On the curvature of the Phillips curve

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Setup

> **i** Note
>
> The `ggforce` package was found on Reddit and provided a better alternative to `facet_wrap`, namely `facet_wrap_paginate`, especially due to the high quantity of different entities (countries) that we face in this dataset.

Motivation for the project

The dynamics in the labor market are the most complicated out of all markets, because it mixes the humane reasoning and the economic one. The Phillips curve has always been a reference tool because of the simplicity of the relationship between prices and unemployment it created. This relationship could not only provide an analytically point of view about the conjuncture, but also a policy tool. In every economics class, the Phillips curve was said to be negatively slopped, but as one progresses, it realizes that this is less and less the case and that multiple other underlying factors actually drive these two forces. This is what we try to explore today.

Data and sources

Sources

Our two sources are from the OECD Data Explorer's 117 Economic Outlook and the World Development Indicator.

Necessary data manipulation

All the data in OECD is annual and there are many unnecessary columns. We keep only the reference area, measure, time period and observation value of the measure, respectively named `country`, `measure`, `time` and `value`. We will also rename all variables such that they will be identifiable from the OECD database, but still recognizable.

WDI data is also annual and we also need to get rid of unnecessary columns. We will also use `pivot_longer` in order to group all the different columns for the different variables in a new `measure` column and their values in the `value` one. Also, `Time` will be renamed as `time`.

We can now merge both datasets and get some useful statistics out of them:
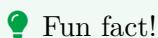
Countries	Variables	Start date	End date
28	57	1960	2026

Therefore, our total dataset comprises 58 countries for 57 variables, and spans 1960 to 2026. Of course, for analysis purposes, we will reduce this dataset and use subsets, which will be specified every time.

Introduction to a bit of history

What is the Phillips curve?

The curve was conceptualized by William Phillips in 1958 in the Economica journal. He apparently observed the negative relationship between the increase in nominal wages and unemployment between 1861 and 1957.

> 💡 Fun fact!
>
> A few people actually know the true story behind this curve, but according to a friend of Phillips who then worked at the ECB and passed down this story in the Financial Analysis department, Phillips actually computed the curve … by hand! In fact, he took the data, created 5 or so averages and observed the negative relationship in the graph. This is the true (yet not so sexy and serious) story behind the "statistical observation" of this curve. Although his work was thereafter taken by other brilliant economist, which found (not so strong though) similar relationships.

He derived the following relationship:

$$\frac{\Delta W}{W} = \alpha \cdot U^{-\beta} - \gamma$$

Where $\frac{\Delta W}{W}$ represents the rate of change of nominal wages and U represents unemployment. The rest are parameters. This equation has a very practical theoretical reasoning which serves as a strong foundation for wage bargaining theory, which itself lies in the center of labor market economics. When unemployment increases, the firm has more "supply" of potential workers, therefore, their bargaining power decreases, and so does their wages, which in turns create a decrease in prices, because firms can supply more. Vice-versa if unemployment decreases.

Historical development

Actual researches on it

The newest researches go as far as to develop an open-economy Phillips curve, where inflation is influenced by both wage levels and the exchange rate:

