



Universidad de Guadalajara,
CUCEI

The Empire Strikes Back

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Think twice, code once

Template

tem.cpp

```
#pragma GCC optimize("Ofast,unroll-loops,no-stack-protector")
#include <bits/stdc++.h>
using namespace std;

#ifdef LOCAL
#include "debug.h"
#else
#define debug(...)
#endif

#define df(b, e) ((b) > (e))
#define fore(i, b, e) for (auto i = (b) - df(b, e); i != e - df(b, e); i += 1 - 2 * df(b, e))
#define sz(x) int(x.size())
#define all(x) begin(x), end(x)
#define f first
#define s second
#define pb push_back

using lli = long long;
using ld = long double;
using ii = pair<int, int>;
using vi = vector<int>;

int main() {
    cin.tie(0) -> sync_with_stdio(0), cout.tie(0);
    // solve the problem here D:
    return 0;
}

debug.h

template <class A, class B>
ostream & operator << (ostream &os, const pair<A, B> &p) {
    return os << "(" << p.first << ", " << p.second << "
        << ")";
}

template <class A, class B, class C>
basic_ostream<A, B> & operator << (basic_ostream<A, B>
    &os, const C &c) {
    os << "[";
    for (const auto &x : c)
        os << ", " + 2 * (&x == &begin(c)) << x;
    return os << "]";
}

void print(string s) { cout << endl; }

template <class H, class... T>
void print(string s, const H &h, const T&... t) {
    const static string reset = "\033[0m";
    bool ok = 1;
    do {
        if (s[0] == '\0') ok = 0;
        else cout << "\033[1;34m" << s[0] << reset;
        s = s.substr(1);
    } while (s.size() && s[0] != ',');
    if (ok) cout << ": " << "\033[3;95m" << h << reset;
    print(s, t...);
}
}
```

Randoms

```
mt19937 rng(chrono::steady_clock::now().
    time_since_epoch().count());
uniform_int_distribution<>(l, r)(rng);
```

Fastio

```
char gc() { return getchar_unlocked(); }

void readInt() {}
template <class H, class... T>
void readInt(H &h, T&&... t) {
    char c, s = 1;
    while (isspace(c = gc()));
    if (c == '-') s = -1, c = gc();
    for (h = c - '0'; isdigit(c = gc()); h = h * 10 + c - '0');
    h *= s;
    readInt(t...);
}

void readFloat() {}
template <class H, class... T>
void readFloat(H &h, T&&... t) {
    int c, s = 1, fp = 0, fpl = 1;
    while (isspace(c = gc()));
    if (c == '-') s = -1, c = gc();
    for (h = c - '0'; isdigit(c = gc()); h = h * 10 + c - '0');
    h *= s;
    if (h == '.')
        for (; isdigit(c = gc()); fp = fp * 10 + c - '0', fpl *= 10);
    h += (double)fp / fpl;
    readFloat(t...);
}
}
```

Compilation (gedit ~/.zshenv)

```
touch a_in{1..9} // make files a_in1, a_in2,..., a_in9
tee {a..m}.cpp < tem.cpp // "" with tem.cpp like base
cat > a_in1 // write on file a_in1
gedit a_in1 // open file a_in1
rm -r a.cpp // deletes file a.cpp :(
```

```
red='\x1B[0;31m'
green='\x1B[0;32m'
noColor='\x1B[0m'
alias flags='-Wall -Wextra -Wshadow -
    D_GLIBCXX_ASSERTIONS -fmax-errors=3 -O2 -w'
go() { g++ --std=c++11 $2 ${flags} $1.cpp && ./a.out }
debug() { go $1 -DLOCAL < $2 }
run() { go $1 "" < $2 }
```

```
random() { // Make small test cases!!!
g++ --std=c++11 $1.cpp -o prog
g++ --std=c++11 gen.cpp -o gen
g++ --std=c++11 brute.cpp -o brute
for ((i = 1; i <= 200; i++)); do
    printf "Test case #$i"
    ./gen > in
    diff -uwi <(. /prog < in) <(. /brute < in) > $1_diff
    if [[ ! $? -eq 0 ]]; then
        printf "${red} Wrong answer ${noColor}\n"
        break
    else
        printf "${green} Accepted ${noColor}\n"
    fi
done
}
```

```
test() {
g++ --std=c++11 $1.cpp -o prog
for ((i = 1; i <= 50; i++)); do
    [[ -f $1_in$i ]] || break
    printf "Test case #$i"
    diff -uwi <(. /prog < $1_in$i) $1_out$i > $1_diff
```

```

if [[ ! $? -eq 0 ]]; then
    printf "${red} Wrong answer ${noColor}\n"
else
    printf "${green} Accepted ${noColor}\n"
fi
done
}

```

Bump allocator

```

static char buf[450 << 20];
void* operator new(size_t s) {
    static size_t i = sizeof buf; assert(s < i);
    return (void *) &buf[i -= s];
}
void operator delete(void *) {}

```

1 Data structures

1.1 Disjoint set with rollback

```

struct Dsu {
    vector<int> pr, tot;
    stack<ii> what;

    Dsu(int n = 0) : pr(n + 5), tot(n + 5, 1) {
        iota(all(pr), 0);
    }

    int find(int u) {
        return pr[u] == u ? u : find(pr[u]);
    }

    void unite(int u, int v) {
        u = find(u), v = find(v);
        if (u == v)
            what.emplace(-1, -1);
        else {
            if (tot[u] < tot[v])
                swap(u, v);
            what.emplace(u, v);
            tot[u] += tot[v];
            pr[v] = u;
        }
    }

    ii rollback() {
        ii last = what.top();
        what.pop();
        int u = last.f, v = last.s;
        if (u != -1) {
            tot[u] -= tot[v];
            pr[v] = v;
        }
        return last;
    }
};

```

1.2 Sparse table

```

int lg[N + 1], sp[1 + __lg(N)][N];

int query(int l, int r) {
    int sum = 0LL;
    for (k, 1 + lg[N], 0)
        if (l + (1 << k) - 1 <= r) {
            sum += sp[k][l];
            l += (1 << k);
        }
    return sum;
}

int query(int l, int r) {
    int k = lg[r - l + 1];

```

```

    return min(sp[k][l], sp[k][r - (1 << k) + 1]);
}

void sparse() {
    for (i, 2, N + 1)
        lg[i] = lg[i >> 1] + 1;
    for (i, 0, n)
        sp[0][i] = a[i];
    for (int k = 1; (1 << k) <= n; k++)
        for (l, 0, n - (1 << k) + 1) {
            int r = l + (1 << (k - 1));
            sp[k][l] = min(sp[k - 1][l], sp[k - 1][r]);
        }
}

```

1.3 Min-Max queue

```

template <class T>
struct MinQueue : deque< pair<T, int> > {
    // add a element to the right {val, pos}
    void add(T val, int pos) {
        while (!empty() && back().f >= val)
            pop_back();
        emplace_back(val, pos);
    }
    // remove all less than pos
    void rem(int pos) {
        while (front().s < pos)
            pop_front();
    }

    T qmin() { return front().f; }
};

```

1.4 Squirtle decomposition

The perfect block size is *squirtle* of N



```

int blo[N], cnt[N][B], a[N];

void update(int i, int x) {
    cnt[blo[i]][x]--;
    a[i] = x;
    cnt[blo[i]][x]++;
}

int query(int l, int r, int x) {
    int tot = 0;
    while (l <= r)
        if (l % B == 0 && l + B - 1 <= r) {
            tot += cnt[blo[l]][x];
            l += B;
        } else {
            tot += (a[l] == x);
            l++;
        }
    return tot;
}

```

1.5 In-Out trick

```

vector<int> in[N], out[N];
vector<Query> queries;

for (x, 0, N) {
    for (int i : in[x])
        add(queries[i]);
    // solve
    for (int i : out[x])
        rem(queries[i]);
}

```

1.6 Parallel binary search

```

int lo[Q], hi[Q];

```

```

queue<int> solve[N];
vector<Query> queries;

fore (it, 0, 1 + __lg(N)) {
    fore (i, 0, sz(queries))
        if (lo[i] != hi[i]) {
            int mid = (lo[i] + hi[i]) / 2;
            solve[mid].emplace(i);
        }
    fore (x, 0, n) {
        // simulate
        while (!solve[x].empty()) {
            int i = solve[x].front();
            solve[x].pop();
            if (can(queries[i]))
                hi[i] = x;
            else
                lo[i] = x + 1;
        }
    }
}

```

1.7 Mo's algorithm

```

vector<Query> queries;
// N = 1e6, so aprox. sqrt(N) +/- C
uniform_int_distribution<int> dis(970, 1030);
const int blo = dis(rng);
sort(all(queries), [&](Query a, Query b) {
    const int ga = a.l / blo, gb = b.l / blo;
    if (ga == gb)
        return (ga & 1) ? a.r < b.r : a.r > b.r;
    return a.l < b.l;
});
int l = queries[0].l, r = 1 - 1;
for (Query &q : queries) {
    while (r < q.r)
        add(++r);
    while (r > q.r)
        rem(r--);
    while (l < q.l)
        rem(l--);
    while (l > q.l)
        add(--l);
    ans[q.i] = solve();
}

```

To make it faster, change the order to *hilbert(l, r)*

```

lli hilbert(int x, int y, int pw = 21, int rot = 0) {
    if (pw == 0)
        return 0;
    int hpw = 1 << (pw - 1);
    int k = ((x < hpw ? y < hpw ? 0 : 3 : y < hpw ? 1 : 2) + rot) & 3;
    const int d[4] = {3, 0, 0, 1};
    lli a = 1LL << ((pw < 1) - 2);
    lli b = hilbert(x & (x ^ hpw), y & (y ^ hpw), pw - 1, (rot + d[k]) & 3);
    return k * a + (d[k] ? a - b - 1 : b);
}

```

Mo's algorithm with updates in $\mathcal{O}(n^{\frac{5}{3}})$

- Choose a *block* of size $n^{\frac{2}{3}}$
- Do a normal Mo's algorithm, in the *Query* definition add an extra variable for the *updatesSoFar*
- Sort the queries by the order $(l/block, r/block, updatesSoFar)$
- If the update lies inside the current query, update the data structure properly

```

struct Update {

```

```

    int pos, prv, nxt;
};

void undo(Update &u) {
    if (l <= u.pos && u.pos <= r) {
        rem(u.pos);
        a[u.pos] = u.prv;
        add(u.pos);
    } else {
        a[u.pos] = u.prv;
    }
}

