

## 1 Interazioni

Interazione	Gravit�	Electromagnetic	Forte	Debole
Between Teoria Bosone Massa bos Source	tutte Newton gravitone massa	tutte QED fotone $\gamma$ 0 electric ch.	quark QCD 8 gluoni $g$ 0 colore	lep, quark, l-q  $W^+, Z^0$ 100 GeV weak ch.
CC	$KM^2$	$\alpha = e^2/4\pi$ $= 1/137$	$\alpha_s \leq 1$	$M^2 G_F$ $= 1/29$
Potenziale	$-GMm/r$	$-\alpha/r$	$-4\alpha_s/3r + kr$	$-f\alpha_s/r$
Lifetime (s)		$10^{-19}$	$10^{-23}$	$10^{-10}$
$\sigma$ (cm $^{-3}$ )			$10^{-26}$	$10^{-38}$

## 2 Simmetrie

Parit�	P	C	CP	G
Espressione	$P\Phi(\mathbf{r}) = \Phi(-\mathbf{r})$	$C\Phi(\pi^+) = \Phi(\pi^-)$ $C\Phi(\pi^0) = \pm\Phi(\pi^0)$	$H = \frac{\mathbf{s} \cdot \mathbf{p}}{sp}$	$G = CR$ $= Ce^{i\pi I_2}$
Conservazione	forte, e.m.	forte, e.m.	debole	

## 3 Leptoni

Leptone		Massa (MeV)	Carica (e)	$L_e$	$L_\mu$	$L_\tau$
Elettrone	$e^-$	0.5	-1	1	0	0
Neutrino	$\nu_e$	<3 eV	0	1	0	0
Muone	$\mu^-$	105	-1	0	1	0
Neutrino	$\nu_\mu$	<0.19	0	0	1	0
Tau	$\tau^-$	1777	-1	0	0	1
Neutrino	$\nu_\tau$	<18.2	0	0	0	1

Particella	I	II	III	Q (e)	Isospin	Ipercarica	Colore
Leptoni LH	$\nu_e$	$\nu_\mu$	$\nu_\tau$	0	1/2	-1	-
	$e$	$\mu$	$\tau$	-1	1/2	-1	-
Leptoni RH	$e$	$\mu$	$\tau$	-1	0	-2	-

## 4 Quarks

$$\text{Ipercarica: } Y = B^* + S \quad \text{Numero barionico: } B^* = \frac{1}{3}$$

$$Q = I_3 + \frac{Y}{2}$$

Quark		$m_q$ (GeV)	$m'_q$ (MeV)	$Q$ (e)	$S$	$C$	$B$	$T$	$I_3$	$Y$
Up	$u$	0.33	1.5–3.3	2/3	0	0	0	0	1/2	1/3
Down	$d$	0.33	3.5–6.0	–1/3	0	0	0	0	–1/2	1/3
Charme	$c$	1.58	1270	2/3	0	1	0	0	1/2	1/3
Strange	$s$	0.47	104	–1/3	–1	0	0	0	0	–2/3
Top	$t$	174	171 200	2/3	0	0	0	1	1/2	1/3
Bottom	$b$	4.58	4200	–1/3	0	0	–1	0	–1/2	1/3

  

Particella	I	II	III	Q (e)	Isospin	Ipercarica	Colore
Quarks LH	$u$	$c$	$t$	+2/3	1/2	+1/3	$r, b, g$
	$d$	$s$	$b$	–1/3	1/2	+1/3	$r, b, g$
Quarks RH	$u$	$c$	$t$	+2/3	0	+4/3	$r, b, g$
	$d$	$s$	$b$	–1/3	0	–2/3	$r, b, g$

#### 4.1 Quark top

$$p + \bar{p} \rightarrow t + \bar{t} + any$$

$$t \rightarrow W + b \rightarrow l + \nu_l + b \quad t \rightarrow W + b \rightarrow \bar{q} + q' + b$$

$$t + \bar{t} \rightarrow W^+ + B + W^- + \bar{b} \rightarrow \bar{l} + \nu_l + b + l + \bar{\nu}_l + \bar{b} \rightarrow l + \bar{l} + \mathbf{p_T} + (\leq 2 \text{ jets})$$

$$t + \bar{t} \rightarrow W^+ + B + W^- + \bar{b} \rightarrow \tau^+ + \nu_\tau + b + \tau^- + \bar{\nu}_\tau + \bar{b} \rightarrow \tau^+ + \tau^- + \mathbf{p_T} + (\leq 2 \text{ jets})$$

