USSR: An UltraSound Sparse Regularization Framework

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Introduction and objectives

1. Ultrafast ultrasound (US) imaging uses unfocused waves to insonify the whole field-of-view at once
2. We present USSR: An UltraSound Sparse Regularization framework composed of:
   - Matrix-free and highly parallel formulations of the measurement and its adjoint in the context of plane-wave (PW) imaging
   - Sparse regularization algorithm with two sparsity priors: ℓ_p-norm to the power of p ($p \geq 1$), ℓ_1-norm in a sparsity averaging (SA) model
3. USSR provides a fast, high-quality and low-memory-footprint image reconstruction method

Notations and model

- Notations:
  - 1D probe composed of N_{el} transducer elements located at $\boldsymbol{p}^i \in \Pi$ recording samples at time instants $t^l = t^0 + l\Delta t$, with $l \in \{1, \ldots, N_t\}$
  - $m\left(\boldsymbol{p}^i, t^l\right)$ signal received at time instant t^l by element located at $\boldsymbol{p}^i$
  - $v_{pe}(t)$ pulse-shape
  - Medium Ω composed of points located at $\boldsymbol{r}^n = \left[x^k, z^l\right]^T$, $(k, l) \in \{1, \ldots, N_x\} \times \{1, \ldots, N_z\}$ and $n = (k-1)N_z + l$
  - Each point characterized by its tissue-reflectivity function $\gamma\left(\boldsymbol{r}^n\right)$

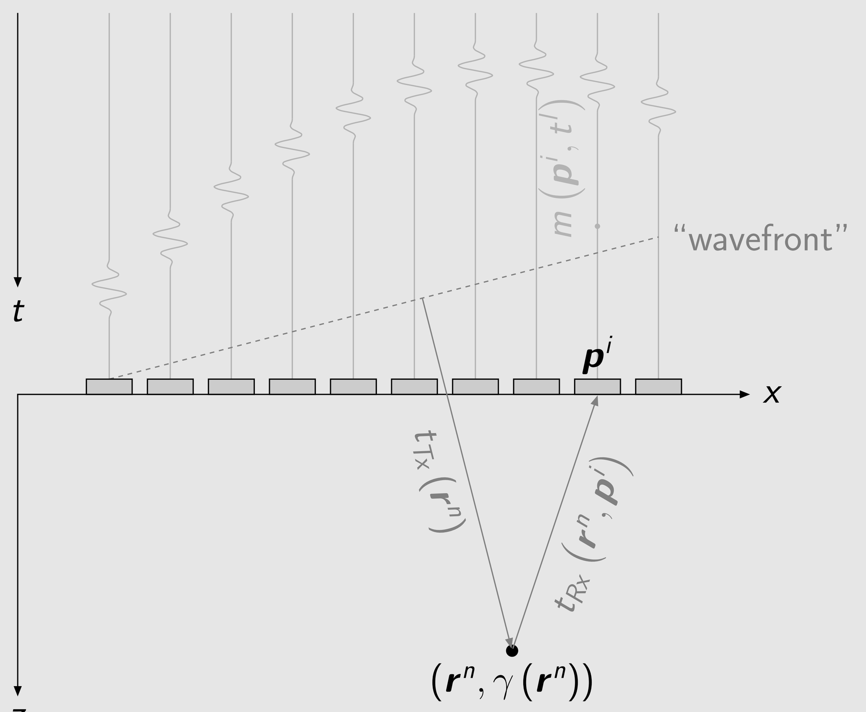

Figure Standard setting for US imaging

- Pulse-echo spatial impulse response model

$$m\left(\boldsymbol{p}^i, t^l\right) = \int_{\boldsymbol{r} \in \Omega} o_d\left(\boldsymbol{r}, \boldsymbol{p}^i\right) \gamma(\boldsymbol{r})\, v_{pe}\left(t^l - t_{Tx}(\boldsymbol{r}) - t_{Rx}\left(\boldsymbol{r}, \boldsymbol{p}^i\right)\right) d\boldsymbol{r}$$

$t_{Tx}(\boldsymbol{r})$ propagation delay on transmit, $t_{Rx}\left(\boldsymbol{r}, \boldsymbol{p}^i\right) = \left\|\boldsymbol{r} - \boldsymbol{p}^i\right\|_2 / c$ propagation delay on receive and $o_d\left(\boldsymbol{r}, \boldsymbol{p}^i\right) = o\left(\boldsymbol{r}, \boldsymbol{p}^i\right) / 2\pi \left\|\boldsymbol{r} - \boldsymbol{p}^i\right\|_2$ where $o\left(\boldsymbol{r}, \boldsymbol{p}^i\right)$ accounts for the element directivity

