USSR: An UltraSound Sparse Regularization Framework

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Introduction and objectives

1. Ultrafast ultrasound (US) imaging uses unfocused waves, such as plane waves (PW), to insonify the whole field-of-view at once
   - High frame rate
   - Degraded image quality
2. Sparse regularization (SR) methods permit reconstruction of high quality images but involve
   - Storage of huge matrices → high-memory footprint
   - Iterative algorithms → slow
3. We present USSR: An UltraSound Sparse Regularization framework, a fast, high-quality and low-memory-footprint image reconstruction method
   - Matrix-free formulations of the measurement model and its adjoint for PW imaging
   - SR algorithm with two sparsity priors: ℓ_p-norm to the power of p ($p \geq 1$), ℓ_1-norm in a sparsity averaging (SA) model

Notations and model

- Notations:
  - 1D probe composed of N_{el} transducer elements located at $\boldsymbol{p}^i \in \Pi$ recording samples at time instants $t^l = t^0 + l\Delta t$, with $l \in \{1, \ldots, N_t\}$
  - $m\left(\boldsymbol{p}^i, t^l\right)$ signal received at time instant t^l by element located at $\boldsymbol{p}^i$
  - $v_{pe}(t)$ pulse-shape
  - Medium Ω composed of points located at $\boldsymbol{r}^n = \left[x^k, z^l\right]^T$, $(k, l) \in \{1, .., N_x\} \times \{1, .., N_z\}$ and $n = (k-1)N_z + l$
  - Each point characterized by its tissue-reflectivity function $\gamma\left(\boldsymbol{r}^n\right)$

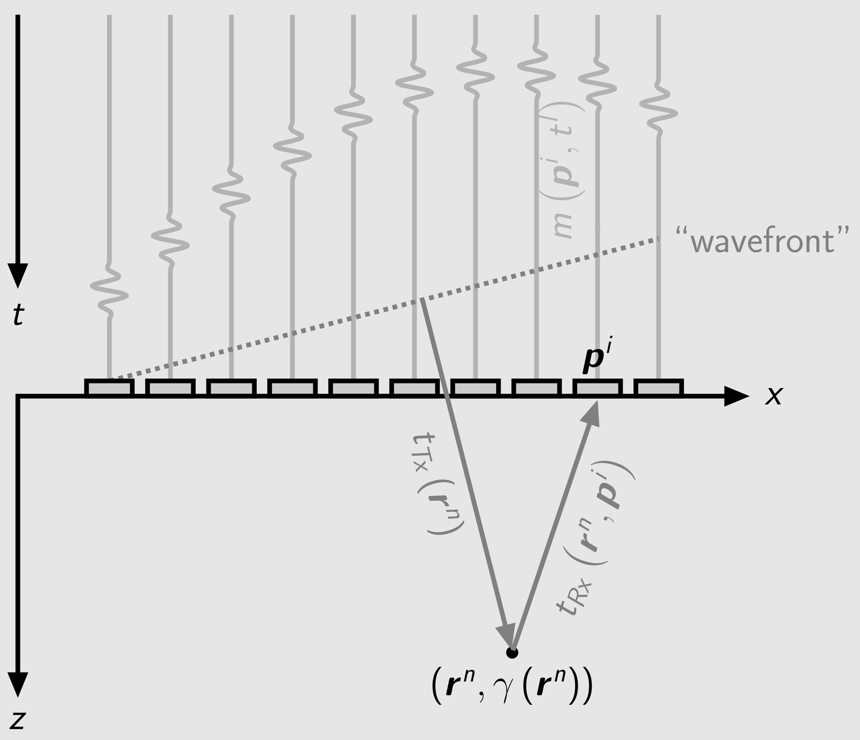

Figure Standard setting for US imaging

- Pulse-echo spatial impulse response model

$$m\left(\boldsymbol{p}^i, t^l\right) = \int_{\boldsymbol{r} \in \Omega} o_d\left(\boldsymbol{r}, \boldsymbol{p}^i\right) \gamma\left(\boldsymbol{r}\right) v_{pe}\left(t^l - t_{Tx}\left(\boldsymbol{r}\right) - t_{Rx}\left(\boldsymbol{r}, \boldsymbol{p}^i\right)\right) d\boldsymbol{r}$$

