, pw <= 1) {
            dp[k].assign(n, 0);
            for (int l = 0; l + pw < n; l++)
                dp[k][l] = min(dp[k - 1][l], dp[k - 1][l + pw]);
        }
    }

    int lcp(int l, int r) {
        if (l == r)
            return n - l;
        tie(l, r) = minmax(pos[l], pos[r]);
        int k = __lg(r - l);
        return min(dp[k][l + 1], dp[k][r - (1 << k) + 1]);
    }

    auto at(int i, int j) {
        return sa[i] + j < n ? s[sa[i] + j] : 'z' + 1;
    }

    int count(T& t) {
        int l = 0, r = n - 1;
        fore (i, 0, sz(t)) {
            int p = l, q = r;
            for (int k = n; k > 0; k >= 1) {
                while (p + k < r && at(p + k, i) < t[i])
                    p += k;
                while (q - k > l && t[i] < at(q - k, i))

```

```

        q -= k;
    }
    l = (at(p, i) == t[i] ? p : p + 1);
    r = (at(q, i) == t[i] ? q : q - 1);
    if (at(l, i) != t[i] && at(r, i) != t[i] || l > r)
        return 0;
    }
    return r - l + 1;
}

bool compare(ii a, ii b) {
    // s[a.f ... a.s] < s[b.f ... b.s]
    int common = lcp(a.f, b.f);
    int szA = a.s - a.f + 1, szB = b.s - b.f + 1;
    if (common >= min(szA, szB))
        return tie(szA, a) < tie(szB, b);
    return s[a.f + common] < s[b.f + common];
}
};

```

15.8 Aho Corasick $\mathcal{O}(\sum s_i)$

```

struct AhoCorasick {
    struct Node : map<char, int> {
        int link = 0, up = 0;
        int cnt = 0, isw = 0;
    };

    vector<Node> trie;

    AhoCorasick(int n = 1) {
        trie.reserve(n), newNode();
    }

    int newNode() {
        trie.pb({});
        return sz(trie) - 1;
    }

    void insert(string& s, int u = 0) {
        for (char c : s) {
            if (!trie[u][c])
                trie[u][c] = newNode();
            u = trie[u][c];
        }
        trie[u].cnt++, trie[u].isw = 1;
    }

    int next(int u, char c) {
        while (u && !trie[u].count(c))
            u = trie[u].link;
        return trie[u][c];
    }

    void pushLinks() {
        queue<int> qu;
        qu.push(0);
        while (!qu.empty()) {
            int u = qu.front();
            qu.pop();
            for (auto& [c, v] : trie[u]) {
                int l = (trie[v].link = u ? next(trie[u].link, c) : 0);
                trie[v].cnt += trie[l].cnt;
                trie[v].up = trie[l].isw ? l : trie[l].up;
                qu.push(v);
            }
        }
    }
};

```

```

template <class F>
void goUp(int u, F f) {
    for (; u != 0; u = trie[u].up)
        f(u);
}

int match(string& s, int u = 0) {
    int ans = 0;
    for (char c : s) {
        u = next(u, c);
        ans += trie[u].cnt;
    }
    return ans;
}

Node& operator[](int u) {
    return trie[u];
}
};

```

15.9 Eertree $\mathcal{O}(\sum s_i)$

```

struct Eertree {
    struct Node : map<char, int> {
        int link = 0, len = 0;
    };

    vector<Node> trie;
    string s = "$";
    int last;

    Eertree(int n = 1) {
        trie.reserve(n), last = newNode(), newNode();
        trie[0].link = 1, trie[1].len = -1;
    }

    int newNode() {
        trie.pb({});
        return sz(trie) - 1;
    }

    int next(int u) {
        while (s[sz(s) - trie[u].len - 2] != s.back())
            u = trie[u].link;
        return u;
    }

    void extend(char c) {
        s.push_back(c);
        last = next(last);
        if (!trie[last][c]) {
            int v = newNode();
            trie[v].len = trie[last].len + 2;
            trie[v].link = trie[next(trie[last].link)][c];
            trie[last][c] = v;
        }
        last = trie[last][c];
    }

