



Universidad de Guadalajara,  
CUCEI

## The Empire Strikes Back

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## Think twice, code once

### Template

```
tem.cpp
#pragma GCC optimize("Ofast,unroll-loops,no-stack-
    protector")
#include <bits/stdc++.h>
using namespace std;

#define fore(i, l, r) for (auto i = (l) - ((l) > (r)); i
    != (r) - ((l) > (r)); i += 1 - 2 * ((l) > (r)))
#define sz(x) int(x.size())
#define all(x) begin(x), end(x)
#define f first
#define s second
#define pb push_back

using ld = long double;
using lli = long long;
using ii = pair<int, int>;
using vi = vector<int>;

#ifdef LOCAL
#include "debug.h"
#else
#define debug(...)
#endif

int main() {
    cin.tie(0)->sync_with_stdio(0), cout.tie(0);
    // solve the problem here D:
    return 0;
}

debug.h
template <class A, class B>
ostream & operator << (ostream &os, const pair<A, B> &p)
{
    return os << "(" << p.first << ", " << p.second << ")"
        ;
}

template <class A, class B, class C>
basic_ostream<A, B> & operator << (basic_ostream<A, B> &
    os, const C &c) {
    os << "[";
    for (const auto &x : c)
        os << ", " + 2 * (&x == &begin(c)) << x;
    return os << "]"
        ;
}

void print(string s) { cout << endl; }

template <class H, class... T>
void print(string s, const H &h, const T&... t) {
    const static string reset = "\033[0m", blue = "\033[1;
        34m", purple = "\033[3;95m";
    bool ok = 1;
    do {
        if (s[0] == '\0') ok = 0;
        else cout << blue << s[0] << reset;
        s = s.substr(1);
    } while (s.size() && s[0] != ',');
    if (ok) cout << ": " << purple << h << reset;
    print(s, t...);
}

Randoms
mt19937 rng(chrono::steady_clock::now().time_since_epoch
    ().count());
template <class T>
T ran(T l, T r) {
    return uniform_int_distribution<T>(l, r)(rng);
}

Compilation (gedit ~/.zshenv)
touch a_in{1..9} // make files a_in1, a_in2,..., a_in9
tee {a..m}.cpp < tem.cpp // "" with tem.cpp like base
cat > a_in1 // write on file a_in1
gedit a_in1 // open file a_in1
rm -r a.cpp // deletes file a.cpp :(

red='\x1B[0;31m'
green='\x1B[0;32m'
noColor='\x1B[0m'
alias flags='-Wall -Wextra -Wshadow -
    _D_GLIBCXX_ASSERTIONS -fmax-errors=3 -O2 -w'
go() { g++ --std=c++11 $2 ${flags} $1.cpp && ./a.out }
debug() { go $1 -DLOCAL < $2 }
run() { go $1 "" < $2 }

random() { // Make small test cases!!!
    g++ --std=c++11 $1.cpp -o prog
    g++ --std=c++11 gen.cpp -o gen
    g++ --std=c++11 brute.cpp -o brute
    for ((i = 1; i <= 200; i++)); do
        printf "Test case #i"
        ./gen > in
        diff -uwi <(. /prog < in) <(. /brute < in) > $1_diff
        if [[ ! $? -eq 0 ]]; then
            printf "${red} Wrong answer ${noColor}\n"
            break
        else
            printf "${green} Accepted ${noColor}\n"
        fi
    done
}

Bump allocator
static char buf[450 << 20];
void* operator new(size_t s) {
    static size_t i = sizeof buf; assert(s < i);
    return (void *) &buf[i -= s];
}
void operator delete(void *) {}

1 Data structures
1.1 Disjoint set with rollback
struct Dsu {
    vi par, tot;
    stack<ii> mem;

    Dsu(int n = 1) : par(n + 1), tot(n + 1, 1) {
        iota(all(par), 0);
    }

    int find(int u) {
```

```

    bool ok = 1;
    do {
        if (s[0] == '\0') ok = 0;
        else cout << blue << s[0] << reset;
        s = s.substr(1);
    } while (s.size() && s[0] != ',');
    if (ok) cout << ": " << purple << h << reset;
    print(s, t...);
}

Randoms
mt19937 rng(chrono::steady_clock::now().time_since_epoch
    ().count());
template <class T>
T ran(T l, T r) {
    return uniform_int_distribution<T>(l, r)(rng);
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    g++ --std=c++11 $1.cpp -o prog
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    g++ --std=c++11 brute.cpp -o brute
    for ((i = 1; i <= 200; i++)); do
        printf "Test case #i"
        ./gen > in
        diff -uwi <(. /prog < in) <(. /brute < in) > $1_diff
        if [[ ! $? -eq 0 ]]; then
            printf "${red} Wrong answer ${noColor}\n"
            break
        else
            printf "${green} Accepted ${noColor}\n"
        fi
    done
}

Bump allocator
static char buf[450 << 20];
void* operator new(size_t s) {
    static size_t i = sizeof buf; assert(s < i);
    return (void *) &buf[i -= s];
}
void operator delete(void *) {}

1 Data structures
1.1 Disjoint set with rollback
struct Dsu {
    vi par, tot;
    stack<ii> mem;

    Dsu(int n = 1) : par(n + 1), tot(n + 1, 1) {
        iota(all(par), 0);
    }

    int find(int u) {
```

```

    return par[u] == u ? u : find(par[u]);
}

void unite(int u, int v) {
    u = find(u), v = find(v);
    if (u != v) {
        if (tot[u] < tot[v])
            swap(u, v);
        mem.emplace(u, v);
        tot[u] += tot[v];
        par[v] = u;
    }
}

void rollback() {
    auto [u, v] = mem.top();
    mem.pop();
    if (u != -1) {
        tot[u] -= tot[v];
        par[v] = v;
    }
}
};

```

## 1.2 Monotone queue

```

template <class T>
struct MonotoneQueue {
    deque<pair<T, int>> q;

    void add(T val, int pos) {
        while (!q.empty() && q.back().f >= val) q.pop_back();
        q.emplace_back(val, pos);
    }

    void remove(int pos) {
        while (q.front().s < pos) q.pop_front();
    }

    T query() {
        return q.front().f;
    }
};

```

## 1.3 Stack queue

```

template <class T, class F = function<T(const T&, const T&)>>
struct Stack : vector<T> {
    vector<T> s;
    F f;

    Stack(const F &f) : f(f) {}

    void push(T x) {
        this->pb(x);
        s.pb(s.empty() ? x : f(s.back(), x));
    }

    T pop() {
        T x = this->back();
        this->pop_back();
        s.pop_back();
        return x;
    }

    T query() {
        return s.back();
    }
};

```

```

template <class T, class F = function<T(const T&, const T&)>>
struct Queue {
    Stack<T> a, b;
    F f;

    Queue(const F &f) : a(f), b(f), f(f) {}

    void push(T x) {
        b.push(x);
    }

    void pop() {
        if (a.empty())
            while (!b.empty())
                a.push(b.pop());
        a.pop();
    }

    T query() {
        if (a.empty()) return b.query();
        if (b.empty()) return a.query();
        return f(a.query(), b.query());
    }
};

```

## 1.4 Mo's algorithm

```

// N = 1e6, so aprox. sqrt(N) +/- C
const int blo = sqrt(N);
sort(all(queries), [&] (Query &a, Query &b) {
    const int ga = a.l / blo, gb = b.l / blo;
    if (ga == gb) return a.r < b.r;
    return a.l < b.l;
});

int l = queries[0].l, r = l - 1;
for (Query &q : queries) {
    while (r < q.r) add(++r);
    while (r > q.r) rem(--r);
    while (l < q.l) rem(--l);
    while (l > q.l) add(--l);
    ans[q.i] = solve();
}

```

To make it faster, change the order to *hilbert(l, r)*

```

lli hilbert(int x, int y, int pw = 21, int rot = 0) {
    if (pw == 0)
        return 0;
    int hpw = 1 << (pw - 1);
    int k = ((x < hpw ? y < hpw ? 0 : 3 : y < hpw ? 1 : 2)
        + rot) & 3;
    const int d[4] = {3, 0, 0, 1};
    lli a = 1LL << ((pw < 1) - 2);
    lli b = hilbert(x & (x ^ hpw), y & (y ^ hpw), pw - 1,
        (rot + d[k]) & 3);
    return k * a + (d[k] ? a - b - 1 : b);
}

```

## 1.5 Static to dynamic

```

template <class Black, class T>
struct StaticDynamic {
    Black box[LogN];
    vector<T> st[LogN];

    void insert(T &x) {
        int p = 0;
        while (p < LogN && !st[p].empty())
            p++;
        st[p].pb(x);
        for (i, 0, p) {
            st[p].insert(st[p].end(), all(st[i]));
        }
    }
};

```

```

    box[i].clear(), st[i].clear();
}
for (auto y : st[p])
    box[p].insert(y);
box[p].init();
}
};

```

## 2 Intervals

### 2.1 Disjoint intervals

```

struct Interval {
    int l, r;
    bool operator < (const Interval &it) const {
        return l < it.l;
    }
};

struct DisjointIntervals : set<Interval> {
    void add(int l, int r) {
        auto it = lower_bound({l, -1});
        if (it != begin() && l <= prev(it)->r)
            l = (--it)->l;
        for (; it != end() && it->l <= r; erase(it++))
            r = max(r, it->r);
        insert({l, r});
    }

    void rem(int l, int r) {
        auto it = lower_bound({l, -1});
        if (it != begin() && l <= prev(it)->r)
            --it;
        int mn = l, mx = r;
        for (; it != end() && it->l <= r; erase(it++))
            mn = min(mn, it->l), mx = max(mx, it->r);
        if (mn < l) insert({mn, l - 1});
        if (r < mx) insert({r + 1, mx});
    }
};

```

### 2.2 Interval tree

```

struct Interval {
    lli l, r, i;
};

struct ITree {
    ITree *left, *right;
    vector<Interval> cur;
    lli m;

    ITree(vector<Interval> &vec, lli l = LLONG_MAX, lli r = LLONG_MIN) : left(0), right(0) {
        if (l > r) { // not sorted yet
            sort(all(vec), [&](Interval a, Interval b) {
                return a.l < b.l;
            });
            for (auto it : vec)
                l = min(l, it.l), r = max(r, it.r);
        }
        m = (l + r) >> 1;
        vector<Interval> lo, hi;
        for (auto it : vec)
            (it.r < m ? lo : m < it.l ? hi : cur).pb(it);
        if (!lo.empty())
            left = new ITree(lo, l, m);
        if (!hi.empty())
            right = new ITree(hi, m + 1, r);
    }

    template <class F>
    void near(lli l, lli r, F f) {

```

```

        if (!cur.empty() && !(r < cur.front().l)) {
            for (auto &it : cur)
                f(it);
        }
        if (left && l <= m)
            left->near(l, r, f);
        if (right && m < r)
            right->near(l, r, f);
    }
};

```

```

template <class F>
void overlapping(lli l, lli r, F f) {
    near(l, r, [&](Interval it) {
        if (l <= it.r && it.l <= r)
            f(it);
    });
}

```

```

template <class F>
void contained(lli l, lli r, F f) {
    near(l, r, [&](Interval it) {
        if (l <= it.l && it.r <= r)
            f(it);
    });
}
};