$$\pi_t = \gamma_1 \cdot \mathbb{E}_t[\pi_{t+1}] + \gamma_2 \cdot x_t + \gamma_3 \cdot q_t$$
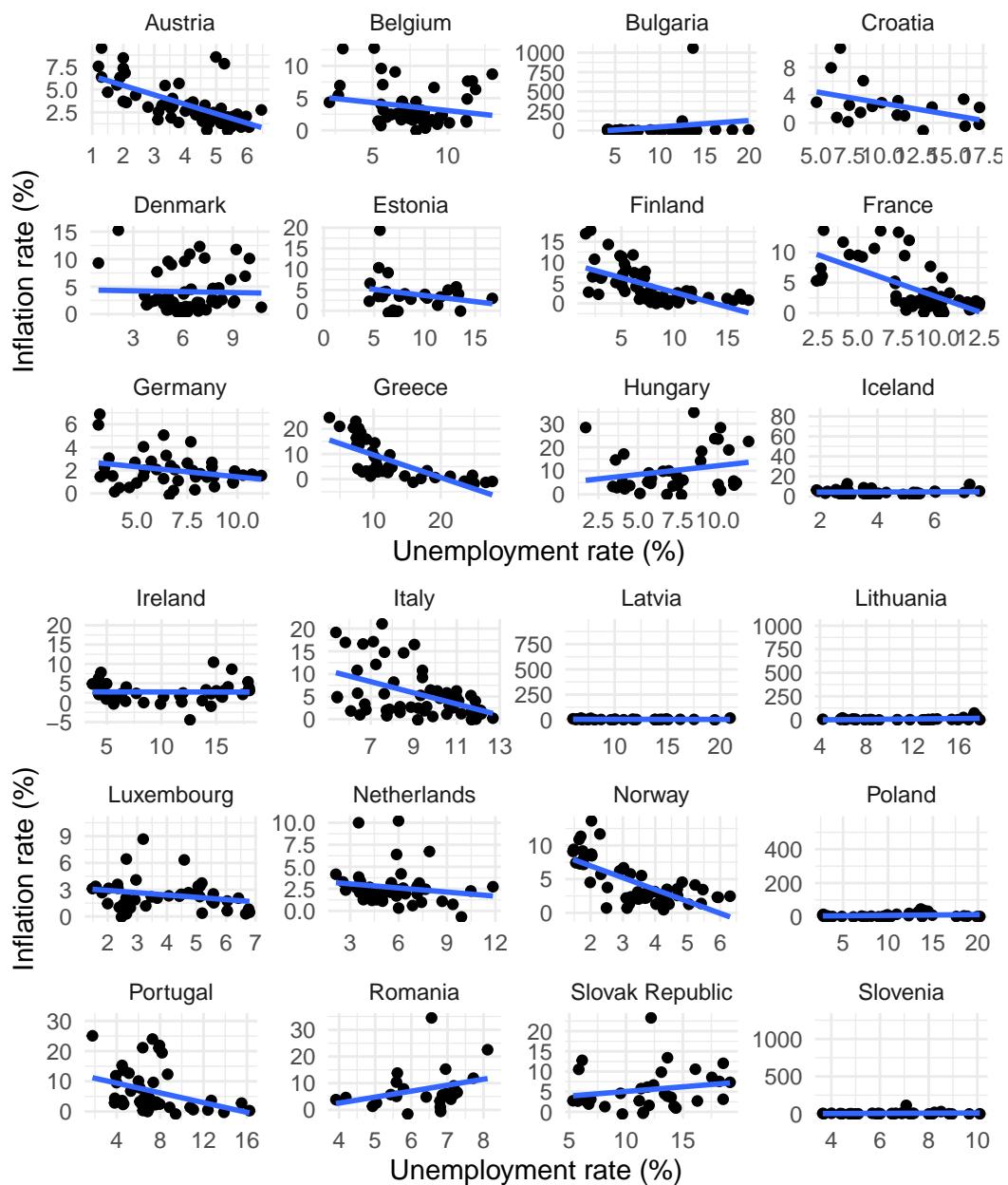
Where γ_1, γ_2 and γ_3 are parameters, π_t represents domestic inflation and x_t is the (real) output gap and q_t the (real) exchange rate. This equation is based on Gali & Monacelli (2005). See Appendix A for full derivation.
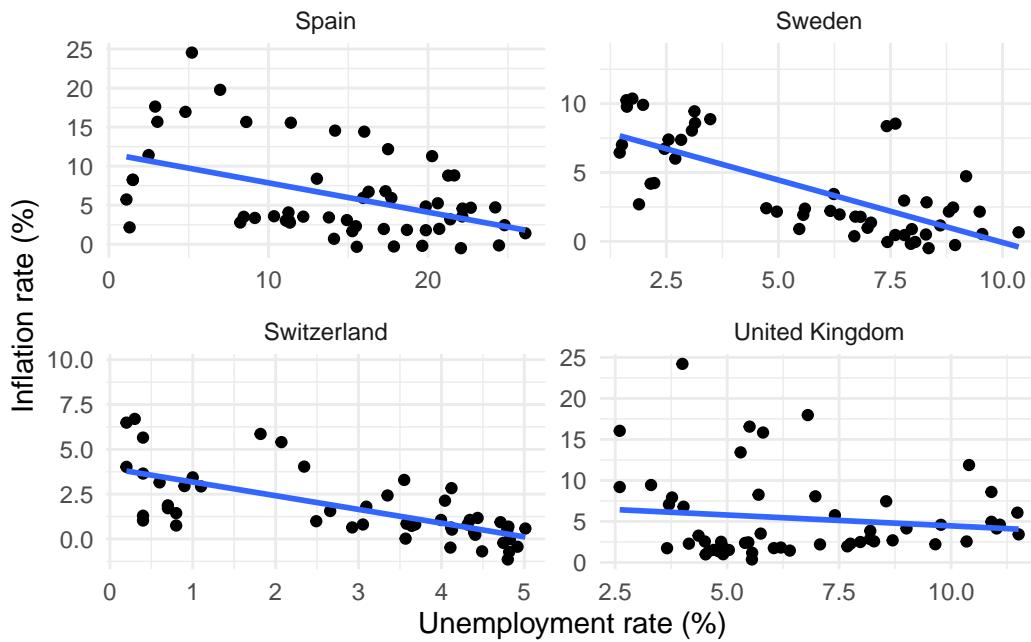
Improvements there could be (according to us)

Data visualization

Let us represent as an "entrée" a basic scatter plot for the Phillips curve for 28 European countries over the period and we are using WDI data because it is more complete:

Phillips Curve

Inflation rate (%)

Unemployment rate (%)

A simple visualization reassures us on theory. We can indeed see that over the complete span 1960-2026, the Phillips curve seems, for most countries, to obey the theory and display a downward slopping tendency. There are though some surprising results, as for Bulgaria, Romania, Hungary or Slovak Republic, with upward slopping curves, or Latvia, Lithuania, Poland, Slovenia, or Iceland with hardly interpretable results.

