UnitelmaSapienza
Università degli Studi di Roma

Sapienza
Università di Roma
Dipartimento di Informatica

Foundational Results

F. Parisi Presicce

UnitelmaSapienza.it

Formal Aspects

- Safety Question
- HRU Model
- Take-Grant Protection Model
- Expressive power
- Typed Access Matrix Model

What Is “Secure”?

- “leaking” is giving a generic right *r* to a subject who did not initially possess it
- If a system *S*, beginning in initial state s_0, cannot leak right *r*, it is *safe* with respect to the right r.
- Leaking a right is not inherently bad
  - Legitimate transfer of rights by owner

Safety Question

- Is there an algorithm for determining whether a protection system *S* with initial state s_0 is *safe* with respect to a generic right *r*?

Formally

Given

- initial state $X_0 = (S_0, O_0, A_0)$ (subjects, objects, matrix)
- Set C of commands

Can we use the commands in C to reach ($X_0 \vdash^* X_n$) a state X_n where $\exists\, s \in S$ and $\exists\, o \in O$ such that A_n[s,o] includes a right r not in A_0[s,o]?

- If so, the system is not safe, but
  - is a "safe" system a secure system?
  - are the commands correctly implemented?

Trust

- Safety does not distinguish a *leak* of a right from an *authorized transfer* of rights
- Subjects authorized to receive transfer of rights deemed "trusted"
  - Eliminate trusted subjects from matrix

Trivial cases of safety

- *r = read*, *own* $\in$ *a[s,o]*, command *can•grant•read•if•own*
- No command includes the *enter* primitive command

How about the general case?

Mono-Operational Commands

Answer: *yes*

Sketch of proof:

Consider minimal sequence of commands c_1, …, c_k to leak the right.

- Can omit **delete**, **destroy**
- Can merge all **create**s into one (since new subjects are all equal)

Worst case: insert every right into every entry; with s subjects and o objects initially, and n rights, upper bound is $k \leq n(s+1)(o+1)$

General Case

Answer: *no*

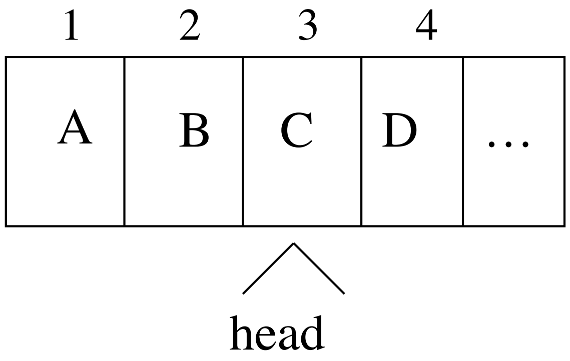
Sketch of proof:

Reduce halting problem to safety problem

Turing Machine review:

- Infinite tape in one direction
- States K, symbols M; distinguished blank b
- Transition function $\delta(k, m) = (k', m', \mathrm{L})$ means in state k, symbol m on tape location replaced by symbol m', head moves to left one square, and enters state k'
- Halting state is q_f; TM halts when it enters this state

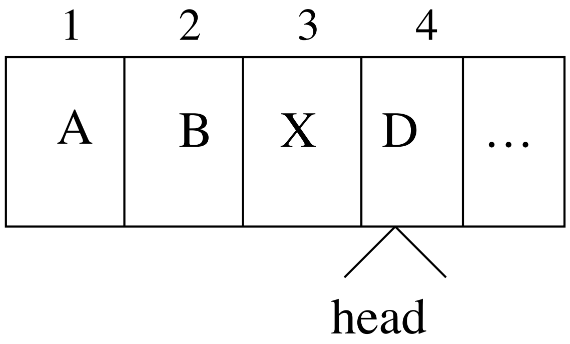
Mapping



Current state is k

	s_1	s_2	s_3	s_4	
s_1	A	*own*			
s_2		B	*own*		
s_3			C k	*own*	
s_4				D end	

Mapping



	s_1	s_2	s_3	s_4	
s_1	A	*own*			
s_2		B	*own*		
s_3			X	*own*	
s_4				D k_1 end	

After $\delta(k, \mathrm{C}) = (k_1, \mathrm{X}, \mathrm{R})$ where k is the current state and k_1 the next state