$$t + \bar{t} \rightarrow W^+ + B + W^- + \bar{b} \rightarrow \tau + \nu_\tau + b + l + \bar{\nu}_l + \bar{b} \rightarrow \tau + l + \mathbf{p_T} + (\leq 2 \text{ jets})$$

$$t + \bar{t} \rightarrow W^+ + B + W^- + \bar{b} \rightarrow \bar{l} + \nu_l + b + q' + \bar{q} + \bar{b} \rightarrow l + \mathbf{p_T} + (\leq 4 \text{ jets})$$

$$t + \bar{t} \rightarrow W^+ + B + W^- + \bar{b} \rightarrow \tau + \nu_\tau + b + q' + \bar{q} + \bar{b} \rightarrow \tau + \mathbf{p_T} + (\leq 4 \text{ jets})$$

$$t + \bar{t} \rightarrow W^+ + B + W^- + \bar{b} \rightarrow \bar{q}' + q + b + q' + \bar{q} + \bar{b} \rightarrow (\leq 6 \text{ jets})$$

### 5 Mesoni

$$\Delta E_{hf}(qq) = \frac{4\pi\alpha_s}{9} |\Psi(0)| \sigma_i \sigma_j = \frac{\Delta E_{hf}(q\bar{q})}{2}$$

$$J^{PC} : \begin{cases} 0 & \text{pseudoscalari} \\ 1 & \text{vettori} \end{cases}$$

Mesone	Composizione	Carica ( $e$ )	Massa (MeV)	Decadimento	$J^{PC}$
$\pi^+$	$u\bar{d}$	+1	140	$\mu^+\nu_\mu$	$0^-$
$\pi^0$	$\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	0	135	$\gamma\gamma$	$0^{-+}$
$\pi^-$	$d\bar{u}$	-1	140	$\mu^-\bar{\nu}_\mu$	$0^-$
$\eta_8$	$\frac{d\bar{d}+u\bar{u}-2s\bar{s}}{\sqrt{6}}$	0	549	$\gamma\gamma$	$0^{-+}$
$\eta_0$	$\frac{d\bar{d}+u\bar{u}+s\bar{s}}{\sqrt{3}}$	0	958	$2\gamma, \eta + 2\pi$	$0^{-+}$
$B^+$	$u\bar{b}$	+1	5279		$0^-$
$B^0$	$d\bar{b}$	0	5279.4		$0^-$
$B^-$	$s\bar{b}$	-1	5369.6		$0^-$
$B_c^+$	$c\bar{b}$	+1	6400		$0^-$
$D^+$	$c\bar{d}$	+1	1869		$0^-$
$D^0$	$c\bar{u}$	0	1864		$0^-$
$D^-$	$s\bar{c}$	-1	1968		$0^-$
$K^+$	$u\bar{s}$	+1	494	$\mu^+\nu_\mu$	$0^-$
$K^0$	$d\bar{s}$	0	498	$\pi^+\pi^-$	$0^-$
$K^-$	$s\bar{u}$	-1	494	$\mu^-\bar{\nu}_\mu$	$0^-$
$\bar{K}^0$	$s\bar{d}$	0	498	$\pi^+\pi^-$	$0^-$
$K^{*+}$	$u\bar{s}$	+1	892	$K\pi$	$1^{--}$
$K^{*0}$	$d\bar{s}$	0	892	$K\pi$	$1^{--}$
$K^{*-}$	$s\bar{u}$	-1	892	$K\pi$	$1^{--}$
$\rho^+$	$d\bar{u}$	+1	776	$2\pi$	$1^-$
$\rho^0$	$\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$	0	776	$2\pi$	$1^{--}$
$\rho^-$	$u\bar{d}$	-1	776	$2\pi$	$1^-$
$\omega$	$\frac{d\bar{d}+u\bar{u}}{\sqrt{2}}$	0	783	$3\pi$	$1^{--}$
$\Phi$	$s\bar{s}$	0	1019	$K\bar{K}$	$1^{--}$
$J/\Psi$	$c\bar{c}$	0	3097		$1^{--}$
$\Upsilon$	$b\bar{b}$	0	9460.3	$B^0\bar{B}^0$	$1^{--}$
$\theta$	$t\bar{t}$	0		$2\pi$	$1^-, 0^+$