```

## 3 Static range queries

### 3.1 Sparse table

```

template <class T, class F = function<T(const T&, const T&>>>
struct Sparse {
    int n;
    vector<vector<T>>> sp;
    F f;

    Sparse(vector<T> &a, const F &f) : n(sz(a)), sp(1 + __lg(n)), f(f) {
        sp[0] = a;
        for (int k = 1; (1 << k) <= n; k++) {
            sp[k].resize(n - (1 << k) + 1);
            for (int i = 0; i + (1 << k) <= n; i++) {
                sp[k][i] = f(sp[k - 1][i], sp[k - 1][i + (1 << k)]);
            }
        }
    }

    T query(int l, int r) {
        int k = __lg(r - l + 1);
        return f(sp[k][l], sp[k][r - (1 << k) + 1]);
    }
};

```

### 3.2 Sqrtle decomposition

The perfect block size is *sqrtle* of N



```

int blo[N], cnt[N][B], a[N];

void update(int i, int x) {
    cnt[blo[i]][x]--;
    a[i] = x;
    cnt[blo[i]][x]++;
}

int query(int l, int r, int x) {
    int tot = 0;
    while (l <= r)
        if (l % B == 0 && l + B - 1 <= r) {

```

```

        tot += cnt[blo[l]][x];
        l += B;
    } else {
        tot += (a[l] == x);
        l++;
    }
}
return tot;
}

```

### 3.3 Parallel binary search

```

int lo[Q], hi[Q];
queue<int> solve[N];
vector<Query> queries;

fore (it, 0, 1 + __lg(N)) {
    fore (i, 0, sz(queries))
        if (lo[i] != hi[i]) {
            int mid = (lo[i] + hi[i]) / 2;
            solve[mid].emplace(i);
        }
    fore (x, 0, n) {
        // simulate
        while (!solve[x].empty()) {
            int i = solve[x].front();
            solve[x].pop();
            if (can(queries[i]))
                hi[i] = x;
            else
                lo[i] = x + 1;
        }
    }
}

```

## 4 Dynamic range queries

### 4.1 D-dimensional Fenwick tree

```

template <class T, int ...N>
struct Fenwick {
    T v = 0;
    void update(T v) { this->v += v; }
    T query() { return v; }
};

template <class T, int N, int ...M>
struct Fenwick<T, N, M...> {
    #define lsb(x) (x & -x)
    Fenwick<T, M...> fenw[N + 1];

    template <typename... Args>
    void update(int i, Args... args) {
        for (; i <= N; i += lsb(i))
            fenw[i].update(args...);
    }

    template <typename... Args>
    T query(int l, int r, Args... args) {
        T v = 0;
        for (; r > 0; r -= lsb(r))
            v += fenw[r].query(args...);
        for (--l; l > 0; l -= lsb(l))
            v -= fenw[l].query(args...);
        return v;
    }
};

```

### 4.2 Dynamic segment tree

```

struct Dyn {
    int l, r;
    Dyn *left, *right;
    lli sum = 0;

    Dyn(int l, int r) : l(l), r(r), left(0), right(0) {}
}

```

```

void pull() {
    sum = (left ? left->sum : 0);
    sum += (right ? right->sum : 0);
}

void update(int p, lli v) {
    if (l == r) {
        sum += v;
        return;
    }
    int m = (l + r) >> 1;
    if (p <= m) {
        if (!left) left = new Dyn(l, m);
        left->update(p, v);
    } else {
        if (!right) right = new Dyn(m + 1, r);
        right->update(p, v);
    }
    pull();
}

lli query(int ll, int rr) {
    if (rr < l || r < ll || r < l)
        return 0;
    if (ll <= l && r <= rr)
        return sum;
    int m = (l + r) >> 1;
    return (left ? left->query(ll, rr) : 0) +
           (right ? right->query(ll, rr) : 0);
}
};

```

### 4.3 Persistent segment tree

```

struct Per {
    int l, r;
    Per *left, *right;
    lli sum = 0;

    Per(int l, int r) : l(l), r(r), left(0), right(0) {}

    Per* pull() {
        sum = left->sum + right->sum;
        return this;
    }

    void build() {
        if (l == r)
            return;
        int m = (l + r) >> 1;
        (left = new Per(l, m))->build();
        (right = new Per(m + 1, r))->build();
        pull();
    }

    Per* update(int p, lli v) {
        if (p < l || r < p)
            return this;
        Per* t = new Per(l, r);
        if (l == r) {
            t->sum = v;
            return t;
        }
        t->left = left->update(p, v);
        t->right = right->update(p, v);
        return t->pull();
    }
}

```

```

lli query(int ll, int rr) {
    if (r < ll || rr < l)

```

```

    return 0;
    if (l1 <= l && r <= rr)
        return sum;
    return left->query(l1, rr) + right->query(l1, rr);
}
};

```

## 4.4 Wavelet tree

```

struct Wav {
    #define iter int* // vector<int>::iterator
    int lo, hi;
    Wav *left, *right;
    vector<int> amt;

    Wav(int lo, int hi, iter b, iter e) : lo(lo), hi(hi) {
        // array 1-indexed
        if (lo == hi || b == e)
            return;
        amt.reserve(e - b + 1);
        amt.pb(0);
        int m = (lo + hi) >> 1;
        for (auto it = b; it != e; it++)
            amt.pb(amt.back() + (*it <= m));
        auto p = stable_partition(b, e, [&](int x) {
            return x <= m;
        });
        left = new Wav(lo, m, b, p);
        right = new Wav(m + 1, hi, p, e);
    }

    int kth(int l, int r, int k) {
        if (r < l)
            return 0;
        if (lo == hi)
            return lo;
        if (k <= amt[r] - amt[l - 1])
            return left->kth(amt[l - 1] + 1, amt[r], k);
        return right->kth(l - amt[l - 1], r - amt[r], k -
            amt[r] + amt[l - 1]);
    }

    int leq(int l, int r, int mx) {
        if (r < l || mx < lo)
            return 0;
        if (hi <= mx)
            return r - l + 1;
        return left->leq(amt[l - 1] + 1, amt[r], mx) +
            right->leq(l - amt[l - 1], r - amt[r], mx);
    }
};

```

## 4.5 Li Chao tree

```

struct Fun {
    lli m = 0, c = inf;
    lli operator()(lli x) const { return m * x + c; }
};

struct LiChao {
    lli l, r;
    LiChao *left, *right;
    Fun f;

    LiChao(lli l, lli r) : l(l), r(r), left(0), right(0) {}

    void add(Fun &g) {
        if (f(l) <= g(l) && f(r) <= g(r))
            return;
        if (g(l) < f(l) && g(r) < f(r)) {
            f = g;
            return;
        }
    }
};

```

```

}
lli m = (l + r) >> 1;
if (g(m) < f(m)) swap(f, g);
if (g(l) <= f(l))
    left = left ? (left->add(g), left) : new LiChao(l,
        m, g);
else
    right = right ? (right->add(g), right) : new LiChao
        (m + 1, r, g);
}

lli query(lli x) {
    if (l == r)
        return f(x);
    lli m = (l + r) >> 1;
    if (x <= m)
        return min(f(x), left ? left->query(x) : inf);
    return min(f(x), right ? right->query(x) : inf);
}
};

```

## 5 Binary trees

### 5.1 Ordered tree

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template <class K, class V = null_type>
using ordered_tree = tree<K, V, less<K>, rb_tree_tag,
    tree_order_statistics_node_update>;
#define rank order_of_key
#define kth find_by_order

```

### 5.2 Unordered tree

```

struct CustomHash {
    const uint64_t C = uint64_t(2e18 * 3) + 71;
    const int R = rng();
    uint64_t operator()(uint64_t x) const {
        return __builtin_bswap64((x ^ R) * C);
    }

    template <class K, class V = null_type>
    using unordered_tree = gp_hash_table<K, V, CustomHash>;

```

### 5.3 Treap

```

typedef struct Node* Treap;
struct Node {
    Treap left = 0, right = 0;
    unsigned pri = rng();
    int val, sz = 1;
    // define more variables here

    void push() {
        // propagate like segtree, values aren't modified!!
    }

    Treap pull() {
        #define sz(t) (t ? t->sz : 0)
        sz = 1 + sz(left) + sz(right);
        // merge(l, this), merge(this, r)
        return this;
    }

    Node(int val) : val(val) {
        pull();
    }
};

template <class F>
pair<Treap, Treap> split(Treap t, const F &leq) { // {<=

```

```

    val, > val}
if (!t) return {t, t};
t->push();
if (leq(t)) {
    auto p = split(t->right, leq);
    t->right = p.f;
    return {t->pull(), p.s};
} else {
    auto p = split(t->left, leq);
    t->left = p.s;
    return {p.f, t->pull()};
}
}

Treap merge(Treap l, Treap r) {
    if (!l || !r) return l ? l : r;
    l->push(), r->push();
    if (l->pri > r->pri)
        return l->right = merge(l->right, r), l->pull();
    else
        return r->left = merge(l, r->left), r->pull();
}

```

## 5.4 Implicit treap (Rope)

```

pair<Treap, Treap> leftmost(Treap t, int k) {
    return split(t, [&](Treap t) {
        int sz = sz(t->left) + 1;
        if (k >= sz) {
            k -= sz;
            return true;
        }
        return false;
    });
}

int pos(Treap t) { // add parent in Node definition
    int sz = sz(t->left);
    for (; t->par; t = t->par) {
        Treap p = t->par;
        if (p->right == t) sz += sz(p->left) + 1;
    }
    return sz + 1;
}

```

# 6 Graphs

## 6.1 Topological sort

```

vi order;
int indeg[N];

void topsort() { // first fill the indeg[]
    queue<int> qu;
    for (u, 1, n + 1)
        if (indeg[u] == 0)
            qu.push(u);
    while (!qu.empty()) {
        int u = qu.front();
        qu.pop();
        order.pb(u);
        for (int v : graph[u])
            if (--indeg[v] == 0)
                qu.push(v);
    }
}

```

## 6.2 Tarjan algorithm (SCC)

```

int tin[N], fup[N];
bitset<N> still;
stack<int> stk;
int timer = 0;

```

```

void tarjan(int u) {
    tin[u] = fup[u] = ++timer;
    still[u] = true;
    stk.push(u);
    for (int v : graph[u]) {
        if (!tin[v])
            tarjan(v);
        if (still[v])
            fup[u] = min(fup[u], fup[v]);
    }
    if (fup[u] == tin[u]) {
        int v;
        do {
            v = stk.top();
            stk.pop();
            still[v] = false;
            // u and v are in the same scc
        } while (v != u);
    }
}

```

## 6.3 Kosaraju algorithm (SCC)

```

int scc[N], k = 0;
char vis[N];
vi order;

void dfs1(int u) {
    vis[u] = 1;
    for (int v : graph[u])
        if (vis[v] != 1)
            dfs1(v);
    order.pb(u);
}

void dfs2(int u, int k) {
    vis[u] = 2, scc[u] = k;
    for (int v : rgraph[u]) // reverse graph
        if (vis[v] != 2)
            dfs2(v, k);
}

void kosaraju() {
    for (u, 1, n + 1)
        if (vis[u] != 1)
            dfs1(u);
    reverse(all(order));
    for (int u : order)
        if (vis[u] != 2)
            dfs2(u, ++k);
}

```

## 6.4 Cutpoints and Bridges