Command Mapping

$\delta(k, \mathrm{C}) = (k_1, \mathrm{X}, \mathrm{R})$ at intermediate becomes

command $\mathtt{c}_{k,\mathtt{C}}(s_3, s_4)$

if *own* **in** $A[s_3, s_4]$ **and** k **in** $A[s_3, s_3]$

and C **in** $A[s_3, s_3]$

then

delete k **from** $A[s_3, s_3]$;

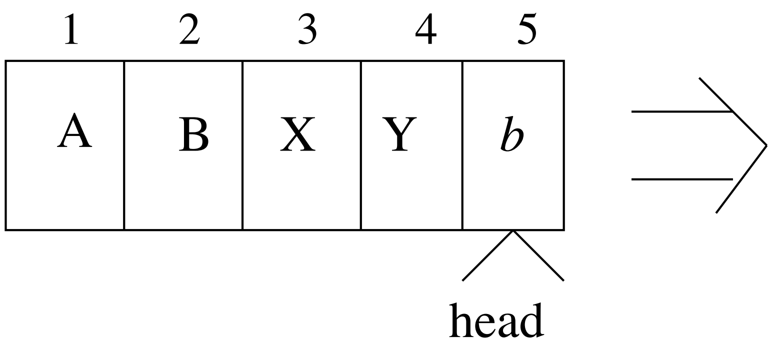
delete C **from** $A[s_3, s_3]$;

enter X **into** $A[s_3, s_3]$;

enter k_1 **into** $A[s_4, s_4]$;

end

Mapping



	s_1	s_2	s_3	s_4	s_5
s_1	A	*own*			
s_2		B	*own*		
s_3			X	*own*	
s_4				Y	*own*
s_5					b k_2 end

After $\delta(k_1, \mathrm{D}) = (k_2, \mathrm{Y}, \mathrm{R})$ where k_1 is the current state and k_2 the next state

Command Mapping

$\delta(k_1, \text{D}) = (k_2, \text{Y}, \text{R})$ at end becomes

```
command crightmost_{k,C}(s_4,s_5)
if end in A[s_4,s_4] and k_1 in A[s_4,s_4]
      and D in A[s_4,s_4]
then
  delete end from A[s_4,s_4];
  create subject s_5;
  enter own into A[s_4,s_5];
  enter end into A[s_5,s_5];
  delete k_1 from A[s_4,s_4];
  delete D from A[s_4,s_4];
  enter Y into A[s_4,s_4];
  enter k_2 into A[s_5,s_5];
end
```

Rest of proof

- Protection system exactly simulates a TM
  - Exactly 1 *end* right in ACM
  - 1 right in entries corresponds to state
  - Thus, at most 1 applicable command
- If TM enters state q_f, then right has leaked
- If safety question decidable, then represent TM as above and determine if q_f leaks
  - Implies halting problem decidable
- Conclusion: safety question undecidable

Other Results

- Set of unsafe systems is recursively enumerable
- Delete **create** primitive; then safety question is complete in **P-SPACE**
- Delete **destroy**, **delete** primitives (this system is called monotonic): safety question is undecidable
- Safety question for mono-conditional, monotonic protection systems is decidable
- Safety question for mono-conditional protection systems with **create**, **enter**, **delete** (and no **destroy**) is decidable.

So ?

- Safety decidable for some models
  - Are they practical?
- Safety only works if total set of rights is known in advance
  - Policy must specify all rights someone could get, not just what they have
- Can the safety of a particular system, with specific rules, be established?

