# Search

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# Objectives

## Specific Objectives

- To understand the role of the search in AI
- Main search algorithms

## Source

- Stuart Russell & Peter Norvig (2009). Artificial Intelligence: A Modern Approach. (3rd Edition). Ed. Pearsons.

# Outline

- **Introduction**
- Problem formulation
- Problem types
- Basic search algorithms
- Conclusions

# Introduction

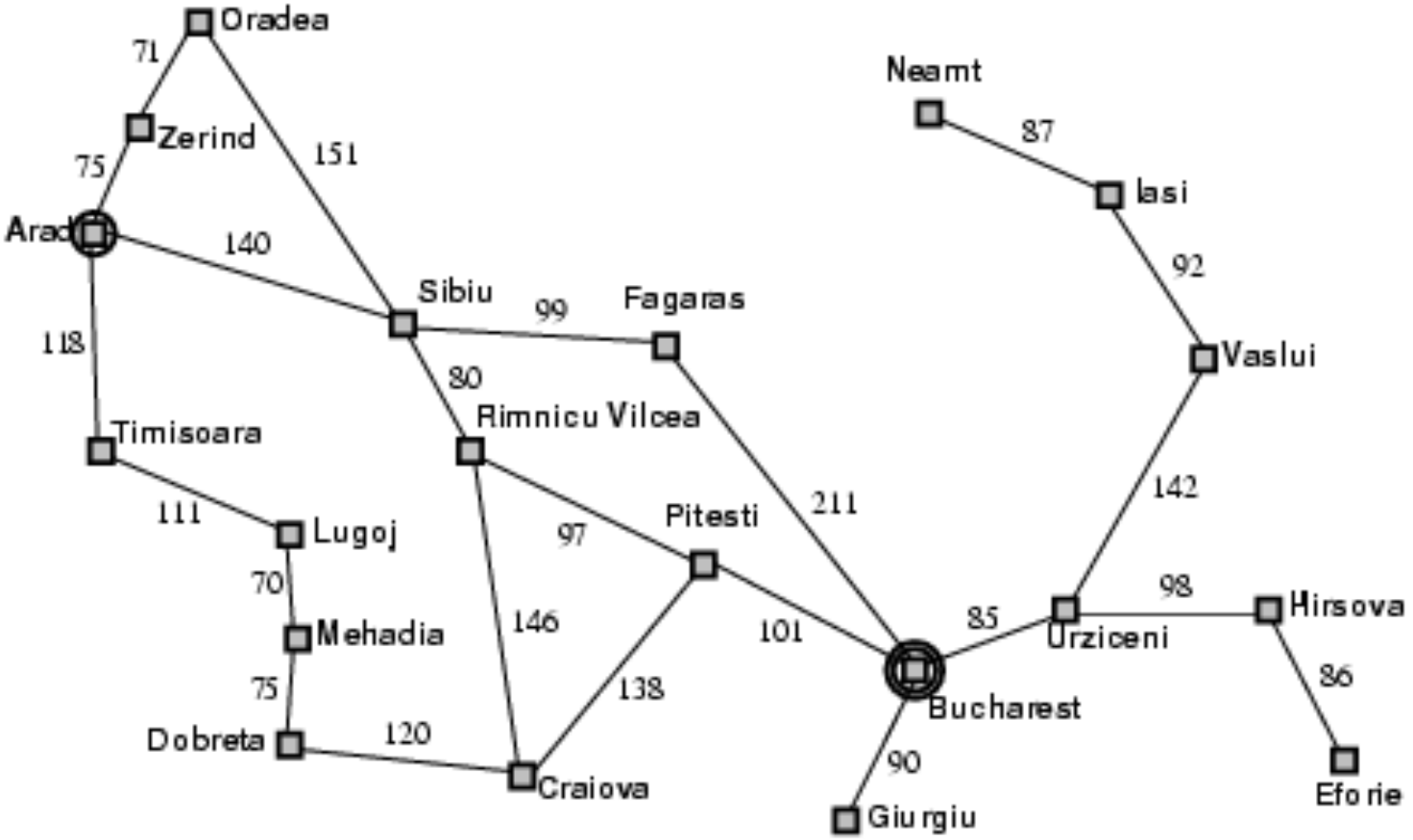
- Early AI works were directed to:
  - Proof of theorems
  - Solving crosswords
  - Games

- All in AI is search
  - Not entirely true (obviously) but more than we can imagine
  - Finding a good/best solution to a problem among several possible solutions

# Outline

- Introduction
- **Problem formulation**
- Problem types
- Basic search algorithms
- Conclusions

# Problem formulation



# Problem formulation (I)

- Agent must maximize its performance measure
- Example: On holiday in Romania; currently in Arad
  Flight leaves tomorrow from Bucharest
- Formulate goal:
  - be in Bucharest
- Formulating the problem:
  - states: multiple cities
  - actions: drive between cities
- Finding a solution:
  - Sequence cities, eg., Arad, Sibiu, Fagaras, Bucharest
- Process of finding such a solution: **Search**

# Problem formulation (II)

- Assumptions of the environment:
  - Static: search and formulation is done without considering changes in the environment
  - Observable: the initial state is known
  - Discrete: the alternative locations are known
  - Deterministic: each state is determined by the current state and the action executed

- The solutions are simple sequences of actions, they are executed without considering perceptions

# Problem formulation (III)

- A problem is defined by four items:
  1. initial state e.g., "at Arad"
  2. actions or successor function S(x) = set of action–state pairs
     e.g., S(Arad) = {<Arad → Zerind, Zerind>, … }
  3. goal test, can be
     - explicit, e.g., x = "at Bucharest"
     - implicit, e.g., Checkmate(x)
  4. path cost (additive)
     - e.g., sum of distances, number of actions executed, etc.
     - c(x,a,y) is the step cost, assumed to be ≥ 0

- A solution is a sequence of actions leading from the initial state to a goal state

# Problem formulation (IV)

- Real world is absurdly complex
  → state space must be abstracted for problem solving
- (Abstract) state = set of real states
- (Abstract) action = complex combination of real actions
  - e.g., "Arad → Zerind" represents a complex set of possible routes, detours, rest stops, etc.
- For guaranteed realizability, any real state "in Arad" must get to some real state "in Zerind"
- (Abstract) solution =
  - set of real paths that are solutions in the real world
- Each abstract action should be "easier" than the original problem

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- Basic search algorithms
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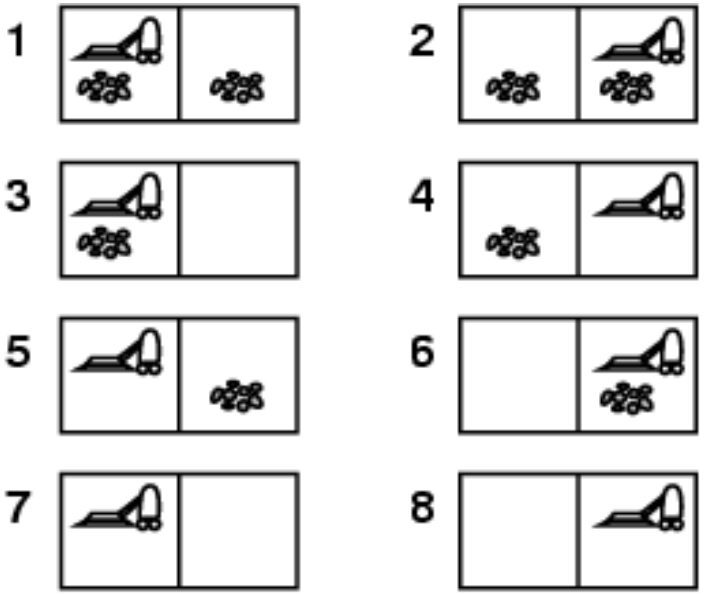
# Problem types

- Deterministic, fully observable → **single-state problem**
  - Agent knows exactly which state it will be in; solution is a sequence
- Non-observable → **sensorless problem** (conformant problem)
  - Agent may have no idea where it is; solution is a sequence
- Nondeterministic and/or partially observable → **contingency problem**
  - percepts provide new information about current state
  - often interleave search with execution
- Unknown state space → **exploration problem**

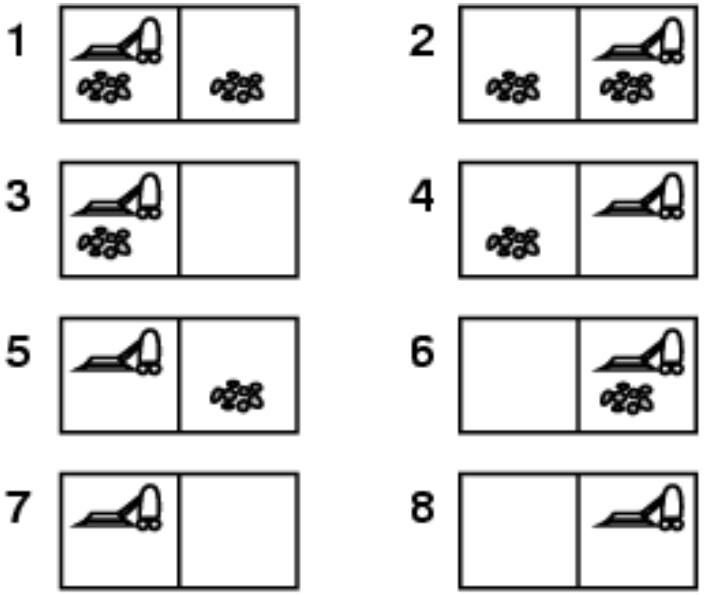
# Problem types: example

- Single-state, start in #5.
  **Solution?**



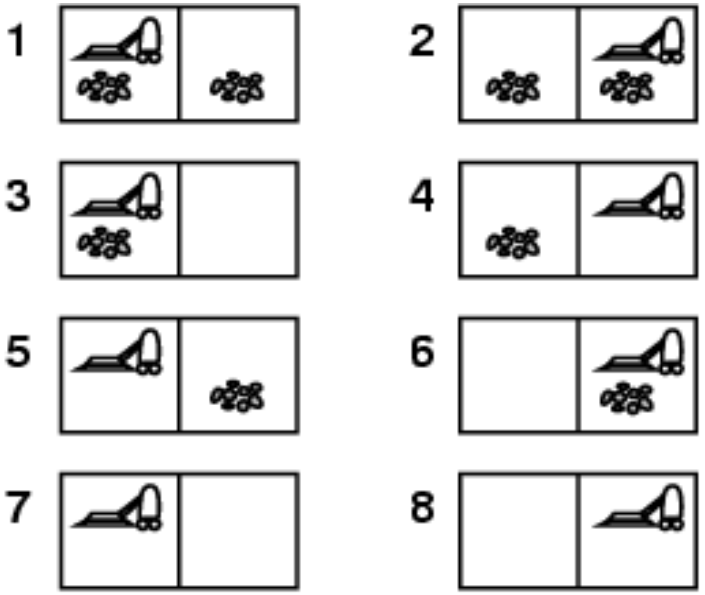
# Problem types: example (I)

- Single-state, start in #5.
  **Solution?***[Right, Suck]*



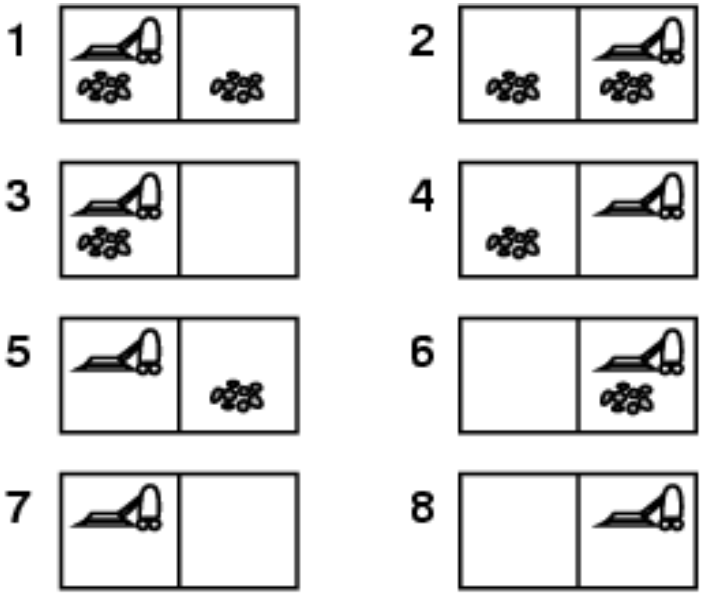
# Problem types: example (II)

- Single-state, start in #5.
  **Solution?***[Right, Suck]*
- Sensorless, start in {1,2,3,4,5,6,7,8} e.g., Right goes to {2,4,6,8}
  **Solution?**



# Problem types: example (III)

- Single-state, start in #5.
  **Solution?***[Right, Suck]*
- Sensorless, start in {1,2,3,4,5,6,7,8} e.g., Right goes to {2,4,6,8}
  **Solution?** *[Right,Suck,Left,Suck]*

