HW5 Problem 3

Pf: d_2 is A-orthogonal to d_1

A is SPD where

$$A = \begin{bmatrix} | & | & & | \\ u_1 & u_2 & \cdots & u_n \\ | & | & & | \end{bmatrix}$$

$d_1 = u_1$ and

$$d_2 = u_2 - \frac{u_2^T A d_1}{d_1^T A d_1} d_1$$

per HW notes.

And we guess that d_2 is A-orthogonal to d_1. that is

$$d_2^T A d_1 = 0.$$

80,

$$\left[u_2 - \frac{u_2^T A d_1}{d_1^T A d_1} \, d_1 \right] A d_1 = 0$$

$$\left[u_2 - u_2^T A d_1 \left(d_1^T A d_1 \right)^{-1} d_1 \right] A d_1 = 0$$

$$\left[u_2 - u_2^T \underbrace{\left(A A^{-1} \right)}_{I} \underbrace{d_1 d_1^{-1}}_{I} \underbrace{\left(d_1 d_1^{-1} \right)}_{I} \right] A d_1 = 0$$

$$\left[u_2 - \underset{0}{\cancel{u_2^T}} \right] A d_1 = 0$$

$$0 = 0$$