    Node& operator[](int u) {
        return trie[u];
    }

    void substringOccurrences() {
        for (u, sz(s), 0)
            trie[trie[u].link].occ += trie[u].occ;
    }

    lli occurrences(string& s, int u = 0) {
        for (char c : s) {
            if (!trie[u].count(c))

```

```

    return 0;
    u = trie[u][c];
}
return trie[u].occ;
}
};

```

15.10 Suffix automaton $\mathcal{O}(\sum s_i)$

- $sam[u].len - sam[sam[u].link].len = \text{distinct strings}$
- Number of different substrings (dp) $\mathcal{O}(\sum s_i)$

$$diff(u) = 1 + \sum_{v \in trie[u]} diff(v)$$

- Total length of all different substrings (2 x dp)

$$totLen(u) = \sum_{v \in trie[u]} diff(v) + totLen(v)$$

- Leftmost occurrence $\mathcal{O}(|s|)$ $trie[u].pos = trie[u].len - 1$ if it is **clone** then $trie[clone].pos = trie[q].pos$
- All occurrence positions
- Smallest cyclic shift $\mathcal{O}(|2 * s|)$ Construct sam of $s + s$, find the lexicographically smallest path of $sz(s)$
- Shortest non-appearing string $\mathcal{O}(|s|)$

$$nonAppearing(u) = \min_{v \in trie[u]} nonAppearing(v) + 1$$

```

struct SuffixAutomaton {
    struct Node : map<char, int> {
        int link = -1, len = 0;
    };

    vector<Node> trie;
    int last;

    SuffixAutomaton(int n = 1) {
        trie.reserve(2 * n), last = newNode();
    }

    int newNode() {
        trie.pb({});
        return sz(trie) - 1;
    }

    void extend(char c) {
        int u = newNode();
        trie[u].len = trie[last].len + 1;
        int p = last;
        while (p != -1 && !trie[p].count(c)) {
            trie[p][c] = u;
            p = trie[p].link;
        }
        if (p == -1)
            trie[u].link = 0;
        else {
            int q = trie[p][c];
            if (trie[p].len + 1 == trie[q].len)
                trie[u].link = q;
            else {
                int clone = newNode();
                trie[clone] = trie[q];
                trie[clone].len = trie[p].len + 1;
                while (p != -1 && trie[p][c] == q) {
                    trie[p][c] = clone;
                    p = trie[p].link;
                }
                trie[q].link = trie[u].link = clone;
            }
        }
    }
}

```

```

last = u;
}

string kthSubstring(lli kth, int u = 0) {
    // number of different substrings (dp)
    string s = "";
    while (kth > 0)
        for (auto& [c, v] : trie[u]) {
            if (kth <= diff(v)) {
                s.pb(c), kth--, u = v;
                break;
            }
            kth -= diff(v);
        }
    return s;
}

void substringOccurrences() {
    // trie[u].occ = 1, trie[clone].occ = 0
    vector<int> who(sz(trie) - 1);
    iota(all(who), 1);
    sort(all(who), [&](int u, int v) {
        return trie[u].len > trie[v].len;
    });
    for (int u : who) {
        int l = trie[u].link;
        trie[l].occ += trie[u].occ;
    }
}

lli occurrences(string& s, int u = 0) {
    for (char c : s) {
        if (!trie[u].count(c))
            return 0;
        u = trie[u][c];
    }
    return trie[u].occ;
}

int longestCommonSubstring(string& s, int u = 0) {
    int mx = 0, len = 0;
    for (char c : s) {
        while (u && !trie[u].count(c)) {
            u = trie[u].link;
            len = trie[u].len;
        }
        if (trie[u].count(c))
            u = trie[u][c], len++;
        mx = max(mx, len);
    }
    return mx;
}

string smallestCyclicShift(int n, int u = 0) {
    string s = "";
    for (i, 0, n) {
        char c = trie[u].begin()->f;
        s += c;
        u = trie[u][c];
    }
    return s;
}

int leftmost(string& s, int u = 0) {
    for (char c : s) {
        if (!trie[u].count(c))
            return -1;
        u = trie[u][c];
    }
    return trie[u].pos - sz(s) + 1;
}

```

```
}  
  
Node& operator[](int u) {  
    return trie[u];  
}  
};
```




The end...