

# HWS Problem 1

(1)

$$f(t_{n+1}) = f(t_n) + h \frac{df}{dt}(t_n) + \frac{h^2}{2} \frac{d^2f}{dt^2}(t_n) + \dots$$

(2)

$$f(t_{n+1}) = f(t_n) + h \frac{df}{dt}(t_n) + \dots$$

$$f(t_{n+1}) - f(t_n) = h \frac{df}{dt}(t_n)$$

$$\frac{df}{dt}(t_n) = \frac{f(t_{n+1}) - f(t_n)}{h}$$

(3)

$$f(t_{n-1}) = f(t_n) - h \frac{df}{dt}(t_n) + \dots$$

(4)

$$f(t_{n-1}) = f(t_n) - h \frac{df}{dt}(t_n) + \dots$$

$$f(t_{n-1}) - f(t_n) = -h \frac{df}{dt}(t_n)$$

$$\frac{df}{dt}(t_n) = \frac{f(t_n) - f(t_{n-1})}{h}$$

(5)

$$f(t_{n+1}) + f(t_{n-1}) =$$

$$(1) \left[ f(t_n) + h \frac{df}{dt}(t_n) + \frac{h^2}{2} \frac{d^2f}{dt^2}(t_n) + \dots \right]$$

$$+ (2) \left[ f(t_n) - h \frac{df}{dt}(t_n) + \frac{h^2}{2} \frac{d^2f}{dt^2}(t_n) - \dots \right]$$

$$= 2f(t_n) + 0 + h^2 \frac{d^2f}{dt^2}(t_n)$$

So,

$$f(t_{n+1}) + f(t_{n-1}) = 2f(t_n) + h^2 \frac{d^2f}{dt^2}(t_n)$$

$$\frac{d^2f}{dt^2} = \frac{f(t_{n-1}) - 2f(t_n) + f(t_{n+1}))}{h^2}$$

(6)

$$\vec{y}' = \mathbb{D} \vec{y}$$

$$\mathbb{D} = \frac{1}{h} \begin{bmatrix} 0 & -1 & 1 & & 0 \\ 0 & 0 & -1 & 1 & \\ & & & \ddots & \\ 0 & & & 0 & -1 & 1 \end{bmatrix}$$

(7) MATLAB

(8)

$$D_1 = \frac{1}{h} \begin{bmatrix} -1 & 0 & 1 & 0 \\ & -1 & 0 & 1 \\ & & \ddots & \\ 0 & & & -1 & 0 & 1 \end{bmatrix}$$

(9)

$$D_2 = \frac{1}{h^2} \begin{bmatrix} 1 & -2 & 1 & & 0 \\ & 1 & -2 & 1 & \\ & & \ddots & \\ 0 & & & 1 & -2 & 1 \end{bmatrix}$$

(10) MATLAB