```

- Solve the problem :D

```

l = queries[0].l, r = 1 - 1, upd = sz(updates) - 1;
for (Query &q : queries) {
    while (upd < q.upd)
        dodo(updates[++upd]);
    while (upd > q.upd)
        undo(updates[upd--]);
    // write down the normal Mo's algorithm
}

```

1.8 Ordered tree

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template <class K, class V = null_type>
using ordered_tree = tree<K, V, less<K>, rb_tree_tag,
    tree_order_statistics_node_update>;
// less_equal<K> for multiset, multimap (?)
#define qrank order_of_key
#define qkth find_by_order

```

1.9 Unordered tree

```

struct chash {
    const uint64_t C = uint64_t(2e18 * 3) + 71;
    const int R = rng();
    uint64_t operator()(uint64_t x) const {
        return __builtin_bswap64((x ^ R) * C);
    }
};

```

```

template <class K, class V = null_type>
using unordered_tree = gp_hash_table<K, V, chash>;

```

1.10 D-dimensional Fenwick tree

```

template <class T, int ...N>
struct Fenwick {
    T v = 0;
    void update(T v) { this->v += v; }
    T query() { return v; }
};

```

```

template <class T, int N, int ...M>
struct Fenwick<T, N, M...> {
    #define lsb(x) (x & -x)
    Fenwick<T, M...> fenw[N + 1];

```

```

    template <typename... Args>
    void update(int i, Args... args) {
        for (; i <= N; i += lsb(i))
            fenw[i].update(args...);
    }

```

```

    template <typename... Args>
    T query(int l, int r, Args... args) {
        T v = 0;
        for (; r > 0; r -= lsb(r))

```

```

    v += fenw[r].query(args...);
    for (--l; l > 0; l -= lsb(l))
        v -= fenw[l].query(args...);
    return v;
}
};

```

1.11 Dynamic segment tree

```

struct Dyn {
    int l, r;
    lli sum = 0;
    Dyn *L, *R;

    Dyn(int l, int r) : l(l), r(r), L(0), R(0) {}

    void pull() {
        sum = (L ? L->sum : 0);
        sum += (R ? R->sum : 0);
    }

    void update(int p, lli v) {
        if (l == r) {
            sum += v;
            return;
        }
        int m = (l + r) >> 1;
        if (p <= m) {
            if (!L)
                L = new Dyn(l, m);
            L->update(p, v);
        } else {
            if (!R)
                R = new Dyn(m + 1, r);
            R->update(p, v);
        }
        pull();
    }

    lli qsum(int ll, int rr) {
        if (rr < l || r < ll || r < l)
            return 0;
        if (ll <= l && r <= rr)
            return sum;
        int m = (l + r) >> 1;
        return (L ? L->qsum(ll, rr) : 0) +
            (R ? R->qsum(ll, rr) : 0);
    }
};

```

1.12 Persistent segment tree

```

struct Per {
    int l, r;
    lli sum = 0;
    Per *L, *R;

    Per(int l, int r) : l(l), r(r), L(0), R(0) {}

    Per* pull() {
        sum = L->sum + R->sum;
        return this;
    }

    void build() {
        if (l == r)
            return;
        int m = (l + r) >> 1;
        (L = new Per(l, m))->build();
        (R = new Per(m + 1, r))->build();
        pull();
    }
};

```

```

Per* update(int p, lli v) {
    if (p < l || r < p)
        return this;
    Per* t = new Per(l, r);
    if (l == r) {
        t->sum = v;
        return t;
    }
    t->L = L->update(p, v);
    t->R = R->update(p, v);
    return t->pull();
}

lli qsum(int ll, int rr) {
    if (r < ll || rr < l)
        return 0;
    if (ll <= l && r <= rr)
        return sum;
    return L->qsum(ll, rr) + R->qsum(ll, rr);
}
};

```

1.13 Wavelet tree

```

struct Wav {
    #define iter int * // vector<int>::iterator
    int lo, hi;
    Wav *L, *R;
    vi amt;

    Wav(int lo, int hi) : lo(lo), hi(hi), L(0), R(0) {}

    void build(iter b, iter e) { // array 1-indexed
        if (lo == hi || b == e)
            return;
        amt.reserve(e - b + 1);
        amt.pb(0);
        int m = (lo + hi) >> 1;
        for (auto it = b; it != e; it++)
            amt.pb(amt.back() + (*it <= m));
        auto p = stable_partition(b, e, [=](int x) {
            return x <= m;
        });
        (L = new Wav(lo, m))->build(b, p);
        (R = new Wav(m + 1, hi))->build(p, e);
    }

    int qkth(int l, int r, int k) {
        if (r < l)
            return 0;
        if (lo == hi)
            return lo;
        if (k <= amt[r] - amt[l - 1])
            return L->qkth(amt[l - 1] + 1, amt[r], k);
        return R->qkth(l - amt[l - 1], r - amt[r], k - amt[r] + amt[l - 1]);
    }

    int qleq(int l, int r, int mx) {
        if (r < l || mx < lo)
            return 0;
        if (hi <= mx)
            return r - l + 1;
        return L->qleq(amt[l - 1] + 1, amt[r], mx) +
            R->qleq(l - amt[l - 1], r - amt[r], mx);
    }
};

```

1.14 Li Chao tree

```

struct Fun {

```

```

lli m = 0, c = inf;
lli operator ()(lli x) const { return m * x + c; }
};

struct LiChao {
    Fun f;
    lli l, r;
    LiChao *L, *R;

    LiChao(lli l, lli r, Fun f = {}) :
        l(l), r(r), f(f), L(0), R(0) {}

    void add(Fun &g) {
        if (f(l) <= g(l) && f(r) <= g(r))
            return;
        if (g(l) < f(l) && g(r) < f(r)) {
            f = g;
            return;
        }
        lli m = (l + r) >> 1;
        if (g(m) < f(m))
            swap(f, g);
        if (g(l) <= f(l))
            L = L ? (L->add(g), L) : new LiChao(l, m, g);
        else
            R = R ? (R->add(g), R) : new LiChao(m + 1, r, g);
    }

    lli query(lli x) {
        if (l == r)
            return f(x);
        lli m = (l + r) >> 1;
        if (x <= m)
            return min(f(x), L ? L->query(x) : inf);
        return min(f(x), R ? R->query(x) : inf);
    }
};

```

1.15 Treap

```

typedef struct Node* Treap;
struct Node {
    uint32_t pri = rng();
    int val;
    Treap ch[2] = {0, 0};
    int sz = 1, flip = 0;
    Node(int val) : val(val) {}
};

void push(Treap t) {
    if (!t)
        return;
    if (t->flip) {
        swap(t->ch[0], t->ch[1]);
        for (Treap ch : t->ch) if (ch)
            ch->flip ^= 1;
        t->flip = 0;
    }
}

Treap pull(Treap t) {
#define gsz(t) (t ? t->sz : 0)
    t->sz = 1;
    for (Treap ch : t->ch)
        push(ch), t->sz += gsz(ch);
    return t;
}

pair<Treap, Treap> split(Treap t, int val) {
    // <= val goes to the left, > val to the right
    if (!t)

```

```

        return {t, t};
    push(t);
    if (val < t->val) {
        auto p = split(t->ch[0], val);
        t->ch[0] = p.s;
        return {p.f, pull(t)};
    }
    auto p = split(t->ch[1], val);
    t->ch[1] = p.f;
    return {pull(t), p.s};
}

pair<Treap, Treap> splitsz(Treap t, int sz) {
    // <= sz goes to the left, > sz to the right
    if (!t)
        return {t, t};
    push(t);
    if (sz <= gsz(t->ch[0])) {
        auto p = splitsz(t->ch[0], sz);
        t->ch[0] = p.s;
        return {p.f, pull(t)};
    }
    auto p = splitsz(t->ch[1], sz - gsz(t->ch[0]) - 1);
    t->ch[1] = p.f;
    return {pull(t), p.s};
}

Treap merge(Treap l, Treap r) {
    if (!l || !r)
        return l ? l : r;
    push(l), push(r);
    if (l->pri > r->pri)
        return l->ch[1] = merge(l->ch[1], r), pull(l);
    else
        return r->ch[0] = merge(l, r->ch[0]), pull(r);
}

Treap qkth(Treap t, int k) { // 0-indexed
    if (!t)
        return t;
    push(t);
    int sz = gsz(t->ch[0]);
    if (sz == k)
        return t;
    return k < sz ? qkth(t->ch[0], k) : qkth(t->ch[1], k - sz - 1);
}

int qrank(Treap t, int val) { // 0-indexed
    if (!t)
        return -1;
    push(t);
    if (val < t->val)
        return qrank(t->ch[0], val);
    if (t->val == val)
        return gsz(t->ch[0]);
    return gsz(t->ch[0]) + qrank(t->ch[1], val) + 1;
}

Treap insert(Treap t, int val) {
    auto p1 = split(t, val);
    auto p2 = split(p1.f, val - 1);
    return merge(p2.f, merge(new Node(val), p1.s));
}

Treap erase(Treap t, int val) {
    auto p1 = split(t, val);
    auto p2 = split(p1.f, val - 1);
    return merge(p2.f, p1.s);
}

```

2 Graphs

2.1 Tarjan algorithm (SCC)

```
vector<vi> scc;
int tin[N], fup[N];
bitset<N> still;
stack<int> stk;
int timer = 0;

void tarjan(int u) {
    tin[u] = fup[u] = ++timer;
    still[u] = true;
    stk.push(u);
    for (int v : graph[u]) {
        if (!tin[v])
            tarjan(v);
        if (still[v])
            fup[u] = min(fup[u], fup[v]);
    }
    if (fup[u] == tin[u]) {
        scc.pb({});
        int v;
        do {
            v = stk.top();
            stk.pop();
            still[v] = false;
            scc.back().pb(v);
        } while (v != u);
    }
}
```

2.2 Kosaraju algorithm (SCC)

```
int scc[N], k = 0;
char vis[N];
vi order;

void dfs1(int u) {
    vis[u] = 1;
    for (int v : graph[u])
        if (vis[v] != 1)
            dfs1(v);
    order.pb(u);
}

void dfs2(int u, int k) {
    vis[u] = 2, scc[u] = k;
    for (int v : rgraph[u]) // reverse graph
        if (vis[v] != 2)
            dfs2(v, k);
}

void kosaraju() {
    for (int u = 1, n + 1)
        if (vis[u] != 1)
            dfs1(u);
    reverse(all(order));
    for (int u : order)
        if (vis[u] != 2)
            dfs2(u, ++k);
}
```

2.3 Two Sat