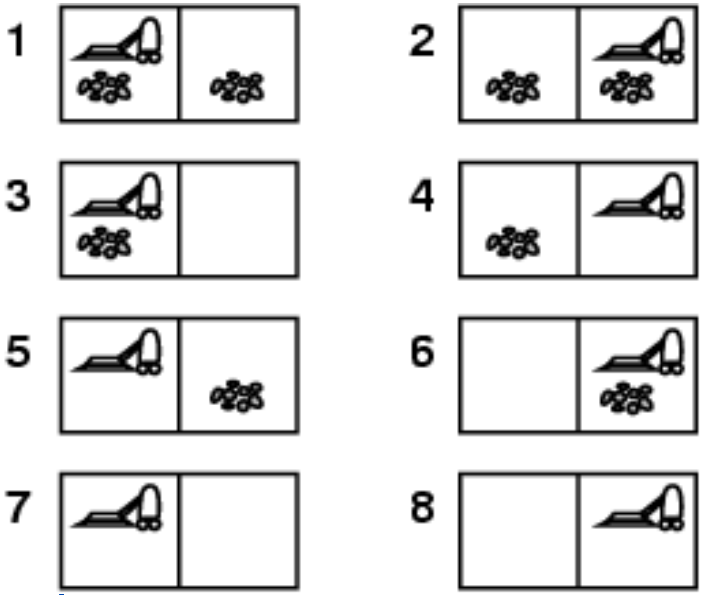


# Problem types: example (IV)

- Single-state, start in #5.
  **Solution?** *[Right, Suck]*

- Sensorless, start in {1,2,3,4,5,6,7,8} e.g.,
  Right goes to {2,4,6,8}
  **Solution?** *[Right,Suck,Left,Suck]*

- Contingency
  - Nondeterministic: suck may dirt the carpet
  - Partially observable: location, dirt at current location
  - Percept: [L, Clean], i.e., start in #5 or #7
    **Solution?**

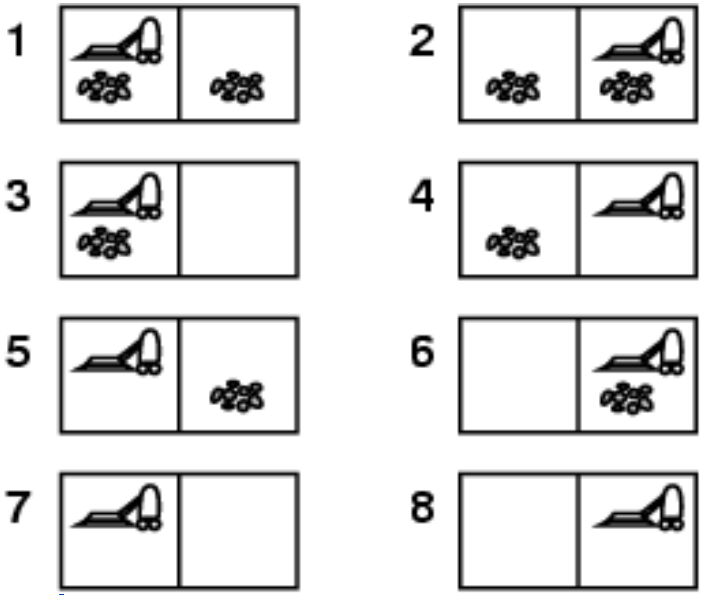


# Problem types: example (V)

- Single-state, start in #5.
  **Solution?** *[Right, Suck]*

- Sensorless, start in {1,2,3,4,5,6,7,8} e.g., Right goes to {2,4,6,8}
  **Solution?** *[Right,Suck,Left,Suck]*

- Contingency
  - Nondeterministic: Suck may dirt the carpet
  - Partially observable: location, dirt at current location
  - Percept: [L, Clean], i.e., start in #5 or #7
    **Solution?** *[Right, **if** dirt **then** Suck]*



# Type of environment

- **Fully observable** (vs. partially observable): an agent's sensors give it access to the complete state of the environment at each point in time
- **Deterministic** (vs. non-deterministic): the next state of the environment is completely determined by the current state and the action executed by the agent
- **Episodic** (vs. sequential): the agent's experience is divided into atomic "episodes" (each episode consists of the agent perceiving and then performing a single action), and the choice of action in each episode depends only on the episode itself
- **Discrete** (vs. continuous): A limited number of distinct, clearly defined percepts and actions
- **Static** (vs. dynamic): if the environment does not change when the agent is deliberating
  - Semidynamic: the environment does not change but the performance of the agent
- **Single agent** (vs. multiagent): an agent operating by itself in an environment

| Environment | Observable | Deterministic | Episódic | Static | Discrete | Agents |
|---|---|---|---|---|---|---|
| *Chess* | | | | | | |
| Chess with clock | | | | | | |
| *Poker* | | | | | | |
| Taxi | | | | | | |
| Medical diagnosis | | | | | | |
| Image analysis | | | | | | |
| *Clasifier*<br>*Robot* | | | | | | |
| *Refinery controller* | | | | | | |
| *Interactive Tutor* | | | | | | |

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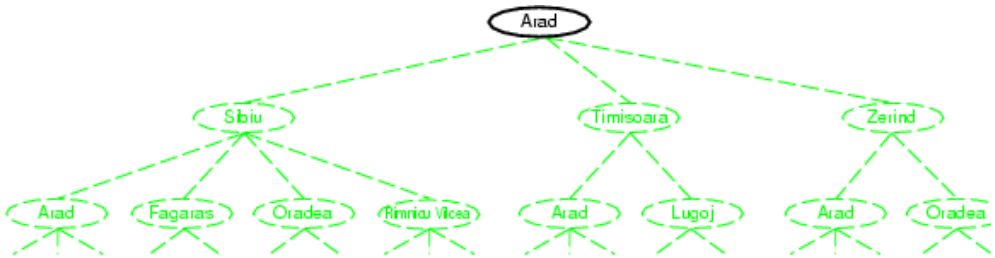
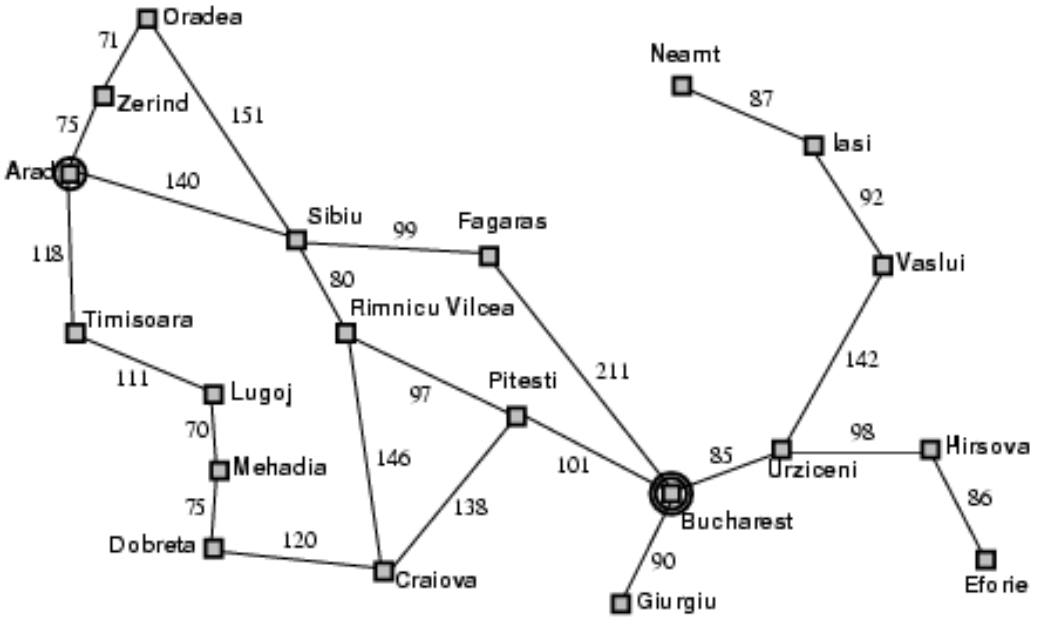
- Problem formulation
- Problem types
- **Search algorithms**
- Conclusions

# Search algorithms

- We have formulated problems, we now need to solve them: search tree
- In general we can have a search graph rather than a tree when the state can be reached from multiple paths
- Basic idea:
  - offline, simulated exploration of state space by generating successors of already-explored states (i.e. expanding states)

# Search algorithms: tree search example



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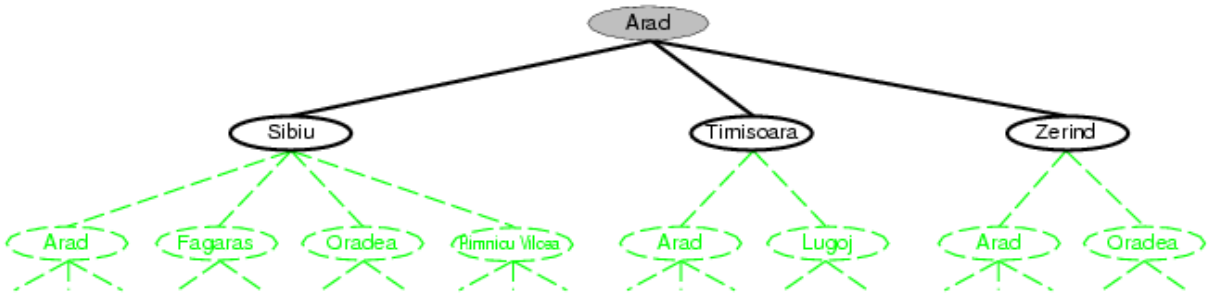

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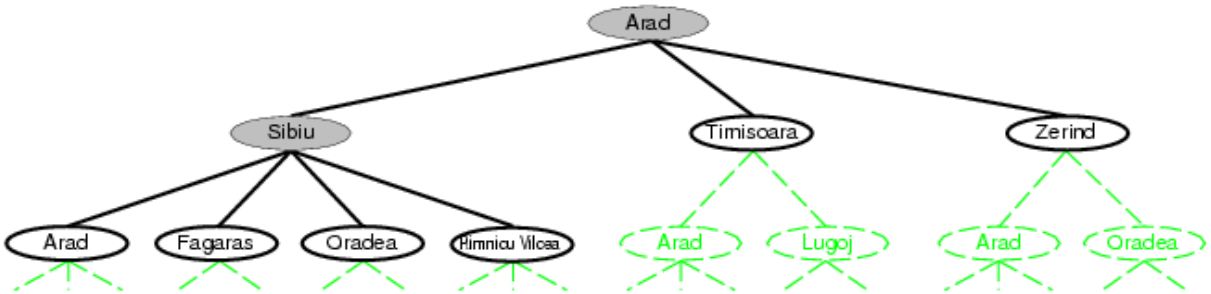
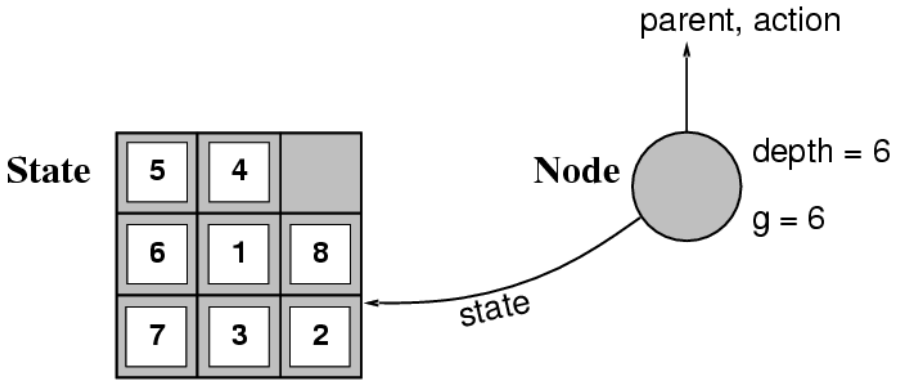