### 5.1 Mesone $K$

$$|K_1\rangle = \frac{|K^0\rangle + |\bar{K}^0\rangle}{\sqrt{2}} \quad |K_2\rangle = \frac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}}$$

$$CP|K_1\rangle = +1|K_1\rangle \quad CP|K_2\rangle = -1|K_2\rangle$$

Produzione		Decadimento		Lifetime (s)
$K^0$	$S = +1$	$K_1 = K_S \rightarrow 2\pi$	$CP = +1$	$\tau_1 = 0.9 \cdot 10^{-10}$
$\bar{K}^0$	$S = -1$	$K_2 = K_L \rightarrow 3\pi$	$CP = -1$	$\tau_2 = 0.5 \cdot 10^{-7}$

$$\pi^- + p \rightarrow \Lambda + K^0 \quad \pi^- + p \rightarrow \bar{K}^0 + K^+ + p \quad \pi^- + p \rightarrow \bar{K}^0 + \bar{\Lambda} + n + n$$

$$S: 0; 0; -1; +1 \quad S: 0; 0; -1; +1; 0 \quad S: 0; 0; -1; +1; 0; 0$$

$$|K_S^0\rangle = \frac{|K_1\rangle + \epsilon|K_2\rangle}{\sqrt{1+\epsilon^2}} \quad |K_L^0\rangle = \frac{|K_2\rangle + \epsilon|K_1\rangle}{\sqrt{1+\epsilon^2}} \quad |\epsilon| = 2.3 \cdot 10^{-3}$$

## 6 Barioni

$$M = M_0 + M_1 Y + M_2 \left[ I(I+1) - \frac{Y^2}{4} \right]$$

$$K = \frac{4\pi\alpha_s}{9} |\Psi(0)|^2$$

$$\Delta E_{fs} = -\alpha^4 m \left( \frac{1}{4n^2} \right) \left( \frac{4n}{2J+1} - \frac{3}{2} \right)$$

$$\Delta E_{hf} = \alpha^4 m \left( \frac{m}{m_p} \right) \left( \frac{\gamma_p}{2n^3} \right) \left( \frac{\pm 4}{(2F+1)(2I+1)} \right)$$

Barione	Quark	Carica (e)	Massa (MeV)	$\Delta E_{hf}/K$	$J^P$
$N$	$qqq$		$\sim 939$	$-3/m_n^2$	
	$uud$	+1			$1/2^+$
	$udd$	0			$1/2^+$
$\Delta$	$qqq$		$\sim 1232$		
	$uuu$	+2			$3/2^+$
	$d uu$	+1			$3/2^+$
	$dd u$	0			$3/2^+$
	$ddd$	-1			$3/2^+$
$\Sigma$	$qqs$		$\sim 1193$	$1/m_n^2 - 4/(m_n m_s)$	
$\Sigma$	$qqs$		$\sim 1384$	$1/m_n^2 + 2/(m_n m_s)$	
	$uus$	+1			$1/2^+$
	$dus$	0			$1/2^+$
	$dds$	-1			$1/2^+$
	$uuc$	+2	2455		$1/2^+$
$\Xi$	$qss$		$\sim 1318$	$1/m_n^2 - 4/(m_n m_s)$	
$\Xi$	$qss$		$\sim 1533$	$1/m_n^2 + 2/(m_n m_s)$	
	$uss$	0			$1/2^+$
	$dss$	-1			$1/2^+$
	$\Lambda$	0	1116	$-3/m_n^2$	$1/2^+$
	$\Lambda_b^0$	0	5624		$1/2^+$
	$\Lambda_c^+$	+1	2285		$1/2^+$
$\Omega^-$	$sss$	-1	1672	$3/m_n^2$	$3/2^+$

## 7 Reazioni e decadimenti

$$N(t) = N(0) e^{-\Gamma t} = \sigma L_{int} \quad \text{Cross section: } \sigma = \frac{W_{if}}{\Phi} \quad \text{Decay rate: } \Gamma = W_{if}$$

$$\text{Event rate: } R = \frac{dN}{dt} = \sigma L \quad \text{Luminosity: } L_{\text{int}} = \int L dt$$

$$\text{Flux: } \Phi = n_a v_i \quad \frac{dN}{dt} = \Phi n_b \sigma = n_a v_i n_b \sigma$$

$$\text{Transition rate: } W_{if} = \Phi \sigma = v_i \sigma \quad M_{if} = \int \Psi L_{int} dV \quad \rho_f = \frac{dN}{dE_0}$$

