

Introduction and objectives

1. Ultrafast ultrasound (US) imaging uses unfocused waves, such as plane waves (PW), to insonify the whole field-of-view at once
 - ▶ High frame rate
 - ▶ Degraded image quality
2. Sparse regularization (SR) methods permit reconstruction of high quality images but involve
 - ▶ Storage of huge matrices \rightarrow high-memory footprint
 - ▶ Iterative algorithms \rightarrow slow
3. We present USSR: An UltraSound Sparse Regularization framework, a fast, high-quality and low-memory-footprint image reconstruction method
 - ▶ Parallel matrix-free formulations of the measurement and its adjoint for PW imaging
 - ▶ SR algorithm with two sparsity priors: ℓ_p -norm to the power of p ($p \geq 1$), ℓ_1 -norm in a sparsity averaging (SA) model

Notations and model

▶ Notations:

- ▶ 1D probe composed of N_{el} transducer elements located at $\mathbf{p}^i \in \Pi$ recording samples at time instants $t^l = t^0 + l\Delta t$, with $l \in \{1, \dots, N_t\}$
- ▶ $m(\mathbf{p}^i, t^l)$ signal received at time instant t^l by element located at \mathbf{p}^i
- ▶ $v_{pe}(t)$ pulse-shape
- ▶ Medium Ω composed of points located at $\mathbf{r}^n = [x^k, z^l]^T$, $(k, l) \in \{1, \dots, N_x\} \times \{1, \dots, N_z\}$ and $n = (k-1)N_z + l$
- ▶ Each point characterized by its tissue-reflectivity function $\gamma(\mathbf{r}^n)$
- ▶ Pulse-echo spatial impulse response model