Take-Grant Protection Model

- A specific (not generic) system
  - System represented as a directed graph
  - Set of graph rewriting rules for state transitions
- Safety is decidable, and in time linear with the size of the system
- Goal: find conditions under which rights can be transferred from one entity to another in the system

System

O objects (files, …)

● subjects (users, processes, …)

⊗ don't care (either a subject or an object)

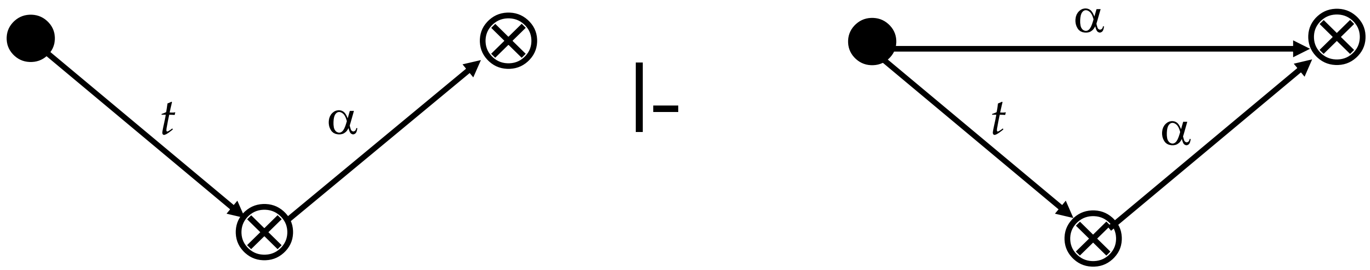
$G \vdash_x G'$ apply a rewriting rule *x* (witness) to G to get G'

$G \vdash^* G'$ apply a sequence of rewriting rules to G to get G'

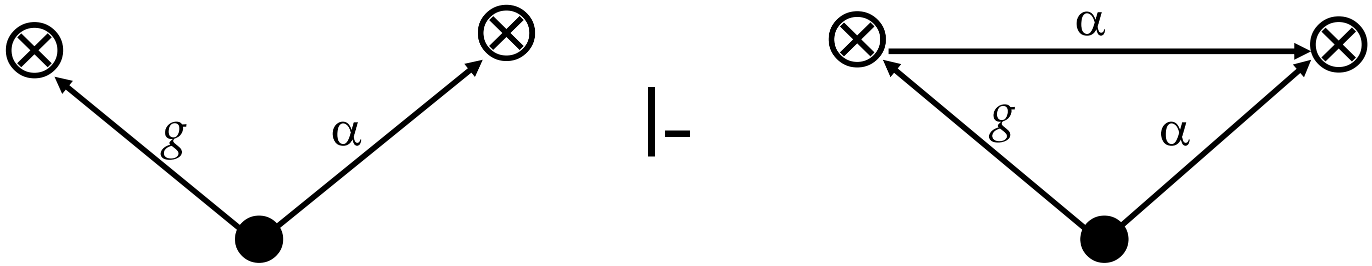
$R = \{ t, g, r, w, \dots \}$ set of rights