Parametric formulation of the model

- The model can be written as:

$$\begin{aligned} m\left(\boldsymbol{p}^i, t^l\right) &= \iint_{\tau \in \mathbb{R}, \boldsymbol{r} \in \Gamma\left(\boldsymbol{p}^i, \tau\right)} \frac{o_d\left(\boldsymbol{r}, \boldsymbol{p}^i\right) \gamma(\boldsymbol{r})}{\left|\nabla_{\boldsymbol{r}} g\right|} d\sigma(\boldsymbol{r})\, v_{pe}\left(t^l - \tau\right) d\tau \\ &= \mathcal{H}\{\gamma\}\left(\boldsymbol{p}^i, t^l\right) \end{aligned} \tag{1}$$

$g\left(\boldsymbol{r}, \boldsymbol{p}^i, t\right) = t - t_{Tx}(\boldsymbol{r}) - t_{Rx}\left(\boldsymbol{r}, \boldsymbol{p}^i\right)$, $\Gamma\left(\boldsymbol{p}^i, t\right) = \left\{\boldsymbol{r} \in \Omega \mid g\left(\boldsymbol{r}, \boldsymbol{p}^i, t\right) = 0\right\}$, $\nabla_{\boldsymbol{r}} g$ denotes the gradient of g w.r.t $\boldsymbol{r}$, $d\sigma(\boldsymbol{r})$ is the measure over the 1D-curve $\Gamma\left(\boldsymbol{p}^i, t\right)$

- To have an efficient way of calculating the integral defined in Equation (1), we derive a parameterization of $\Gamma\left(\boldsymbol{p}^i, t\right)$ as follows:

$$\boldsymbol{r} = [x, z]^T \in \Gamma\left(\boldsymbol{p}^i, t\right) \Leftrightarrow \boldsymbol{r}\left(\alpha, \boldsymbol{p}^i, t\right) = \left[\alpha, f\left(\alpha, \boldsymbol{p}^i, t\right)\right]^T, \ \alpha \in \mathbb{R} \tag{2}$$

- This leads us to the parametric formulation of the model

$$m\left(\boldsymbol{p}^i, t^l\right) = \iint_{\tau \in \mathbb{R}, \alpha \in \mathbb{R}} o_d\left(\boldsymbol{r}\left(\alpha, \boldsymbol{p}^i, t^l\right), \boldsymbol{p}^i\right) \gamma\left(\boldsymbol{r}\left(\alpha, \boldsymbol{p}^i, t^l\right)\right) \frac{\left|J_\alpha\right|}{\left|\nabla_{\boldsymbol{r}} g\right|} d\alpha\, v_{pe}\left(t^l - \tau\right) d\tau \tag{3}$$

$|J_\alpha|$ Jacobian associated with the change of variable

Parametric equations for plane wave imaging

- Parametric equations obtained by finding the roots of the following function:

$$f(z) = \sqrt{\left(x - p_x^i\right)^2 + \left(z - p_z^i\right)^2} + z\cos(\theta) + x\sin(\theta) - ct \tag{4}$$

which gives the following solution:

$$z = \frac{1}{\sin(\theta)^2}\left(p_z^i - ct\cos(\theta) + x\sin(\theta)\cos(\theta) \pm \sqrt{\Delta}\right) \tag{5}$$

$$\Delta = \left(ct - p_z^i\cos(\theta) - p_x^i\sin(\theta)\right)\left(ct - p_z^i\cos(\theta) + \left(p_x^i - 2x\right)\sin(\theta)\right)$$

