HW5 Problem 1

(1)

$$f(t_{n+1}) = f(t_n) + h\frac{df}{dt}(t_n) + \frac{h^2}{2}\frac{d^2f}{dt^2}(t_n) + \cdots$$

(2)

$$f(t_{n+1}) = f(t_n) + h\frac{df}{dt}(t_n) + \cancel{\cdots}$$

$$f(t_{n+1}) - f(t_n) = h\frac{df}{dt}(t_n)$$

$$\frac{df}{dt}(t_n) = \frac{f(t_{n+1}) - f(t_n)}{h}$$

(3)

$$f(t_{n-1}) = f(t_n) - h\frac{df}{dt}(t_n) + \cdots$$

(4)

$$f(t_{n-1}) = f(t_n) - h\frac{df}{dt}(t_n) + \cancel{\cdots}$$

$$f(t_{n-1}) - f(t_n) = -h\frac{df}{dt}(t_n)$$

$$\frac{f(t_{n-1}) - f(t_n)}{-h} = \frac{df}{dt}(t_n)$$

$$\frac{f(t_n) - f(t_{n-1})}{h}$$

$$\frac{df}{dt}(t_n) = \frac{f(t_n) - f(\dots)}{h}$$

(5)

$$f(t_{n+1}) + f(t_{n-1}) =$$

(1) $\left[f(t_n) + h\frac{df}{dt}(t_n) + \frac{h^2}{2}\frac{d^2f}{dt^2}(t_n) + \dots \right]$

+(3) $\left[f(t_n) - h\frac{df}{dt}(t_n) + \frac{h^2}{2}\frac{d^2f}{dt}(t_n) - \dots \right]$

$$= 2f(t_n) + 0 + h^2\frac{d^2f}{dt^2}(t_n)$$

So,

$$f(t_{n+1}) + f(t_{n-1}) = 2f(t_n) + h^2\frac{d^2f}{dt^2}(t_n)$$

$$\frac{d^2f}{dt^2} = \frac{f(t_{n+1}) + f(t_{n-1}) - 2f(t_n)}{h^2}$$

(6)

$$\vec{y}' = D_1\vec{y}$$

$$D = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 2 & 0 & \cdots & 0 \\ 0 & 0 & 3 & 0 \cdots & 0 \\ & & & \ddots & 1 \\ 0 & \cdots & & 0 & n \end{bmatrix}$$

(7) MATLAB

(8) $\frac{df}{dt} = \frac{f(t_n) - f(t_{n-1})}{h}$

$y = x^2$

$y^2 = 2x$

$\frac{x^2 - (x-h)^2}{h} = \frac{x^2}{h}(x-h)(x-h)$

$= \frac{x^2}{h}(x^2 - 2xh + h^2)$

$= x^4 - 2x^3 + xh$

$\begin{bmatrix} & & \end{bmatrix} \begin{bmatrix} 1 \\ x^2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & x^4 - 2x^3 & 0 \end{bmatrix}$

(9)

$D_2 = D^2 = DD$

$D_2 =$