$$\text{Fermi golden rule: } W_{if} = \frac{2\pi}{\hbar} |M_{if}|^2 \rho_f$$

Decadimento	$a \rightarrow b + c$	$a + b \rightarrow c + d$
Diff decay rate	$\frac{1}{8\pi m_a} \int  M_{if} ^2 \frac{ \mathbf{p} }{E^*} dE^* \delta(m_a - E^*)$	
Decay rate	$\frac{ \mathbf{p}   M_{if} ^2}{8\pi m_a^2}$	$\sigma \Phi$
Diff cross section	$\frac{\Gamma}{\Phi}$	$\frac{1}{(8\pi)^2} \frac{ M_{if} ^2}{(E_a + E_b)^2} \frac{\mathbf{p}_f}{\mathbf{p}_i}$

$$\text{Breit-Wigner formula relativistica: } P(m) \propto \frac{1}{(m - m_0)^2 + \frac{\Gamma}{4}}$$

## 8 QED

$$\text{Schrödinger: } \left( -\frac{1}{2m} \right) \nabla^2 \Psi = i \frac{\partial \Psi}{\partial t}$$

$$\text{Klein-Gordon spin 0: } -\frac{\partial^2 \Psi}{\partial t^2} + \nabla^2 \Psi = m^2 \Psi \quad \text{Dirac spin } \frac{1}{2}: i\gamma^\mu \partial_\mu \Psi = m\Psi$$

$$(\gamma^\mu p_\mu \pm m) u = 0 \quad E = \pm \sqrt{\mathbf{p}^2 + m^2} \quad g_e = \sqrt{4\pi e^2} = \sqrt{4\pi\alpha}$$

$$\text{Vertice: } i\gamma_\mu g_e (2\pi)^4 \delta^4(k_1 + k_2 + k_3)$$

$$\text{Leptoni, quarks: } \frac{i(\gamma^\mu q_\mu + m)}{q^2 - m^2} \frac{d^4 q}{(2\pi)^4} \quad \text{Fotoni: } \frac{ig^{\mu\nu}}{q^2} \frac{d^4 q}{(2\pi)^4}$$

$$M_{if} = -\frac{g_e^2}{(p_1 - p_3)^2} [\bar{u}_{s_3}(p_3) \gamma^\mu u_{s_1}(p_1)] [\bar{u}_{s_4}(p_4) \gamma_\mu u_{s_2}(p_2)]$$

Scattering	Nome
$e^- + \mu^- \rightarrow e^- + \mu^-$	elastic scattering (particelle di Dirac)
$e^- + e^- \rightarrow e^- + e^-$	scattering
$e^- + e^+ \rightarrow e^- + e^+$	scattering
$e^- + \gamma \rightarrow e^- + \gamma$	Compton scattering
$e^- + p \rightarrow e^- + p$	elastic scattering (esclusiva)
$e^- + p \rightarrow e^- + X$	inelastic scattering (inclusiva)
$e^- + e^+ \rightarrow \gamma, Z^0 \rightarrow q + \bar{q} \rightarrow \text{adroni}$	annihilation into hadrons

$$g_R = g_e \sqrt{1 - \frac{g_e^2}{12\pi^2} \left[ \ln \left( \frac{M_{\text{cutoff}}^2}{m^2} \right) \right]} = \sqrt{4\pi\alpha(q^2)}$$

$$\alpha(q^2) \equiv \alpha(q_0^2) \left[ 1 + \frac{\alpha(q_0^2)}{3\pi} f \left( -\frac{q^2}{m^2} \right) \right]$$

$$M_{if} = -g_R^2 [\bar{u}(3) \gamma^\mu u(1)] \frac{g_{\mu\nu}}{q^2} \left[ 1 + \frac{g_R^2}{12\pi^2} f \left( -\frac{q^2}{m^2} \right) \right] [\bar{u}(4) \gamma^\nu u(2)]$$

