exo 1 a)

exo 5 a), b), h), i), n), r), e), f)

exo 1 a) $U_n = e^{n^2+1}$

$f : [0, +\infty[\longrightarrow \mathbb{R}$

$x \longmapsto e^{x^2+1}$

$(e^u)' = u' e^u$

$f'(x) = 2x \underbrace{e^{x^2+1}}_{} \geq 0$ pour $x \in [0, +\infty[$

Donc $f \nearrow$, donc $(U_n) \nearrow$

exo 5 a) $U_n = \frac{1}{n+3}$

$$\lim_{n \to +\infty} \frac{1}{n+3} = 0$$

$$\frac{2000}{1001} \simeq 2$$

b) $U_n = \frac{2n}{n+1}$

$$\lim_{n \to +\infty} \frac{2n}{n+1} = \lim_{n \to +\infty} \frac{2n}{n} = 2$$

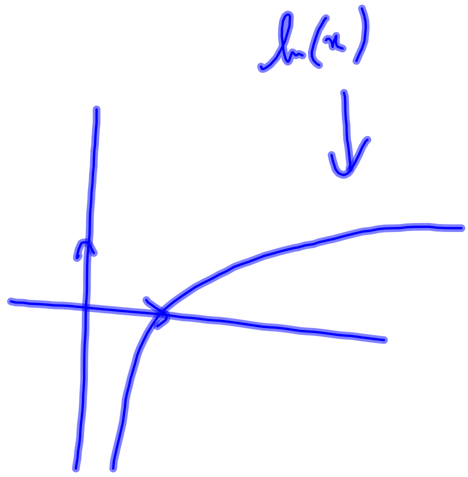
à l'infini, un polynôme se comporte comme son terme de + haut degré.

h) $U_n = \ln\left(\frac{1}{1+n}\right)$

$$\lim_{n \to +\infty} \frac{1}{1+n} = 0$$

$$\lim_{x \to 0} \ln(x) = -\infty$$

$$\lim_{n \to +\infty} U_n = -\infty$$



i) $U_n = \dfrac{n + e^n}{2n + e^n}$

$$\lim_{n \to +\infty} U_n = \lim_{n \to +\infty} \frac{\cancel{e^n} \cdot \left(\frac{n}{e^n} + 1\right)}{\cancel{e^n} \cdot \left(\frac{2n}{e^n} + 1\right)}$$

$$\lim_{n \to +\infty} \frac{e^n}{n^\alpha} = +\infty \qquad (\alpha > 0)$$

$$\lim_{n \to +\infty} \frac{n}{e^n} = \lim_{n \to +\infty} \frac{2n}{e^n} = 0 \qquad \text{donc } \lim_{n \to +\infty} U_n = 1$$

m) $U_n = 5^n - 4^n = 5^n\left(1 - \frac{4^n}{5^n}\right)$

$= 5^n\left[1 - \left(\frac{4}{5}\right)^n\right]$

$$\left.\begin{array}{l}\lim\limits_{n\to+\infty}\left(\frac{4}{5}\right)^n = 0 \\ \\ \text{or } \lim\limits_{n\to+\infty} 5^n = +\infty\end{array}\right\} \text{ donc } \lim\limits_{n\to+\infty} U_n = +\infty$$

r) $U_n = \dfrac{n + \cos n}{n - \sin n} = \dfrac{\cancel{n}\left(1 + \frac{\cos n}{n}\right)}{\cancel{n}\left(1 - \frac{\sin n}{n}\right)}$

$\forall n \in \mathbb{N}$, $-1 \leq \cos n \leq 1$ donc $\lim\limits_{n \to +\infty} \dfrac{\cos n}{n} = 0$

$-1 \leq \sin n \leq 1$ donc $\lim\limits_{n \to +\infty} \dfrac{\sin n}{n} = 0$

Donc $\lim\limits_{n \to +\infty} U_n = 1$

e) $U_n = \sqrt{n+1} + \sqrt{n}$

$$\lim_{n \to +\infty} U_n = +\infty$$

f) $U_n = \sqrt{n+1} - \sqrt{n} = \sqrt{n+1}\left(1 - \frac{\sqrt{n}}{\sqrt{n+1}}\right)$

FI : $+\infty \times 0$

$$U_n = \frac{\left(\sqrt{n+1} - \sqrt{n}\right)\left(\sqrt{n+1} + \sqrt{n}\right)}{\sqrt{n+1} + \sqrt{n}}$$

$(a+b)(a-b)$
$= a^2 - b^2$

$$= \frac{n+1-n}{\sqrt{n+1}+\sqrt{n}} = \frac{1}{\sqrt{n+1}+\sqrt{n}}$$

$$\lim_{n \to +\infty} U_n = 0$$

exo 1) b)

$$(\ln u)' = \frac{1}{u} \times u' = \frac{u'}{u}$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$a^x = e^{x \ln a}$$

$a > 0$

$$(e^x)' = e^x$$

$$(e^u)' = e^u \times u'$$

$(-3)^n$ $(-3)^{x^2}$