Rules

take



grant



More rules

create $\bullet \quad \vdash \quad \bullet \xrightarrow{\alpha} \otimes$

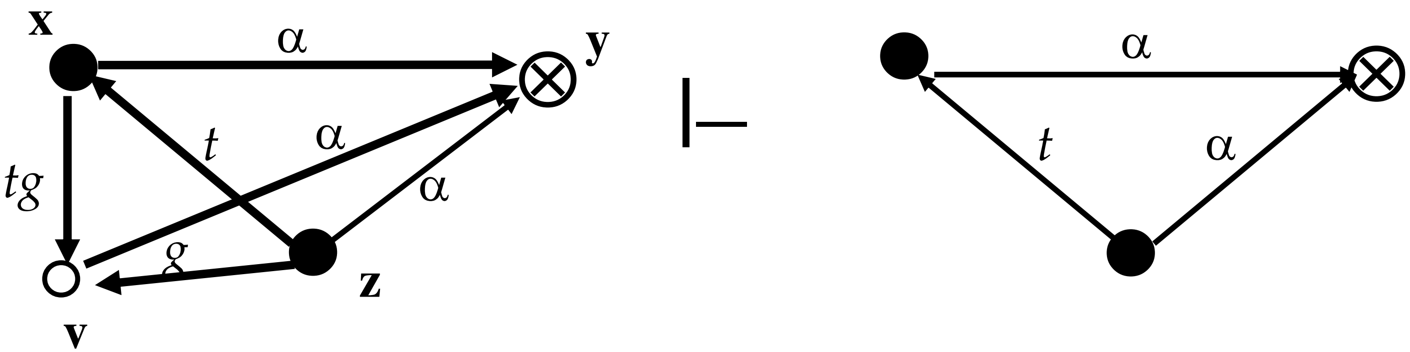
remove $\bullet \xrightarrow{\alpha} \otimes \quad \vdash \quad \bullet \xrightarrow{\alpha - \beta} \otimes$

These four rules are called the *de jure* rules

Example: Shared Buffer

- Initially s has grant rights for processes p and q.
- s sets up a shared buffer for p,q with the following steps
  - s creates new object b
  - s grants ({r,w} to b) to p
  - s grants ({r,w} to b) to q

Symmetry



1. **x** creates (*tg* to new) **v**
2. **z** takes (*g* to **v**) from **x**
3. **z** grants (α to **y**) to **v**
4. **x** takes (α to **y**) from **v**

Similar result for grant

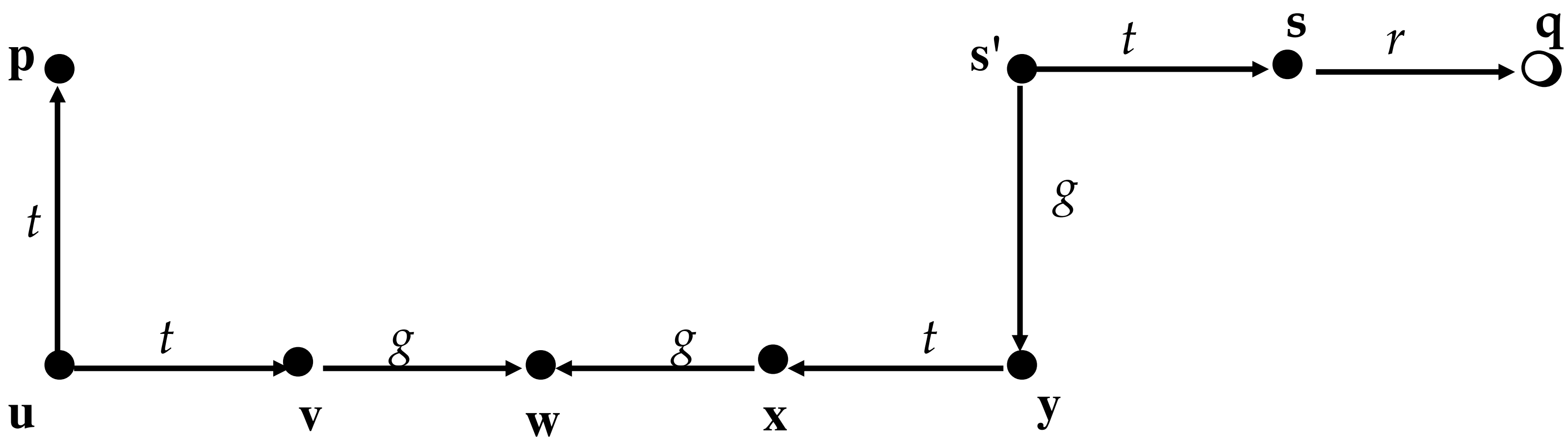
Islands

tg-path: path of distinct vertices connected by edges labeled *t* or *g*

- Call them "tg-connected"

island: maximal *tg*-connected subject-only subgraph

- Any right one vertex has can be shared with any other vertex



can•share

Definition:

can•share(r, **x**, **y**, G_0) if and only if there is a sequence of protection graphs $G_0, \ldots, G_n$ such that $G_0 \vdash^* G_n$ using only *de jure* rules and in G_n there is an edge from **x** to **y** labeled r.