## 9 Interazione elettromagnetica

### 9.1 $e^- + \mu^- \rightarrow e^- + \mu^-$

$$M_{if} = -\frac{g_e^2}{(p_1 - p_3)^2} [\bar{u}_{s_3}(p_3) \gamma^\mu u_{s_1}(p_1)] [\bar{u}_{s_4}(p_4) \gamma_\mu u_{s_2}(p_2)] = -\frac{g_e^2}{(p_1 - p_3)^2} J_e^\lambda J_{\mu,\lambda}$$

$$\langle |M_{if}|^2 \rangle = \frac{8g_e^4}{(p_1 - p_3)^4} [(p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_3 \cdot p_2) - (p_1 \cdot p_3)m_2^2]$$

$$\text{Formula di Dirac: } \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4\left(\frac{\theta}{2}\right)} \frac{E'}{E} \left[ \cos^2\left(\frac{\theta}{2}\right) - \frac{q^2}{2m_\mu^2} \sin^2\left(\frac{\theta}{2}\right) \right]$$

### 9.2 $e^- + p \rightarrow e^- + p$

$$J_{p,\lambda} \equiv \bar{u}(4) \left[ F_1(q^2) \gamma_\lambda + \frac{k}{2M_p} F_2(q^2) i\sigma_{\lambda\nu} q^\nu \right] u(2) \quad k = 1.79$$

Fattore di forma elettrico:  $F_1$       Fattore di forma magnetico:  $F_2$

$$G_E = F_1 + \frac{kq^2}{4M^2} F_2 \quad G_M = F_1 + kF_2$$

$$\text{Rosenbluth: } \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4\left(\frac{\theta}{2}\right)} \frac{E'}{E} \left\{ \left[ F_1^2 - \frac{kq^2}{4M_p^2} F_2^2 \right] \cos^2 \frac{\theta}{2} - \frac{q^2}{2M_p^2} [F_1 + kF_2]^2 \sin^2 \left( \frac{\theta}{2} \right) \right\}$$

### 9.3 $e^- + p \rightarrow e^- + X$

$$\frac{d\sigma}{dE'd\Omega} = \frac{\alpha^2}{4E^2 \sin^4\left(\frac{\theta}{2}\right)} \left[ 2W_1(q^2, x) \sin^2\left(\frac{\theta}{2}\right) + W_2(q^2, x) \cos^2\left(\frac{\theta}{2}\right) \right]$$

$$\text{Fattori di struttura: } W_1 = \frac{[F_1 + kF_2]^2}{2M_p} \delta(x-1) \quad W_2 = -\frac{2M}{q^2} \left[ F_1^2 - \frac{kq^2}{4M_p^2} F_2^2 \right] \delta(x-1)$$

## 9.4 Deep inelastic scattering

Relazione Callan-Gross:  $2xF_1(x) = F_2(x)$

Funzione di struttura:  $F_1 = W_1 M_p$       Funzione di struttura:  $F_2 = -\frac{q^2}{2M_p x} W_2$

$$W_1^i = \frac{Q_i^2}{2M_p} \delta(x - z_i) \quad W_2^i = -\frac{2x^2 M_p Q_i^2}{q^2} \delta(x - z_i)$$

Considerando il protone come  $uud$ :

$$F_1(x) = \frac{1}{2} \left[ 2 \left( \frac{2}{3} \right)^2 \delta \left( \frac{m_u}{M_p} - x \right) + \left( -\frac{1}{3} \right)^2 \delta \left( \frac{m_d}{M_p} - x \right) \right] = \frac{1}{2} \delta \left( \frac{m_u}{M_p} - x \right)$$

$$F_2(x) = x \delta \left( \frac{m_u}{M_p} - x \right)$$

## 9.5 $e^- + e^+ \rightarrow \gamma, Z^0 \rightarrow q + \bar{q} \rightarrow \text{adroni}$

$$M_{if} = -\frac{Qg_e^2}{(p_1 + p_2)^2} [\bar{v}(2) \gamma_\mu u(1)] [\bar{u}(3) \gamma^\mu v(4)]$$

$$\sigma = \frac{\pi}{3} \left( \frac{Q\alpha}{E} \right)^2 \sqrt{1 - \frac{M^2}{E^2}} \sqrt{1 - \frac{m^2}{E^2}} \left( 1 + \frac{M^2}{2E^2} \right) \left( 1 + \frac{m^2}{2E^2} \right)$$