$t_{Tx}\left(\boldsymbol{r}\right)$ propagation time on transmit, $t_{Rx}\left(\boldsymbol{r}, \boldsymbol{p}^i\right) = \left\|\boldsymbol{r} - \boldsymbol{p}^i\right\|_2 / c$ propagation time on receive and $o_d\left(\boldsymbol{r}, \boldsymbol{p}^i\right) = o\left(\boldsymbol{r}, \boldsymbol{p}^i\right) / 2\pi \left\|\boldsymbol{r} - \boldsymbol{p}^i\right\|_2$ where $o\left(\boldsymbol{r}, \boldsymbol{p}^i\right)$ accounts for the element directivity

Parametric formulation of the model

- The model can be written as

$$m\left(\boldsymbol{p}^i, t^l\right) = \iint_{\tau \in \mathbb{R}, \boldsymbol{r} \in \Gamma\left(\boldsymbol{p}^i, \tau\right)} \frac{o_d\left(\boldsymbol{r}, \boldsymbol{p}^i\right) \gamma\left(\boldsymbol{r}\right)}{\mid \nabla_{\boldsymbol{r}} g \mid} d\sigma\left(\boldsymbol{r}\right) v_{pe}\left(t^l - \tau\right) d\tau = \mathcal{H}\left\{\gamma\right\}\left(\boldsymbol{p}^i, t^l\right) \quad (1)$$

$g\left(\boldsymbol{r}, \boldsymbol{p}^i, t\right) = t - t_{Tx}\left(\boldsymbol{r}\right) - t_{Rx}\left(\boldsymbol{r}, \boldsymbol{p}^i\right)$, $\Gamma\left(\boldsymbol{p}^i, t\right) = \left\{\boldsymbol{r} \in \Omega \mid g\left(\boldsymbol{r}, \boldsymbol{p}^i, t\right) = 0\right\}$, $\nabla_{\boldsymbol{r}} g$ denotes the gradient of g w.r.t. $\boldsymbol{r}$, $d\sigma\left(\boldsymbol{r}\right)$ is the measure over $\Gamma\left(\boldsymbol{p}^i, t\right)$

- We derive a parameterization of $\Gamma\left(\boldsymbol{p}^i, t\right)$

$$\boldsymbol{r} = [x, z]^T \in \Gamma\left(\boldsymbol{p}^i, t\right) \Leftrightarrow \boldsymbol{r}\left(\alpha, \boldsymbol{p}^i, t\right) = \left[\alpha, f\left(\alpha, \boldsymbol{p}^i, t\right)\right]^T, \ \alpha \in \mathbb{R} \quad (2)$$

- This leads us to the parametric formulation of the model

$$m\left(\boldsymbol{p}^i, t^l\right) = \iint_{\tau \in \mathbb{R}, \alpha \in \mathbb{R}} o_d\left(\boldsymbol{r}\left(\alpha, \boldsymbol{p}^i, t^l\right), \boldsymbol{p}^i\right) \gamma\left(\boldsymbol{r}\left(\alpha, \boldsymbol{p}^i, t^l\right)\right) \frac{\mid J_\alpha \mid}{\mid \nabla_{\boldsymbol{r}} g \mid} d\alpha v_{pe}\left(t^l - \tau\right) d\tau \quad (3)$$

$|J_\alpha|$ Jacobian associated with the change of variable

Parametric equations for plane wave imaging

- Parametric equations obtained by finding the roots of the following function:

$$f(z) = \sqrt{\left(x - p_x^i\right)^2 + \left(z - p_z^i\right)^2} + z\cos(\theta) + x\sin(\theta) - ct \quad (4)$$

which gives the following solution:

$$z = \sin(\theta)^{-2}\left(p_z^i - ct\cos(\theta) + x\sin(\theta)\cos(\theta) \pm \sqrt{\Delta}\right) \quad (5)$$

$$\Delta = \left(ct - p_z^i\cos(\theta) - p_x^i\sin(\theta)\right)\left(ct - p_z^i\cos(\theta) + \left(p_x^i - 2x\right)\sin(\theta)\right)$$

Dicretization of the model

- Equation (3) is discretized as

$$m\left(\boldsymbol{p}^i, t^l\right) = \mathcal{H}_d\left\{\gamma\right\}\left(\boldsymbol{p}^i, t^l\right) = \left(\tilde{\boldsymbol{m}}\left(\boldsymbol{p}^i\right) *_t \boldsymbol{v_{pe}}\right)\left(t^l\right) \quad (6)$$

where $*_t$ is the 1D-convolution and $\tilde{\boldsymbol{m}}\left(\boldsymbol{p}^i\right) = \left(\tilde{m}\left(\boldsymbol{p}^i, t^l\right)\right)_{t^l \in T_d}$ defined by:

$$\tilde{m}\left(\boldsymbol{p}^i, t^l\right) = \sum_{k=1}^{N_x} w^k o_d\left(\boldsymbol{r}\left(\alpha^k, \boldsymbol{p}^i, t^l\right), \boldsymbol{p}^i\right) \varphi\left(\boldsymbol{r}\left(\alpha^k, \boldsymbol{p}^i, t^l\right)\right) \gamma \quad (7)$$

where w^k is the integration weight and φ is a 1D-interpolation kernel

Parametric formulation of the adjoint operator of the model

- Adjoint operator of the linear operator $\mathcal{H}\left\{\gamma\right\}$ described in (1) is defined as:

$$\mathcal{H}^\dagger\left\{m\right\}\left(\boldsymbol{r}^n\right) = \sum_{\boldsymbol{p}^i \in \Pi} o_d\left(\boldsymbol{r}^n, \boldsymbol{p}^i\right) \int_{\tau \in \mathbb{R}} m\left(\boldsymbol{p}^i, \tau\right) u\left(t_{Tx}\left(\boldsymbol{r}^n\right) + t_{Rx}\left(\boldsymbol{r}^n, \boldsymbol{p}^i\right) - \tau\right) d\tau \quad (8)$$

where $u(t) = v_{pe}(-t)$ is the matched filter of the pulse shape

- Discretization of the adjoint operator expressed as:

$$\mathcal{H}_d^\dagger\left\{\boldsymbol{m}\right\}\left(\boldsymbol{r}^n\right) = \sum_{\boldsymbol{p}^i \in \Pi} \omega^n o_d\left(\boldsymbol{r}^n, \boldsymbol{p}^i\right) \psi\left(t_{Tx}\left(\boldsymbol{r}^n\right) + t_{Rx}\left(\boldsymbol{r}^n, \boldsymbol{p}^i\right)\right) \hat{\boldsymbol{m}}, \quad (9)$$

where $\hat{\boldsymbol{m}} = \boldsymbol{m} *_t \boldsymbol{u}$, ψ is a 1D-interpolation kernel and ω^n accounts for the integration weight

Image reconstruction procedure

- Linear measurement operator $\mathcal{H}_d\left\{\gamma\right\}$ defines the following inverse problem:

$$\boldsymbol{m} = \mathrm{H}_d \gamma + \boldsymbol{\nu} \quad (10)$$

where $\mathrm{H}_d \in \mathbb{R}^{N_{el}N_t \times N_x N_z}$ matrix associated with the linear measurement model and $\boldsymbol{\nu} \in \mathbb{R}^{N_{el}N_t}$ noise due to model discrepancy and discretization

- Sparse regularization to solve the problem

$$\min_{\gamma \in \mathbb{R}^{N_x N_z}} \lambda \mathcal{R}\left(\gamma\right) + \frac{1}{2}\left\|\mathrm{H}_d \gamma - \boldsymbol{m}\right\|_2^2 \quad (11)$$

where $\mathcal{R}\left(\gamma\right)$ prior term and $\lambda \in \mathbb{R}_+$ regularization parameter

- Two priors considered in this study:
  - ℓ_p-norm to the power of p: $\mathcal{R}\left(\gamma\right) = \left\|\gamma\right\|_p^p$, $p \geq 1$;
  - ℓ_1-norm in the SA model: $\mathcal{R}\left(\gamma\right) = \left\|\Psi^\dagger \gamma\right\|_1$, where $\Psi = \frac{1}{\sqrt{q}}\left[\Psi_1, \ldots, \Psi_q\right]$, with Ψ_i the i-th Daubechies wavelet.

Implementation and performance of USSR

- USSR is implemented on GPU and multi-threaded CPU platforms
- 200 iterations of the fast iterative shrinkage thresholding algorithm used for reconstruction
- Reconstructions of images from PICMUS dataset (1 PW insonification) take around 4.5 s on an NVIDIA Titan X GPU card
- Reproducible code available on GitHub: `https://github.com/LTS5/USSR-IUS2017`