- If x and y are subjects in an island, then *can•share*(r, **x**, **y**, G_0)
  - Proof by induction using the properties of tg-connected subjects
- General result: *can•share*(r, **x**, **y**, G_0) is decidable using an algorithm of complexity O(|V| + |E|) where V and E are the vertices and edges in the graph
  - Proof omitted (Exercise)

Key Question

- Characterize class of models for which safety is decidable
  - Existence: Take-Grant Protection Model is a member of such a class
  - Universality: in general, the question undecidable, so for some models it is not decidable
- What is the dividing line?

Typed Access Matrix Model

Like ACM, but with set of types T

- All subjects, objects have types
- Set of types for subjects TS

Protection state is (S, O, τ, A)

- $\tau: O \to T$ specifies type of each object
- If $\mathbf{X}$ subject, $\tau(\mathbf{X})$ in TS
- If $\mathbf{X}$ object, $\tau(\mathbf{X})$ in $T - TS$

Same rules as ACM except for create

Create Rules

Subject creation

- **create subject** *s* **of type** *ts*
- *s* must not exist as subject or object when operation executed
- $ts \in TS$

Object creation

- **create object** *o* **of type** *to*
- o must not exist as object when operation executed
- $to \in T - TS$

Definitions

MTAM (Monotonic TAM) Model: TAM model without **delete**, **destroy**

$\alpha(x_1{:}t_1, \ldots, x_n{:}t_n)$ create command

- t_i child type in α if any of **create subject** x_i **of type** t_i or **create object** x_i **of type** t_i occur in body of α
- t_i parent type otherwise

Creation graph: nodes for types, and arc from parent type to child type

Cyclic Creates

command *havoc*($s_1 : u$, $s_2 : u$, $o_1 : v$, $o_2 : v$, $o_3 : w$, o_4 : w)

  create subject s_1 **of type** u ;

  create object o_1 **of type** v ;

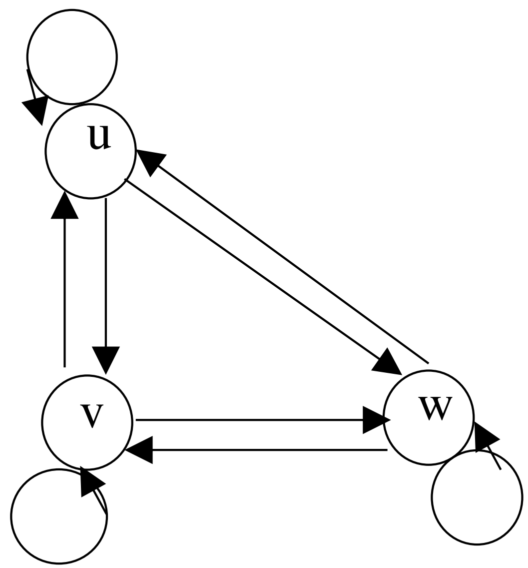
  create object o_3 **of type** w ;

  enter r **into** $a[s_2, s_1]$;

  enter r **into** $a[s_2, o_2]$;

  enter r **into** $a[s_2, o_4]$;

end



What kind of types are u, v and w?

Theorems

- Safety decidable for systems with *acyclic* MTAM schemes
- Safety for acyclic ternary MTAM decidable in time polynomial in the size of the initial ACM
  - “ternary” means commands have no more than 3 parameters
  - Equivalent in expressive power to MTAM

Key Points

- Safety problem is undecidable
  - Important to discuss the limits of the formalism
- Limiting the scope of systems can make the problem decidable
- Types are critical to the analysis of the safety problem