```

int tin[N], fup[N], timer = 0;

void findWeakness(int u, int p = 0) {
    tin[u] = fup[u] = ++timer;
    int children = 0;
    for (int v : graph[u]) if (v != p) {
        if (!tin[v]) {
            ++children;
            findWeakness(v, u);
            fup[u] = min(fup[u], fup[v]);
            if (fup[v] >= tin[u] && p) // u is a cutpoint
                if (fup[v] > tin[u]) // bridge u -> v
                    ;
        }
        fup[u] = min(fup[u], tin[v]);
    }
    if (!p && children > 1) // u is a cutpoint
        ;
}

```

## 6.5 Two Sat



```

struct TwoSat {
    int n;
    vector<vi> imp;

    TwoSat(int _n) : n(_n + 1), imp(2 * n) {}

    void either(int a, int b) {
        a = max(2 * a, -1 - 2 * a);
        b = max(2 * b, -1 - 2 * b);
        imp[a ^ 1].pb(b);
        imp[b ^ 1].pb(a);
    }

    void implies(int a, int b) { either(~a, b); }
    void setVal(int a) { either(a, a); }

    vi solve() {
        int k = sz(imp);
        vi s, b, id(sz(imp));

        function<void(int)> dfs = [&](int u) {
            b.pb(id[u] = sz(s));
            s.pb(u);
            for (int v : imp[u]) {
                if (!id[v]) dfs(v);
                else while (id[v] < b.back()) b.pop_back();
            }
            if (id[u] == b.back())
                for (b.pop_back(), ++k; id[u] < sz(s); s.
                    pop_back())
                    id[s.back()] = k;
        };

        fore (u, 0, sz(imp))
            if (!id[u]) dfs(u);

        vi val(n);
        fore (u, 0, n) {
            int x = 2 * u;
            if (id[x] == id[x ^ 1])
                return {};
            val[u] = id[x] < id[x ^ 1];
        }
        return val;
    }
};

```

## 6.6 Detect a cycle

```

bool cycle(int u) {
    vis[u] = 1;
    for (int v : graph[u]) {
        if (vis[v] == 1)
            return true;
        if (!vis[v] && cycle(v))
            return true;
    }
    vis[u] = 2;
    return false;
}

```

## 6.7 Euler tour for Mo's in a tree

Mo's in a tree, extended euler tour  $tin[u] = ++timer$ ,  $tout[u] = ++timer$

- $u = lca(u, v)$ ,  $query(tin[u], tin[v])$
- $u \neq lca(u, v)$ ,  $query(tout[u], tin[v]) + query(tin[lca], tin[lca])$

## 6.8 Isomorphism

```

lli f(lli x) {
    // K * n <= 9e18
    static uniform_int_distribution<lli> uid(1, K);
    if (!mp.count(x))

```

```

        mp[x] = uid(rng);
        return mp[x];
    }

    lli hsh(int u, int p = 0) {
        dp[u] = h[u] = 0;
        for (int v : graph[u]) {
            if (v == p)
                continue;
            dp[u] += hsh(v, u);
        }
        return h[u] = f(dp[u]);
    }
}

```

## 6.9 Dynamic connectivity

```

struct DynamicConnectivity {
    struct Query {
        int op, u, v, at;
    };

    Dsu dsu; // with rollback
    vector<Query> queries;
    map<ii, int> mp;
    int timer = -1;

    DynamicConnectivity(int n = 0) : dsu(n) {}

    void add(int u, int v) {
        mp[minmax(u, v)] = ++timer;
        queries.pb({'+', u, v, INT_MAX});
    }

    void rem(int u, int v) {
        int in = mp[minmax(u, v)];
        queries.pb({'-', u, v, in});
        queries[in].at = ++timer;
        mp.erase(minmax(u, v));
    }

    void query() {
        queries.push_back({'?', -1, -1, ++timer});
    }

    void solve(int l, int r) {
        if (l == r) {
            if (queries[l].op == '?') // solve the query here
                return;
        }
        int m = (l + r) >> 1;
        int before = sz(dsu.mem);
        for (int i = m + 1; i <= r; i++) {
            Query &q = queries[i];
            if (q.op == '-' && q.at < l)
                dsu.unite(q.u, q.v);
        }
        solve(l, m);
        while (sz(dsu.mem) > before)
            dsu.rollback();
        for (int i = l; i <= m; i++) {
            Query &q = queries[i];
            if (q.op == '+' && q.at > r)
                dsu.unite(q.u, q.v);
        }
        solve(m + 1, r);
        while (sz(dsu.mem) > before)
            dsu.rollback();
    }
};

```

## 7 Tree queries

### 7.1 Lowest common ancestor (LCA)

```
const int LogN = 1 + __lg(N);
int par[LogN][N], dep[N];

void dfs(int u, int par[]) {
    for (int v : graph[u])
        if (v != par[u]) {
            par[v] = u;
            dep[v] = dep[u] + 1;
            dfs(v, par);
        }
}

int lca(int u, int v) {
    if (dep[u] > dep[v])
        swap(u, v);
    for (k, LogN, 0)
        if (dep[v] - dep[u] >= (1 << k))
            v = par[k][v];
    if (u == v)
        return u;
    for (k, LogN, 0)
        if (par[k][v] != par[k][u])
            u = par[k][u], v = par[k][v];
    return par[0][u];
}

int dist(int u, int v) {
    return dep[u] + dep[v] - 2 * dep[lca(u, v)];
}

void init(int r) {
    dfs(r, par[0]);
    for (k, 1, LogN)
        for (u, 1, n + 1)
            par[k][u] = par[k - 1][par[k - 1][u]];
}
```

### 7.2 Virtual tree

```
vi virt[N];

int virtualTree(vi &ver) {
    auto byDfs = [&](int u, int v) {
        return tin[u] < tin[v];
    };
    sort(all(ver), byDfs);
    for (i, sz(ver), 1)
        ver.pb(lca(ver[i - 1], ver[i]));
    sort(all(ver), byDfs);
    ver.erase(unique(all(ver)), ver.end());
    for (int u : ver)
        virt[u].clear();
    for (i, 1, sz(ver))
        virt[lca(ver[i - 1], ver[i])].pb(ver[i]);
    return ver[0];
}
```

### 7.3 Guni

```
int cnt[C], color[N];
int sz[N];

int guni(int u, int p = 0) {
    sz[u] = 1;
    for (int &v : graph[u]) if (v != p) {
        sz[u] += guni(v, u);
        if (sz[v] > sz[graph[u][0]] || p == graph[u][0])
            swap(v, graph[u][0]);
    }
    return sz[u];
}
```

```
}

void add(int u, int p, int x, bool skip) {
    cnt[color[u]] += x;
    for (int i = skip; i < sz(graph[u]); i++) // don't
        change it with a fore!!!
        if (graph[u][i] != p)
            add(graph[u][i], u, x, 0);
}

void solve(int u, int p, bool keep = 0) {
    fore (i, sz(graph[u]), 0)
        if (graph[u][i] != p)
            solve(graph[u][i], u, !i);
    add(u, p, +1, 1); // add
    // now cnt[i] has how many times the color i appears
    // in the subtree of u
    if (!keep) add(u, p, -1, 0); // remove
}
```

### 7.4 Centroid decomposition

```
int cdp[N], sz[N];
bitset<N> rem;

int dfsz(int u, int p = 0) {
    sz[u] = 1;
    for (int v : graph[u])
        if (v != p && !rem[v])
            sz[u] += dfsz(v, u);
    return sz[u];
}

int centroid(int u, int n, int p = 0) {
    for (int v : graph[u])
        if (v != p && !rem[v] && 2 * sz[v] > n)
            return centroid(v, n, u);
    return u;
}

void solve(int u, int p = 0) {
    cdp[u] = centroid(u, dfsz(u)) = p;
    rem[u] = true;
    for (int v : graph[u])
        if (!rem[v])
            solve(v, u);
}
```

### 7.5 Heavy-light decomposition

```
int par[N], dep[N], sz[N], head[N], pos[N], who[N],
timer = 0;
Lazy* tree;

int dfs(int u) {
    sz[u] = 1, head[u] = 0;
    for (int &v : graph[u]) if (v != par[u]) {
        par[v] = u;
        dep[v] = dep[u] + 1;
        sz[u] += dfs(v);
        if (graph[u][0] == par[u] || sz[v] > sz[graph[u][0]])
            swap(v, graph[u][0]);
    }
    return sz[u];
}

void hld(int u, int h) {
    head[u] = h, pos[u] = ++timer, who[timer] = u;
    for (int &v : graph[u])
        if (v != par[u])
            hld(v, v == graph[u][0] ? h : v);
}
```

```

template <class F>
void processPath(int u, int v, F f) {
    for (; head[u] != head[v]; v = par[head[v]]) {
        if (dep[head[u]] > dep[head[v]]) swap(u, v);
        f(pos[head[v]], pos[v]);
    }
    if (dep[u] > dep[v]) swap(u, v);
    if (u != v) f(pos[graph[u][0]], pos[v]);
    f(pos[u], pos[u]); // only if hld over vertices
}

void updatePath(int u, int v, lli z) {
    processPath(u, v, [&](int l, int r) {
        tree->update(l, r, z);
    });
}

lli queryPath(int u, int v) {
    lli sum = 0;
    processPath(u, v, [&](int l, int r) {
        sum += tree->query(l, r);
    });
    return sum;
}

```

## 7.6 Link-Cut tree

```

typedef struct Node* Splay;
struct Node {
    Splay left = 0, right = 0, par = 0;
    bool rev = 0;
    int sz = 1;
    int sub = 0, vsub = 0; // subtree
    int path = 0; // path
    int self = 0; // node info

    void push() {
        if (rev) {
            swap(left, right);
            if (left) left->rev ^= 1;
            if (right) right->rev ^= 1;
            rev = 0;
        }
    }

    void pull() {
#define sub(u) (u ? u->sub : 0)
#define path(u) (u ? u->path : 0)
#define sz(u) (u ? u->sz : 0)
        sz = 1 + sz(left) + sz(right);
        sub = vsub + sub(left) + sub(right) + self;
        path = path(left) + self + path(right);
    }

    void virSub(Splay v, int add) {
        vsub += 1LL * add * sub(v);
    }
};

void splay(Splay u) {
    auto assign = [&](Splay u, Splay v, bool d) {
        (d == 0 ? u->left : u->right) = v;
        if (v) v->par = u;
    };
    auto dir = [&](Splay u) {
        Splay p = u->par;
        if (!p) return -1;
        return p->left == u ? 0 : (p->right == u ? 1 : -1);
    };
    auto rotate = [&](Splay u) {

```

```

        Splay p = u->par, g = p->par;
        int d = dir(u);
        assign(p, d ? u->left : u->right, d);
        if (dir(p) == -1) u->par = g;
        else assign(g, u, dir(p));
        assign(u, p, !d);
        p->pull(), u->pull();
    };
    while (~dir(u)) {
        Splay p = u->par, g = p->par;
        if (~dir(p)) g->push();
        p->push(), u->push();
        if (~dir(p)) rotate(dir(p) == dir(u) ? p : u);
        rotate(u);
    }
    u->push(), u->pull();
}

```

```

void access(Splay u) {
    Splay last = 0;
    for (Splay v = u; v; last = v, v = v->par) {
        splay(v);
        v->virSub(v->right, +1);
        v->virSub(v->right = last, -1);
        v->pull();
    }
    splay(u);
}

```

```

void reroot(Splay u) {
    access(u);
    u->rev ^= 1;
}

```

```

void link(Splay u, Splay v) {
    reroot(v), access(u);
    u->virSub(v, +1);
    v->par = u;
    u->pull();
}

```

```

void cut(Splay u, Splay v) {
    reroot(v), access(u);
    u->left = 0, v->par = 0;
    u->pull();
}

```

```

Splay lca(Splay u, Splay v) {
    if (u == v) return u;
    access(u), access(v);
    if (!u->par) return 0;
    return splay(u), u->par ? u : 0;
}

```

```

Splay queryPath(Splay u, Splay v) {
    return reroot(u), access(v), v; // path
}

```

```

Splay querySubtree(Splay u, Splay x) {
    // query subtree of u where x is outside
    return reroot(x), access(u), u; // vsub + self
}

```

## 8 Flows

### 8.1 Dinic $\mathcal{O}(\min(E \cdot \text{flow}, V^2 E))$

If the network is massive, try to compress it by looking for patterns.