Dicretization of the model

- Equation (3) is discretized as

$$m\left(\boldsymbol{p}^i, t^l\right) = \mathcal{H}_d\{\gamma\}\left(\boldsymbol{p}^i, t^l\right) = \left(\tilde{\boldsymbol{m}}\left(\boldsymbol{p}^i\right) *_t \boldsymbol{v_{pe}}\right)\left(t^l\right) \tag{6}$$

where $*_t$ is the 1D-convolution and $\tilde{\boldsymbol{m}}\left(\boldsymbol{p}^i\right) = \left(\tilde{m}\left(\boldsymbol{p}^i, t^l\right)\right)_{t^l \in T_d}$ defined by:

$$\tilde{m}\left(\boldsymbol{p}^i, t^l\right) = \sum_{k=1}^{N_x} w^k o_d\left(\boldsymbol{r}\left(\alpha^k, \boldsymbol{p}^i, t^l\right), \boldsymbol{p}^i\right) \varphi\left(\boldsymbol{r}\left(\alpha^k, \boldsymbol{p}^i, t^l\right)\right) \gamma \tag{7}$$

where w^k is the integration weight and φ is a 1D-interpolation kernel

Parametric formulation of the adjoint operator of the model

- Adjoint operator of the linear operator $\mathcal{H}\{\gamma\}$ described in (1) is defined as:

$$\mathcal{H}^\dagger\{m\}\left(\boldsymbol{r}^n\right) = \sum_{\boldsymbol{p}^i \in \Pi} o_d\left(\boldsymbol{r}^n, \boldsymbol{p}^i\right) \int_{\tau \in \mathbb{R}} m\left(\boldsymbol{p}^i, \tau\right) u\left(t_{Tx}\left(\boldsymbol{r}^n\right) + t_{Rx}\left(\boldsymbol{r}^n, \boldsymbol{p}^i\right) - \tau\right) d\tau \tag{8}$$

where $u(t) = v_{pe}(-t)$ is the matched filter of the pulse shape

- Discretization of the adjoint operator expressed as:

$$\mathcal{H}_d^\dagger\{\boldsymbol{m}\}\left(\boldsymbol{r}^n\right) = \sum_{\boldsymbol{p}^i \in \Pi} \omega^n o_d\left(\boldsymbol{r}^n, \boldsymbol{p}^i\right) \psi\left(t_{Tx}\left(\boldsymbol{r}^n\right) + t_{Rx}\left(\boldsymbol{r}^n, \boldsymbol{p}^i\right)\right) \hat{\boldsymbol{m}}, \tag{9}$$

where $\hat{\boldsymbol{m}} = \boldsymbol{m} *_t \boldsymbol{u}$, ψ is a 1D-interpolation kernel and ω^n accounts for the integration weight

Image reconstruction procedure

- Linear measurement operator $\mathcal{H}_d\{\gamma\}$ defines the following inverse problem:

$$\boldsymbol{m} = \mathrm{H}_d \gamma + \boldsymbol{\nu} \tag{10}$$

where $\mathrm{H}_d \in \mathbb{R}^{N_{el}N_t \times N_x N_z}$ matrix associated with the linear measurement model and $\boldsymbol{\nu} \in \mathbb{R}^{N_{el}N_t}$ noise due to model discrepancy and discretization

- Sparse regularization to solve the problem

$$\min_{\gamma \in \mathbb{R}^{N_x N_z}} \lambda \mathcal{R}(\gamma) + \frac{1}{2}\left\|\mathrm{H}_d \gamma - \boldsymbol{m}\right\|_2^2 \tag{11}$$

where $\mathcal{R}(\gamma)$ prior term and $\lambda \in \mathbb{R}_+$ regularization parameter

- Two priors considered in this study:
  - ℓ_p-norm to the power of p: $\mathcal{R}(\gamma) = \|\gamma\|_p^p$, $p \geq 1$;
  - ℓ_1-norm in the SA model: $\mathcal{R}(\gamma) = \left\|\Psi^\dagger \gamma\right\|_1$, where $\Psi = \frac{1}{\sqrt{q}}\left[\Psi_1, \ldots, \Psi_q\right]$, with Ψ_i the i-th Daubechies wavelet.

Implementation of USSR

- USSR implemented on GPU platforms and on multi-threaded CPU platforms
- 200 iterations of the fast iterative shrinkage thresholding algorithm used for reconstruction
- Reconstructions of images from PICMUS dataset (1 PW insonification) take around 4.5 s on an NVIDIA Titan X GPU card
- Code available on GitHub: `https://github.com/LTS5/USSR`