```
void add(int u, int v) {
    graph[u].pb(v);
    rgraph[v].pb(u);
}

void implication(int u, int v) {
    #define neg(u) ((n) + (u))
    add(u, v);
    add(neg(v), neg(u));
}
```

```
}

pair<bool, vi> satisfy(int n) {
    kosaraju(2 * n); // size of the two-sat is 2 * n
    vi ans(n + 1, 0);
    for (int u = 1, n + 1) {
        if (scc[u] == scc[neg(u)])
            return {0, ans};
        ans[u] = scc[u] > scc[neg(u)];
    }
    return {1, ans};
}
```

2.4 Topological sort

2.5 Cutpoints and Bridges

```
int tin[N], fup[N], timer = 0;

void findWeakness(int u, int p = 0) {
    tin[u] = fup[u] = ++timer;
    int children = 0;
    for (int v : graph[u]) if (v != p) {
        if (!tin[v]) {
            ++children;
            findWeakness(v, u);
            fup[u] = min(fup[u], fup[v]);
            if (fup[v] >= tin[u] && p) // u is a cutpoint
                if (fup[v] > tin[u]) // bridge u -> v
                    ;
        }
        fup[u] = min(fup[u], tin[v]);
    }
    if (!p && children > 1) // u is a cutpoint
        ;
}
```

2.6 Detect a cycle

```
bool cycle(int u) {
    vis[u] = 1;
    for (int v : graph[u]) {
        if (vis[v] == 1)
            return true;
        if (!vis[v] && cycle(v))
            return true;
    }
    vis[u] = 2;
    return false;
}
```

2.7 Euler tour for Mo's in a tree

Mo's in a tree, extended euler tour $tin[u] = ++timer$, $tout[u]$

- $u = lca(u, v)$, query($tin[u]$, $tin[v]$)
- $u \neq lca(u, v)$, query($tout[u]$, $tin[v]$) + query($tin[lca]$, $tin[lca]$)

2.8 Lowest common ancestor (LCA)

```
const int LogN = 1 + __lg(N);
int pr[LogN][N], dep[N];

void dfs(int u, int pr[]) {
    for (int v : graph[u])
        if (v != pr[u]) {
            pr[v] = u;
            dep[v] = dep[u] + 1;
            dfs(v, pr);
        }
}

int lca(int u, int v) {
    if (dep[u] > dep[v])
        swap(u, v);
    for (int k = LogN, 0)
        if (dep[v] - dep[u] >= (1 << k))
```



```

    v = pr[k][v];
    if (u == v)
        return u;
    for (k, LogN, 0)
        if (pr[k][v] != pr[k][u])
            u = pr[k][u], v = pr[k][v];
    return pr[0][u];
}

int dist(int u, int v) {
    return dep[u] + dep[v] - 2 * dep[lca(u, v)];
}

void init(int r) {
    dfs(r, pr[0]);
    for (k, 1, LogN)
        for (u, 1, n + 1)
            pr[k][u] = pr[k - 1][pr[k - 1][u]];
}

```

2.9 Guni

```

int tin[N], tout[N], who[N], sz[N], heavy[N], color[N];
int timer = 0;

int dfs(int u, int pr = 0) {
    sz[u] = 1, tin[u] = ++timer, who[timer] = u;
    for (int v : graph[u]) if (v != pr) {
        sz[u] += dfs(v, u);
        if (sz[v] > sz[heavy[u]])
            heavy[u] = v;
    }
    return tout[u] = timer, sz[u];
}

void guni(int u, int pr = 0, bool keep = 0) {
    for (int v : graph[u])
        if (v != pr && v != heavy[u])
            guni(v, u, 0);
    if (heavy[u])
        guni(heavy[u], u, 1);
    for (int v : graph[u])
        if (v != pr && v != heavy[u])
            for (i, tin[v], tout[v] + 1)
                add(color[who[i]]);
    add(color[u]);
    // Solve the subtree queries here
    if (keep == 0)
        for (i, tin[u], tout[u] + 1)
            rem(color[who[i]]);
}

```

2.10 Centroid decomposition

```

int cdp[N], sz[N];
bitset<N> rem;

int dfsz(int u, int p = 0) {
    sz[u] = 1;
    for (int v : graph[u])
        if (v != p && !rem[v])
            sz[u] += dfsz(v, u);
    return sz[u];
}

int centroid(int u, int n, int p = 0) {
    for (int v : graph[u])
        if (v != p && !rem[v] && 2 * sz[v] > n)
            return centroid(v, n, u);
    return u;
}

```

```

void solve(int u, int p = 0) {
    cdp[u] = centroid(u, dfsz(u));
    rem[u] = true;
    for (int v : graph[u])
        if (!rem[v])
            solve(v, u);
}

```

2.11 Heavy-light decomposition

```

int pr[N], dep[N], sz[N], heavy[N], head[N], pos[N],
    who[N], timer = 0;
Lazy* tree; // generally a lazy segtree

int dfs(int u) {
    sz[u] = 1, heavy[u] = head[u] = 0;
    for (int v : graph[u]) if (v != pr[u]) {
        pr[v] = u;
        dep[v] = dep[u] + 1;
        sz[u] += dfs(v);
        if (sz[v] > sz[heavy[u]])
            heavy[u] = v;
    }
    return sz[u];
}

void hld(int u, int h) {
    head[u] = h, pos[u] = ++timer, who[timer] = u;
    if (heavy[u] != 0)
        hld(heavy[u], h);
    for (int v : graph[u])
        if (v != pr[u] && v != heavy[u])
            hld(v, v);
}

template <class F>
void processPath(int u, int v, F fun) {
    for (; head[u] != head[v]; v = pr[head[v]]) {
        if (dep[head[u]] > dep[head[v]])
            swap(u, v);
        fun(pos[head[v]], pos[v]);
    }
    if (dep[u] > dep[v])
        swap(u, v);
    if (u != v)
        fun(pos[heavy[u]], pos[v]);
    fun(pos[u], pos[u]); // process lca(u, v) too?
}

```

```

void updatePath(int u, int v, lli z) {
    processPath(u, v, [&](int l, int r) {
        tree->update(l, r, z);
    });
}

lli queryPath(int u, int v) {
    lli sum = 0;
    processPath(u, v, [&](int l, int r) {
        sum += tree->qsum(l, r);
    });
    return sum;
}

```

2.12 Link-Cut tree

```

typedef struct Node* Splay;
struct Node {
    int val, mx = 0;
    Splay ch[2] = {0, 0}, p = 0;
    int sz = 1, flip = 0;
    Node(int val) : val(val), mx(val) {}
};

```

```

void push(Splay u) {
    if (!u || !u->flip)
        return;
    swap(u->ch[0], u->ch[1]);
    for (Splay v : u->ch)
        if (v) v->flip ^= 1;
    u->flip = 0;
}

void pull(Splay u) {
    #define gsz(t) (t ? t->sz : 0)
    u->sz = 1, u->mx = u->val;
    for (Splay v : u->ch) if (v) {
        push(v);
        u->sz += gsz(v);
        u->mx = max(u->mx, v->mx);
    }
}

int dir(Splay u) {
    if (!u->p) return -2; // root of the LCT component
    if (u->p->ch[0] == u) return 0; // left child
    if (u->p->ch[1] == u) return 1; // right child
    return -1; // root of current splay tree
}

void add(Splay u, Splay v, int d) {
    if (v) v->p = u;
    if (d >= 0) u->ch[d] = v;
}

void rot(Splay u) { // assume p and p->p propagated
    int x = dir(u);
    Splay g = u->p;
    add(g->p, u, dir(g));
    add(g, u->ch[x ^ 1], x);
    add(u, g, x ^ 1);
    pull(g), pull(u);
}

void splay(Splay u) {
    #define isRoot(u) (dir(u) < 0)
    while (!isRoot(u) && !isRoot(u->p)) {
        push(u->p->p), push(u->p), push(u);
        rot(dir(u) == dir(u->p) ? u->p : u);
        rot(u);
    }
    if (!isRoot(u)) push(u->p), push(u), rot(u);
    push(u);
}

// puts u on the preferred path, splay (right subtree
// is empty)
void access(Splay u) {
    for (Splay v = u, last = 0; v; v = v->p) {
        splay(v); // now switch virtual children, i don't
        // know what this means!!
        // if (last) v->vsub -= last->sub;
        // if (v->ch[1]) v->vsub += v->ch[1]->sub;
        v->ch[1] = last, pull(v), last = v;
    }
    splay(u);
}

void rootify(Splay u) {
    access(u), u->flip ^= 1, access(u);
}

Splay lca(Splay u, Splay v) {
    if (u == v) return u;

```

```

    access(u), access(v);
    if (!u->p) return 0;
    return splay(u), u->p ? u;
}

bool connected(Splay u, Splay v) {
    return lca(u, v);
}

void link(Splay u, Splay v) { // make u parent of v,
    // make sure they aren't connected
    if (!connected(u, v)) {
        rootify(v), access(u);
        add(v, u, 0), pull(v);
    }
}

void cut(Splay u) { // cut u from its parent
    access(u);
    u->ch[0] = u->ch[0]->p = 0;
    pull(u);
}

void cut(Splay u, Splay v) { // if u, v adj in tree
    rootify(u), access(v), cut(v);
}

Splay getRoot(Splay u) {
    access(u);
    while (u->ch[0])
        u = u->ch[0], push(u);
    return access(u), u;
}

Splay lift(Splay u, int k) {
    push(u);
    int sz = gsz(u->ch[0]);
    if (sz == k)
        return splay(u), u;
    return k < sz ? lift(u->ch[0], k) : lift(u->ch[1], k
        - sz - 1);
}

Splay ancestor(Splay u, int k) {
    return access(u), lift(u, gsz(u->ch[0]) - k);
}

Splay query(Splay u, Splay v) {
    return rootify(u), access(v), v;
}

```

Splay lct[N];

3 Flows

3.1 Dinic $\mathcal{O}(\min(E \cdot \text{flow}, V^2 E))$

If the network is massive, try to compress it by looking for patterns. Dinic with scaling works in $\mathcal{O}(EV \cdot \log(\max Cap))$.