```

template <class F>
struct Dinic {
    struct Edge {
        int v, inv;

```

```

F cap, flow;
Edge(int v, F cap, int inv) : v(v), cap(cap), flow(0), inv(inv) {}
};

F eps = (F) 1e-9;
int s, t, n, m = 0;
vector< vector<Edge> > g;
vi dist, ptr;

Dinic(int n) : n(n), g(n), dist(n), ptr(n), s(n - 2), t(n - 1) {}

void add(int u, int v, F cap) {
    g[u].pb(Edge(v, cap, sz(g[v])));
    g[v].pb(Edge(u, 0, sz(g[u]) - 1));
    m += 2;
}

bool bfs() {
    fill(all(dist), -1);
    queue<int> qu({s});
    dist[s] = 0;
    while (sz(qu) && dist[t] == -1) {
        int u = qu.front();
        qu.pop();
        for (Edge &e : g[u] if (dist[e.v] == -1)
            if (e.cap - e.flow > eps) {
                dist[e.v] = dist[u] + 1;
                qu.push(e.v);
            }
        }
    }
    return dist[t] != -1;
}

F dfs(int u, F flow = numeric_limits<F>::max()) {
    if (flow <= eps || u == t)
        return max<F>(0, flow);
    for (int &i = ptr[u]; i < sz(g[u]); i++) {
        Edge &e = g[u][i];
        if (e.cap - e.flow > eps && dist[u] + 1 == dist[e.v]) {
            F pushed = dfs(e.v, min<F>(flow, e.cap - e.flow));
            if (pushed > eps) {
                e.flow += pushed;
                g[e.v][e.inv].flow -= pushed;
                return pushed;
            }
        }
    }
    return 0;
}

F maxFlow() {
    F flow = 0;
    while (bfs()) {
        fill(all(ptr), 0);
        while (F pushed = dfs(s))
            flow += pushed;
    }
    return flow;
}
};

```

## 8.2 Min cost flow $\mathcal{O}(\min(E \cdot \text{flow}, V^2 E))$

If the network is massive, try to compress it by looking for patterns.

```

template <class C, class F>
struct MCMF {
    struct Edge {
        int u, v, inv;

```

```

F cap, flow;
C cost;
Edge(int u, int v, C cost, F cap, int inv) : u(u), v(v), cost(cost), cap(cap), flow(0), inv(inv) {}
};

F eps = (F) 1e-9;
int s, t, n, m = 0;
vector< vector<Edge> > g;
vector<Edge*> prev;
vector<C> cost;
vector<int> state;

MCMF(int n) : n(n), g(n), cost(n), state(n), prev(n), s(n - 2), t(n - 1) {}

void add(int u, int v, C cost, F cap) {
    g[u].pb(Edge(u, v, cost, cap, sz(g[v])));
    g[v].pb(Edge(v, u, -cost, 0, sz(g[u]) - 1));
    m += 2;
}

bool bfs() {
    fill(all(state), 0);
    fill(all(cost), numeric_limits<C>::max());
    deque<int> qu;
    qu.push_back(s);
    state[s] = 1, cost[s] = 0;
    while (sz(qu)) {
        int u = qu.front(); qu.pop_front();
        state[u] = 2;
        for (Edge &e : g[u] if (e.cap - e.flow > eps)
            if (cost[u] + e.cost < cost[e.v]) {
                cost[e.v] = cost[u] + e.cost;
                prev[e.v] = &e;
                if (state[e.v] == 2 || (sz(qu) && cost[qu.front()] > cost[e.v]))
                    qu.push_front(e.v);
                else if (state[e.v] == 0)
                    qu.push_back(e.v);
                state[e.v] = 1;
            }
        }
    }
    return cost[t] != numeric_limits<C>::max();
}

pair<C, F> minCostFlow() {
    C cost = 0; F flow = 0;
    while (bfs()) {
        F pushed = numeric_limits<F>::max();
        for (Edge* e = prev[t]; e != nullptr; e = prev[e->u])
            pushed = min(pushed, e->cap - e->flow);
        for (Edge* e = prev[t]; e != nullptr; e = prev[e->u]) {
            e->flow += pushed;
            g[e->v][e->inv].flow -= pushed;
            cost += e->cost * pushed;
        }
        flow += pushed;
    }
    return make_pair(cost, flow);
}
};

```

## 8.3 Hopcroft-Karp $\mathcal{O}(E\sqrt{V})$

```

struct HopcroftKarp {
    int n, m = 0;
    vector<vi> g;
    vi dist, match;

```

```

HopcroftKarp(int _n) : n(_n + 5), g(n), dist(n), match
(n, 0) {} // 1-indexed!!

void add(int u, int v) {
    g[u].pb(v), g[v].pb(u);
    m += 2;
}

bool bfs() {
    queue<int> qu;
    fill(all(dist), -1);
    for (u, 1, n)
        if (!match[u])
            dist[u] = 0, qu.push(u);
    while (!qu.empty()) {
        int u = qu.front(); qu.pop();
        for (int v : g[u])
            if (dist[match[v]] == -1) {
                dist[match[v]] = dist[u] + 1;
                if (match[v])
                    qu.push(match[v]);
            }
    }
    return dist[0] != -1;
}

bool dfs(int u) {
    for (int v : g[u])
        if (!match[v] || (dist[u] + 1 == dist[match[v]] &&
            dfs(match[v]))) {
            match[u] = v, match[v] = u;
            return 1;
        }
    dist[u] = 1 << 30;
    return 0;
}

int maxMatching() {
    int tot = 0;
    while (bfs())
        for (u, 1, n)
            tot += match[u] ? 0 : dfs(u);
    return tot;
}
};

```

## 8.4 Hungarian $\mathcal{O}(N^3)$

$n$  jobs,  $m$  people

```

template <class C>
pair<C, vector<int>> Hungarian(vector<vector<C>> &a) {
    int n = sz(a), m = sz(a[0]), p, q, j, k; // n <= m
    vector<C> fx(n, numeric_limits<C>::min()), fy(m, 0);
    vector<int> x(n, -1), y(m, -1);
    for (i, 0, n)
        for (j, 0, m)
            fx[i] = max(fx[i], a[i][j]);
    for (i, 0, n) {
        vector<int> t(m, -1), s(n + 1, i);
        for (p = q = 0; p <= q && x[i] < 0; p++)
            for (k = s[p], j = 0; j < m && x[i] < 0; j++)
                if (abs(fx[k] + fy[j] - a[k][j]) < eps && t[j] < 0) {
                    s[++q] = y[j], t[j] = k;
                    if (s[q] < 0) for (p = j; p >= 0; j = p)
                        y[j] = k = t[j], p = x[k], x[k] = j;
                }
        if (x[i] < 0) {
            C d = numeric_limits<C>::max();
            for (k, 0, q + 1)
                for (j, 0, m) if (t[j] < 0)

```

```

                    d = min(d, fx[s[k]] + fy[j] - a[s[k]][j]);
            for (j, 0, m)
                fy[j] += (t[j] < 0 ? 0 : d);
            for (k, 0, q + 1)
                fx[s[k]] -= d;
            i--;
        }
    }
    C cost = 0;
    for (i, 0, n) cost += a[i][x[i]];
    return make_pair(cost, x);
}

```

## 9 Strings

### 9.1 Hash

```

vi mod = {999727999, 999992867, 1000000123, 1000002193,
1000003211, 1000008223, 1000009999, 1000027163, 1070
777777};

```

```

struct H : array<int, 2> {
    #define oper(op) friend H operator op (H a, H b) { \
        for (i, 0, sz(a)) a[i] = (1LL * a[i] op b[i] + mod[i]
        ) % mod[i]; \
        return a; }
    oper(+) oper(-) oper(*)
} pw[N], ipw[N];

```

```

struct Hash {
    vector<H> h;

    Hash(string &s) : h(sz(s) + 1) {
        for (i, 0, sz(s)) {
            int x = s[i] - 'a' + 1;
            h[i + 1] = h[i] + pw[i] * H{x, x};
        }
    }

    H cut(int l, int r) {
        return (h[r + 1] - h[l]) * ipw[l];
    }
};

```

```

int inv(int a, int m) {
    a %= m;
    return a == 1 ? 1 : int(m - lli(inv(m, a)) * lli(m) /
        a);
}

```

```

const int P = uniform_int_distribution<int>(MaxAlpha + 1
, min(mod[0], mod[1]) - 1)(rng);
pw[0] = ipw[0] = {1, 1};
H Q = {inv(P, mod[0]), inv(P, mod[1])};
for (i, 1, N) {
    pw[i] = pw[i - 1] * H{P, P};
    ipw[i] = ipw[i - 1] * Q;
}

```

// Save len in the struct and when you do a cut

```

H merge(vector<H> &cuts) {
    H f = {0, 0};
    for (i, sz(cuts), 0) {
        H g = cuts[i];
        f = g + f * pw[g.len];
    }
    return f;
}

```

### 9.2 KMP

period =  $n - p[n - 1]$ , period(abcabc) = 3,  $n \bmod \text{period} \equiv 0$

```

vi lps(string &s) {

```

```

vi p(sz(s), 0);
int j = 0;
fore (i, 1, sz(s)) {
    while (j && s[i] != s[j])
        j = p[j - 1];
    j += (s[i] == s[j]);
    p[i] = j;
}
return p;
}
// how many times t occurs in s
int kmp(string &s, string &t) {
vi p = lps(t);
int j = 0, tot = 0;
fore (i, 0, sz(s)) {
    while (j && s[i] != t[j])
        j = p[j - 1];
    if (s[i] == t[j])
        j++;
    if (j == sz(t))
        tot++; // pos: i - sz(t) + 1;
}
return tot;
}

```

### 9.3 KMP automaton

```

int go[N][A];

void kmpAutomaton(string &s) {
s += "$";
vi p = lps(s);
fore (i, 0, sz(s))
    fore (c, 0, A) {
        if (i && s[i] != 'a' + c)
            go[i][c] = go[p[i - 1]][c];
        else
            go[i][c] = i + ('a' + c == s[i]);
    }
s.pop_back();
}

```

### 9.4 Z algorithm

```

vi zf(string &s) {
vi z(sz(s), 0);
for (int i = 1, l = 0, r = 0; i < sz(s); i++) {
    if (i <= r)
        z[i] = min(r - i + 1, z[i - l]);
    while (i + z[i] < sz(s) && s[i + z[i]] == s[z[i]])
        ++z[i];
    if (i + z[i] - 1 > r)
        l = i, r = i + z[i] - 1;
}
return z;
}

```

### 9.5 Manacher algorithm

```

vector<vi> manacher(string &s) {
vector<vi> pal(2, vi(sz(s), 0));
fore (k, 0, 2) {
    int l = 0, r = 0;
    fore (i, 0, sz(s)) {
        int t = r - i + !k;
        if (i < r)
            pal[k][i] = min(t, pal[k][l + t]);
        int p = i - pal[k][i], q = i + pal[k][i] - !k;
        while (p >= 1 && q + 1 < sz(s) && s[p - 1] == s[q + 1])
            ++pal[k][i], --p, ++q;
        if (q > r)
            l = p, r = q;
    }
}
}

```

```

return pal;
}

```

### 9.6 Suffix array

- Duplicates  $\sum_{i=1}^n lcp[i]$
- Longest Common Substring of various strings  
Add *notUsed* characters between strings, i.e.  $a + \$ + b + \# + c$   
Use two-pointers to find a range  $[l, r]$  such that all *notUsed* characters are present, then  $query(lcp[l + 1], \dots, lcp[r])$  for that window is the common length.