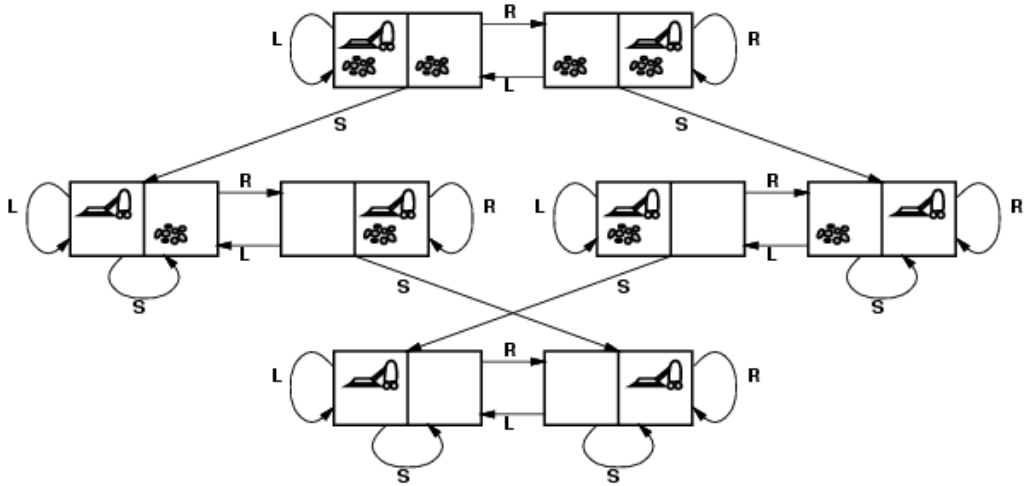
# Search algorithms: states vs. nodes

- A state is a (representation of) a physical configuration
- A node is a data structure constituting part of a search tree includes state, parent node, action, path cost *g(x)*, depth



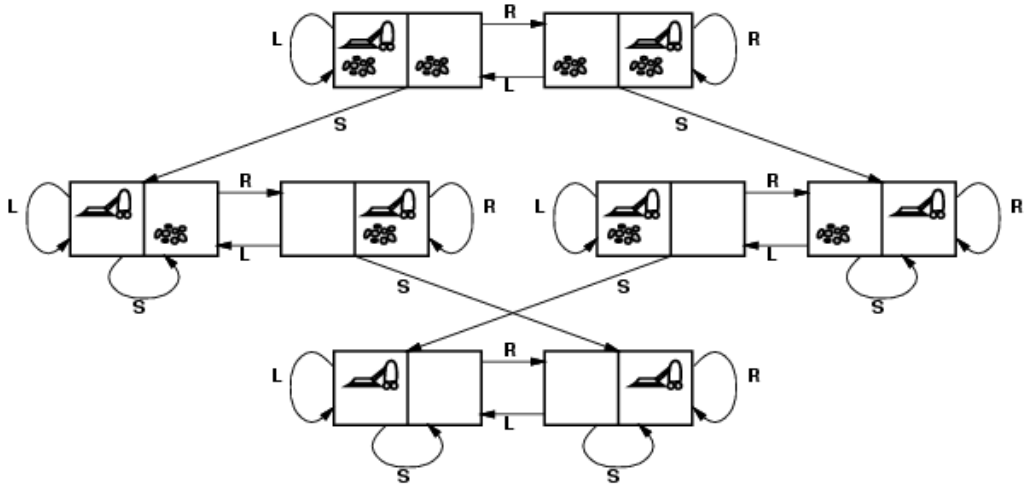

- The `Expand` function to create the corresponding states

# Search algorithms: Vacuum world (I)



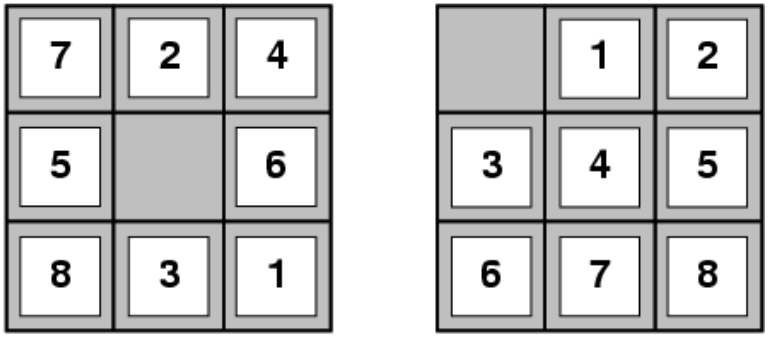

- states?
- actions?
- goal test?
- path cost?

# Search algorithms: Vacuum world (II)



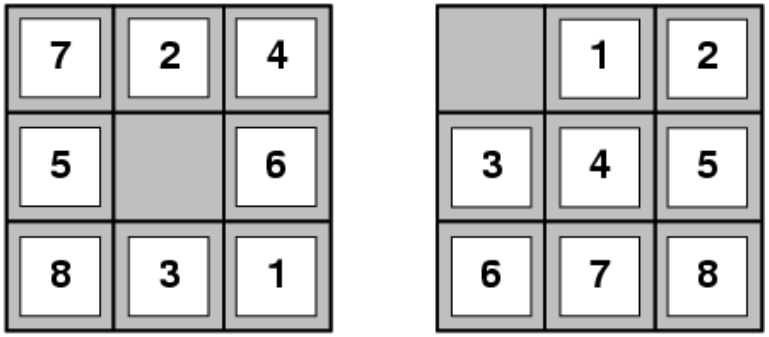

- states? integer dirt and robot location
- actions? *Left*, *Right*, *Suck*
- goal test? no dirt at all locations
- path cost? 1 per action

# Search algorithms: The 8-puzzle (I)




- states?
- actions?
- goal test?
- path cost?

# Search algorithms: The 8-puzzle (II)




- states? locations of tiles
- actions? move blank left, right, up, down
- goal test? = goal state (given)
- path cost? 1 per move

[Note: optimal solution of *n*-Puzzle family is NP-hard]

# Search strategies

- A search strategy is defined by picking the order of node expansion
- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?

- Time and space complexity are measured in terms of
  - b: maximum branching factor of the search tree
  - d: depth of the least-cost solution
  - m: maximum depth of the state space (may be ∞)

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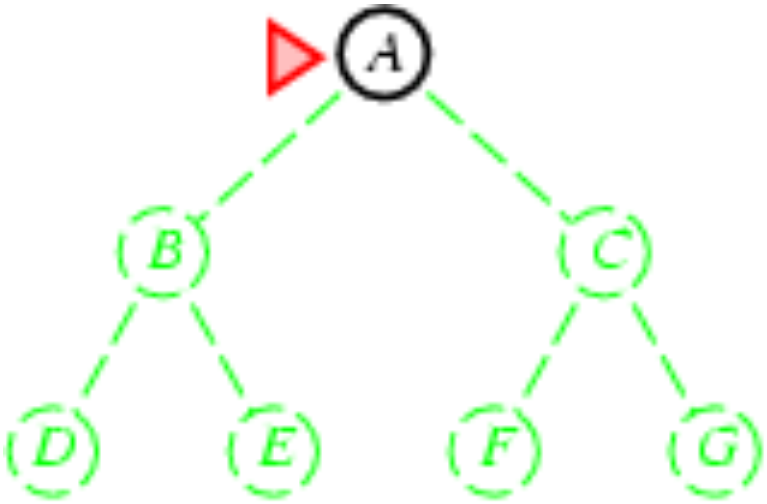
- Introduction
- Problem formulation
- Problem types
- Basic search algorithms
  - **Uninformed search**
  - Informed search
- Conclusions

# Introduction

- Uninformed search strategies use only the information available in the problem definition
  - Breadth-first search/ Búsqueda en anchura
  - Uniform-cost search/ Búsqueda de coste uniforme
  - Depth-first search/ Búsqueda en profundidad
  - Depth-limited search/ Búsqueda en profundidad limitada
  - Iterative deepening search/Búsqueda de profundización iterativa

# Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end



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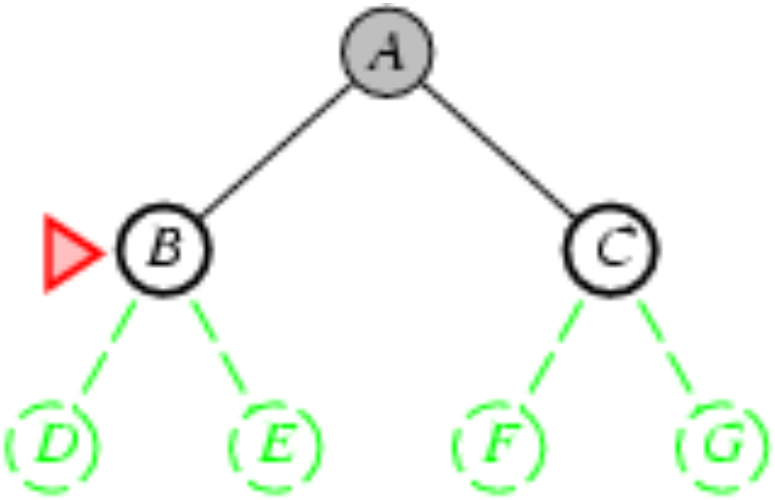

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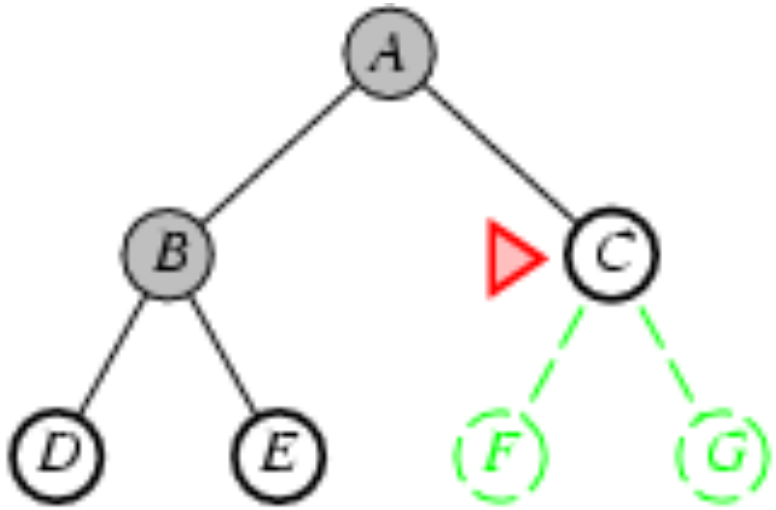

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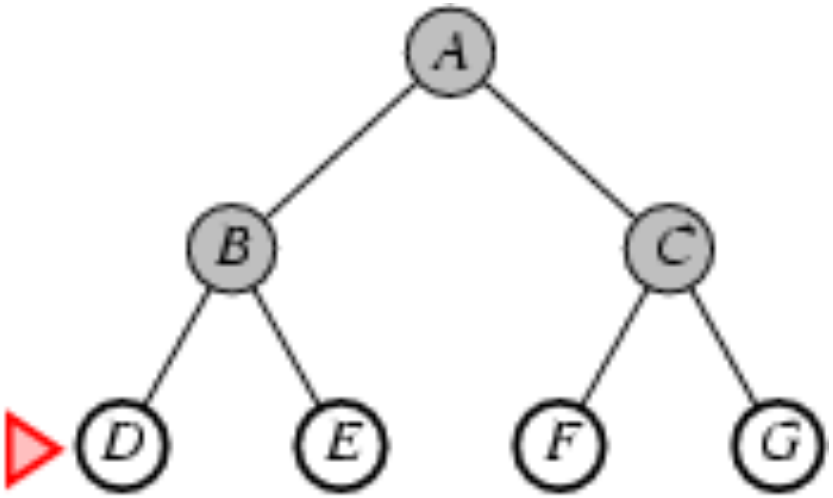


# Properties of breadth-first search

- Complete? Yes (if $b$ is finite)
- Time? $1+b+b^2+b^3+\ldots+b^d+b(b^d-1) = O(b^{d+1})$
- Space? $O(b^{d+1})$ (keeps every node in memory)
- Optimal? Yes (if cost = 1 per step)

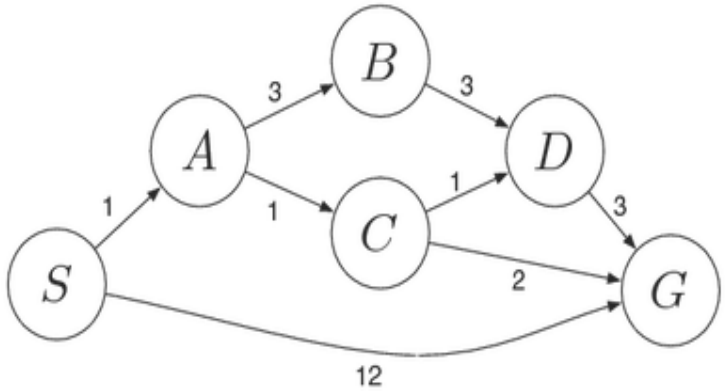
Space is the bigger problem (more than time)