## 10 Interazione debole

Corrente	Carica CC	Neutra NC
Mediatore	$W^\pm$	$Z^0$
$\Delta Q$	$\neq 0$	$= 0$

$$W^\pm \text{ propagatore: } -\frac{i \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2} \right)}{q^2 - M_W^2} \quad Z^0 \text{ propagatore: } -\frac{i \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{M_Z^2} \right)}{q^2 - M_Z^2}$$

$$\text{Se } q^2 \ll M_{W,Z}^2 \implies \frac{ig_{\mu\nu}}{M_{W,Z}^2}$$

$$\text{Vertice CC: } -\frac{ig_W}{\sqrt{8}} [\gamma^\mu (1 - \gamma^5)] \quad \text{Vertice NC: } -\frac{ig_Z}{2} [\gamma^\mu (C_V - C_A \gamma^5)]$$

$$C_V = (T_f^3) - 2 \sin^2(\theta_W) Q_f \quad \sin^2(\theta_W) = 0.23153 \pm 0.00016$$

$$M_W = 80.419 \pm 0.056 \text{ GeV} \quad M_Z = 91.1882 \pm 0.0022 \text{ GeV}$$

	$Q_f (e)$	$(T_f^3)_{LH}$	$(T_f^3)_{RH}$	$C_{V,f}$	$C_{A,f}$
$\nu_e, \nu_\mu, \nu_\tau$	0	+1/2		+1/2	+1/2
$e, \mu, \tau$	-1	-1/2	0	$-1/2 + 2 \sin^2 \theta_W$	-1/2
$u, c, t$	+2/3	+1/2	0	$+1/2 - 4/3 \sin^2 \theta_W$	+1/2
$d, s, b$	-1/3	-1/2	0	$-1/2 + 2/3 \sin^2 \theta_W$	-1/2

Mediatore	Decadimento
$W^-$	$e^- \bar{\nu}_e, \mu^- \bar{\nu}_\mu, \tau^- \bar{\nu}_\tau$
$W^+$	$e^+ \nu_e, \mu^+ \nu_\mu, \tau^+ \nu_\tau$
$W^\pm$	$q' \bar{q}$
$Z^0$	$e^+ e^-, \mu^+ \mu^-, \tau^+ \tau^-, \nu \bar{\nu}, q \bar{q}$

  

Scattering	Nome
$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$	quasi-elastic scattering
$\mu^- \rightarrow \bar{\nu}_e + \nu_\mu + e^-$	muon decay
$n \rightarrow \bar{\nu}_e + p + e^-$	neutron $\beta$ decay
$\pi \rightarrow l + \nu_l$	$\pi$ meson decay
$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$	elastic scattering
$e^- + e^+ \rightarrow \gamma, Z^0 \rightarrow f + \bar{f}$	electron-positron annihilation at $Z^0$ pole
$\nu_e + e^- \rightarrow e^- + \nu_e$	elastic scattering
$\bar{\nu}_e + e^- \rightarrow e^- + \bar{\nu}_e$	elastic scattering
$\nu_\mu + q \rightarrow \mu^- + q'$	scattering

### 10.1 $\nu_\mu + e^- \rightarrow \mu^- + \nu_e$

$$M_{if} = \frac{g_W^2}{8M_W^2} [\bar{u}(3) \gamma^\mu (1 - \gamma^5) u(1)] [\bar{u}(4) \gamma_\mu (1 - \gamma^5) u(2)]$$

$$\langle |M_{if}|^2 \rangle = 2 \left( \frac{g_W}{M_W} \right)^4 (p_1 \cdot p_2) (p_3 \cdot p_4) \quad \sigma = \frac{1}{8\pi} \left[ \left( \frac{g_W}{M_W} \right)^2 E \right]^2 \left[ 1 - \left( \frac{m_\mu}{2E} \right)^2 \right]^2$$

### 10.2 $\mu^- \rightarrow \bar{\nu}_e + \nu_\mu + e^-$

$$M_{if} = \frac{g_W^2}{8M_W^2} [\bar{u}(3) \gamma^\mu (1 - \gamma^5) u(1)] [\bar{u}(4) \gamma_\mu (1 - \gamma^5) u(2)]$$

$$\langle |M_{if}|^2 \rangle = 2 \left( \frac{g_W}{M_W} \right)^4 (p_1 \cdot p_2) (p_3 \cdot p_4) \quad \Gamma = \left( \frac{g_W}{M_W} \right)^4 \frac{m_\mu^5}{12(8\pi)^3} = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

$$\text{Fermi CC: } G_F = \frac{\sqrt{2}}{8} \left( \frac{g_W}{M_W} \right)^2 = 1.166 \cdot 10^{-5} \text{ GeV}^{-2}$$