Reconstructed B-mode images

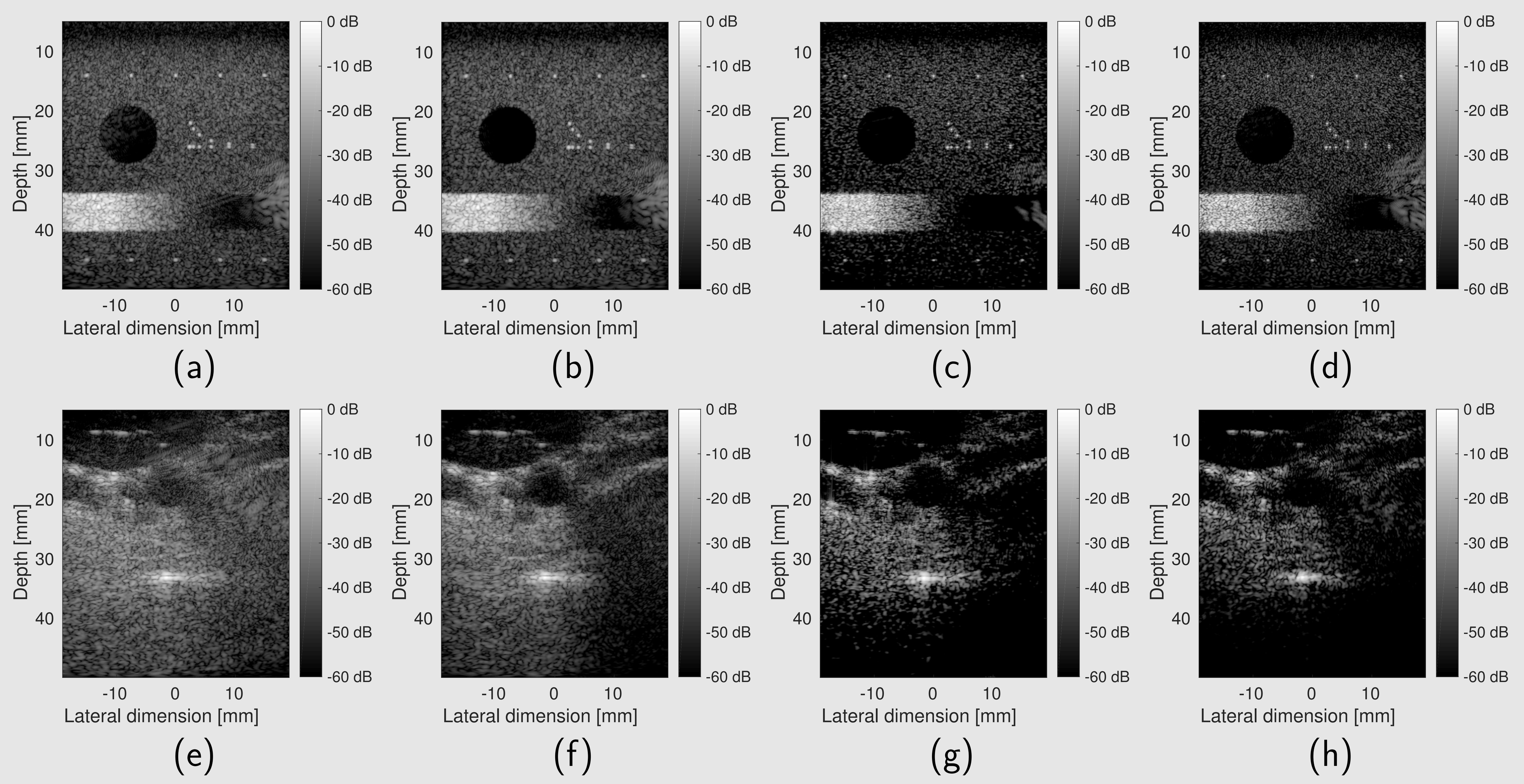

Figure Image of the numerical phantom of PICMUS dataset reconstructed with (a) DAS - 1 PW insonification, (b) DAS - 5 PW insonifications, (c) USSR-SA - 1 PW insonification, (c) USSR - ℓ_p - 1 PW insonification; Image of the *in-vivo* carotid reconstructed with (e) DAS - 1 PW insonification, (f) DAS - 5 PW insonifications, (g) USSR-SA - 1 PW insonification, (h) USSR-ℓ_p - 1 PW insonification.

Conclusion and perspectives

1. We propose USSR: an UltraSound Sparse Regularization framework
   - Matrix-free and highly parallelizable formulations of measurement model and adjoint
   - Two priors: ℓ_p-norm in the image domain and ℓ_1-norm in a sparsity averaging model
2. The proposed approach leads to high-quality at fast rates, with low-memory footprint
3. Current work focuses on optimizing the code

Acknowledgments

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