```

struct SuffixArray {
int n;
string s;
vector<int> sa, lcp;

SuffixArray(string &s) : n(sz(s) + 1), s(s), sa(n),
    lcp(n) {
vector<int> top(max(256, n), rk(n));
fore (i, 0, n) top[rk[i] = s[i] & 255]++;
partial_sum(all(top), top.begin());
fore (i, 0, n) sa[--top[rk[i]]] = i;
vector<int> sb(n);
for (int len = 1, j; len < n; len <= 1) {
    fore (i, 0, n) {
        j = (sa[i] - len + n) % n;
        sb[top[rk[j]]++] = j;
    }
    sa[sb[top[0] = 0]] = j = 0;
    fore (i, 1, n) {
        if (rk[sb[i]] != rk[sb[i - 1]] || rk[sb[i] + len]
            != rk[sb[i - 1] + len])
            top[++j] = i;
        sa[sb[i]] = j;
    }
    copy(all(sa), rk.begin());
    copy(all(sb), sa.begin());
    if (j >= n - 1)
        break;
}
for (int i = 0, j = rk[lcp[0] = 0], k = 0; i < n - 1; i++, k++)
    while (k >= 0 && s[i] != s[sa[j - 1] + k])
        lcp[j] = k--, j = rk[sa[j] + 1];
}

char at(int i, int j) {
int k = sa[i] + j;
return k < n ? s[k] : 'z' + 1;
}

int count(string &t) {
ii lo(0, n - 1), hi(0, n - 1);
fore (i, 0, sz(t)) {
    while (lo.f + 1 < lo.s) {
        int mid = (lo.f + lo.s) / 2;
        (at(mid, i) < t[i] ? lo.f : lo.s) = mid;
    }
    while (hi.f + 1 < hi.s) {
        int mid = (hi.f + hi.s) / 2;
        (t[i] < at(mid, i) ? hi.s : hi.f) = mid;
    }
    int p1 = (at(lo.f, i) == t[i] ? lo.f : lo.s);
    int p2 = (at(hi.s, i) == t[i] ? hi.s : hi.f);
    if (at(p1, i) != t[i] || at(p2, i) != t[i] || p1 > p2)
        return 0;
    lo = hi = ii(p1, p2);
}
return lo.s - lo.f + 1;
}

```

```
};
```

## 9.7 Suffix automaton

- $sam[u].len - sam[sam[u].link].len = \text{distinct strings}$
- Number of different substrings (dp)

$$diff(u) = 1 + \sum_{v \in trie[u]} diff(v)$$

- Total length of all different substrings (2 x dp)

$$totLen(u) = \sum_{v \in trie[u]} diff(v) + totLen(v)$$

- Leftmost occurrence  $trie[u].pos = trie[u].len - 1$   
if it is **clone** then  $trie[clone].pos = trie[q].pos$
- All occurrence positions
- Smallest cyclic shift  
Construct sam of  $s + s$ , find the lexicographically smallest path of  $sz(s)$
- Shortest non-appearing string

$$nonAppearing(u) = \min_{v \in trie[u]} nonAppearing(v) + 1$$

```
struct SuffixAutomaton {
    struct Node : map<char, int> {
        int link = -1, len = 0;
    };

    vector<Node> trie;
    int last;

    SuffixAutomaton() { last = newNode(); }

    int newNode() {
        trie.pb({});
        return sz(trie) - 1;
    }

    void extend(char c) {
        int u = newNode();
        trie[u].len = trie[last].len + 1;
        int p = last;
        while (p != -1 && !trie[p].count(c)) {
            trie[p][c] = u;
            p = trie[p].link;
        }
        if (p == -1)
            trie[u].link = 0;
        else {
            int q = trie[p][c];
            if (trie[p].len + 1 == trie[q].len)
                trie[u].link = q;
            else {
                int clone = newNode();
                trie[clone] = trie[q];
                trie[clone].len = trie[p].len + 1;
                while (p != -1 && trie[p][c] == q) {
                    trie[p][c] = clone;
                    p = trie[p].link;
                }
                trie[q].link = trie[u].link = clone;
            }
        }
        last = u;
    }

    string kthSubstring(lli kth, int u = 0) {
        // number of different substrings (dp)
        string s = "";
        while (kth > 0)
            for (auto &[c, v] : trie[u]) {
```

```
                if (kth <= diff(v)) {
                    s.pb(c), kth--, u = v;
                    break;
                }
                kth -= diff(v);
            }
        return s;
    }

    void occurs() {
        // trie[u].occ = 1, trie[clone].occ = 0
        vi who;
        for (u, 1, sz(trie))
            who.pb(u);
        sort(all(who), [&](int u, int v) {
            return trie[u].len > trie[v].len;
        });
        for (int u : who) {
            int l = trie[u].link;
            trie[l].occ += trie[u].occ;
        }
    }

    lli queryOccurrences(string &s, int u = 0) {
        for (char c : s) {
            if (!trie[u].count(c))
                return 0;
            u = trie[u][c];
        }
        return trie[u].occ;
    }

    int longestCommonSubstring(string &s, int u = 0) {
        int mx = 0, clen = 0;
        for (char c : s) {
            while (u && !trie[u].count(c)) {
                u = trie[u].link;
                clen = trie[u].len;
            }
            if (trie[u].count(c))
                u = trie[u][c], clen++;
            mx = max(mx, clen);
        }
        return mx;
    }

    string smallestCyclicShift(int n, int u = 0) {
        string s = "";
        for (i, 0, n) {
            char c = trie[u].begin()->f;
            s += c;
            u = trie[u][c];
        }
        return s;
    }

    int leftmost(string &s, int u = 0) {
        for (char c : s) {
            if (!trie[u].count(c))
                return -1;
            u = trie[u][c];
        }
        return trie[u].pos - sz(s) + 1;
    }

    Node& operator [](int u) {
        return trie[u];
    }
};
```

## 9.8 Aho corasick

```

struct AhoCorasick {
    struct Node : map<char, int> {
        int link = 0, out = 0;
        int cnt = 0, isw = 0;
    };

    vector<Node> trie;

    AhoCorasick() { newNode(); }

    int newNode() {
        trie.pb({});
        return sz(trie) - 1;
    }

    void insert(string &s, int u = 0) {
        for (char c : s) {
            if (!trie[u][c])
                trie[u][c] = newNode();
            u = trie[u][c];
        }
        trie[u].cnt++, trie[u].isw = 1;
    }

    int go(int u, char c) {
        while (u && !trie[u].count(c))
            u = trie[u].link;
        return trie[u][c];
    }

    void pushLinks() {
        queue<int> qu;
        qu.push(0);
        while (!qu.empty()) {
            int u = qu.front();
            qu.pop();
            for (auto &[c, v] : trie[u]) {
                int l = (trie[v].link = u ? go(trie[u].link, c) : 0);
                trie[v].cnt += trie[l].cnt;
                trie[v].out = trie[l].isw ? l : trie[l].out;
                qu.push(v);
            }
        }
    }

    int match(string &s, int u = 0) {
        int ans = 0;
        for (char c : s) {
            u = go(u, c);
            ans += trie[u].cnt;
            for (int x = u; x != 0; x = trie[x].out)
                // pass over all elements of the implicit vector
        }
        return ans;
    }

    Node& operator [](int u) {
        return trie[u];
    }
};

```

## 9.9 Eertree

```

struct Eertree {
    struct Node : map<char, int> {
        int link = 0, len = 0;
    };

    vector<Node> trie;
    string s = "$";

```

```

    int last;

    Eertree() {
        last = newNode(), newNode();
        trie[0].link = 1, trie[1].len = -1;
    }

    int newNode() {
        trie.pb({});
        return sz(trie) - 1;
    }

    int go(int u) {
        while (s[sz(s) - trie[u].len - 2] != s.back())
            u = trie[u].link;
        return u;
    }

    void extend(char c) {
        s += c;
        int u = go(last);
        if (!trie[u][c]) {
            int v = newNode();
            trie[v].len = trie[u].len + 2;
            trie[v].link = trie[go(trie[u].link)][c];
            trie[u][c] = v;
        }
        last = trie[u][c];
    }

    Node& operator [](int u) {
        return trie[u];
    }
};

```

## 10 Dynamic Programming

### 10.1 All submasks of a mask

```
for (int B = A; B > 0; B = (B - 1) & A)
```

### 10.2 Matrix Chain Multiplication

```

int dp(int l, int r) {
    if (l > r)
        return 0LL;
    int &ans = mem[l][r];
    if (!done[l][r]) {
        done[l][r] = true, ans = inf;
        for (k, l, r + 1) // split in [l, k] [k + 1, r]
            ans = min(ans, dp(l, k) + dp(k + 1, r) + add);
    }
    return ans;
}

```

### 10.3 Digit DP

Counts the amount of numbers in  $[l, r]$  such are divisible by  $k$ . (flag *nonzero* is for different lengths)  
It can be reduced to  $dp(i, x, small)$ , and has to be solve like  $f(r) - f(l - 1)$

```

#define state [i][x][small][big][nonzero]
int dp(int i, int x, bool small, bool big, bool nonzero)
{
    if (i == sz(r))
        return x % k == 0 && nonzero;
    int &ans = mem state;
    if (done state != timer) {
        done state = timer;
        ans = 0;
        int lo = small ? 0 : l[i] - '0';
        int hi = big ? 9 : r[i] - '0';
        for (y, lo, max(lo, hi) + 1) {

```



```

    bool small2 = small | (y > lo);
    bool big2 = big | (y < hi);
    bool nonzero2 = nonzero | (x > 0);
    ans += dp(i + 1, (x * 10 + y) % k, small2, big2,
        nonzero2);
}
}
return ans;
}

```

## 10.4 Knapsack 0/1