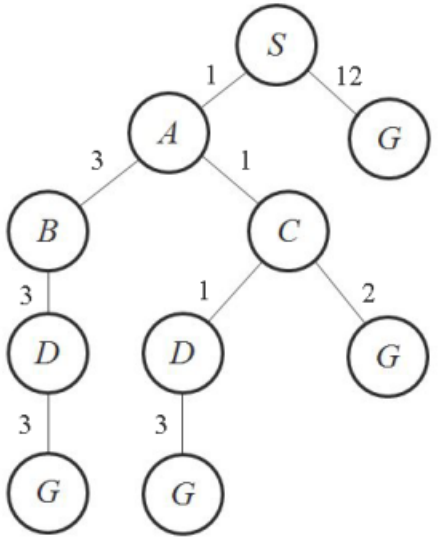
Each state has b successors (branching factor)

d is the shallower depth

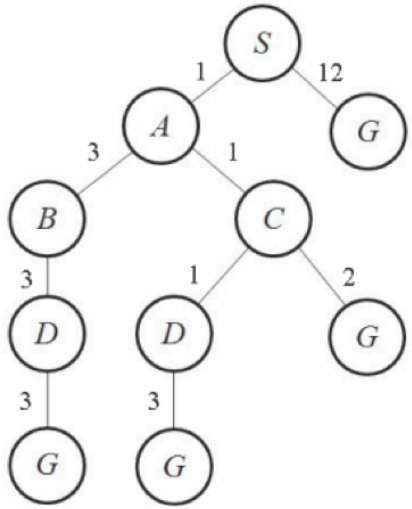
# Uniform-cost search

- Expand least-cost unexpanded node
- Implementation:
  - *fringe* = queue ordered by path cost
- Find the solution with minimum cumulative cost, i.e. an optimal solution



# Uniform-cost search (Solution)




**Initialization: { [ S , 0 ] }**

**Iteration1: { [ S->A , 1 ] , [ S->G , 12 ] }**

**Iteration2: { [ S->A->C , 2 ] , [ S->A->B , 4 ] , [ S->G , 12] }**

**Iteration3: { [ S->A->C->D , 3 ] , [ S->A->C->G , 4 ] , [ S->A->B->D , 7 ] , [ S->G , 12 ] }**

**Iteration 4: {[ S->A->C->D->G , 6 ], [ S->A->C->G , 4 ] , [ S->A->B->D , 7 ] , [ S->G , 12 ] }**

**Iteration 5: {[S->A->C->G , 4 ], [S->A->C->D->G , 6 ], [ S->A->B->D->G, 10] , [ S->G , 12 ]}**

**Solution: S->A->C->G.**

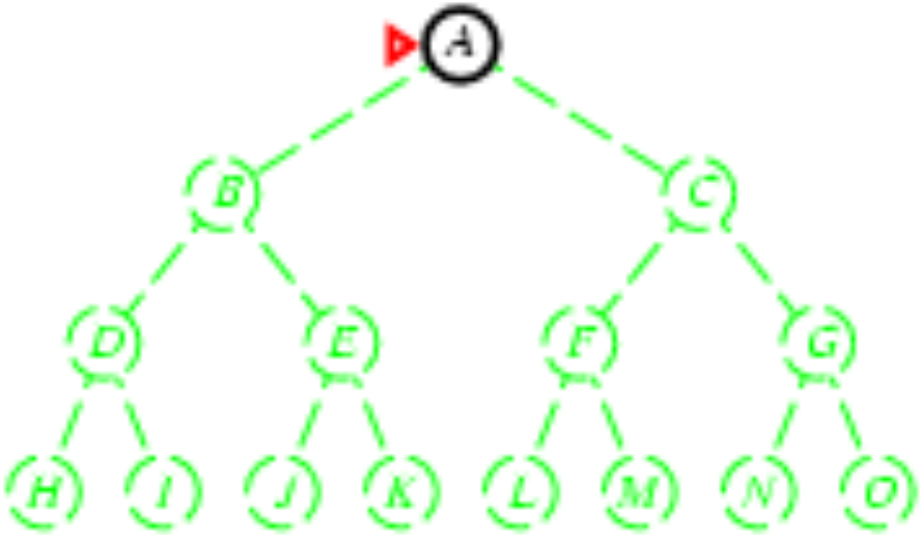
# Uniform-cost search

- Complete? Yes, if step cost $\geq \varepsilon$
- Time? # of nodes with $g \leq$ cost of optimal solution, $O(b^{ceiling(C^*/\varepsilon)})$ where $C^*$ is the cost of the optimal solution
- Space? # of nodes with $g \leq$ cost of optimal solution, $O(b^{ceiling(C^*/\varepsilon)})$
- Optimal? Yes – nodes expanded in increasing order of $g(n)$

*If all costs are equal* → $O(b^d)$

# Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



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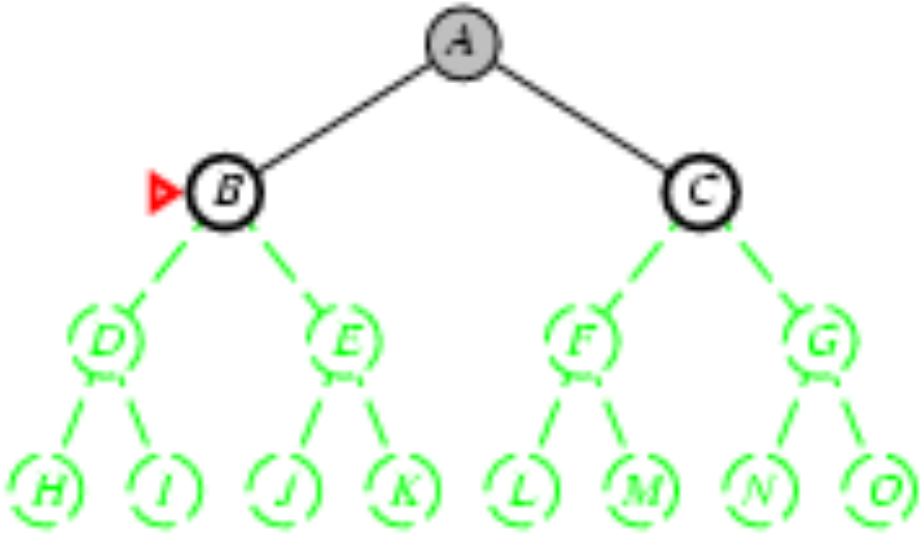

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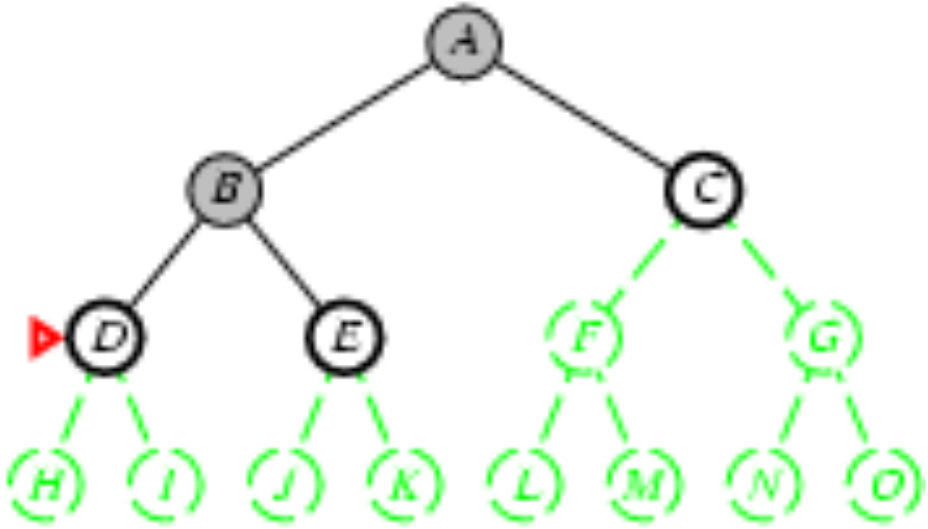

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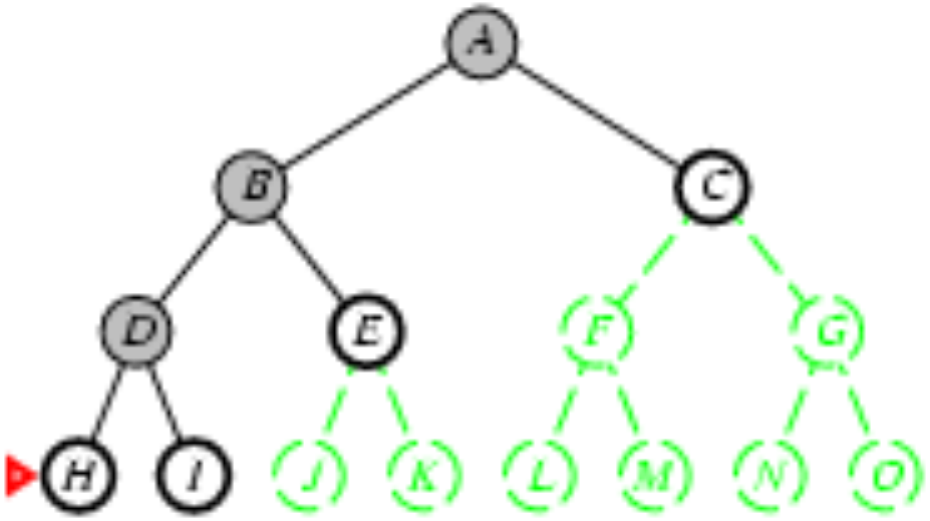

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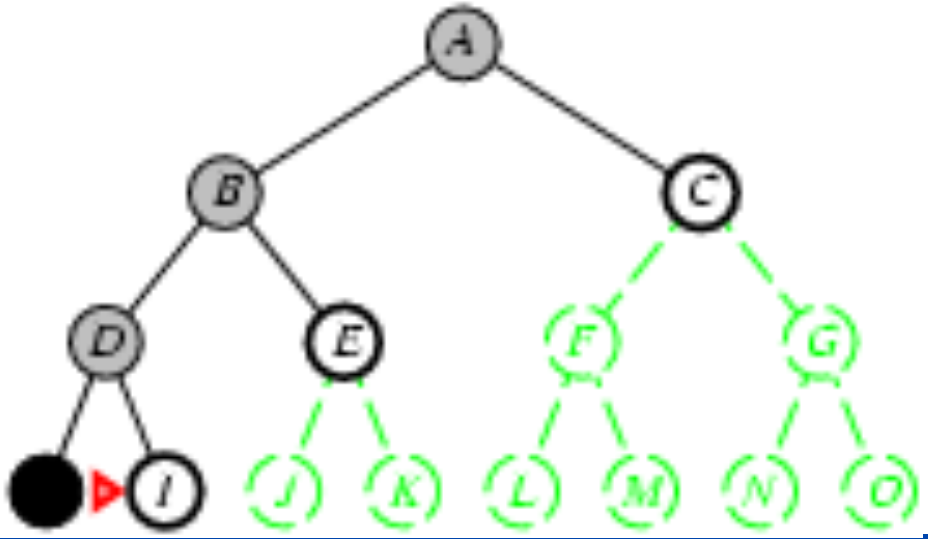

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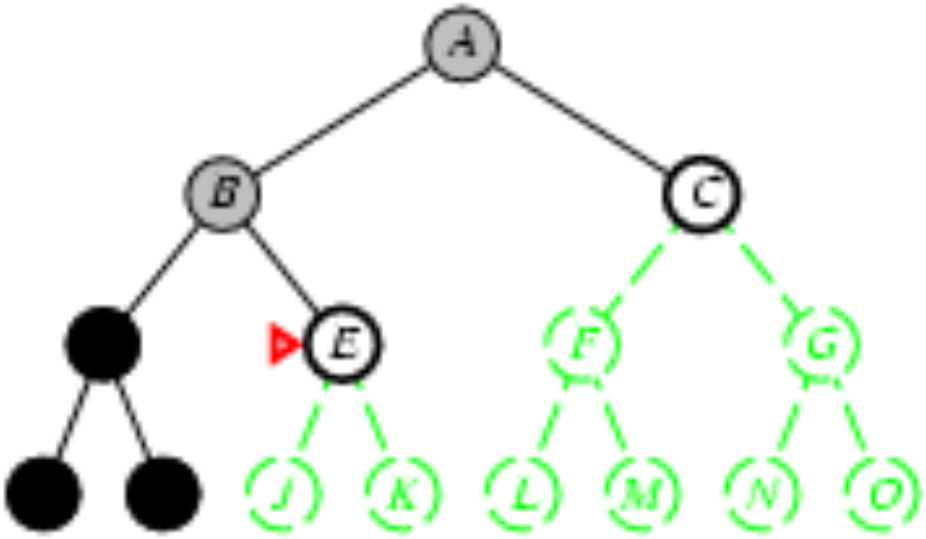