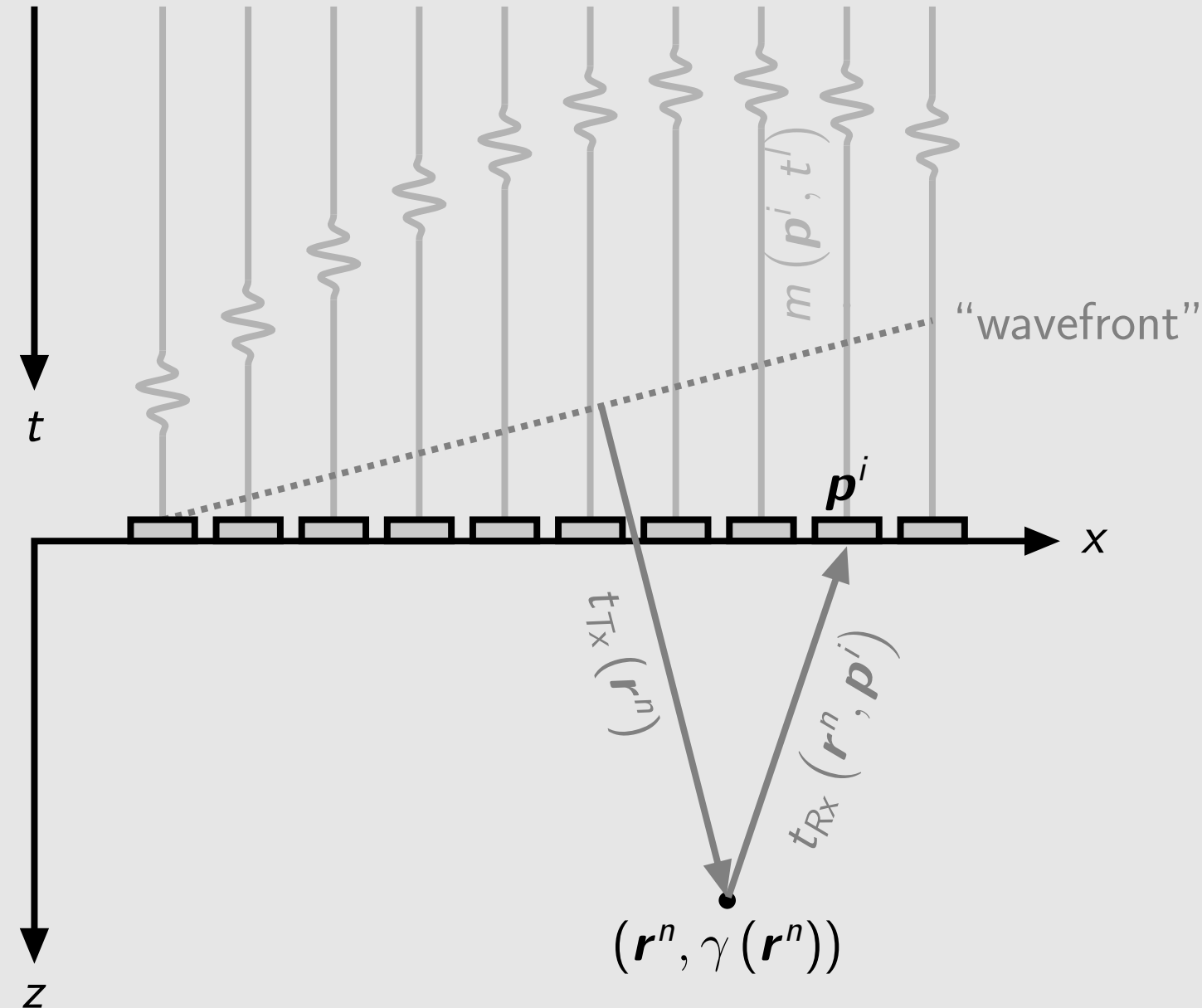


Figure Standard setting for US imaging

$$m(\mathbf{p}^i, t^l) = \int_{\mathbf{r} \in \Omega} o_d(\mathbf{r}, \mathbf{p}^i) \gamma(\mathbf{r}) v_{pe}(t^l - t_{Tx}(\mathbf{r}) - t_{Rx}(\mathbf{r}, \mathbf{p}^i)) d\mathbf{r}$$

$t_{Tx}(\mathbf{r})$ propagation time on transmit, $t_{Rx}(\mathbf{r}, \mathbf{p}^i) = \|\mathbf{r} - \mathbf{p}^i\|_2 / c$ propagation time on receive and $o_d(\mathbf{r}, \mathbf{p}^i) = o(\mathbf{r}, \mathbf{p}^i) / 2\pi \|\mathbf{r} - \mathbf{p}^i\|_2$ where $o(\mathbf{r}, \mathbf{p}^i)$ accounts for the element directivity

Parametric formulation of the model

- ▶ The model can be written as

$$m(\mathbf{p}^i, t^l) = \iint_{\tau \in \mathbb{R}, \mathbf{r} \in \Gamma(\mathbf{p}^i, \tau)} \frac{o_d(\mathbf{r}, \mathbf{p}^i) \gamma(\mathbf{r})}{|\nabla_{\mathbf{r}} g|} d\sigma(\mathbf{r}) v_{pe}(t^l - \tau) d\tau = \mathcal{H}\{\gamma\}(\mathbf{p}^i, t^l) \quad (1)$$

$g(\mathbf{r}, \mathbf{p}^i, t) = t - t_{Tx}(\mathbf{r}) - t_{Rx}(\mathbf{r}, \mathbf{p}^i)$, $\Gamma(\mathbf{p}^i, t) = \{\mathbf{r} \in \Omega \mid g(\mathbf{r}, \mathbf{p}^i, t) = 0\}$, $\nabla_{\mathbf{r}} g$ denotes the gradient of g w.r.t. \mathbf{r} , $d\sigma(\mathbf{r})$ is the measure over $\Gamma(\mathbf{p}^i, t)$

- ▶ We derive a parameterization of $\Gamma(\mathbf{p}^i, t)$

$$\mathbf{r} = [x, z]^T \in \Gamma(\mathbf{p}^i, t) \Leftrightarrow \mathbf{r}(\alpha, \mathbf{p}^i, t) = [\alpha, f(\alpha, \mathbf{p}^i, t)]^T, \alpha \in \mathbb{R} \quad (2)$$

- ▶ This leads us to the parametric formulation of the model

$$m(\mathbf{p}^i, t^l) = \iint_{\tau \in \mathbb{R}, \alpha \in \mathbb{R}} o_d(\mathbf{r}(\alpha, \mathbf{p}^i, t^l), \mathbf{p}^i) \gamma(\mathbf{r}(\alpha, \mathbf{p}^i, t^l)) \frac{|J_{\alpha}|}{|\nabla_{\mathbf{r}} g|} d\alpha v_{pe}(t^l - \tau) d\tau \quad (3)$$

$|J_{\alpha}|$ Jacobian associated with the change of variable

Parametric equations for plane wave imaging

- ▶ Parametric equations obtained by finding the roots of the following function:

$$f(z) = \sqrt{(x - p_x^i)^2 + (z - p_z^i)^2} + z \cos(\theta) + x \sin(\theta) - ct \quad (4)$$

which gives the following solution:

$$z = \sin(\theta)^{-2} \left(p_z^i - ct \cos(\theta) + x \sin(\theta) \cos(\theta) \pm \sqrt{\Delta} \right) \quad (5)$$

$$\Delta = (ct - p_z^i \cos(\theta) - p_x^i \sin(\theta)) (ct - p_z^i \cos(\theta) + (p_x^i - 2x) \sin(\theta))$$