```

for (auto &cur : items)
    fore (w, W + 1, cur.w) // [cur.w, W]
        umax(dp[w], dp[w - cur.w] + cur.cost);

```

## 10.5 Convex Hull Trick $\mathcal{O}(n^2) \Rightarrow \mathcal{O}(n)$

```

dp[i] = min_{j < i} (dp[j] + b[j] * a[i])
dp[i][j] = min_{k < j} (dp[i - 1][k] + b[k] * a[j])
b[j] ≥ b[j + 1] optionally a[i] ≤ a[i + 1]

// for doubles, use inf = 1/.0, div(a,b) = a / b
struct Line {
    mutable lli m, c, p;
    bool operator < (const Line &l) const { return m < l.m; }
    ; }

    bool operator < (lli x) const { return p < x; }
    lli operator ()(lli x) const { return m * x + c; }
};

struct DynamicHull : multiset<Line, less<>> {
    lli div(lli a, lli b) {
        return a / b - ((a ^ b) < 0 && a % b);
    }

    bool isect(iterator x, iterator y) {
        if (y == end()) return x->p = inf, 0;
        if (x->m == y->m) x->p = x->c > y->c ? inf : -inf;
        else x->p = div(x->c - y->c, y->m - x->m);
        return x->p >= y->p;
    }

    void add(lli m, lli c) {
        auto z = insert({m, c, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y))
            isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }

    lli query(lli x) {
        if (empty()) return 0LL;
        auto f = *lower_bound(x);
        return f(x);
    }
};

```

## 10.6 Divide and conquer $\mathcal{O}(kn^2) \Rightarrow \mathcal{O}(k \cdot n \log n)$

Split the array of size  $n$  into  $k$  continuous groups.  $k \leq n$   
 $cost(a, c) + cost(b, d) \leq cost(a, d) + cost(b, c)$  with  $a \leq b \leq c \leq d$

```

void dc(int cut, int l, int r, int optl, int opttr) {
    if (r < l)
        return;
    int mid = (l + r) / 2;
    pair<lli, int> best = {inf, -1};
    fore (p, optl, min(mid, opttr) + 1)
        best = min(best, {dp[~cut & 1][p - 1] + cost(p, mid)
            , p});
}

```

```

dp[cut & 1][mid] = best.f;
dc(cut, l, mid - 1, optl, best.s);
dc(cut, mid + 1, r, best.s, opttr);
}

fore (i, 1, n + 1)
    dp[1][i] = cost(1, i);
fore (cut, 2, k + 1)
    dc(cut, cut, n, cut, n);

```

## 10.7 Knuth optimization $\mathcal{O}(n^3) \Rightarrow \mathcal{O}(n^2)$

$dp[l][r] = \min_{l \leq k \leq r} \{dp[l][k] + dp[k][r]\} + cost(l, r)$

```

fore (len, 1, n + 1)
    fore (l, 0, n) {
        int r = l + len - 1;
        if (r > n - 1)
            break;
        if (len <= 2) {
            dp[l][r] = 0;
            opt[l][r] = l;
            continue;
        }
        dp[l][r] = inf;
        fore (k, opt[l][r - 1], opt[l + 1][r] + 1) {
            lli cur = dp[l][k] + dp[k][r] + cost(l, r);
            if (cur < dp[l][r]) {
                dp[l][r] = cur;
                opt[l][r] = k;
            }
        }
    }
}

```

# 11 Game Theory

## 11.1 Grundy Numbers

If the moves are consecutive  $S = \{1, 2, 3, \dots, x\}$  the game can be solved like  $stackSize \pmod{x+1} \neq 0$

```

int mem[N];

int mex(set<int> &st) {
    int x = 0;
    while (st.count(x))
        x++;
    return x;
}

int grundy(int n) {
    if (n < 0)
        return inf;
    if (n == 0)
        return 0;
    int &g = mem[n];
    if (g == -1) {
        set<int> st;
        for (int x : {a, b})
            st.insert(grundy(n - x));
        g = mex(st);
    }
    return g;
}

```

## 12 Combinatorics

Combinatorics table		
Number	Factorial	Catalan
0	1	1
1	1	1
2	2	2
3	6	5
4	24	14
5	120	42
6	720	132
7	5,040	429
8	40,320	1,430
9	362,880	4,862
10	3,628,800	16,796
11	39,916,800	58,786
12	479,001,600	208,012
13	6,227,020,800	742,900

### 12.1 Factorial

```

fac[0] = 1LL;
for (i, 1, N)
    fac[i] = lli(i) * fac[i - 1] % mod;
ifac[N - 1] = fpow(fac[N - 1], mod - 2);
for (i, N - 1, 0)
    ifac[i] = lli(i + 1) * ifac[i + 1] % mod;

```

### 12.2 Factorial mod *smallPrime*

```

lli facMod(lli n, int p) {
    lli r = 1LL;
    for (; n > 1; n /= p) {
        r = (r * ((n / p) % 2 ? p - 1 : 1)) % p;
        for (i, 2, n % p + 1)
            r = r * i % p;
    }
    return r % p;
}

```

### 12.3 Lucas theorem

Changes  $\binom{n}{k} \bmod p$ , with  $n \geq 2e6, k \geq 2e6$  and  $p \leq 1e7$

$$\binom{n}{k} \equiv \prod_{i=0}^n \binom{n_i}{k_i} \bmod p$$

```

lli lucas(lli n, lli k) {
    if (k == 0)
        return 1LL;
    return lucas(n / mod, k / mod) * choose(n % mod, k % mod) % mod;
}

```

### 12.4 Stars and bars

Enclosing  $n$  objects in  $k$  boxes

$$\binom{n+k-1}{k-1} = \binom{n+k-1}{n}$$

### 12.5 N choose K

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\binom{n}{k_1, k_2, \dots, k_m} = \frac{n!}{k_1! * k_2! * \dots * k_m!}$$

```

lli choose(int n, int k) {
    if (n < 0 || k < 0 || n < k)
        return 0LL;
    return fac[n] * ifac[k] % mod * ifac[n - k] % mod;
}

lli choose(int n, int k) {

```

```

double r = 1;
for (i, 1, k + 1)
    r = r * (n - k + i) / i;
return lli(r + 0.01);
}

```

### 12.6 Catalan

```

catalan[0] = 1LL;
for (i, 0, N) {
    catalan[i + 1] = catalan[i] * lli(4 * i + 2) % mod *
        fpow(i + 2, mod - 2) % mod;
}

```

### 12.7 Burnside's lemma

$$|classes| = \frac{1}{|G|} \cdot \sum_{x \in G} f(x)$$

### 12.8 Prime factors of N!

```

vector< pair<lli, int> > factorialFactors(lli n) {
    vector< pair<lli, int> > fac;
    for (lli p : primes) {
        if (n < p)
            break;
        lli mul = 1LL, k = 0;
        while (mul <= n / p) {
            mul *= p;
            k += n / mul;
        }
        fac.emplace_back(p, k);
    }
    return fac;
}

```

## 13 Number Theory

### 13.1 Goldbach conjecture

- All number  $\geq 6$  can be written as sum of 3 *primes*
- All even number  $> 2$  can be written as sum of 2 *primes*

### 13.2 Prime numbers distribution

Amount of primes approximately  $\frac{n}{\ln(n)}$

### 13.3 Sieve of Eratosthenes

To factorize divide  $x$  by  $factor[x]$  until is equal to 1

```

void factorizeSieve() {
    iota(factor, factor + N, 0);
    for (int i = 2; i * i < N; i++) if (factor[i] == i)
        for (int j = i * i; j < N; j += i)
            factor[j] = i;
}

map<int, int> factorize(int n) {
    map<int, int> cnt;
    while (n > 1) {
        cnt[factor[n]]++;
        n /= factor[n];
    }
    return cnt;
}

```

Use it if you need a huge amount of  $\phi[x]$  up to some N

```

void phiSieve() {
    isPrime.set();
    iota(phi, phi + N, 0);
    for (i, 2, N) if (isPrime[i])
        for (int j = i; j < N; j += i) {
            isPrime[j] = (i == j);
            phi[j] = phi[j] / i * (i - 1);
        }
}

```

```

    }
}

```

### 13.4 Phi of euler

```

lli phi(lli n) {
    if (n == 1) return 0;
    lli r = n;
    for (lli i = 2; i * i <= n; i++)
        if (n % i == 0) {
            while (n % i == 0) n /= i;
            r -= r / i;
        }
    if (n > 1) r -= r / n;
    return r;
}

```

### 13.5 Miller-Rabin

```

bool miller(lli n) {
    if (n < 2 || n % 6 % 4 != 1)
        return (n | 1) == 3;
    int k = __builtin_ctzll(n - 1);
    lli d = n >> k;
    auto compo = [&](lli p) {
        lli x = fpow(p % n, d, n), i = k;
        while (x != 1 && x != n - 1 && p % n && i--)
            x = mul(x, x, n);
        return x != n - 1 && i != k;
    };
    for (lli p : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}) {
        if (compo(p)) return 0;
        if (compo(2 + rng() % (n - 3))) return 0;
    }
    return 1;
}

```

### 13.6 Pollard-Rho

```

lli rho(lli n) {
    while (1) {
        lli x = 2 + rng() % (n - 3), c = 1 + rng() % 20;
        auto f = [&](lli x) { return (mul(x, x, n) + c) % n; };
        lli y = f(x), g;
        while ((g = __gcd(n + y - x, n)) == 1)
            x = f(x), y = f(f(y));
        if (g != n) return g;
    }
    return -1;
}

void pollard(lli n, map<lli, int> &fac) {
    if (n == 1) return;
    if (n % 2 == 0) {
        fac[2]++;
        pollard(n / 2, fac);
        return;
    }
    if (miller(n)) {
        fac[n]++;
        return;
    }
    lli x = rho(n);
    pollard(x, fac);
    pollard(n / x, fac);
}

```

### 13.7 Amount of divisors

```

lli amountOfDivisors(lli n) {
    lli cnt = 1LL;
    for (int p : primes) {
        if (1LL * p * p * p > n) break;

```

```

        if (n % p == 0) {
            lli k = 0;
            while (n > 1 && n % p == 0) n /= p, ++k;
            cnt *= (k + 1);
        }
    }
    lli sq = mysqrt(n); // A binary search, the last x * x
    <= n
    if (miller(n)) cnt *= 2;
    else if (sq * sq == n && miller(sq)) cnt *= 3;
    else if (n > 1) cnt *= 4;
    return cnt;
}

```

### 13.8 Bézout's identity

$a_1 * x_1 + a_2 * x_2 + \dots + a_n * x_n = g$   
 $g = \gcd(a_1, a_2, \dots, a_n)$

### 13.9 GCD

$a \leq b; \gcd(a + k, b + k) = \gcd(b - a, a + k)$

### 13.10 LCM

$x = p * lcm(a_1, a_2, \dots, a_k) + q, 0 \leq q < lcm(a_1, a_2, \dots, a_k)$   
 $x \pmod{a_i} \equiv q \pmod{a_i}$  as  $a_i \mid lcm(a_1, a_2, \dots, a_k)$

### 13.11 Euclid

```

pair<lli, lli> euclid(lli a, lli b) {
    if (b == 0)
        return {1, 0};
    auto p = euclid(b, a % b);
    return {p.s, p.f - a / b * p.s};
}

```

### 13.12 Chinese remainder theorem