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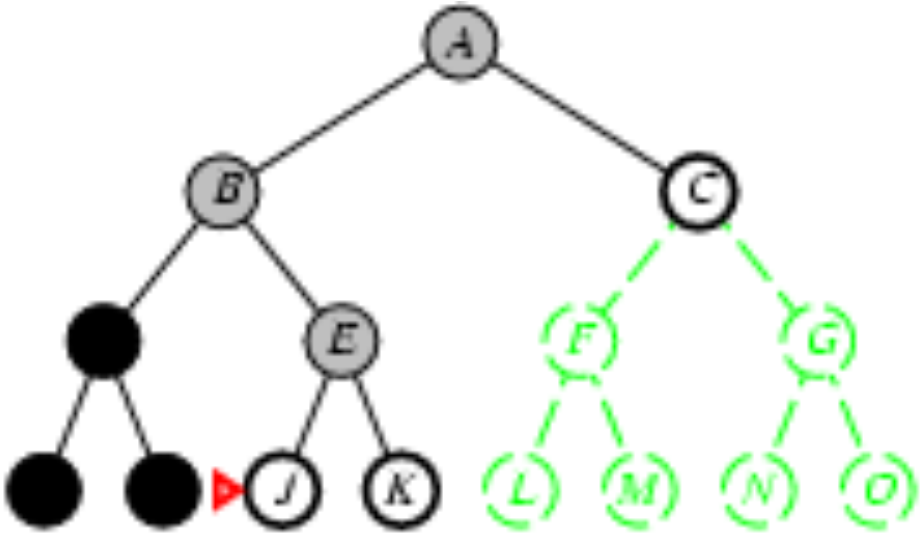

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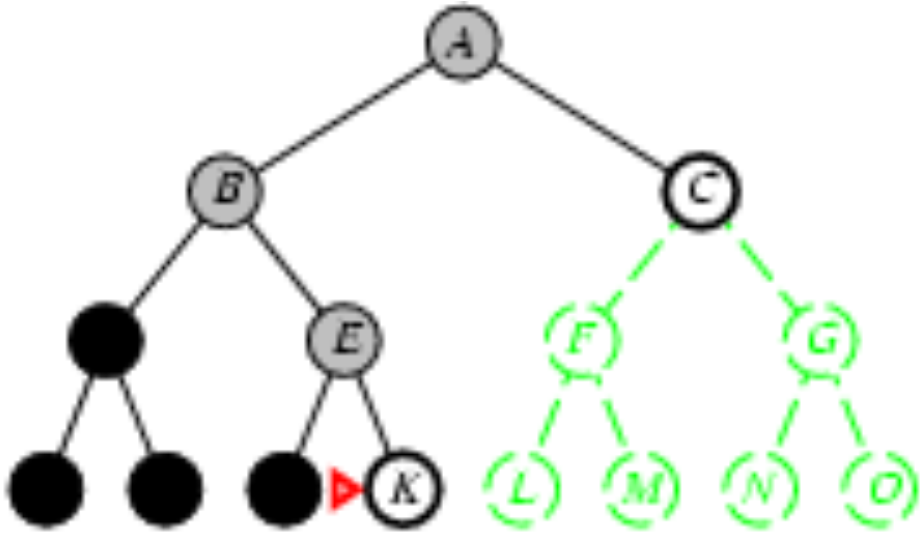

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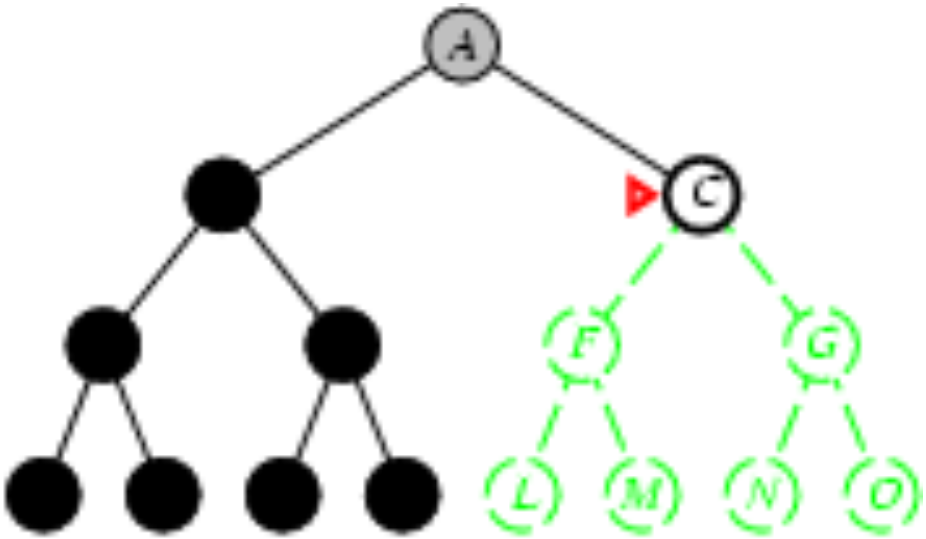
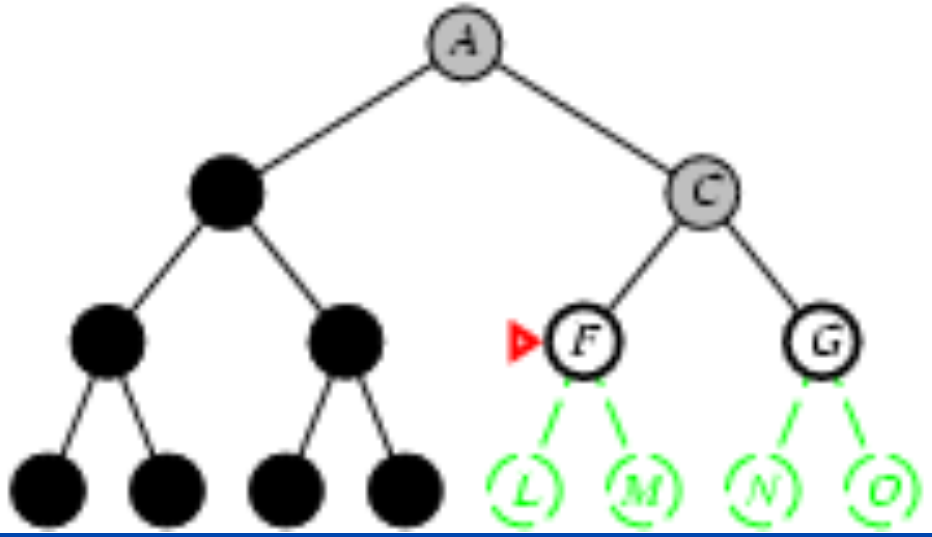


# Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



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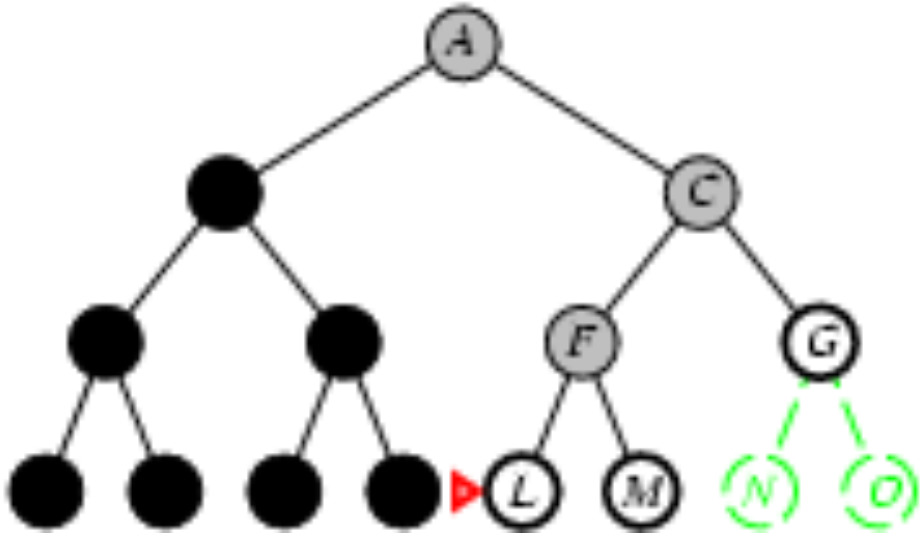

# Depth-first search

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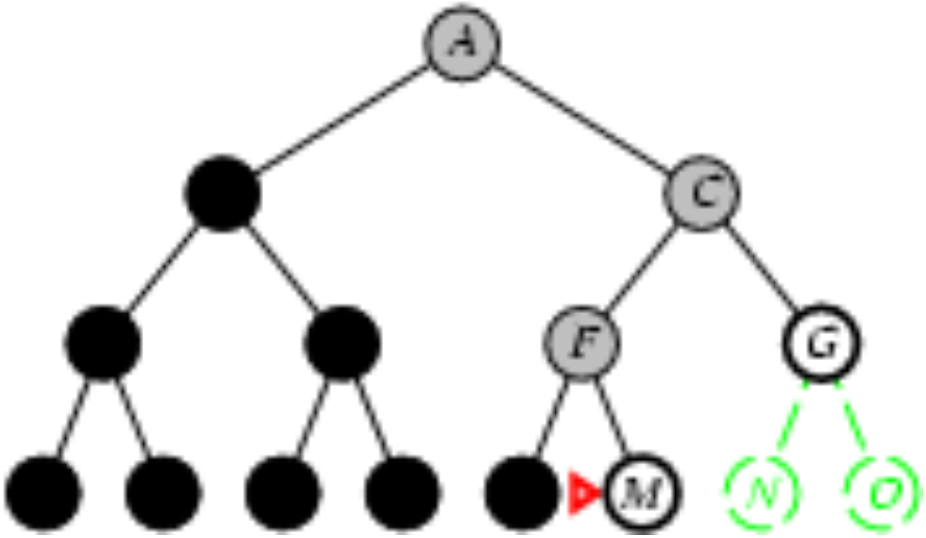


# Properties of depth-first search

- Complete? No: fails in infinite-depth spaces, spaces with loops
  - Modify to avoid repeated states along path
    → complete in finite spaces
- Time? $O(b^m)$: terrible if $m$ is much larger than $d$
  - but if solutions are dense, may be much faster than breadth-first
- Space? $O(bm)$
- Optimal? No

# Depth-limited search

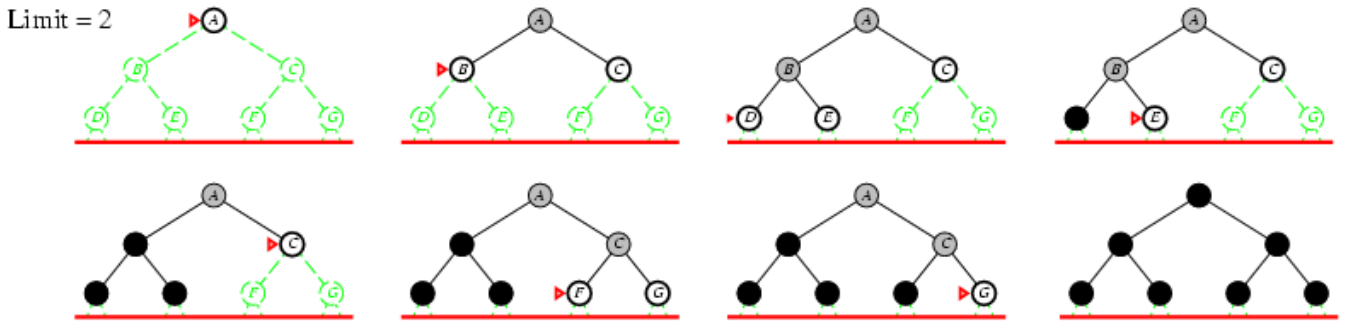
- = depth-first search with depth limit L
- i.e., nodes at depth L have no successor