```

template <class F>
struct Dinic {
    struct Edge {
        int v, inv;
        F cap, flow;
        Edge(int v, F cap, int inv) :
            v(v), cap(cap), flow(0), inv(inv){}
    };

    F eps = (F) 1e-9, lim = (F) 1e-9;
    const bool scaling = 0;
    int s, t, n, m = 0;
    vector< vector<Edge> > g;

```

```

vi dist, ptr;

Dinic(int n, int ss = -1, int tt = -1) :
    n(n), g(n + 5), dist(n + 5), ptr(n + 5) {
    s = ss == -1 ? n + 1 : ss;
    t = tt == -1 ? n + 2 : tt;
}

void add(int u, int v, F cap) {
    g[u].pb(Edge(v, cap, sz(g[v])));
    g[v].pb(Edge(u, 0, sz(g[u]) - 1));
    lim = (scaling ? max(lim, cap) : lim);
    m += 2;
}

bool bfs() {
    fill(all(dist), -1);
    queue<int> qu({s});
    dist[s] = 0;
    while (sz(qu) && dist[t] == -1) {
        int u = qu.front(); qu.pop();
        for (Edge &e : g[u] if (dist[e.v] == -1)
            if (scaling ? e.cap - e.flow >= lim : e.cap -
                e.flow > eps) {
                dist[e.v] = dist[u] + 1;
                qu.push(e.v);
            }
    }
    return dist[t] != -1;
}

F dfs(int u, F flow = numeric_limits<F>::max()) {
    if (flow <= eps || u == t)
        return max<F>(0, flow);
    for (int &i = ptr[u]; i < sz(g[u]); i++) {
        Edge &e = g[u][i];
        if (e.cap - e.flow > eps && dist[u] + 1 == dist[
            e.v]) {
            F pushed = dfs(e.v, min<F>(flow, e.cap - e.flow
                ));
            if (pushed > eps) {
                e.flow += pushed;
                g[e.v][e.inv].flow -= pushed;
                return pushed;
            }
        }
    }
    return 0;
}

F maxFlow() {
    F flow = 0;
    for (lim = scaling ? lim : 1; lim > eps; lim /= 2)
        while (bfs()) {
            fill(all(ptr), 0);
            while (F pushed = dfs(s))
                flow += pushed;
        }
    return flow;
}
};

```

3.2 Min cost flow $\mathcal{O}(\min(E \cdot \text{flow}, V^2 E))$

If the network is massive, try to compress it by looking for patterns.

```

template <class C, class F>
struct MCMF {
    static constexpr F eps = (F) 1e-9;
    struct Edge {
        int u, v, inv;
        F cap, flow;
    };
};

```

```

C cost;
Edge(int u, int v, C cost, F cap, int inv) :
    u(u), v(v), cost(cost), cap(cap), flow(0), inv(
        inv) {}

};

int s, t, n, m = 0;
vector< vector<Edge> > g;
vector<Edge*> prev;
vector<C> cost;
vi state;

MCMF(int n, int ss = -1, int tt = -1):
    n(n), g(n + 5), cost(n + 5), state(n + 5), prev(n
        + 5) {
    s = ss == -1 ? n + 1 : ss;
    t = tt == -1 ? n + 2 : tt;
}

void add(int u, int v, C cost, F cap) {
    g[u].pb(Edge(u, v, cost, cap, sz(g[v])));
    g[v].pb(Edge(v, u, -cost, 0, sz(g[u]) - 1));
    m += 2;
}

bool bfs() {
    fill(all(state), 0);
    fill(all(cost), numeric_limits<C>::max());
    deque<int> qu;
    qu.push_back(s);
    state[s] = 1, cost[s] = 0;
    while (sz(qu)) {
        int u = qu.front(); qu.pop_front();
        state[u] = 2;
        for (Edge &e : g[u] if (e.cap - e.flow > eps)
            if (cost[u] + e.cost < cost[e.v]) {
                cost[e.v] = cost[u] + e.cost;
                prev[e.v] = &e;
                if (state[e.v] == 2 || (sz(qu) && cost[qu.
                    front()] > cost[e.v]))
                    qu.push_front(e.v);
                else if (state[e.v] == 0)
                    qu.push_back(e.v);
                state[e.v] = 1;
            }
    }
    return cost[t] != numeric_limits<C>::max();
}

pair<C, F> minCostFlow() {
    C cost = 0; F flow = 0;
    while (bfs()) {
        F pushed = numeric_limits<F>::max();
        for (Edge* e = prev[t]; e != nullptr; e = prev[e
            ->u])
            pushed = min(pushed, e->cap - e->flow);
        for (Edge* e = prev[t]; e != nullptr; e = prev[e
            ->u]) {
            e->flow += pushed;
            g[e->v][e->inv].flow -= pushed;
            cost += e->cost * pushed;
        }
        flow += pushed;
    }
    return make_pair(cost, flow);
}
};

```

3.3 Hopcroft-Karp $\mathcal{O}(E\sqrt{V})$

```

struct HopcroftKarp {
    int n, m = 0;
};

```

```

vector<vi> g;
vi dist, match;

HopcroftKarp(int _n) : n(5 + _n), g(n + 5), dist(n + 5), match(n + 5, 0) {}

void add(int u, int v) {
    g[u].pb(v), g[v].pb(u);
    m += 2;
}

bool bfs() {
    queue<int> qu;
    fill(all(dist), -1);
    for (u, 1, n + 1)
        if (!match[u])
            dist[u] = 0, qu.push(u);
    while (!qu.empty()) {
        int u = qu.front(); qu.pop();
        for (int v : g[u])
            if (dist[match[v]] == -1) {
                dist[match[v]] = dist[u] + 1;
                if (match[v])
                    qu.push(match[v]);
            }
    }
    return dist[0] != -1;
}

bool dfs(int u) {
    for (int v : g[u])
        if (!match[v] || (dist[u] + 1 == dist[match[v]] && dfs(match[v]))) {
            match[u] = v, match[v] = u;
            return 1;
        }
    dist[u] = 1 << 30;
    return 0;
}

int maxMatching() {
    int tot = 0;
    while (bfs())
        for (u, 1, n + 1)
            tot += match[u] ? 0 : dfs(u);
    return tot;
}
};

```

3.4 Hungarian $\mathcal{O}(N^3)$

n jobs, m people

```

template <class C>
pair<C, vi> Hungarian(vector< vector<C> > &a) {
    int n = sz(a), m = sz(a[0]), p, q, j, k; // n <= m
    vector<C> fx(n, numeric_limits<C>::min()), fy(m, 0);
    vi x(n, -1), y(m, -1);
    for (i, 0, n)
        for (j, 0, m)
            fx[i] = max(fx[i], a[i][j]);
    for (i, 0, n) {
        vi t(m, -1), s(n + 1, i);
        for (p = q = 0; p <= q && x[i] < 0; p++)
            for (k = s[p], j = 0; j < m && x[i] < 0; j++)
                if (abs(fx[k] + fy[j] - a[k][j]) < eps && t[j] < 0) {
                    s[++q] = y[j], t[j] = k;
                    if (s[q] < 0) for (p = j; p >= 0; j = p)
                        y[j] = k = t[j], p = x[k], x[k] = j;
                }
        if (x[i] < 0) {

```

```

            C d = numeric_limits<C>::max();
            for (k, 0, q + 1)
                for (j, 0, m) if (t[j] < 0)
                    d = min(d, fx[s[k]] + fy[j] - a[s[k]][j]);
            for (j, 0, m)
                fy[j] += (t[j] < 0 ? 0 : d);
            for (k, 0, q + 1)
                fx[s[k]] -= d;
            i--;
        }
    }
    C cost = 0;
    for (i, 0, n) cost += a[i][x[i]];
    return make_pair(cost, x);
}

```

4 Strings

4.1 Hash

```

vi p = {10006793, 1777771, 10101283, 10101823, 1013635
        9, 10157387, 10166249};
vi mod = {999992867, 107077777, 999727999, 1000008223
        , 1000009999, 1000003211, 1000027163, 1000002193,
        1000000123};
int pw[2][N];

struct Hash {
    vector<vi> h;

    Hash(string &s) : h(2, vi(sz(s) + 1, 0)) {
        for (i, 0, 2)
            for (j, 0, sz(s))
                h[i][j + 1] = (h[i][j] + lli(s[j] - 'a' + 1) *
                    pw[i][j]) % mod[i];
    }

    lli query(int l, int r) {
        lli f[2] = {0, 0};
        for (i, 0, 2)
            f[i] = 1LL * (h[i][r + 1] - h[i][l] + mod[i]) %
                mod[i] * pw[i][N - l - 1] % mod[i];
        return (f[0] << 32) | f[1];
    }
};

shuffle(all(p), rng), shuffle(all(mod), rng);
for (i, 0, 2) {
    pw[i][0] = 1LL;
    for (j, 1, N)
        pw[i][j] = 1LL * p[i] * pw[i][j - 1] % mod[i];
}

```

4.2 KMP

period = $n - \text{lps}[n - 1]$, period(abcabc) = 3, $n \bmod \text{period} \equiv 0$

```

int lps[N];

void pre(string &s) {
    int j = (lps[0] = 0);
    for (i, 1, sz(s)) {
        while (j && s[i] != s[j])
            j = lps[j - 1];
        if (s[i] == s[j])
            j++;
        lps[i] = j;
    }
}

// how many times t occurs in s
int kmp(string &s, string &t) {
    pre(t);
    int j = 0, tot = 0;
    for (i, 0, sz(s)) {
        while (j && s[i] != t[j])

```

```

    j = lps[j - 1];
    if (s[i] == t[j])
        j++;
    if (j == sz(t))
        tot++; // pos: i - sz(t) + 1;
}
return tot;
}

```

4.3 KMP automaton

```

int go[N][A];

void kmpAutomaton(string &s) {
    s += "$";
    for (i, 0, sz(s))
        for (c, 0, A) {
            if (i && s[i] != 'a' + c)
                go[i][c] = go[lps[i - 1]][c];
            else
                go[i][c] = i + ('a' + c == s[i]);
        }
    s.pop_back();
}

```

4.4 Z algorithm

```

int z[N];

void zf(string &s) {
    for (int i = 1, l = 0, r = 0; i < sz(s); i++) {
        if (i <= r)
            z[i] = min(r - i + 1, z[i - l]);
        while (i + z[i] < sz(s) && s[i + z[i]] == s[z[i]])
            ++z[i];
        if (i + z[i] - 1 > r)
            l = i, r = i + z[i] - 1;
    }
}

```

4.5 Manacher algorithm

```

int aba[2][N];

void manacher(string &s) {
    for (k, 0, 2) {
        int l = 0, r = 0;
        for (i, 0, sz(s)) {
            if (i < r)
                aba[k][i] = min(r - i + 1, aba[k][l + r - i + 1]);
            int p = i - aba[k][i], q = i + aba[k][i] - 1;
            while (p >= 1 && q + 1 < sz(s) && s[p - 1] == s[q + 1])
                ++aba[k][i], --p, ++q;
            if (q > r)
                l = p, r = q;
        }
    }
}

```

4.6 Suffix array

- Duplicates $\sum_{i=1}^n lcp[i]$
- Longest Common Substring of various strings
Add *notUsed* characters between strings, i.e. $a+\$+b+\#+c$
Use two-pointers to find a range $[l, r]$ such that all *notUsed* characters are present, then $query(lcp[l + 1], \dots, lcp[r])$ for that window is the common length.