### 10.3 Decadimento $\beta$ : $n \rightarrow p + e^- + \bar{\nu}_e$

$$M_{if} = \frac{g_W^2}{8M_W^2} [\bar{u}(3) \gamma^\mu (C_V - C_A \gamma^5) u(1)] [\bar{u}(4) \gamma_\mu (1 - \gamma^5) v(2)]$$

$$\langle |M_{if}|^2 \rangle = \left( \frac{g_W}{M_W} \right)^4 M_n E_\nu (M_n^2 - M_p^2 - m_e^2 - 2M_n E_\nu)$$

$$C_V = 1.0 \pm 0.003 \quad C_A = 1.26 \pm 0.02$$



**10.4**  $\pi \rightarrow l + \nu_l$

$$M_{if} = \frac{g_W^2}{8M_W^2} [\bar{u}(3) \gamma_\mu (1 - \gamma_5) v(2)] f_\pi p^\mu \quad f_\pi = m_\pi \cos(\theta_C)$$

$$\langle |M_{if}|^2 \rangle = \left( \frac{g_W}{2M_W} \right)^4 f_\pi^2 m_l^2 (m_\pi^2 - m_l^2) \quad \Gamma = \left( \frac{g_W}{4M_W} \right)^4 \frac{f_\pi^2}{\pi m_\pi} m_l^2 (m_\pi^2 - m_l^2)^2$$

$$\theta_C = 13.1^\circ$$

**10.5**  $\nu_\mu + e^- \rightarrow \nu_\mu + e^-$

$$M_{if} = \frac{g_Z^2}{8M_Z^2} [\bar{u}(3) \gamma_\mu (1 - \gamma_5) u(1)] [\bar{u}(4) \gamma^\mu (C_V - C_A \gamma_5) u(2)]$$

$$\sigma = \frac{2}{3\pi} \left[ \left( \frac{g_Z}{2M_Z} \right)^2 E_\nu \right]^2 (C_V^2 + C_A^2 + C_V C_A)$$

**10.6**  $e^- + e^+ \rightarrow \gamma, Z^0 \rightarrow f + \bar{f}$

$$\sigma_\gamma = \frac{g_e^4 Q_f^2}{48\pi E^2} \quad \sigma_Z = \frac{(g_Z^2 E)^2}{48\pi} \left\{ \frac{\left[ (C_V^f)^2 + (C_A^f)^2 \right] \left[ (C_V^e)^2 + (C_A^e)^2 \right]}{(4E^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \right\}$$

$$M_{if} = -\frac{g_Z^2}{4(q^2 - M_Z^2)} [\bar{u}(4) \gamma^\mu (C_V^f - C_A^f \gamma_5) v(3)] \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{M_Z^2} \right) [\bar{v}(2) \gamma^\mu (C_V^e - C_A^e \gamma_5) u(1)]$$

**10.7**  $\nu_e + e^- \rightarrow e^- + \nu_e$

$$\langle |M_{if}|^2 \rangle = 2 \left( \frac{g_W}{M_W} \right)^4 (p_1 \cdot p_2) (p_3 \cdot p_4) \quad \sigma = \frac{G_F^2}{\pi} s \quad G_F = \frac{\sqrt{2}}{8} \left( \frac{g_W}{M_W} \right)^2$$

**10.8**  $\bar{\nu}_e + e^- \rightarrow e^- + \bar{\nu}_e$

$$\langle |M_{if}|^2 \rangle = 2 \left( \frac{g_W}{M_W} \right)^4 (p_1 \cdot p_2) (p_3 \cdot p_4) \quad \sigma = \frac{G_F^2}{3\pi} s \quad G_F = \frac{\sqrt{2}}{8} \left( \frac{g_W}{M_W} \right)^2$$

### 10.9 Neutrino-nucleon deep inelastic scattering. $\nu_\mu (\bar{\nu}_\mu) + N \rightarrow \mu^- (\mu^+) + X$

$$q^2 = -4EE' \sin^2 \left( \frac{\theta}{2} \right) \quad x = -\frac{q^2}{2M(E-E')} \quad y = \frac{(E-E')}{E}$$

$$\frac{d\sigma^{eN}}{dq^2 dx} = \frac{4\pi\alpha^2}{q^4} \left[ \frac{y^2}{2} \frac{2xF_1^{eN}(x)}{x} + (1-y) \frac{F_2^{eN}(x)}{x} \right]$$