# Iterative deepening search $L=0$

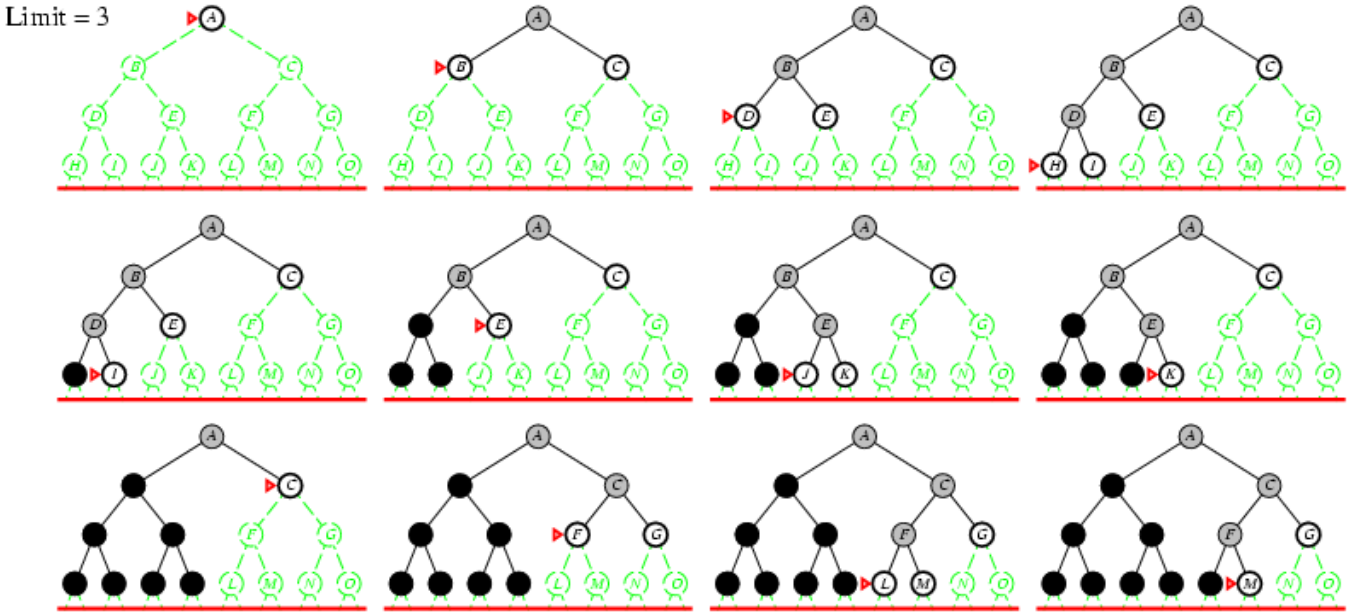
Limit = 0

# Iterative deepening search $L=1$

# Iterative deepening search $L=2$



# Iterative deepening search $L=3$

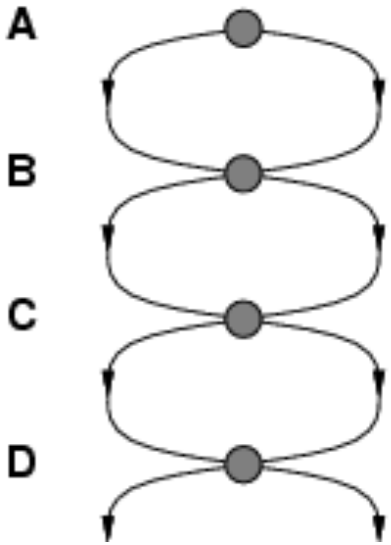


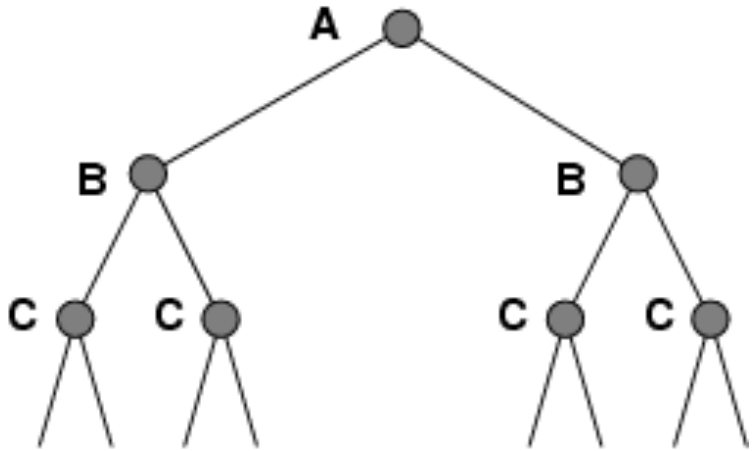
# Properties of iterative deepening search

- Complete? Yes
- Time? $(d+1)b^0 + d\,b^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
- Space? $O(bd)$
- Optimal? Yes, if step cost = 1

# Repeated states

- Failure to detect repeated states can turn a linear problem into an exponential one!



# Summary of algorithms

| Criterion | Breadth-First | Uniform-Cost | Depth-First | Depth-Limited | Iterative Deepening |
|---|---|---|---|---|---|
| Complete? | Yes | Yes | No | No | Yes |
| Time | $O(b^{d+1})$ | $O(b^{\lceil C^*/\epsilon \rceil})$ | $O(b^m)$ | $O(b^l)$ | $O(b^d)$ |
| Space | $O(b^{d+1})$ | $O(b^{\lceil C^*/\epsilon \rceil})$ | $O(bm)$ | $O(bl)$ | $O(bd)$ |
| Optimal? | Yes | Yes | No | No | Yes |

# Outline

- Introduction
- Problem formulation
- Problem types
- Basic search algorithms
  - Uninformed search
  - **Informed search**
- Conclusions

# Informed search strategies

- Use the problem-specific knowledge beyond the definition of the problem itself to find more efficient solutions that uninformed strategies
  - Best-first search
    - Greedy best-first search/Búsqueda voraz primero el mejor
    - A*
  - Heuristics
  - Local search algorithms
    - Hill-climbing search/Búsqueda de escalada
    - Simulated annealing search/Búsqueda de temple simulado
    - Local beam search/Búsqueda de haz local
    - Genetic algorithms/Algoritmos genéticos

# Best-first search

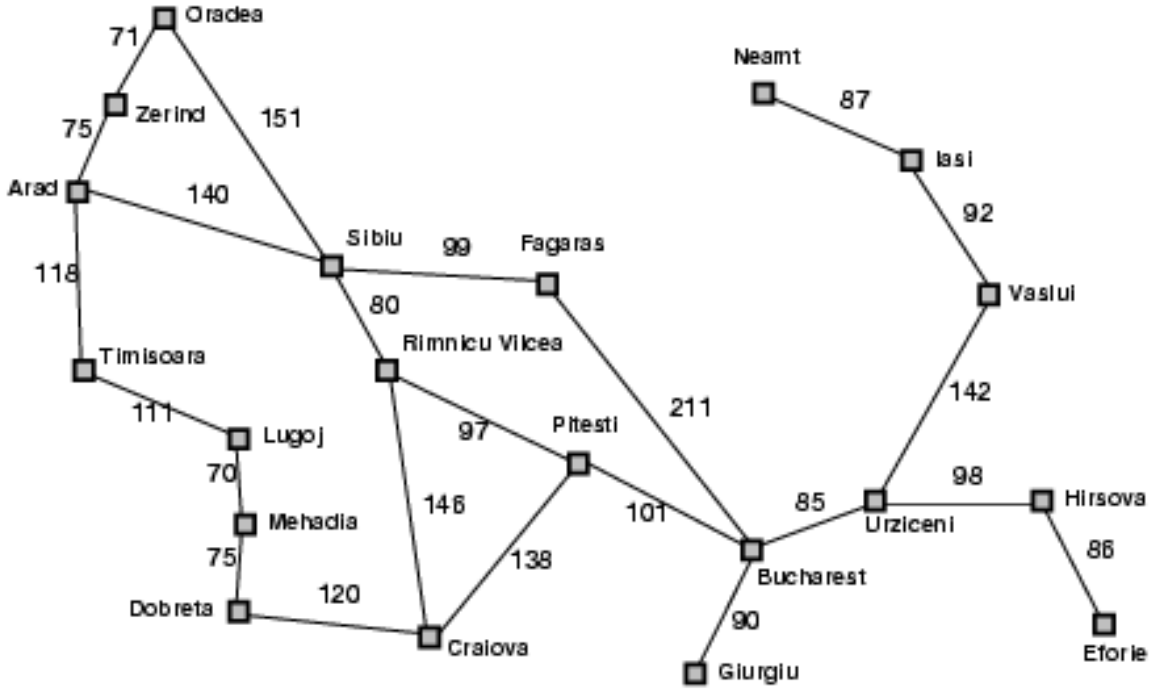
- Idea: use an evaluation function *f(n)* for each node
  - estimate of "desirability"
  - Expand most desirable unexpanded node

- Implementation:

  Order the nodes in fringe in decreasing order of desirability

- Special cases:
  - greedy best-first search
  - A* search

# Romania with step costs in km




| Straight–line distance to Bucharest | |
|---|---|
| **Arad** | 366 |
| **Bucharest** | 0 |
| **Craiova** | 160 |
| **Dobreta** | 242 |
| **Eforie** | 161 |
| **Fagaras** | 176 |
| **Giurgiu** | 77 |
| **Hirsova** | 151 |
| **Iasi** | 226 |
| **Lugoj** | 244 |
| **Mehadia** | 241 |
| **Neamt** | 234 |
| **Oradea** | 380 |
| **Pitesti** | 10 |
| **Rimnicu Vilcea** | 193 |
| **Sibiu** | 253 |
| **Timisoara** | 329 |
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| **Zerind** | 374 |

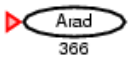
# Informed search strategies

- Best-first search
  - **Greedy best-first search**
  - A* search
- Heuristics
- Local search algorithms
  - Hill-climbing search
  - Simulated annealing search
  - Local beam search
  - Genetic algorithms

# Greedy best-first search

- Evaluation function $f(n) = h(n)$ (heuristic) = estimate of cost from $n$ to *goal*
- Greedy best-first search expands the node that appears to be closest to the goal
- Implementation: as a priority queue to keep the fringe in ascending order of f-values
- e.g., $h_{SLD}(n)$ = straight-line distance from $n$ to Bucharest

# Greedy best-first search example



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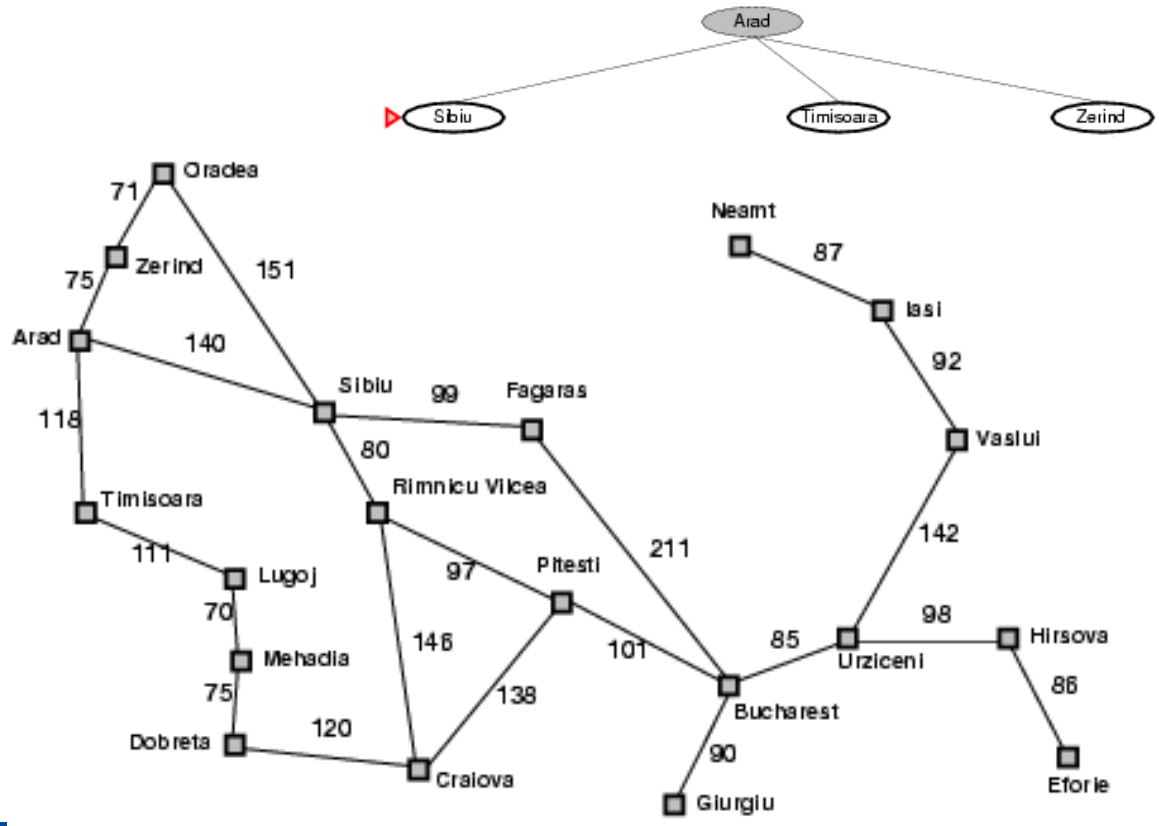


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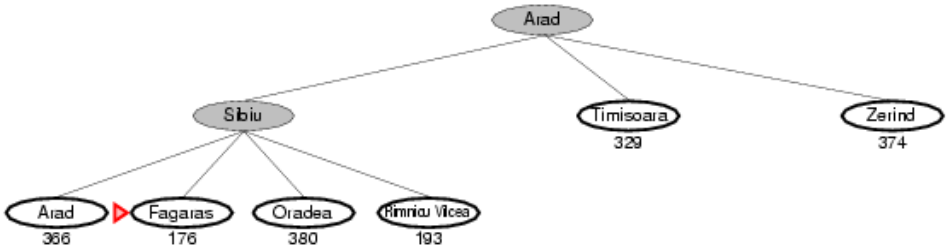
# Greedy best-first search example