```

struct SuffixArray {
    int n;
    string s;
    vi sa, lcp;

```

```

    SuffixArray(string &s) : n(sz(s) + 1), s(s), sa(n),
        lcp(n) {
        vi top(max(256, n)), rk(n);
        for (i, 0, n)
            top[rk[i] = s[i] & 255]++;
        partial_sum(all(top), top.begin());
        for (i, 0, n)
            sa[--top[rk[i]]] = i;
        vi sb(n);
        for (int len = 1, j; len < n; len <= 1) {
            for (i, 0, n) {
                j = (sa[i] - len + n) % n;
                sb[top[rk[j]]++] = j;
            }
            sa[sb[top[0] = 0]] = j = 0;
            for (i, 1, n) {
                if (rk[sb[i]] != rk[sb[i - 1]] || rk[sb[i] + len] != rk[sb[i - 1] + len])
                    top[++j] = i;
                sa[sb[i]] = j;
            }
            copy(all(sa), rk.begin());
            copy(all(sb), sa.begin());
            if (j >= n - 1)
                break;
        }
        for (int i = 0, j = rk[lcp[0] = 0], k = 0; i < n - 1; i++, k++)
            while (k >= 0 && s[i] != s[sa[j - 1] + k])
                lcp[j] = k--, j = rk[sa[j] + 1];
    }

    char at(int i, int j) {
        int k = sa[i] + j;
        return k < n ? s[k] : 'z' + 1;
    }

    bool count(string &t) {
        ii lo(0, n - 1), hi(0, n - 1);
        for (i, 0, sz(t)) {
            while (lo.f + 1 < lo.s) {
                int mid = (lo.f + lo.s) / 2;
                if (at(mid, i) < t[i])
                    lo.f = mid;
                else
                    lo.s = mid;
            }
            while (hi.f + 1 < hi.s) {
                int mid = (hi.f + hi.s) / 2;
                if (t[i] < at(mid, i))
                    hi.s = mid;
                else
                    hi.f = mid;
            }
            int p1 = (at(lo.f, i) == t[i] ? lo.f : lo.s);
            int p2 = (at(hi.s, i) == t[i] ? hi.s : hi.f);
            if (at(p1, i) != t[i] || at(p2, i) != t[i] || p1 > p2)
                return 0;
            lo = hi = ii(p1, p2);
        }
        return lo.s - lo.f + 1;
    }
};

```

4.7 Suffix automaton

- $len[u] - len[link[u]] = \text{distinct strings}$
- Number of different substrings (dp)

$$diff(u) = 1 + \sum_{v \in trie[u]} diff(v)$$

- Total length of all different substrings (2 x dp)

$$totLen(u) = \sum_{v \in trie[u]} diff(v) + totLen(v)$$

- Leftmost occurrence $pos[u] = len[u] - 1$
if is **clone** then $pos[clone] = pos[q]$
- All occurrence positions
- Smallest cyclic shift
Construct sam of $s + s$, find the lexicographically smallest path of $sz(s)$
- Shortest non-appearing string

$$nonAppearing(u) = \min_{v \in trie[u]} nonAppearing(v) + 1$$

```
struct SuffixAutomaton {
    vector< map<char, int> > trie;
    vi link, len;
    int last;

    SuffixAutomaton() { last = newNode(); }

    int newNode() {
        trie.pb({}), link.pb(-1), len.pb(0);
        return sz(trie) - 1;
    }

    void extend(char c) {
        int u = newNode();
        len[u] = len[last] + 1;
        int p = last;
        while (p != -1 && !trie[p].count(c)) {
            trie[p][c] = u;
            p = link[p];
        }
        if (p == -1)
            link[u] = 0;
        else {
            int q = trie[p][c];
            if (len[p] + 1 == len[q])
                link[u] = q;
            else {
                int clone = newNode();
                len[clone] = len[p] + 1;
                trie[clone] = trie[q];
                link[clone] = link[q];
                while (p != -1 && trie[p][c] == q) {
                    trie[p][c] = clone;
                    p = link[p];
                }
                link[q] = link[u] = clone;
            }
        }
        last = u;
    }

    string qkthSubstring(lli kth, int u = 0) {
        // number of different substrings (dp)
        string s = "";
        while (kth > 0)
            for (auto &[c, v] : trie[u]) {
                if (kth <= diff(v)) {
                    s.pb(c), kth--, u = v;
                    break;
                }
            }
            kth -= diff(v);
        return s;
    }

    void occurs() {
```

```
// occ[u] = 1, occ[clone] = 0
vi who;
for (u, 1, sz(trie))
    who.pb(u);
sort(all(who), [&](int u, int v) {
    return len[u] > len[v];
});
for (int u : who)
    occ[link[u]] += occ[u];
}

int qocc(string &s, int u = 0) {
    for (char c : s) {
        if (!trie[u].count(c))
            return 0;
        u = trie[u][c];
    }
    return occ[u];
}

int longestCommonSubstring(string &s, int u = 0) {
    int mx = 0, clen = 0;
    for (char c : s) {
        while (u && !trie[u].count(c)) {
            u = link[u];
            clen = len[u];
        }
        if (trie[u].count(c))
            u = trie[u][c], clen++;
        mx = max(mx, clen);
    }
    return mx;
}

string smallestCyclicShift(int n, int u = 0) {
    string s = "";
    for (i, 0, n) {
        char c = trie[u].begin()->f;
        s += c;
        u = trie[u][c];
    }
    return s;
}

int leftmost(string &s, int u = 0) {
    for (char c : s) {
        if (!trie[u].count(c))
            return -1;
        u = trie[u][c];
    }
    return pos[u] - sz(s) + 1;
}
} sam;
```

4.8 Aho corasick

```
struct AhoCorasick {
    vector< map<char, int> > trie;
    vi link, cnt;

    AhoCorasick() { newNode(); }

    int newNode() {
        trie.pb({}), link.pb(0), cnt.pb(0);
        return sz(trie) - 1;
    }

    void insert(string &s, int u = 0) {
        for (char c : s) {
            if (!trie[u].count(c))
                trie[u][c] = newNode();
            u = trie[u][c];
        }
```

```

    }
    cnt[u]++;
}

int go(int u, char c) {
    while (u && !trie[u].count(c))
        u = link[u];
    return trie[u][c];
}

void pushLinks() {
    queue<int> qu;
    qu.push(0);
    while (!qu.empty()) {
        int u = qu.front();
        qu.pop();
        for (auto &[c, v] : trie[u]) {
            if (v == 0) {
                v = trie[link[u]][c];
                continue;
            }
            link[v] = u ? go(link[u], c) : 0;
            cnt[v] += cnt[link[v]];
            qu.push(v);
        }
    }
}

int match(string &s, int u = 0) {
    int ans = 0;
    for (char c : s)
        u = go(u, c), ans += cnt[u];
    return ans;
}
} aho;

```

4.9 Eertree

```

struct Eertree {
    vector< map<char, int> > trie;
    vi link, len;
    string s = "$";
    int last;

    Eertree() {
        last = newNode(), newNode();
        link[0] = 1, len[1] = -1;
    }

    int newNode() {
        trie.pb({}), link.pb(0), len.pb(0);
        return sz(trie) - 1;
    }

    int go(int u) {
        while (s[sz(s) - len[u] - 2] != s.back())
            u = link[u];
        return u;
    }

    void extend(char c) {
        s += c;
        int u = go(last);
        if (!trie[u][c]) {
            int v = newNode();
            len[v] = len[u] + 2;
            link[v] = trie[go(link[u])][c];
            trie[u][c] = v;
        }
        last = trie[u][c];
    }
}

```

```

} eert;

```

5 Dynamic Programming

5.1 Matrix Chain Multiplication

```

int dp(int l, int r) {
    if (l > r)
        return 0LL;
    int &ans = mem[l][r];
    if (!done[l][r]) {
        done[l][r] = true, ans = inf;
        for (k, l, r + 1) // split in [l, k] [k + 1, r]
            ans = min(ans, dp(l, k) + dp(k + 1, r) + add);
    }
    return ans;
}

```

5.2 Digit DP

Counts the amount of numbers in $[l, r]$ such are divisible by k . (flag *nonzero* is for different lengths)

It can be reduced to $dp(i, x, small)$, and has to be solve like $f(r) - f(l - 1)$

```

#define state [i][x][small][big][nonzero]
int dp(int i, int x, bool small, bool big, bool
    nonzero) {
    if (i == sz(r))
        return x % k == 0 && nonzero;
    int &ans = mem state;
    if (done state != timer) {
        done state = timer;
        ans = 0;
        int lo = small ? 0 : l[i] - '0';
        int hi = big ? 9 : r[i] - '0';
        for (y, lo, max(lo, hi) + 1) {
            bool small2 = small | (y > lo);
            bool big2 = big | (y < hi);
            bool nonzero2 = nonzero | (x > 0);
            ans += dp(i + 1, (x * 10 + y) % k, small2, big2,
                nonzero2);
        }
    }
    return ans;
}

```

5.3 Knapsack 0/1