Discretization of the model

- ▶ Equation (3) is discretized as

$$m(\mathbf{p}^i, t^l) = \mathcal{H}_d\{\gamma\}(\mathbf{p}^i, t^l) = (\tilde{\mathbf{m}}(\mathbf{p}^i) *_{\mathbf{t}} \mathbf{v}_{pe})(t^l) \quad (6)$$

where $*_{\mathbf{t}}$ is the 1D-convolution and $\tilde{\mathbf{m}}(\mathbf{p}^i) = (\tilde{m}(\mathbf{p}^i, t^l))_{t^l \in T_d}$ defined by:

$$\tilde{m}(\mathbf{p}^i, t^l) = \sum_{k=1}^{N_x} w^k o_d(\mathbf{r}(\alpha^k, \mathbf{p}^i, t^l), \mathbf{p}^i) \varphi(\mathbf{r}(\alpha^k, \mathbf{p}^i, t^l)) \gamma \quad (7)$$

where w^k is the integration weight and φ is a 1D-interpolation kernel

Parametric formulation of the adjoint operator of the model

- ▶ Adjoint operator of the linear operator $\mathcal{H}\{\gamma\}$ described in (1) is defined as:

$$\mathcal{H}^{\dagger}\{m\}(\mathbf{r}^n) = \sum_{\mathbf{p}^i \in \Pi} o_d(\mathbf{r}^n, \mathbf{p}^i) \int_{\tau \in \mathbb{R}} m(\mathbf{p}^i, \tau) u(t_{Tx}(\mathbf{r}^n) + t_{Rx}(\mathbf{r}^n, \mathbf{p}^i) - \tau) d\tau \quad (8)$$

where $u(t) = v_{pe}(-t)$ is the matched filter of the pulse shape

- ▶ Discretization of the adjoint operator expressed as:

$$\mathcal{H}_d^{\dagger}\{\mathbf{m}\}(\mathbf{r}^n) = \sum_{\mathbf{p}^i \in \Pi} \omega^n o_d(\mathbf{r}^n, \mathbf{p}^i) \psi(t_{Tx}(\mathbf{r}^n) + t_{Rx}(\mathbf{r}^n, \mathbf{p}^i)) \hat{\mathbf{m}}, \quad (9)$$

where $\hat{\mathbf{m}} = \mathbf{m} *_{\mathbf{t}} \mathbf{u}$, ψ is a 1D-interpolation kernel and ω^n accounts for the integration weight

Image reconstruction procedure

- ▶ Linear measurement operator $\mathcal{H}_d\{\gamma\}$ defines the following inverse problem:

$$\mathbf{m} = \mathbf{H}_d \gamma + \nu \quad (10)$$

where $\mathbf{H}_d \in \mathbb{R}^{N_{el}N_t \times N_xN_z}$ matrix associated with the linear measurement model and $\nu \in \mathbb{R}^{N_{el}N_t}$ noise due to model discrepancy and discretization

- ▶ Sparse regularization to solve the problem

$$\min_{\gamma \in \mathbb{R}^{N_xN_z}} \lambda \mathcal{R}(\gamma) + \frac{1}{2} \|\mathbf{H}_d \gamma - \mathbf{m}\|_2^2 \quad (11)$$

where $\mathcal{R}(\gamma)$ prior term and $\lambda \in \mathbb{R}_+$ regularization parameter

- ▶ Two priors considered in this study:

- ▶ ℓ_p -norm to the power of p : $\mathcal{R}(\gamma) = \|\gamma\|_p^p$, $p \geq 1$;
- ▶ ℓ_1 -norm in the SA model: $\mathcal{R}(\gamma) = \|\Psi^{\dagger} \gamma\|_1$, where $\Psi = \frac{1}{\sqrt{q}} [\Psi_1, \dots, \Psi_q]$, with Ψ_i the i -th Daubechies wavelet.