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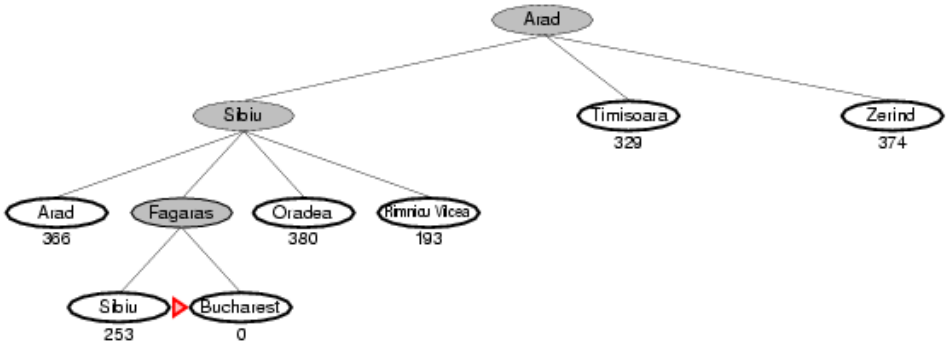


# Properties of greedy best-first search

- Complete? No – can get stuck in loops, e.g., Iasi → Neamt → Iasi → Neamt →
- Time? $O(b^m)$, but a good heuristic can give dramatic improvement
- Space? $O(b^m)$ -- keeps all nodes in memory
- Optimal? No

Similar to depth-first search

Each state has b successors (branching factor)

d is the depth of the shallowest solution

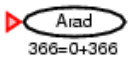
# Informed search strategies

- Best-first search
  - Greedy best-first search
  - **A* search**
- Heuristics
- Local search algorithms
  - Hill-climbing search
  - Simulated annealing search
  - Local beam search
  - Genetic algorithms

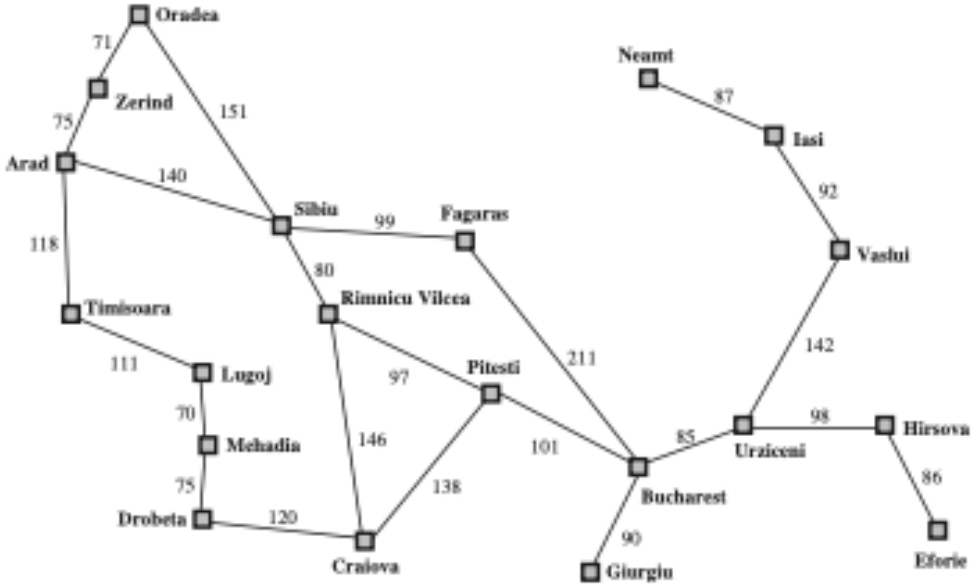
# A* search

- Idea: avoid expanding paths that are already expensive
- Evaluation function f(n) = g(n) + h(n)
  - g(n) = cost so far to reach n
  - h(n) = estimated cost from n to goal
  - f(n) = estimated total cost of path through n to goal
- A * is optimal if h(n) is an admissible heuristic such that h(n) never overestimates the cost to reach the goal

# A* search example



# A* search example


| | | | |
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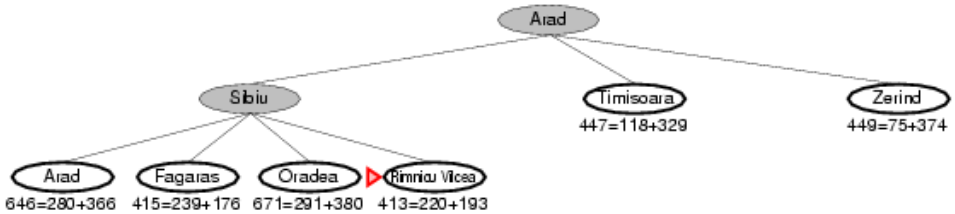


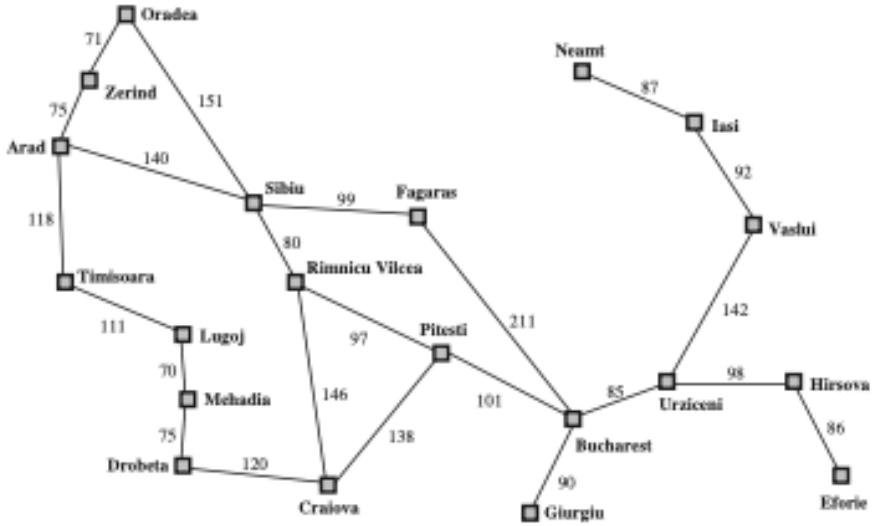


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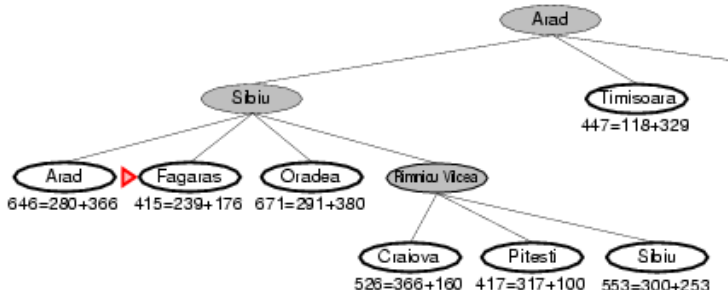


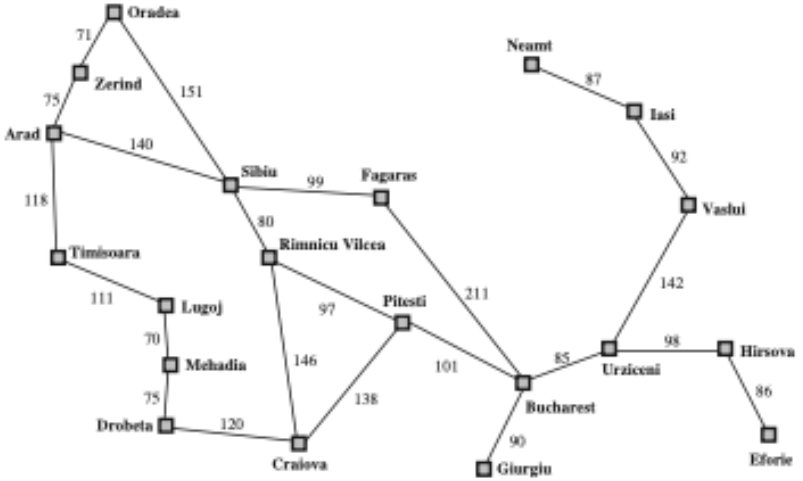


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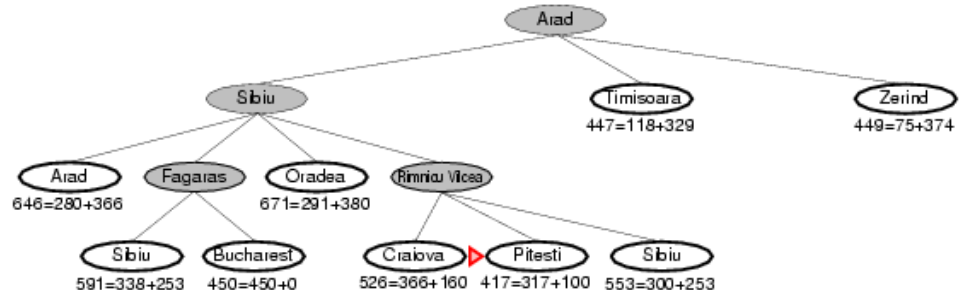


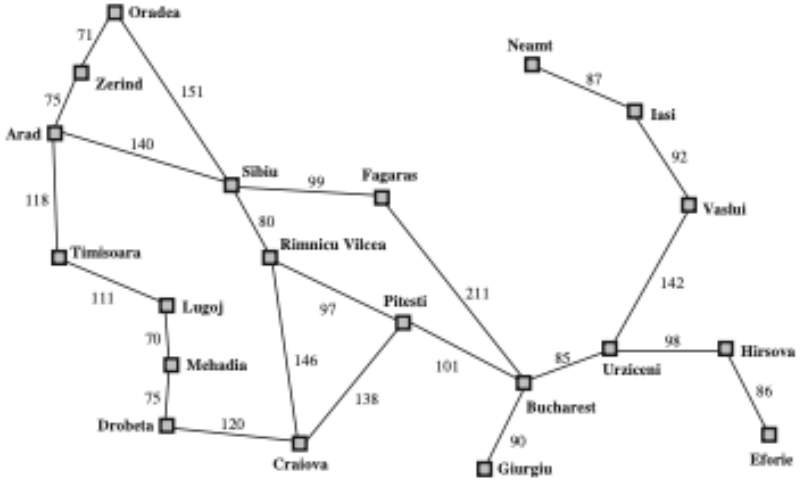


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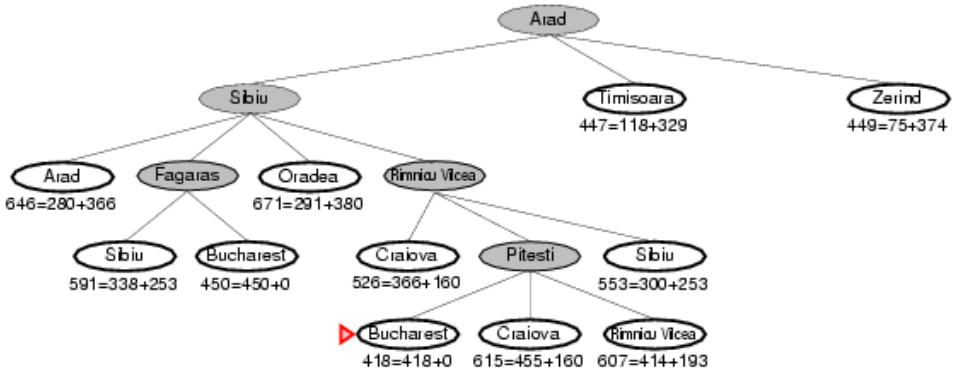


# Properties of A*

- Complete? Yes (unless there are infinitely many nodes with f $\leq f(G)$)
- Time? Exponential
- Space? Keeps all nodes in memory
- Optimal? Yes

# Informed search strategies

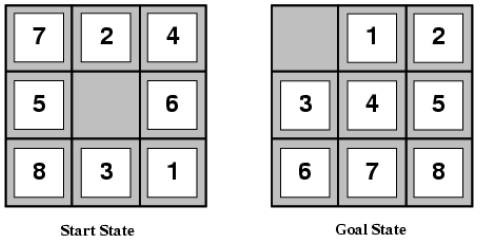
- Best-first search
- Greedy best-first search
- A* search
- **Heuristics**
- Local search algorithms
  - Hill-climbing search
  - Simulated annealing search
  - Local beam search
  - Genetic algorithms

# Admissible heuristics

- Objective:
  - produce a solution that is good enough for solving the problem at hand
  - This solution may not be the best but approximate the exact solution
- $h(n)$ is admissible if for node $n$, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from $n$
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)
- Theorem: If $h(n)$ is admissible, A* using TREE-SEARCH is optimal

# Admissible heuristics for 8-puzzle

- Heuristic: produce a solution in a reasonable time frame, good enough to solve the problem
- The average cost for the 8-puzzle are approx. 22 steps. Here are 26 steps.
- Branching factor is approx. 3
  - Empty in the middle, 4 mov
  - Empty in the corner, 2 mov
  - Rest cases, 3 mov
- Depth-first search will look $3^{22}$ states
- If we keep track of repeated states, we could reduce to 170.000
- In the15-puzle = $10^{13}$