```

for (auto &cur : items)
    fore (w, W + 1, cur.w) // [cur.w, W]
        umax(dp[w], dp[w - cur.w] + cur.cost);

```

5.4 Convex Hull Trick $\mathcal{O}(n^2) \Rightarrow \mathcal{O}(n)$

$dp[i] = \min_{j < i} (dp[j] + b[j] * a[i])$
 $dp[i][j] = \min_{k < j} (dp[i - 1][k] + b[k] * a[j])$
 $b[j] \geq b[j + 1]$ optionally $a[i] \leq a[i + 1]$

```

// for doubles, use inf = 1/.0, div(a,b) = a / b
struct Line {
    mutable lli m, c, p;
    bool operator < (const Line &l) const { return m < l
        .m; }
    bool operator < (lli x) const { return p < x; }
    lli operator ()(lli x) const { return m * x + c; }
};

lli bet(const Line &a, const Line &b) {
    if (a.m == b.m)
        return a.c > b.c ? -inf : -inf;
    // can just be a / b?
    return (b.c - a.c) / (a.m - b.m);
    // lli divi(lli a, lli b) { return a / b - ((a ^ b)
        < 0 && a % b); }
    // return divi(b.c - a.c, a.m - b.m);
}

```

```

struct DynamicHull : multiset<Line, less<>> {
    bool isect(iterator x, iterator y) {
        if (y == end()) {
            x->p = inf;
            return 0;
        }
        x->p = bet(*x, *y);
        return x->p >= y->p;
    }

    void add(lli m, lli c) {
        auto z = insert({m, c, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y))
            isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }

    lli query(lli x) {
        if (empty()) return 0LL;
        auto f = *lower_bound(x);
        return f(x);
    }
}

```

5.5 Divide and conquer $\mathcal{O}(kn^2) \Rightarrow \mathcal{O}(k \cdot n \log n)$

Split the array of size n into k continuous groups. $k \leq n$
 $cost(a, c) + cost(b, d) \leq cost(a, d) + cost(b, c)$ with $a \leq b \leq c \leq d$

```

void dc(int cut, int l, int r, int optl, int opttr) {
    if (r < l)
        return;
    int mid = (l + r) / 2;
    pair<lli, int> best = {inf, -1};
    fore (p, optl, min(mid, opttr) + 1) {
        lli nxtGroup = dp[~cut & 1][p - 1] + cost(p, mid);
        if (nxtGroup < best.f)
            best = {nxtGroup, p};
    }
    dp[cut & 1][mid] = best.f;
    int opt = best.s;
    dc(cut, l, mid - 1, optl, opt);
    dc(cut, mid + 1, r, opt, opttr);
}

fore (i, 1, n + 1)
    dp[1][i] = cost(1, i);
fore (cut, 2, k + 1)
    dc(cut, cut, n, cut, n);

```

5.6 Knuth optimization $\mathcal{O}(n^3) \Rightarrow \mathcal{O}(n^2)$

$dp[l][r] = \min_{l \leq k \leq r} \{dp[l][k] + dp[k][r]\} + cost(l, r)$

```

fore (len, 1, n + 1)
    fore (l, 0, n) {
        int r = l + len - 1;
        if (r > n - 1)
            break;
        if (len <= 2) {
            dp[l][r] = 0;
            opt[l][r] = l;
            continue;
        }
        dp[l][r] = inf;
        fore (k, opt[l][r - 1], opt[l + 1][r] + 1) {
            lli cur = dp[l][k] + dp[k][r] + cost(l, r);
            if (cur < dp[l][r]) {
                dp[l][r] = cur;
            }
        }
    }

```

```

        opt[l][r] = k;
    }
}

```

5.7 Do all submasks of a mask

```
for (int B = A; B > 0; B = (B - 1) & A)
```

6 Game Theory

6.1 Grundy Numbers

If the moves are consecutive $S = \{1, 2, 3, \dots, x\}$ the game can be solved like $stackSize \pmod{x+1} \neq 0$

```

int mem[N];

int mex(set<int> &st) {
    int x = 0;
    while (st.count(x))
        x++;
    return x;
}

int grundy(int n) {
    if (n < 0)
        return inf;
    if (n == 0)
        return 0;
    int &g = mem[n];
    if (g == -1) {
        set<int> st;
        for (int x : {a, b})
            st.insert(grundy(n - x));
        g = mex(st);
    }
    return g;
}

```

7 Combinatorics

Combinatorics table		
Number	Factorial	Catalan
0	1	1
1	1	1
2	2	2
3	6	5
4	24	14
5	120	42
6	720	132
7	5,040	429
8	40,320	1,430
9	362,880	4,862
10	3,628,800	16,796
11	39,916,800	58,786
12	479,001,600	208,012
13	6,227,020,800	742,900

7.1 Factorial

```

fac[0] = 1LL;
fore (i, 1, N)
    fac[i] = lli(i) * fac[i - 1] % mod;
ifac[N - 1] = fpow(fac[N - 1], mod - 2);
fore (i, N - 1, 0)
    ifac[i] = lli(i + 1) * ifac[i + 1] % mod;

```

7.2 Factorial mod *smallPrime*

```

lli facMod(lli n, int p) {
    lli r = 1LL;
    for (; n > 1; n /= p) {
        r = (r * ((n / p) % 2 ? p - 1 : 1)) % p;
    }
}

```



```

    fore (i, 2, n % p + 1)
        r = r * i % p;
}
return r % p;
}

```

7.3 Lucas theorem

Changes $\binom{n}{k} \bmod p$, with $n \geq 2e6, k \geq 2e6$ and $p \leq 1e7$

$$\binom{n}{k} \equiv \prod_{i=0}^n \binom{n_i}{k_i} \bmod p$$

7.4 Stars and bars

Enclosing n objects in k boxes

$$\binom{n+k-1}{k-1} = \binom{n+k-1}{n}$$

7.5 N choose K

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\binom{n}{k_1, k_2, \dots, k_m} = \frac{n!}{k_1! * k_2! * \dots * k_m!}$$

```

lli choose(int n, int k) {
    if (n < 0 || k < 0 || n < k)
        return 0LL;
    return fac[n] * ifac[k] % mod * ifac[n - k] % mod;
}

```

```

lli choose(int n, int k) {
    double r = 1;
    fore (i, 1, k + 1)
        r = r * (n - k + i) / i;
    return lli(r + 0.001);
}

```

7.6 Catalan

```

catalan[0] = 1LL;
fore (i, 0, N) {
    catalan[i + 1] = catalan[i] * lli(4 * i + 2) % mod *
        fpow(i + 2, mod - 2) % mod;
}

```

7.7 Burnside's lemma

$$|classes| = \frac{1}{|G|} \cdot \sum_{x \in G} f(x)$$

7.8 Prime factors of N!

```

vector< pair<lli, int> > factorialFactors(int n) {
    vector< pair<lli, int> > fac;
    for (lli p : primes) {
        if (n < p)
            break;
        lli mul = 1LL, k = 0;
        while (mul <= n / p) {
            mul *= p;
            k += n / mul;
        }
        fac.emplace_back(p, k);
    }
    return fac;
}

```

8 Number Theory

8.1 Goldbach conjecture

- All number ≥ 6 can be written as sum of 3 *primes*
- All even number > 2 can be written as sum of 2 *primes*

8.2 Sieve of Eratosthenes

Numbers up to $2e8$

```

int erat[N >> 6];
#define bit(i) ((i >> 1) & 31)
#define prime(i) !(erat[i >> 6] >> bit(i) & 1)

void bitSieve() {
    for (int i = 3; i * i < N; i += 2) if (prime(i))
        for (int j = i * i; j < N; j += (i << 1))
            erat[j >> 6] |= 1 << bit(j);
}

```

To factorize divide x by $factor[x]$ until is equal to 1

```

void factorizeSieve() {
    iota(factor, factor + N, 0);
    for (int i = 2; i * i < N; i++) if (factor[i] == i)
        for (int j = i * i; j < N; j += i)
            factor[j] = i;
}

```

Use it if you need a huge amount of $phi[x]$ up to some N

```

void phiSieve() {
    isp.set(); // bitset<N> is faster
    iota(phi, phi + N, 0);
    fore (i, 2, N) if (isp[i]) {
        for (int j = i; j < N; j += i) {
            isp[j] = (i == j);
            phi[j] /= i;
            phi[j] *= i - 1;
        }
    }
}

```

8.3 Phi of euler

```

lli phi(lli n) {
    if (n == 1)
        return 0;
    lli r = n;
    for (lli i = 2; i * i <= n; i++)
        if (n % i == 0) {
            while (n % i == 0)
                n /= i;
            r -= r / i;
        }
    if (n > 1)
        r -= r / n;
    return r;
}

```

8.4 Miller-Rabin

```

bool compo(lli p, lli d, lli n, lli k) {
    lli x = fpow(p % n, d, n), i = k;
    while (x != 1 && x != n - 1 && p % n && i--)
        x = mul(x, x, n);
    return x != n - 1 && i != k;
}

bool miller(lli n) {
    if (n < 2 || n % 6 % 4 != 1)
        return (n | 1) == 3;
    int k = __builtin_ctzll(n - 1);
    lli d = n >> k;
    for (lli p : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}) {
        if (compo(p, d, n, k))
            return 0;
        if (compo(2 + rng() % (n - 3), d, n, k))
            return 0;
    }
    return 1;
}

```

8.5 Pollard-Rho

```
lli f(lli x, lli c, lli mod) {
    return (mul(x, x, mod) + c) % mod;
}

lli rho(lli n) {
    while (1) {
        lli x = 2 + rng() % (n - 3), c = 1 + rng() % 20, y
            = f(x, c, n), g;
        while ((g = __gcd(n + y - x, n)) == 1)
            x = f(x, c, n), y = f(f(y, c, n), c, n);
        if (g != n) return g;
    }
    return -1;
}

void pollard(lli n, map<lli, int> &fac) {
    if (n == 1) return;
    if (n % 2 == 0) {
        fac[2]++;
        pollard(n / 2, fac);
        return;
    }
    if (miller(n)) {
        fac[n]++;
        return;
    }
    lli x = rho(n);
    pollard(x, fac);
    pollard(n / x, fac);
}
```

8.6 Amount of divisors