$$\frac{4\pi\alpha^2}{q^4} = \frac{G_F^2}{2\pi} \quad dq^2 = 2M_N E x dy$$

$$\frac{d\sigma^{\nu N}}{dx dy} = \frac{G_F^2 M_N E}{2\pi} \left[ (F_2^{\nu N}(x) + xF_3^{\nu N}(x)) (F_2^{\nu N}(x) - xF_3^{\nu N}(x)) (1-y)^2 \right]$$

$$F_2^{\nu N} = 2x [Q(x) + \bar{Q}(x)] \quad F_2^{\nu N} = \frac{18}{5} F_2^{eN} \quad xF_3^{\nu N} = 2x [Q(x) - \bar{Q}(x)]$$

$$Q(x) = d^p(x) + d^n(x) = d(x) + u(x) \quad \bar{Q}(x) = \bar{u}^p(x) + \bar{u}^n(x) = \bar{u}(x) + \bar{d}(x)$$

$$\text{Gross-Llewellyn-Smith: } \int_0^1 F_3^{\nu N} dx = 3$$

## 11 QCD

$$g_s = \sqrt{4\pi\alpha_s}$$

$$\langle 1| = r\bar{b} \quad \langle 2| = r\bar{g} \quad \langle 3| = b\bar{r} \quad \langle 4| = b\bar{g} \quad \langle 5| = g\bar{r} \quad \langle 6| = g\bar{b}$$

$$\langle 7| = \frac{r\bar{r} - g\bar{g}}{\sqrt{2}} \quad \langle 8| = \frac{r\bar{r} + g\bar{g} - 2b\bar{b}}{\sqrt{6}} \quad \langle 9| = \frac{r\bar{r} + g\bar{g} + b\bar{b}}{\sqrt{3}}$$

$$f = \frac{1}{4} [c_3^+ \lambda^\alpha c_1] [c_2^+ \lambda^\alpha c_4] \quad \text{Vertice quark-gluone: } \frac{-ig_s \lambda^\alpha \gamma^\mu}{2}$$

$$\text{Propagatore quark: } \frac{i(\gamma^\mu q_\mu + m)}{q^2 - m^2} \quad \text{Propagatore gluone: } -\frac{ig^{\mu\nu} \delta^{\alpha\beta}}{q^2}$$

$$M_{if} = -\frac{g_s^2}{(p_1 - p_3)^2} [\bar{u}(3) \gamma^\mu u(1)] [\bar{v}(2) \gamma_\mu v(4)] f$$

$$\alpha_s(q^2) = \frac{\alpha_s(\mu^2)}{1 + \left( \frac{\alpha_s(\mu^2)}{12\pi} \right) (11n - 2f) \ln \left( \frac{q^2}{\mu^2} \right)}$$

$$\alpha_s(q) = \frac{12\pi}{(11n - 2f) \ln \left( \frac{q^2}{\Lambda^2} \right)} \quad 100 \text{ MeV} < \Lambda < 500 \text{ MeV}$$

### 11.1 Produzione di jet

$$\frac{d\sigma}{dx_1 dx_2 d(\cos(\theta^*))} = \sum_{l,m} \frac{F_l(x_1)}{x_1} \frac{F_m(x_2)}{x_2} \frac{d\sigma_{l,m}}{d(\cos(\theta^*))} K$$

Cross section elementare:  $\frac{d\sigma_{l,m}}{d(\cos(\theta^*))}$       Densità partoni:  $\frac{F_{l,m}(x_{1,2})}{x_{1,2}}$        $K \simeq 2.5$

$$F(x) = G(x) + \frac{4}{9} (Q(x) + \bar{Q}(x))$$

### 11.2 Drell-Yan production

$$\begin{aligned} p + p &\rightarrow l^+ + l^- + X & q + \bar{q} &\rightarrow l^+ + l^- \\ e^+ + e^- &\rightarrow \text{adroni}, e^+ + e^- \rightarrow \bar{q} + q & l^\pm + p &\rightarrow l^\pm + X, l^\pm + q \rightarrow l^\pm + q \\ \sigma(q\bar{q} \rightarrow l^+ l^-) &= \frac{4\pi\alpha^2}{9M^2} Q_f^2 K & \frac{d^2\sigma}{dx_1 dx_2} &= \frac{4\pi\alpha^2}{9M^2} K S(x_1, x_2) \\ S(x_1, x_2) &= \sum_f Q_f^2 [q_B(x_1) \bar{q}_T(x_2) + \bar{q}_B(x_1) q_T(x_2)] \end{aligned}$$