```

pair<lli, lli> crt(pair<lli, lli> a, pair<lli, lli> b) {
    if (a.s < b.s) swap(a, b);
    auto p = euclid(a.s, b.s);
    lli g = a.s * p.f + b.s * p.s, l = a.s / g * b.s;
    if ((b.f - a.f) % g != 0)
        return {-1, -1}; // no solution
    p.f = a.f + (b.f - a.f) % b.s * p.f % b.s / g * a.s;
    return {p.f + (p.f < 0) * l, l};
}

```

## 14 Math

### 14.1 Progressions

#### Arithmetic progressions

$$a_n = a_1 + (n - 1) * diff$$

$$\sum_{i=1}^n a_i = n * \frac{a_1 + a_n}{2}$$

#### Geometric progressions

$$a_n = a_1 * r^{n-1}$$

$$\sum_{k=1}^n a_1 * r^k = a_1 * \left( \frac{r^{n+1} - 1}{r - 1} \right) : r \neq 1$$

### 14.2 Fpow

```

template <class T>
T fpow(T x, lli n) {
    T r(1);
    for (; n > 0; n >>= 1) {
        if (n & 1) r = r * x;
        x = x * x;
    }
    return r;
}

```

### 14.3 Fibonacci

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n = \begin{bmatrix} fib_{n+1} & fib_n \\ fib_n & fib_{n-1} \end{bmatrix}$$

## 15 Bit tricks

Bits++	
Operations on <i>int</i>	Function
$x \& -x$	Least significant bit in $x$
$\_lg(x)$	Most significant bit in $x$
$c = x \& -x, r = x + c;$ $((r \ll x) \gg 2) / c \mid r$	Next number after $x$ with same number of bits set
builtin	Function
<code>popcount(x)</code>	Amount of 1's in $x$
<code>clz(x)</code>	0's to the <b>left</b> of biggest bit
<code>ctz(x)</code>	0's to the <b>right</b> of smallest bit

### 15.1 Bitset

Bitset<Size>	
Operation	Function
<code>_Find_first()</code>	Least significant bit
<code>_Find_next(idx)</code>	First set bit after index $idx$
<code>any()</code> , <code>none()</code> , <code>all()</code>	Just what the expression says
<code>set()</code> , <code>reset()</code> , <code>flip()</code>	Just what the expression says x2
<code>to_string('.', 'A')</code>	Print 011010 like .AA.A.

### 15.2 Geometry

```
const ld eps = 1e-20;
#define eq(a, b) (abs((a) - (b)) <= +eps)
#define neq(a, b) (!eq(a, b))
#define geq(a, b) ((a) - (b) >= -eps)
#define leq(a, b) ((a) - (b) <= +eps)
#define ge(a, b) ((a) - (b) > +eps)
#define le(a, b) ((a) - (b) < -eps)
```

```
enum {ON = -1, OUT, IN, OVERLAP, INF};
```

## 16 Points

### 16.1 Points

```
int sgn(ld a) { return (a > eps) - (a < -eps); }

struct Pt {
    ld x, y;
    explicit Pt(ld x = 0, ld y = 0) : x(x), y(y) {}
    Pt operator + (Pt p) const { return Pt(x + p.x, y + p.y); }
    Pt operator - (Pt p) const { return Pt(x - p.x, y - p.y); }
    Pt operator * (ld k) const { return Pt(x * k, y * k); }
    Pt operator / (ld k) const { return Pt(x / k, y / k); }

    ld dot(Pt p) const {
        // 0 if vectors are orthogonal
        // - if vectors are pointing in opposite directions
        // + if vectors are pointing in the same direction
        return x * p.x + y * p.y;
    }

    ld cross(Pt p) const {
        // 0 if collinear
        // - if b is to the right of a
        // + if b is to the left of a
        // gives you 2 * area
        return x * p.y - y * p.x;
    }

    ld norm() const { return x * x + y * y; }
    ld length() const { return sqrt1(norm()); }

    ld angle() const {
        ld ang = atan2(y, x);
        return ang + (ang < 0 ? 2 * acos(-1) : 0);
    }
}
```

```
}

Pt perp() const { return Pt(-y, x); }
Pt unit() const { return (*this) / length(); }
Pt rotate(ld angle) const {
    // counter-clockwise rotation in radians
    // degree = radian * 180 / pi
    return Pt(x * cos(angle) - y * sin(angle), x * sin(
        angle) + y * cos(angle));
}

int dir(Pt a, Pt b) const {
    return sgn((a - *this).cross(b - *this));
}

int cuad() const {
    if (x > 0 && y >= 0) return 0;
    if (x <= 0 && y > 0) return 1;
    if (x < 0 && y <= 0) return 2;
    if (x >= 0 && y < 0) return 3;
    return -1;
}
```

### 16.2 Angle between vectors

```
double angleBetween(Pt a, Pt b) {
    double x = a.dot(b) / a.length() / b.length();
    return acos1(max(-1.0, min(1.0, x)));
}
```

### 16.3 Closest pair of points

```
pair<Pt, Pt> closestPairOfPoints(Poly &pts) {
    sort(all(pts), [&](Pt a, Pt b) {
        return le(a.y, b.y);
    });
    set<Pt> st;
    ld ans = inf;
    Pt p, q;
    int pos = 0;
    for (i, 0, sz(pts)) {
        while (pos < i && geq(pts[i].y - pts[pos].y, ans))
            st.erase(pts[pos++]);
        auto lo = st.lower_bound(Pt(pts[i].x - ans - eps, -inf));
        auto hi = st.upper_bound(Pt(pts[i].x + ans + eps, -inf));
        for (auto it = lo; it != hi; ++it) {
            ld d = (pts[i] - *it).length();
            if (le(d, ans))
                ans = d, p = pts[i], q = *it;
        }
        st.insert(pts[i]);
    }
    return {p, q};
}
```

### 16.4 Projection

```
ld proj(Pt a, Pt b) {
    return a.dot(b) / b.length();
}
```

### 16.5 KD-Tree

```
struct KDTree {
    // p.pos(0) = x, p.pos(1) = y, p.pos(2) = z
    #define iter Pt* // vector<Pt>::iterator
    KDTree *left, *right;
    Pt p;
    ld val;
    int k;

    KDTree(iter b, iter e, int k = 0) : k(k), left(0),
        right(0) {
        int n = e - b;
    }
}
```

```

if (n == 1) {
    p = *b;
    return;
}
nth_element(b, b + n / 2, e, [&](Pt a, Pt b) {
    return a.pos(k) < b.pos(k);
});
val = (b + n / 2) ->pos(k);
left = new KDTree(b, b + n / 2, (k + 1) % 2);
right = new KDTree(b + n / 2, e, (k + 1) % 2);
}

pair<ld, Pt> nearest(Pt q) {
    if (!left && !right) // take care if is needed a
        different one
        return make_pair((p - q).norm(), p);
    pair<ld, Pt> best;
    if (q.pos(k) <= val) {
        best = left->nearest(q);
        if (geq(q.pos(k) + sqrt(best.f), val))
            best = min(best, right->nearest(q));
    } else {
        best = right->nearest(q);
        if (leq(q.pos(k) - sqrt(best.f), val))
            best = min(best, left->nearest(q));
    }
    return best;
}
};

```

## 17 Lines and segments

### 17.1 Line

```

struct Line {
    Pt a, b, v;

    Line() {}
    Line(Pt a, Pt b) : a(a), b(b), v((b - a).unit()) {}

    bool contains(Pt p) {
        return eq((p - a).cross(b - a), 0);
    }

    int intersects(Line l) {
        if (eq(v.cross(l.v), 0))
            return eq((l.a - a).cross(v), 0) ? INF : 0;
        return 1;
    }

    int intersects(Seg s) {
        if (eq(v.cross(s.v), 0))
            return eq((s.a - a).cross(v), 0) ? INF : 0;
        return sgn(v.cross(s.a - a)) != sgn(v.cross(s.b - a))
            ? 1 : 0;
    }

    template <class Line>
    Pt intersection(Line l) { // can be a segment too
        return a + v * ((l.a - a).cross(l.v) / v.cross(l.v))
            ;
    }

    Pt projection(Pt p) {
        return a + v * proj(p - a, v);
    }

    Pt reflection(Pt p) {
        return a * 2 - p + v * 2 * proj(p - a, v);
    }
};

```

### 17.2 Segment

```

struct Seg {
    Pt a, b, v;

    Seg() {}
    Seg(Pt a, Pt b) : a(a), b(b), v(b - a) {}

    bool contains(Pt p) {
        return eq(v.cross(p - a), 0) && leq((a - p).dot(b -
            p), 0);
    }

    int intersects(Seg s) {
        int t1 = sgn(v.cross(s.a - a)), t2 = sgn(v.cross(s.b
            - a));
        if (t1 == t2)
            return t1 == 0 && (contains(s.a) || contains(s.b)
                || s.contains(a) || s.contains(b)) ? INF : 0;
        return sgn(s.v.cross(a - s.a)) != sgn(s.v.cross(b -
            s.a));
    }

    template <class Seg>
    Pt intersection(Seg s) { // can be a line too
        return a + v * ((s.a - a).cross(s.v) / v.cross(s.v))
            ;
    }
};

```

### 17.3 Distance point-line

```

ld distance(Pt p, Line l) {
    Pt q = l.projection(p);
    return (p - q).length();
}

```

### 17.4 Distance point-segment

```

ld distance(Pt p, Seg s) {
    if (le((p - s.a).dot(s.b - s.a), 0))
        return (p - s.a).length();
    if (le((p - s.b).dot(s.a - s.b), 0))
        return (p - s.b).length();
    return abs((s.a - p).cross(s.b - p) / (s.b - s.a).
        length());
}

```

### 17.5 Distance segment-segment

```

ld distance(Seg a, Seg b) {
    if (a.intersects(b))
        return 0.L;
    return min({distance(a.a, b), distance(a.b, b),
        distance(b.a, a), distance(b.b, a)});
}

```

## 18 Circles

### 18.1 Circle

```

struct Cir {
    Pt o;
    ld r;
    Cir() {}
    Cir(ld x, ld y, ld r) : o(x, y), r(r) {}
    Cir(Pt o, ld r) : o(o), r(r) {}

    int inside(Cir c) {
        ld l = c.r - r - (o - c.o).length();
        return ge(l, 0) ? IN : eq(l, 0) ? ON : OVERLAP;
    }

    int outside(Cir c) {
        ld l = (o - c.o).length() - r - c.r;
        return ge(l, 0) ? OUT : eq(l, 0) ? ON : OVERLAP;
    }
};

```

```

int contains(Pt p) {
    ld l = (p - o).length() - r;
    return le(l, 0) ? IN : eq(l, 0) ? ON : OUT;
}