```
lli divs(lli n) {
    lli cnt = 1LL;
    for (lli p : primes) {
        if (p * p * p > n)
            break;
        if (n % p == 0) {
            lli k = 0;
            while (n > 1 && n % p == 0)
                n /= p, ++k;
            cnt *= (k + 1);
        }
    }
    lli sq = mysqrt(n); // A binary search, the last x *
        x <= n
    if (miller(n))
        cnt *= 2;
    else if (sq * sq == n && miller(sq))
        cnt *= 3;
    else if (n > 1)
        cnt *= 4;
    return cnt;
}
```

8.7 Bézout's identity

$a_1 * x_1 + a_2 * x_2 + \dots + a_n * x_n = g$
 $g = \gcd(a_1, a_2, \dots, a_n)$

8.8 GCD

$a \leq b; \gcd(a + k, b + k) = \gcd(b - a, a + k)$

8.9 LCM

$x = p * \text{lcm}(a_1, a_2, \dots, a_k) + q, 0 \leq q \leq \text{lcm}(a_1, a_2, \dots, a_k)$
 $x \pmod{a_i} \equiv q \pmod{a_i}$ as $a_i \mid \text{lcm}(a_1, a_2, \dots, a_k)$

8.10 Euclid

```
pair<lli, lli> euclid(lli a, lli b) {
    if (b == 0)
        return {1, 0};
```

```
    auto p = euclid(b, a % b);
    return {p.s, p.f - a / b * p.s};
}
```

8.11 Chinese remainder theorem

```
pair<lli, lli> crt(pair<lli, lli> a, pair<lli, lli> b)
{
    if (a.s < b.s)
        swap(a, b);
    auto p = euclid(a.s, b.s);
    lli g = a.s * p.f + b.s * p.s, l = a.s / g * b.s;
    if ((b.f - a.f) % g != 0)
        return {-1, -1}; // no solution
    p.f = a.f + (b.f - a.f) % b.s * p.f % b.s / g * a.s;
    return {p.f + (p.f < 0) * l, l};
}
```

9 Math

9.1 Progressions

Arithmetic progressions

$$a_n = a_1 + (n - 1) * diff$$

$$\sum_{i=1}^n a_i = n * \frac{a_1 + a_n}{2}$$

Geometric progressions

$$a_n = a_1 * r^{n-1}$$

$$\sum_{k=1}^n a_1 * r^k = a_1 * \left(\frac{r^{n+1} - 1}{r - 1} \right) : r \neq 1$$

9.2 Mod multiplication

```
lli mul(lli x, lli y, lli mod) {
    lli r = 0LL;
    for (x %= mod; y > 0; y >= 1) {
        if (y & 1) r = (r + x) % mod;
        x = (x + x) % mod;
    }
    return r;
}
```

9.3 Fpow

```
lli fpow(lli x, lli y, lli mod) {
    lli r = 1;
    for (; y > 0; y >= 1) {
        if (y & 1) r = mul(r, x, mod);
        x = mul(x, x, mod);
    }
    return r;
}
```

9.4 Fibonacci

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n = \begin{bmatrix} fib_{n+1} & fib_n \\ fib_n & fib_{n-1} \end{bmatrix}$$

10 Geometry

10.1 Point

```
const ld eps = 1e-9;
#define eq(a, b) fabs((a) - (b)) <= eps
#define neq(a, b) fabs((a) - (b)) > eps
#define geq(a, b) (a) - (b) >= -eps
#define ge(a, b) (a) - (b) > eps
#define le(a, b) (b) - (a) > eps
#define leq(a, b) (b) - (a) >= -eps
#define sq(x) (x) * (x)
```

```
int sgn(ld x) {
    return x > 0 ? 1 : (x < 0 ? -1 : 0);
}
```

```
template <class T>
```

```

struct Point {
    typedef Point<T> P;
    T x, y;
    explicit Point(T x = 0, T y = 0) : x(x), y(y) {}
    P operator + (const P &p) const {
        return P(x + p.x, y + p.y); }
    P operator - (const P &p) const {
        return P(x - p.x, y - p.y); }
    P operator * (T k) const {
        return P(x * k, y * k); }
    P operator / (T k) const {
        return P(x / k, y / k); }

    T dot(const P &p) { return x * p.x + y * p.y; }
    T cross(const P &p) { return x * p.y - y * p.x; }
    double length() const { return sqrt(norm()); }
    T norm() const { return sq(x) + sq(y); } // double ?
    double angle() { return atan2(y, x); }

    P perp() const { return P(-y, x); }
    P unit() const { return (*this) / length(); }
    P rotate (double angle) const {
        return P(x * cos(angle) - y * sin(angle),
                x * sin(angle) + y * cos(angle)); }

    bool operator == (const P &p) const {
        return eq(x, p.x) && eq(y, p.y); }
    bool operator != (const P &p) const {
        return neq(x, p.x) || neq(y, p.y); }

    friend ostream & operator << (ostream &os, P &p) {
        return os << "(" << p.x << ", " << p.y << ")";
    }
};
typedef Point<double> P;

double ccw(P a, P b, P c) {
    return (b - a).cross(c - a);
}

```

10.2 Angle Between Vectors

```

double angleBetween(P a, P b) {
    double x = a.dot(b) / a.length() / b.length();
    return acos(max(ld(-1.0), min(ld(1.0), ld(x))));
}

```

10.3 Area Poligon

```

double area(vector<P> &p) {
    double sum = 0;
    for (i, 0, n)
        sum += p[i].cross(p[(i + 1) % sz(p)]);
    return abs(sum / 2);
}

```

10.4 Area Poligon In Circle

```

vector<P> intersectLineCircle(const P &a, const P &b,
    const P &c, ld r) {
    ld h2 = r * r - v.cross(c - a) * v.cross(c - a) / v.
        norm();
    P p = a + v * v.dot(c - a) / v.norm();
    if (eq(h2, 0))
        return {p}; // line tangent to circle
    else if (le(h2, 0))
        return {}; // no intersection
    else {
        point u = v.unit() * sqrt(h2);
        return {p - u, p + u}; // two points of
            intersection (chord)
    }
}

bool pointInLine(const P &a, const P &b, const P &p) {

```

```

    return eq((p - a).cross(v), 0);
}

bool pointInSegment(const P &a, const P &b, const P &p)
    {
        return pointInLine(a, b - a, p) && leq((a - p).dot(b
            - p), 0);
    }

int pointInCircle(const P &c, ld r, const P &p) {
    ld l = (p - c).length() - r;
    return (le(l, 0) ? 1 : (eq(l, 0) ? -1 : 0));
}

vector<P> intersectSegmentCircle(const P &a, const P &
    b, const point &c, ld r) {
    vector<P> points = intersectLineCircle(a, b - a, c,
        r), ans;
    for (const P &p : points) {
        if (pointInSegment(a, b, p)) ans.pb(p);
    }
    return ans;
}

ld signed_angle(const P &a, const P &b) {
    return sgn(a.cross(b)) * acosl(a.dot(b) / (a.length
        () * b.length()));
}

ld intersectPolygonCircle(const vector<P> &points,
    const P &c, ld r) {
    int n = points.size();
    ld ans = 0;
    for (int i = 0; i < n; ++i) {
        P p = points[i], q = points[(i + 1) % n];
        bool p_inside = (pointInCircle(c, r, p) != 0);
        bool q_inside = (pointInCircle(c, r, q) != 0);
        if (p_inside && q_inside) {
            ans += (p - c).cross(q - c);
        } else if (p_inside && !q_inside) {
            P s1 = intersectSegmentCircle(p, q, c, r)[0];
            P s2 = intersectSegmentCircle(c, q, c, r)[0];
            ans += (p - c).cross(s1 - c) + r * r *
                signed_angle(s1 - c, s2 - c);
        } else if (!p_inside && q_inside) {
            P s1 = intersectSegmentCircle(c, p, c, r)[0];
            P s2 = intersectSegmentCircle(p, q, c, r)[0];
            ans += (s2 - c).cross(q - c) + r * r *
                signed_angle(s1 - c, s2 - c);
        } else {
            auto info = intersectSegmentCircle(p, q, c, r);
            if (info.size() <= 1) {
                ans += r * r * signed_angle(p - c, q - c);
            } else {
                P s2 = info[0], s3 = info[1];
                P s1 = intersectSegmentCircle(c, p, c, r)[0];
                P s4 = intersectSegmentCircle(c, q, c, r)[0];
                ans += (s2 - c).cross(s3 - c) + r * r * (
                    signed_angle(s1 - c, s2 - c) +
                    signed_angle(s3 - c, s4 - c));
            }
        }
    }
    return abs(ans) / 2;
}

```

10.5 Closest Pair Of Points

```

pair<P, P> cpp(vector<P> points) {
    sort(all(points), [&](P a, P b) {
        return le(a.y, b.y);
    });
    set<P> st;
    ld ans = inf;
    P p, q;
    int pos = 0, n = sz(points);

```

```

fore (i, 0, n) {
    while (pos < i && geq(points[i].y - points[pos].y,
        ans))
        st.erase(points[pos++]);
    auto lo = st.lower_bound({points[i].x - ans - eps,
        -inf});
    auto hi = st.upper_bound({points[i].x + ans + eps,
        -inf});
    for (auto it = lo; it != hi; ++it) {
        ld d = (points[i] - *it).length();
        if (le(d, ans))
            ans = d, p = points[i], q = *it;
    }
    st.insert(points[i]);
}
return {p, q};
}

```

10.6 Convex Hull

```

vector<P> convexHull(vector<P> &pts) {
    int n = sz(pts);
    vector<P> low, up;
    sort(all(pts), [&](P a, P b) {
        return a.x == b.x ? a.y < b.y : a.x < b.x;
    });
    pts.erase(unique(all(pts)), pts.end());
    if (n <= 2)
        return pts;
    fore (i, 0, n) {
        while(sz(low) >= 2 && (low.end()[-1] - low.end()[-2]).cross(pts[i] - low.end()[-1]) <= 0)
            low.pop_back();
        low.pb(pts[i]);
    }
    fore (i, n, 0) {
        while(sz(up) >= 2 && (up.end()[-1] - up.end()[-2]).cross(pts[i] - up.end()[-1]) <= 0)
            up.pop_back();
        up.pb(pts[i]);
    }
    low.pop_back(), up.pop_back();
    low.insert(low.end(), all(up));
    return low;
}

```

10.7 Distance Point Line

```

double distance_P_line(P a, P v, P p){
    return (proj(p - a, v) - (p - a)).length();
}

```

10.8 Get Circle

```

pair<P, double> getCircle(P m, P n, P p){
    P c = intersectLines((n + m) / 2, (n - m).perp(), (p
        + m) / 2, (p - m).perp());
    double r = (c - m).length();
    return {c, r};
}

```

10.9 Intersects Line

