

# Homework 2

PROBLEM 2

$$(1) f(t) = \frac{1}{2}a_0 + \sum_{n=1}^N a_n \cos\left(\frac{n\pi t}{L}\right)$$

$$(2) a_n = \frac{2}{L} \int_0^{\frac{1}{2}} f(t) \cos\left(\frac{n\pi t}{L}\right) dt$$

For (2), we use wolfram alpha

$$\int_0^{\frac{1}{2}} \sqrt{\frac{1}{4} - t^2} \cos\left(\frac{n\pi t}{L}\right) dt$$

$$= \frac{J_1\left(\frac{n\pi}{2}\right)}{4n} \text{ for } n \in \mathbb{R}$$

And we plug  $a_n$  into (1) to get

$$(1)^* f(t) =$$

$$\frac{a_0}{2} + \sum_{n=1}^N \left( \frac{2}{L} \frac{J_1\left(\frac{n\pi}{2}\right)}{4n} \right) \cos\left(\frac{n\pi t}{L}\right)$$

where  $L = 1$ .

So our approximation takes the form

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^N \frac{J_1\left(\frac{n\pi}{2}\right)}{2n} \cos(n\pi t)$$