Reconstructed B-mode images

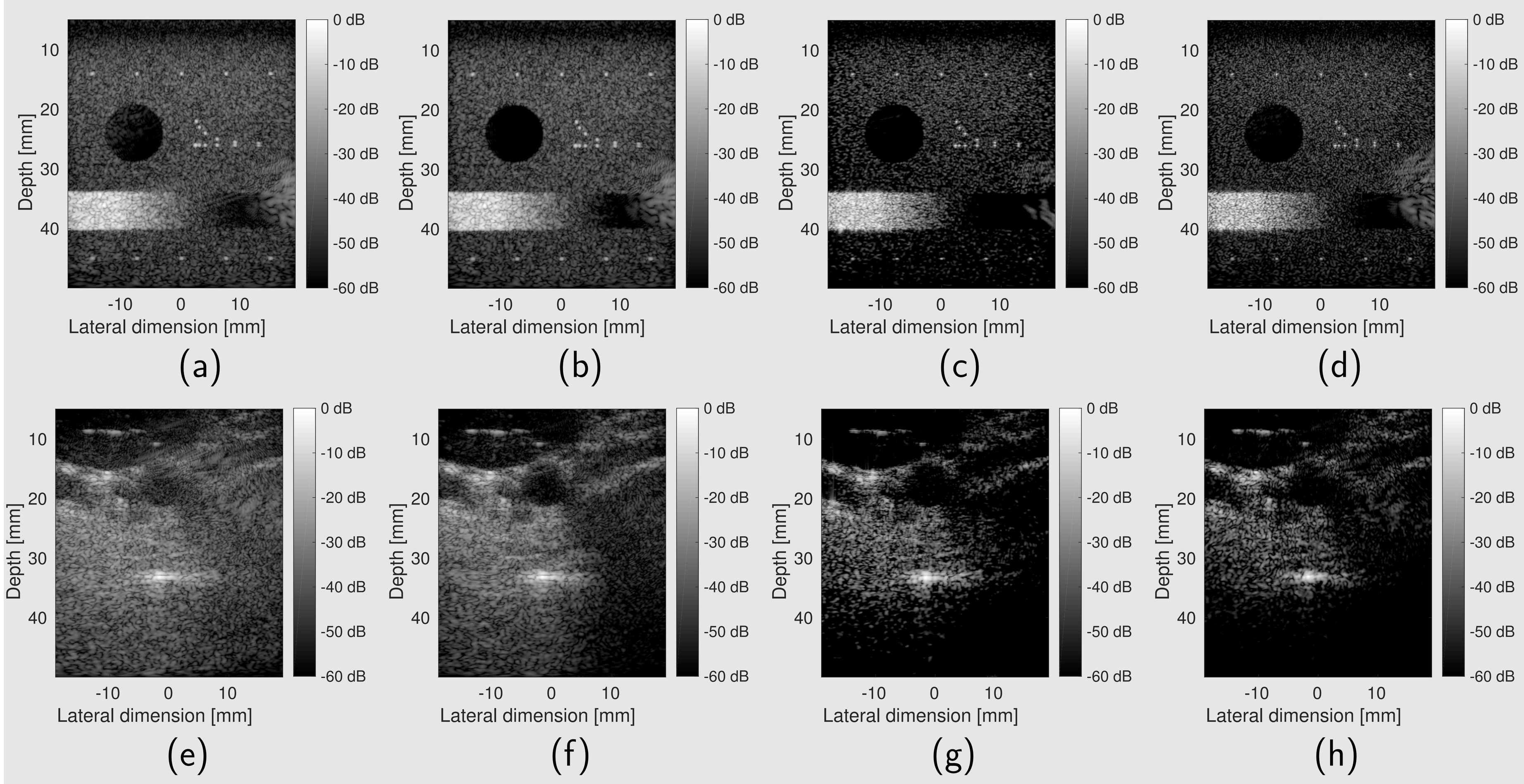

Figure Image of the numerical phantom of PICMUS dataset reconstructed with (a) DAS - 1 PW insonification, (b) DAS - 5 PW insonifications, (c) USSR-SA - 1 PW insonification, (c) USSR - ℓ_p - 1 PW insonification; Image of the *in-vivo* carotid reconstructed with (e) DAS - 1 PW insonification, (f) DAS - 5 PW insonifications, (g) USSR-SA - 1 PW insonification, (h) USSR-ℓ_p - 1 PW insonification.

Conclusion and perspectives

1. We propose USSR: an UltraSound Sparse Regularization framework
   - Matrix-free and highly parallelizable formulations of measurement model and adjoint
   - Two priors: ℓ_p-norm in the image domain and ℓ_1-norm in a sparsity averaging model
2. The proposed approach leads to high-quality at fast rates, with low-memory footprint
3. Current work focuses on extending and optimizing the code

Acknowledgments

This work was supported in part by the UltrasoundToGo RTD project (no. 20NA21 145911), evaluated by the Swiss NSF and funded by Nano-Tera.ch with Swiss Confederation financing.