```

int intersectLinesInfo(P a1, P v1, P a2, P v2) { // v
    1 = b - a, v2 = d - c
    if (v1.cross(v2) == 0)
        return (a2 - a1).cross(v1) == 0 ? -1 : 0; // -1:
        infinity Ps, 0: no Ps
    else
        return 1; // single P
}

P intersectLines(P a1, P v1, P a2, P v2) {
    return a1 + v1 * ((a2 - a1).cross(v2) / v1.cross(v2)
        );
}

```

```

}

```

10.10 Intersects Line Segment

```

int intersectLineSegmentInfo(P a, P v, P c, P d) {
    P v2 = d - c;
    ld det = v.cross(v2);
    if (det == 0) {
        if ((c - a).cross(v) == 0)
            return -1; // infinity points
        else
            return 0; //no points
    } else
        return sgn(v.cross(c - a)) != sgn(v.cross(d - a))
            ;
}

```

10.11 Intersects Segment

```

int intersectSegmentsInfo(const P &a, const P &b,
    const P &c, const P &d) {
    P v1 = b - a, v2 = d - c;
    int t = sgn(v1.cross(c - a)), u = sgn(v1.cross(d - a
        ));
    if (t == u) {
        if (t == 0) {
            if (PInSegment(a, b, c) || PInSegment(a, b, d)
                || PInSegment(c, d, a) || PInSegment(c, d,
                    b))
                return -1; // infinity Ps
            else
                return 0; // no P
        } else
            return 0; // no P
    } else
        return sgn(v2.cross(a - c)) != sgn(v2.cross(b -
            c)); // 1: single P 0: no P
}

```

10.12 Is Convex

```

bool isConvex(vector<P> points) {
    int n = sz(points);
    bool hasPos = false, hasNeg = false;
    fore (i, 0, n) {
        P first = points[(i+1)%n] - points[i];
        P second = points[(i+2)%n] - points[(i+1)%n];
        double sign = first.cross(second);
        if (sign > 0) hasPos = true;
        if (sign < 0) hasNeg = true;
    }
    return !(hasPos && hasNeg);
}

```

10.13 Perimeter

```

double perimeter(vector<P> points){
    int n = sz(points);
    double sum = 0;
    fore (i, 0, n)
        sum += (points[(i + 1) % n] - points[i]).length();
    return sum;
}

```

10.14 Point In Convex Polygon logN

```

// log(n)
// first preprocess: seg = process(points)
// for each query: PInConvexPolygon(seg, p - Ps[0])
vector<P> process(const vector<P> &Ps) {
    int n = sz(Ps);
    rotate(Ps.begin(), min_element(all(Ps), [&](P a, P b
        ) {
            return a.x == b.x ? a.y < b.y : a.x < b.x;
        }), Ps.end());
    vector<P> seg(n - 1);
}

```

```

    fore (i, 0, n - 1)
        seg[i] = Ps[i + 1] - Ps[0];
    return seg;
}

bool PInConvexPolygon(const vector<P> &seg, const P &p)
{
    int n = sz(seg);
    if (neq(seg[0].cross(p), 0) && sgn(seg[0].cross(p))
        != sgn(seg[0].cross(seg[n - 1])))
        return false;
    if (neq(seg[n - 1].cross(p), 0) && sgn(seg[n - 1].
        cross(p)) != sgn(seg[n - 1].cross(seg[0])))
        return false;
    if (eq(seg[0].cross(p), 0))
        return geq(seg[0].length(), p.length());
    int l = 0, r = n - 1;
    while (r - l > 1) {
        int m = l + ((r - l) >> 1);
        if (geq(seg[m].cross(p), 0))
            l = m;
        else
            r = m;
    }
    return eq(fabs(seg[l].cross(seg[l + 1])), fabs((p -
        seg[l]).cross(p - seg[l + 1])) +
        fabs(p.cross(seg[l])) + fabs(p.cross(seg[l +
            1])));
}

```

10.15 Point In Polygon

```

int pointInPolygon(const vector<P> &points, P p) { //
    O(N)
    int n = sz(points), ans = 0;
    fore (i, 0, n) {
        P a = points[i], b = points[(i + 1) % n];
        if (pointInSegment(a, b, p))
            return -1; // on perimeter
        if (a.y > b.y)
            swap(a, b);
        if (a.y <= p.y && b.y > p.y && (a - p).cross(b - p)
            > 0)
            ans ^= 1;
    }
    return ans ? 1 : 0; // inside, outside
}

```

10.16 Point In Segment

```

bool pointInSegment(P a, P b, P p){
    return (b - a).cross(p - a) == 0 && (a - p).dot(b -
        p) <= 0;
}

```

10.17 Points Of Tangency

```

pair<P, P> PsOfTangency(P c, double r, P p){
    P v = (p - c).unit() * r;
    double cos_theta = r / (p - c).length();
    double theta = acos(max(-1.0, min(1.0, cos_theta)));
    return {c + v.rotate(-theta), c + v.rotate(theta)};
}

```

10.18 Projection

```

P proj(P a, P v){
    v = v / v.unit();
    return v * a.dot(v);
}

```

10.19 Projection Line

```

P proj_line(P a, P v, P p){
    return a + proj(p - a, v);
}

```

10.20 Reflection Line

```

P reflection_line(P a, P v, P p){
    return a*2 - p + proj(p - a, v)*2;
}

```

10.21 Signed Distance Point Line

```

double signed_distance_P_line(P a, P v, P p){
    return v.cross(p - a) / v.length();
}

```

10.22 Sort Along Line

```

void sort_along_line(P a, P v, vector<P> &Ps){
    sort(Ps.begin(), Ps.end(), [](P u, P w){
        return u.dot(v) < w.dot(v);
    });
}

```

10.23 Intersects Line Circle

```

vector<P> intersectLineCircle(P a, P v, P c, double r)
{
    P p = proj_line(a, v, c);
    double d = (p - c).length();
    double h = sq(r) - sq(d);
    if(h == 0)
        return {p}; //line tangent to circle
    else if(h < 0)
        return {}; //no intersection
    else {
        P u = v.unit() * sqrt(h);
        return {p - u, p + u}; //two Ps of intersection (
            chord)
    }
}

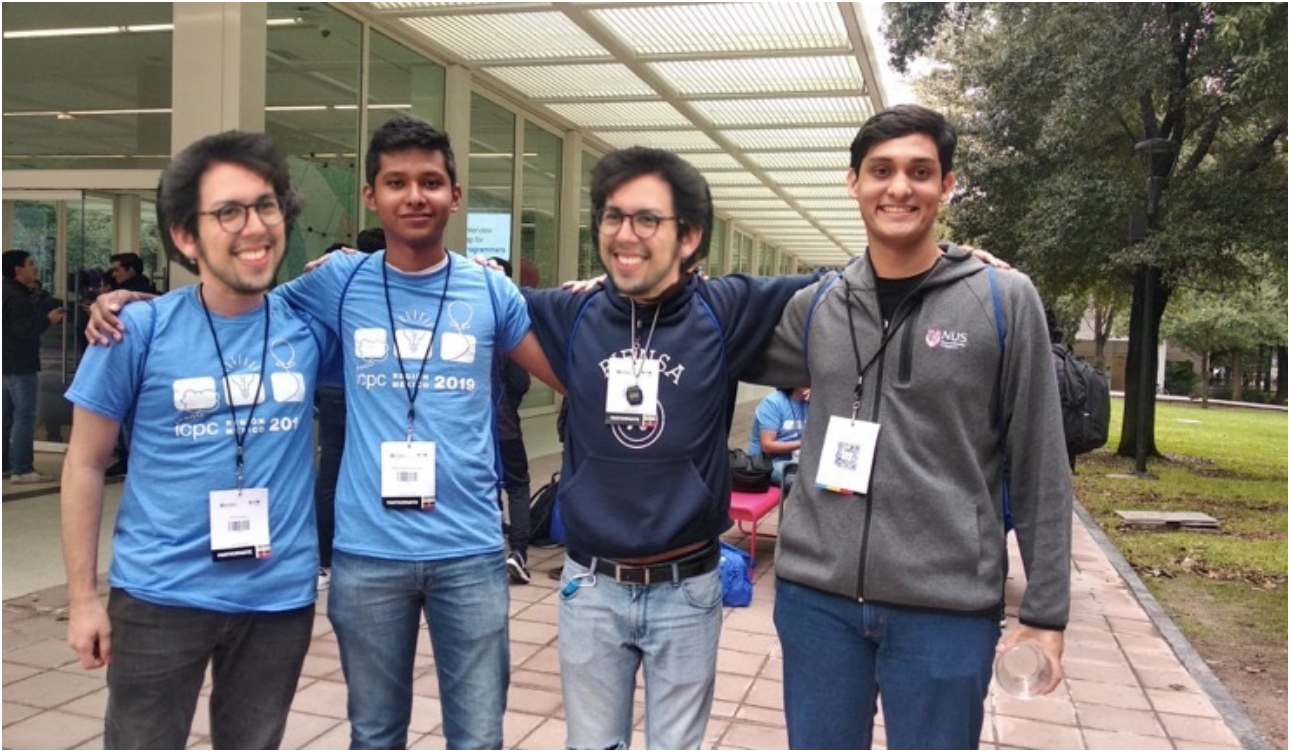
```

11 Bit tricks

Bits++	
Operations on <i>int</i>	Function
<code>x & -x</code>	Least significant bit in <i>x</i>
<code>__lg(x)</code>	Most significant bit in <i>x</i>
<code>c = x&-x, r = x+c;</code> <code>((r^x) >> 2)/c r</code>	Next number after <i>x</i> with same number of bits set
__builtin__	
<code>popcount(x)</code>	Amount of 1's in <i>x</i>
<code>clz(x)</code>	0's to the left of biggest bit
<code>ctz(x)</code>	0's to the right of smallest bit

11.1 Bitset

Bitset<Size>	
Operation	Function
<code>_Find_first()</code>	Least significant bit
<code>_Find_next(idx)</code>	First set bit after index <i>idx</i>
<code>any()</code> , <code>none()</code> , <code>all()</code>	Just what the expression says
<code>set()</code> , <code>reset()</code> , <code>flip()</code>	Just what the expression says x2
<code>to_string('.', 'A')</code>	Print 011010 like .AA.A.



The end...