Whether these results are due to true economic tendencies or simply a lack of data or data mismatch will be determined with the analysis.

The question of the curvature of the Phillips curve, however, requires the differentiation between periods, say decades or key dates, which will be our next observation.

Data analysis

Numerical model

Conclusion

Appendix

Appendix A: Derivation of the open economy New Keynesian Phillips Curve (NKPC)

In the classical New Keynesian model, the demand side of the economy yields the CES consumption function:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} \cdot C_t$$

For $i \in [0, 1]$ firms. Market clears at $Y_t(i) = C_t(i)$. Firms produce according to a linear production function $Y_t(i) = A_t N_t(i)$ where $a_t \equiv \log A_t$ follows an AR(1) process with MA(∞) representation $a_t = \sum_{k=0}^{+\infty} \rho_a^k \epsilon_{t-k}$. Moreover, firms price *a la Calvo* (1983), where $1 - \theta$ (oligopolist) firms can reset prices at level $P_t^*(i)$ in each period, which implies maximizing:

$$\max_{P_t(i)} \mathbb{E}_t \left[\sum_{k=0}^{+\infty} (\beta\theta)^k \Lambda_{t,t+k} Y_{t+k|t} \left(P_t^*(i) - MC_{t+k} P_{t+k} \right) \right]$$

Under the demand constraint $Y_{t+k|t} = \left(\frac{P_t^*(i)}{P_t}\right)^{-\varepsilon} \cdot Y_{t+k}$ and the marginal cost of the firm defined as:

$$MC_t = (1 - \chi)\frac{W_t}{P_t} + \chi S_t P_t^f$$

Where S_t is the nominal exchange rate and P_t^f is the foreign price index. <mark>χ represents home bias preference, and therefore is a natural index for openness to trade</mark> The optimal consumer price index (CPI) inflation condition, where inflation is defined as $\pi_t \equiv p_t - p_{t-1}$, is given by:

$$\pi_t = \beta \cdot \mathbb{E}_t[\pi_{t+1}] + \kappa \cdot \widehat{mc}_t$$

Where $\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$. Real marginal cost $\widehat{mc}_t$ can be found by log-linearizing the firms' nominal marginal cost, which holds for every firm due to the linearity of the production function:

$$\widehat{mc}_t = (1-\chi) \cdot (\hat{w}_t - \hat{p}_t - \hat{a}_t) + \chi \cdot (\hat{s}_t + \hat{p}_t^f - \hat{p}_t) \Leftrightarrow \widehat{mc}_t^* = (\hat{w}_t - \hat{p}_t - \hat{a}_t) + \alpha \cdot \hat{q}_t$$

Where $\hat{s}_t + \hat{p}_t^f - \hat{p}_t \equiv \hat{q}_t$ is exactly the formula for the real exchange rate, $\widehat{mc}_t^* \equiv \frac{\widehat{mc}_t}{1-\chi}$ and $\alpha \equiv \frac{\chi}{1-\chi}$. This allows to derive the final New Keynesian Phillips Curve (NKPC):

$$\pi_t = \beta \cdot \mathbb{E}_t[\pi_{t+1}] + \kappa \cdot (\hat{w}_t - \hat{p}_t - \hat{a}_t) + \kappa\alpha \cdot \hat{q}_t$$

Which allow us to derive three different possible NKPC:

$$\text{NKPC (1):} \quad \pi_t = \gamma_1 \cdot \mathbb{E}_t[\pi_{t+1}] + \gamma_2 \cdot (\hat{w}_t - \hat{p}_t - \hat{a}_t) + \gamma_3 \cdot \hat{q}_t$$

With parameters $\gamma_1 \equiv \beta$, $\gamma_2 \equiv \kappa$ and $\gamma_3 \equiv \kappa\alpha$.

$$\text{NKPC (2):} \quad \pi_t = \gamma_1 \cdot \mathbb{E}_t[\pi_{t+1}] + \gamma_2 \cdot (\hat{y}_t - \hat{y}_t^n) + \gamma_3 \cdot \hat{q}_t$$

Using the fact that ==under imperfect competition, the real unit labor cost is equal to the output gap== such that $\hat{w}_t - \hat{p}_t - \hat{a}_t = \hat{y}_t - \hat{y}_t^n$, and with parameters $\gamma_1 \equiv \beta$, $\gamma_2 \equiv \kappa\varphi$ and $\gamma_3 \equiv \kappa\alpha$.

$$\text{NKPC (3):} \quad \pi_t = \gamma_1 \cdot \mathbb{E}_t[\pi_{t+1}] + \gamma_2 \cdot (\hat{u}_t - \hat{u}_t^n) + \gamma_3 \cdot \hat{q}_t$$

Using ==Okun's law==, which can be expressed as $-\delta(\hat{y}_t - \hat{y}_t^n) = \hat{u}_t - \hat{u}_t^n$, and with parameters $\gamma_1 \equiv \beta$, $\gamma_2 \equiv -\frac{\kappa}{\delta}$ and $\gamma_3 \equiv \kappa\alpha$.