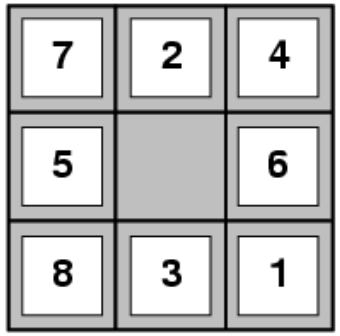


# Admissible heuristics for 8-puzzle

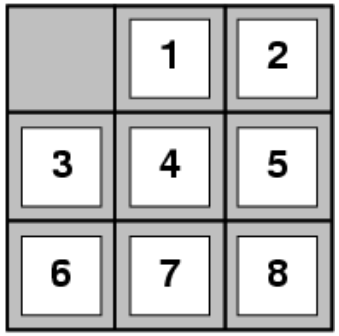
E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., + horizontal and vertical distance from desired location of each tile)




Start State




Goal State

- $h_1(S) = ?$
- $h_2(S) = ?$

# Admissible heuristics for 8-puzzle

E.g., for the 8-puzzle:

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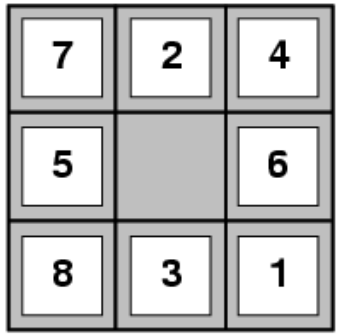
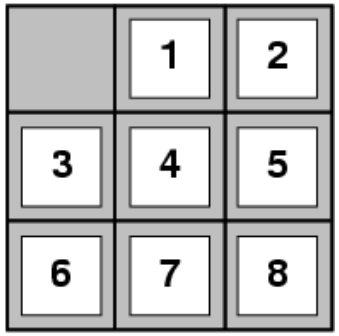


Start State



Goal State

- $h_1(S) = ?$ 8
- $h_2(S) = ?$ 3+1+2+2+2+3+3+2 = 18

# Dominance

- If $h_2(n) \geq h_1(n)$ for all $n$ (both admissible) then $h_2$ dominates $h_1$
- $h_2$ is better for search

- Typical search costs (average number of nodes expanded):
- *d=12* IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes
- *d=24* IDS = too many nodes
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes

# Relaxed problems

- A problem with fewer restrictions on the actions is called a relaxed problem
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution

# Informed search strategies

- Best-first search
  - Greedy best-first search
  - A* search
- Heuristics
- **Local search algorithms**
  - Hill-climbing search
  - Simulated annealing search
  - Local beam search

# Local search algorithms

- In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution
- State space = set of "complete" configurations. Find configuration satisfying constraints, e.g., n-queens
- In such cases, we can use local search algorithms
- Keep a single "current" state, try to improve it
- Work with one current state and generally moves to the neighboring state
- The paths followed by the search are not retained
  - They use little memory
  - You can find reasonable solutions in large state spaces or infinite

# Example: *n*-queens

- Put *n* queens on an *n* × *n* board with no two queens on the same row, column, or diagonal

# Informed search strategies

- Best-first search
- Greedy best-first search
- A* search
- Heuristics
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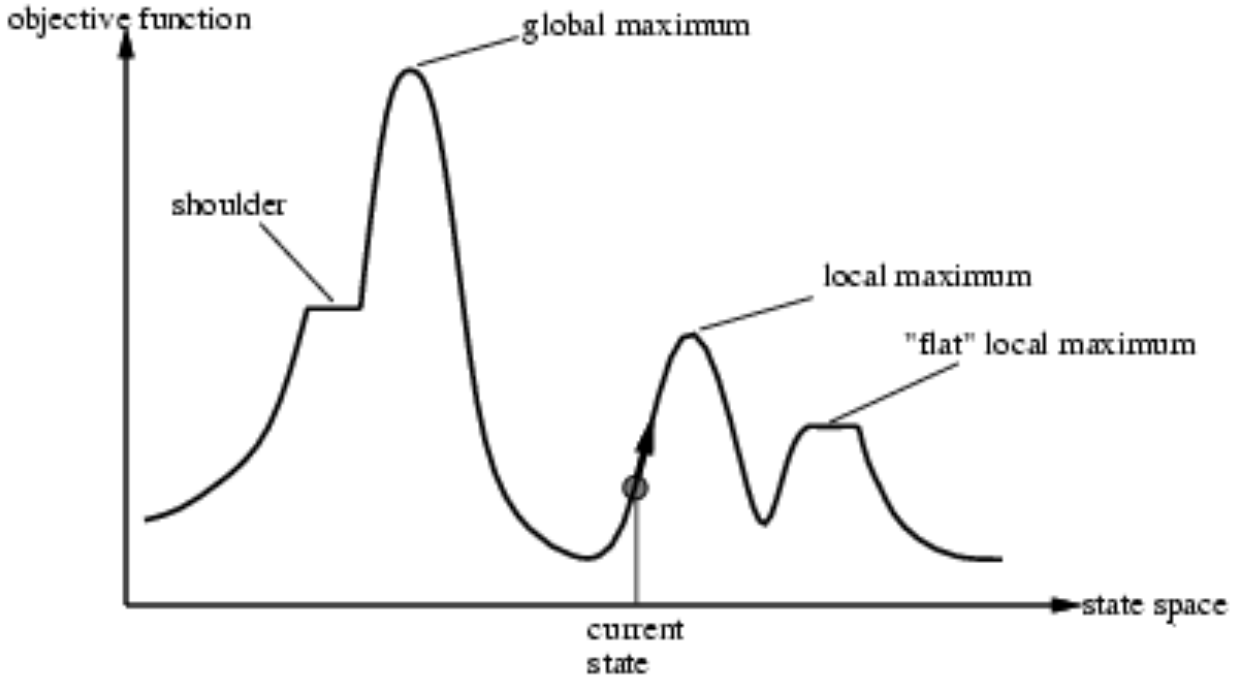
  - **Hill-climbing search**
  - Simulated annealing search
  - Local beam search

# Hill-climbing search

- It's just a loop that moves in the direction of increasing value
  - Ends when it reaches a peak where no neighbor has a higher value
  - The search tree is not kept, just a data structure of the current node to check the goal condition and its objective function value
- "Like climbing Everest in thick fog with amnesia"

# Hill-climbing search

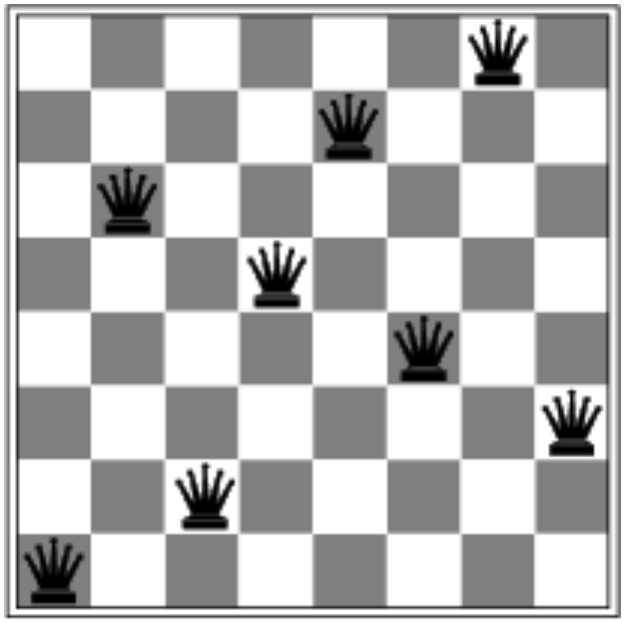
- Problem: depending on initial state, can get stuck in local maxima



# Hill-climbing search: 8-queens problem


- *h* = number of pairs of queens that are attacking each other, either directly or indirectly
- *h* = *17* for the above state
- The figure also shows the values of all successors, top successors have h = 12

# Hill-climbing search: 8-queens problem



- A local minimum with h = 1 (obtained in 5 steps)

# Hill-climbing search

- The algorithm gets stuck for several reasons::
  - Local Maximum: it is a peak that is higher than each of its neighbours, but lower than the maximum overall
  - Ridges: cause a sequence of local maxima that make navigation difficult
  - Plateau (flat): can lead to a local maximum  where there is no ascendant exit or a terrace to advance
- In the 8-queens, it gets stack in 86% and solve 14% cases
- If we allow lateral movements with the hope that we find a terrace (limiting them if reach a local maximum, e.g.100):→ 94% success
- Variants:
  - Stochastically (randomly chooses upward movements)
  - Random restart (the initial states are generated randomly)

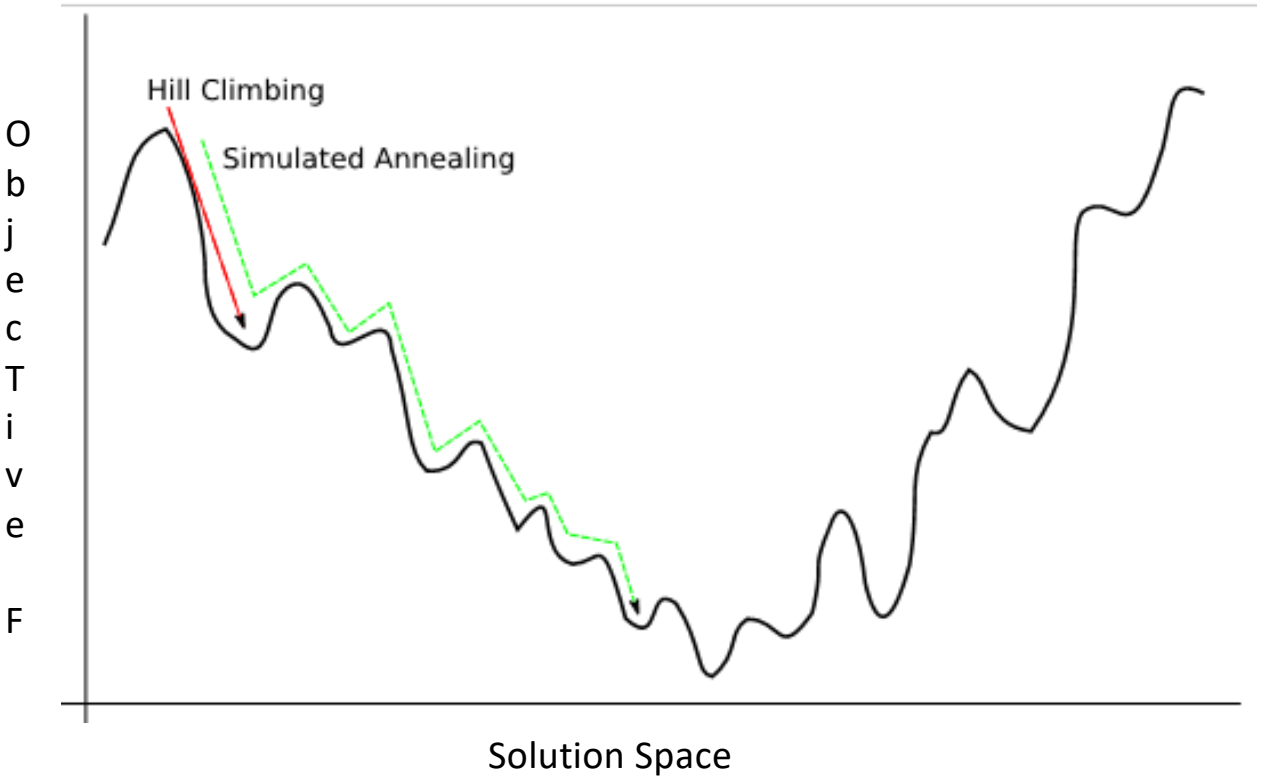
# Informed search strategies

- Best-first search
- Greedy best-first search
- A* search
- Heuristics
- Local search algorithms
  - Hill-climbing search
  - **Simulated annealing search**
  - Local beam search

# Simulated annealing search

- Process of tempering or hardening metals by heating and then cooling them gradually
- Idea: escape local maxima by allowing some "bad" moves but gradually decrease their frequency
- It combines hill-climbing with random generation successor
- Good for problems with a large search space,  optimum is surrounded by many local optima
- Problem: determine the values of the parameters, and requires an important work of experimentation that depends on each problem
- Widely used in VLSI layout, airline scheduling, etc

# Simulated annealing search



# Informed search strategies

- Best-first search
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  - **Local beam search**

# Local beam search

- Idea: Keep track of *k* states rather than just one
  - Start with *k* randomly generated states
  - At each iteration, all the successors of all *k* states are generated
  - If any one is a goal state, stop; else select the *k* best successors from the complete list and repeat
  - Alternatively stochastic LBS randomly choose k successors, with the probability of choosing a successor as an increasing function of its value

# Outline

- Introduction
- Problem formulation
- Problem types
- Basic search algorithms
- **Conclusions**

# Conclusions

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored

- Variety of uninformed and informed search strategies