Pt projection(Pt p) {
    return o + (p - o).unit() * r;
}

vector<Pt> tangency(Pt p) {
    // point outside the circle
    Pt v = (p - o).unit() * r;
    ld d2 = (p - o).norm(), d = sqrt(d2);
    if (leq(d, 0)) return {}; // on circle, no tangent
    Pt v1 = v * (r / d), v2 = v.perp() * (sqrt(d2 - r * r) / d);
    return {o + v1 - v2, o + v1 + v2};
}

vector<Pt> intersection(Cir c) {
    ld d = (c.o - o).length();
    if (eq(d, 0) || ge(d, r + c.r) || le(d, abs(r - c.r))) return {}; // circles don't intersect
    Pt v = (c.o - o).unit();
    ld a = (r * r + d * d - c.r * c.r) / (2 * d);
    Pt p = o + v * a;
    if (eq(d, r + c.r) || eq(d, abs(r - c.r))) return {p}; // circles touch at one point
    ld h = sqrt(r * r - a * a);
    Pt q = v.perp() * h;
    return {p - q, p + q}; // circles intersects twice
}

template <class Line>
vector<Pt> intersection(Line l) {
    // for a segment you need to check that the point
    // lies on the segment
    ld h2 = r * r - l.v.cross(o - l.a) * l.v.cross(o - l.a) / l.v.norm();
    Pt p = l.a + l.v * l.v.dot(o - l.a) / l.v.norm();
    if (eq(h2, 0)) return {p}; // line tangent to circle
    if (le(h2, 0)) return {}; // no intersection
    Pt q = l.v.unit() * sqrt(h2);
    return {p - q, p + q}; // two points of intersection (chord)
}

Cir(Pt a, Pt b, Pt c) {
    // find circle that passes through points a, b, c
    Pt mab = (a + b) / 2, mcb = (b + c) / 2;
    Seg ab(mab, mab + (b - a).perp());
    Seg cb(mcb, mcb + (b - c).perp());
    o = ab.intersection(cb);
    r = (o - a).length();
}

ld commonArea(Cir c) {
    if (le(r, c.r))
        return c.commonArea(*this);
    ld d = (o - c.o).length();
    if (leq(d + c.r, r)) return c.r * c.r * pi;
    if (geq(d, r + c.r)) return 0.0;
    auto angle = [&](ld a, ld b, ld c) {
        return acos((a * a + b * b - c * c) / (2 * a * b));
    };
    auto cut = [&](ld a, ld r) {
        return (a - sin(a)) * r * r / 2;
    };

```

```

        ld a1 = angle(d, r, c.r), a2 = angle(d, c.r, r);
        return cut(a1 * 2, r) + cut(a2 * 2, c.r);
    }
}

```

## 18.2 Distance point-circle

```

ld distance(Pt p, Cir c) {
    return max(0.L, (p - c.o).length() - c.r);
}

```

## 18.3 Minimum enclosing circle

```

Cir minEnclosing(vector<Pt> &pts) { // a bunch of points
    shuffle(all(pts), rng);
    Cir c(0, 0, 0);
    for (i, 0, sz(pts)) if (c.contains(pts[i]) != OUT) {
        c = Cir(pts[i], 0);
        for (j, 0, i) if (c.contains(pts[j]) != OUT) {
            c = Cir((pts[i] + pts[j]) / 2, (pts[i] - pts[j]).length() / 2);
            for (k, 0, j) if (c.contains(pts[k]) != OUT)
                c = Cir(pts[i], pts[j], pts[k]);
        }
    }
    return c;
}

```

## 18.4 Common area circle-polygon

```

ld commonArea(const Cir &c, const Poly &poly) {
    auto arg = [&](Pt p, Pt q) {
        return atan2(p.cross(q), p.dot(q));
    };
    auto tri = [&](Pt p, Pt q) {
        Pt d = q - p;
        ld a = d.dot(p) / d.norm(), b = (p.norm() - c.r * c.r) / d.norm();
        ld det = a * a - b;
        if (leq(det, 0)) return arg(p, q) * c.r * c.r;
        ld s = max(0.L, -a - sqrt(det)), t = min(1.L, -a + sqrt(det));
        if (t < 0 || 1 <= s) return arg(p, q) * c.r * c.r;
        Pt u = p + d * s, v = p + d * t;
        return u.cross(v) + (arg(p, u) + arg(v, q)) * c.r * c.r;
    };
    ld sum = 0;
    for (i, 0, sz(poly))
        sum += tri(poly[i] - c.o, poly[(i + 1) % sz(poly)] - c.o);
    return abs(sum / 2);
}

```

# 19 Polygons

## 19.1 Area of polygon

```

ld area(const Poly &pts) {
    ld sum = 0;
    for (i, 0, sz(pts))
        sum += pts[i].cross(pts[(i + 1) % sz(pts)]);
    return abs(sum / 2);
}

```

## 19.2 Convex-Hull

```

Poly convexHull(Poly pts) {
    Poly low, up;
    sort(all(pts), [&](Pt a, Pt b) {
        return a.x == b.x ? a.y < b.y : a.x < b.x;
    });
    pts.erase(unique(all(pts)), pts.end());
    if (sz(pts) <= 2)
        return pts;
    for (i, 0, sz(pts)) {
        while(sz(low) >= 2 && (low.end()[-1] - low.end()[-2]

```

```

    ]).cross(pts[i] - low.end()[-1]) <= 0)
    low.pop_back();
    low.pb(pts[i]);
}
fore (i, sz(pts), 0) {
    while(sz(up) >= 2 && (up.end()[-1] - up.end()[-2]).
        cross(pts[i] - up.end()[-1]) <= 0)
        up.pop_back();
    up.pb(pts[i]);
}
low.pop_back(), up.pop_back();
low.insert(low.end(), all(up));
return low;
}

```

### 19.3 Cut polygon by a line

```

Poly cut(const Poly &pts, Line l) {
    Poly ans;
    int n = sz(pts);
    fore (i, 0, n) {
        int j = (i + 1) % n;
        if (geq(l.v.cross(pts[i] - l.a), 0)) // left
            ans.pb(pts[i]);
        Seg s(pts[i], pts[j]);
        if (l.intersects(s) == 1) {
            Pt p = l.intersection(s);
            if (p != pts[i] && p != pts[j])
                ans.pb(p);
        }
    }
    return ans;
}

```

### 19.4 Perimeter

```

ld perimeter(const Poly &pts){
    ld sum = 0;
    fore (i, 0, sz(pts))
        sum += (pts[(i + 1) % sz(pts)] - pts[i]).length();
    return sum;
}

```

### 19.5 Point in polygon

```

int contains(const Poly &pts, Pt p) {
    int rays = 0, n = sz(pts);
    fore (i, 0, n) {
        Pt a = pts[i], b = pts[(i + 1) % n];
        if (ge(a.y, b.y))
            swap(a, b);
        if (Seg(a, b).contains(p))
            return ON;
        rays ^= (leq(a.y, p.y) && le(p.y, b.y) && ge((a - p)
            .cross(b - p), 0));
    }
    return rays & 1 ? IN : OUT;
}

```

### 19.6 Point in convex-polygon

```

bool contains(const Poly &a, Pt p) {
    int lo = 1, hi = sz(a) - 1;
    if (a[0].dir(a[lo], a[hi]) > 0)
        swap(lo, hi);
    if (p.dir(a[0], a[lo]) >= 0 || p.dir(a[0], a[hi]) <= 0)
        return false;
    while (abs(lo - hi) > 1) {
        int mid = (lo + hi) >> 1;
        (p.dir(a[0], a[mid]) > 0 ? hi : lo) = mid;
    }
    return p.dir(a[lo], a[hi]) < 0;
}

```

### 19.7 Is convex

```

bool isConvex(const Poly &pts) {
    int n = sz(pts);
    bool pos = 0, neg = 0;
    fore (i, 0, n) {
        Pt a = pts[(i + 1) % n] - pts[i];
        Pt b = pts[(i + 2) % n] - pts[(i + 1) % n];
        int dir = sgn(a.cross(b));
        if (dir > 0) pos = 1;
        if (dir < 0) neg = 1;
    }
    return !(pos && neg);
}

```

## 20 Geometry misc

### 20.1 Radial order

```

struct Radial {
    Pt c;
    Radial(Pt c) : c(c) {}

    bool operator()(Pt a, Pt b) const {
        Pt p = a - c, q = b - c;
        if (p.cuad() == q.cuad())
            return p.y * q.x < p.x * q.y;
        return p.cuad() < q.cuad();
    }
};

```

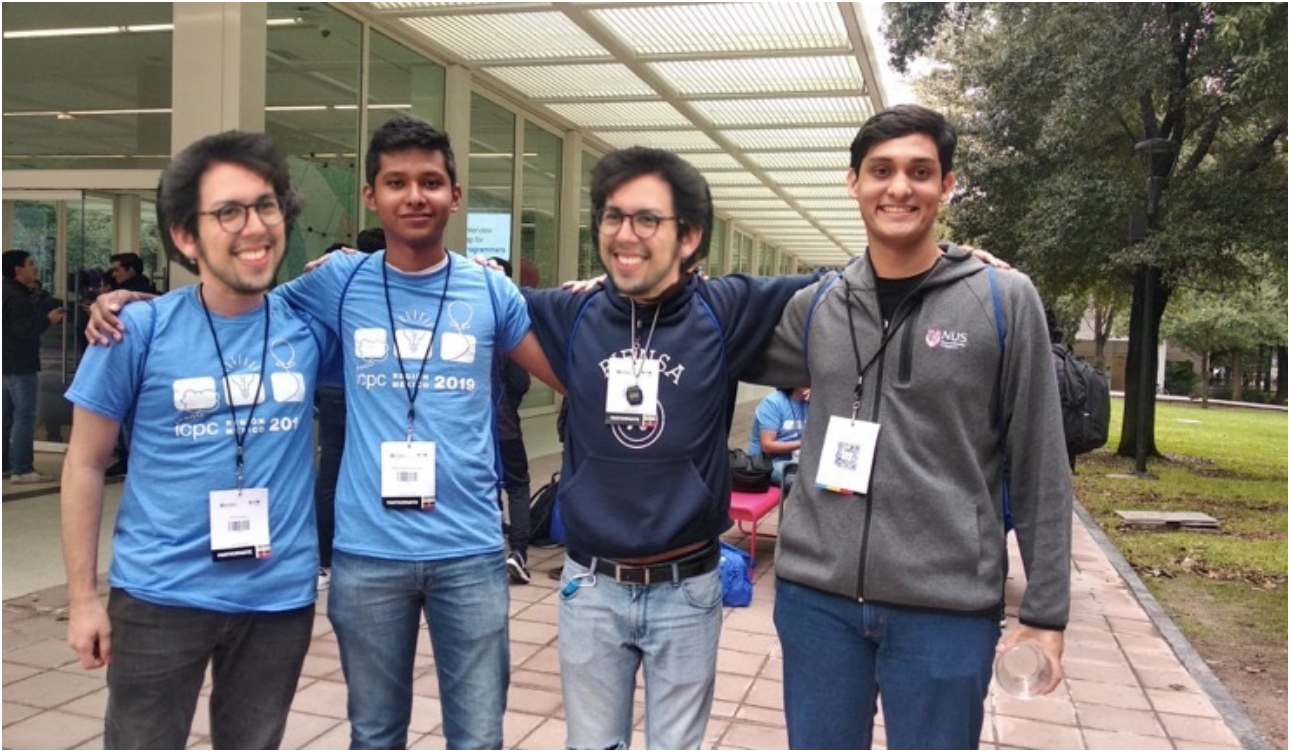
### 20.2 Sort along a line

```

void sortAlongLine(vector<Pt> &pts, Line l){
    sort(all(pts), [&](Pt a, Pt b){
        return a.dot(l.v) < b.dot(l.v);
    });
}

```





The end...