Implementation and performance of USSR

- ▶ USSR is implemented on GPU and multi-threaded CPU platforms
- ▶ 200 iterations of the fast iterative shrinkage thresholding algorithm used for reconstruction
- ▶ Reconstructions of images from PICMUS dataset (1 PW insonification) take around 4.5s on an NVIDIA Titan X GPU card
- ▶ Reproducible code available on GitHub:
<https://github.com/LTS5/USSR-IUS2017>

Reconstructed B-mode images

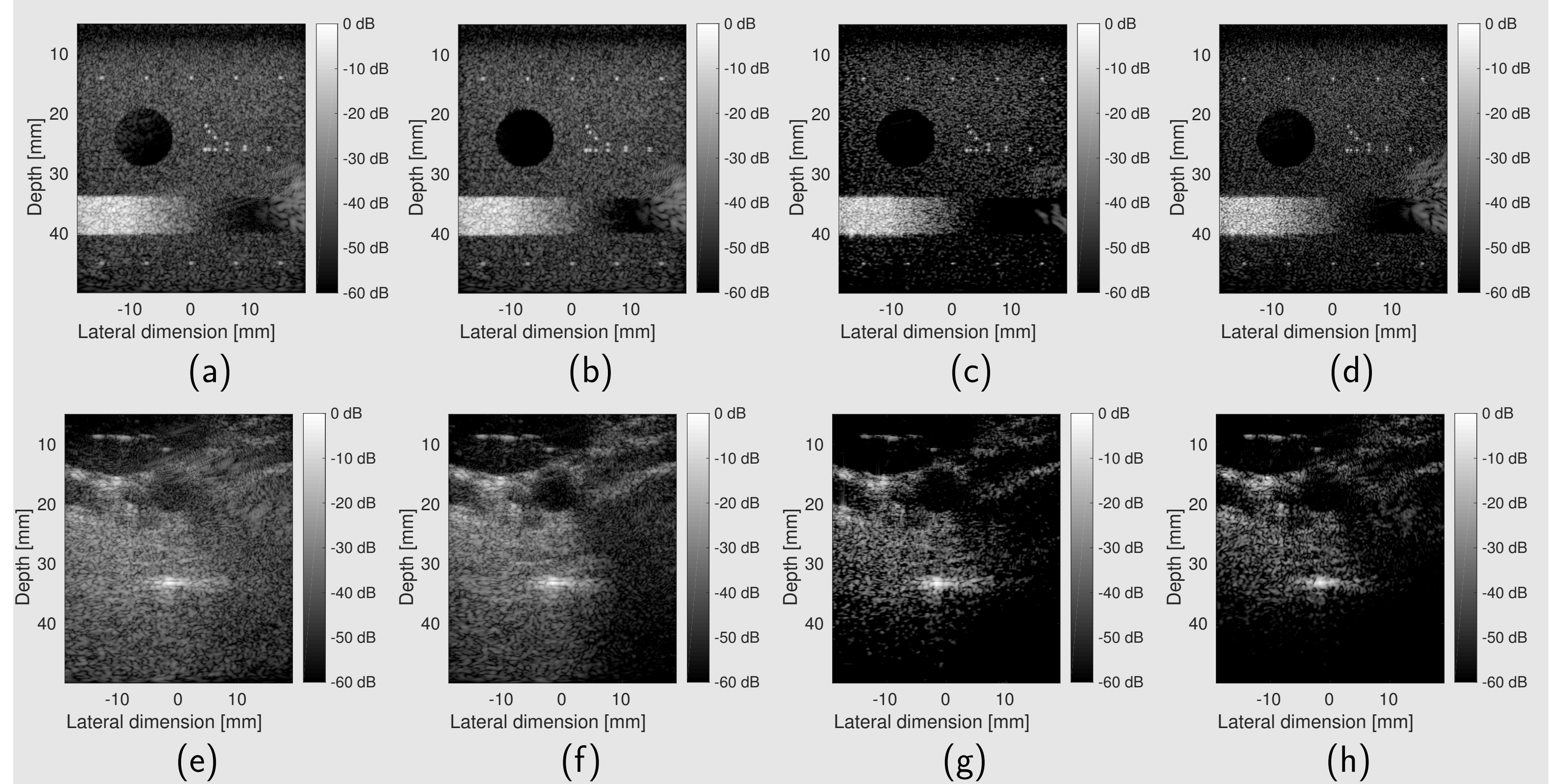


Figure Image of the numerical phantom of PICMUS dataset reconstructed with (a) DAS - 1 PW insonification, (b) DAS - 5 PW insonifications, (c) USSR-SA - 1 PW insonification, (c) USSR - ℓ_p - 1 PW insonification; Image of the *in-vivo* carotid reconstructed with (e) DAS - 1 PW insonification, (f) DAS - 5 PW insonifications, (g) USSR-SA - 1 PW insonification, (h) USSR- ℓ_p - 1 PW insonification.

Conclusion and perspectives

1. We propose USSR: an UltraSound Sparse Regularization framework
 - ▶ Matrix-free and highly parallelizable formulations of measurement model and adjoint
 - ▶ Two priors: ℓ_p -norm in the image domain and ℓ_1 -norm in a sparsity averaging model
2. The proposed approach leads to high-quality at fast rates, with low-memory footprint
3. Current work focuses on extending and optimizing the code

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