On the curvature of the Phillips curve

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Setup

> **Note**
>
> The `ggforce` package was found on Reddit and provided a better alternative to `facet_wrap`, namely `facet_wrap_paginate`, especially due to the high quantity of different entities (countries) that we face in this dataset.

Motivation for the project

The dynamics in the labor market are the most complicated out of all markets, because it mixes the humane reasoning and the economic one. The Phillips curve has always been a reference tool because of the simplicity of the relationship between prices and unemployment it created. This relationship could not only provide an analytically point of view about the conjuncture, but also a policy tool. In every economics class, the Phillips curve was said to be negatively slopped, but as one progresses, it realizes that this is less and less the case and that multiple other underlying factors actually drive these two forces. This is what we try to explore today.

Data and sources

Sources

Our two sources are from the OECD Data Explorer's 117 Economic Outlook and the World Development Indicator.

Necessary data manipulation

All the data in OECD is annual and there are many unnecessary columns. We keep only the reference area, measure, time period and observation value of the measure, respectively named `country`, `measure`, `time` and `value`. We will also rename all variables such that they will be identifiable from the OECD database, but still recognizable.

WDI data is also annual and we also need to get rid of unnecessary columns. We will also use `pivot_longer` in order to group all the different columns for the different variables in a new `measure` column and their values in the `value` one. Also, `Time` will be renamed as `time`.

We can now merge both datasets and get some useful statistics out of them:

Countries	Variables	Start date	End date
28	57	1960	2026

Therefore, our total dataset comprises 28 countries for 57 variables, and spans 1960 to 2026. Of course, for analysis purposes, we will reduce this dataset and use subsets, which will be specified every time.

Introduction to a bit of history

What is the Phillips curve?

The curve was conceptualized by William Phillips in 1958 in the Economica journal. He apparently observed the negative relationship between the increase in nominal wages and unemployment between 1861 and 1957.

> Fun fact!
>
> A few people actually know the true story behind this curve, but according to a friend of Phillips who then worked at the ECB and passed down this story in the Financial Analysis department, Phillips actually computed the curve by hand! In fact, he took the data, created 5 or so averages and observed the negative relationship in the graph. This is the true (yet not so sexy and serious) story behind the "statistical observation" of this curve. Although his work was thereafter taken by other brilliant economist, which found (not so strong though) similar relationships.

He derived the following relationship:

$$\frac{\Delta W}{W} = \alpha \cdot U^{-\beta} - \gamma$$

Where $\frac{\Delta W}{W}$ represents the rate of change of nominal wages and U represents unemployment. The rest are parameters. This equation has a very practical theoretical reasoning which serves as a strong foundation for wage bargaining theory, which itself lies in the center of labor market economics. When unemployment increases, the firm has more "supply" of potential workers, therefore, their bargaining power decreases, and so does their wages, which in turns create a decrease in prices, because firms can supply more. Vice-versa if unemployment decreases.

Historical development

The study of British data confirmed the existence of an inverse relationship between unemployment and the rate of change in nominal wages. Popularized as the "Phillips curve," this relationship quickly became fundamental for two reasons. It allowed Keynesian macroeconomics to move from a static to a dynamic framework by introducing a mechanism that links output, employment, and price evolution over time. In the original Keynesian model, with fixed prices, fluctuations in activity were explained by changes in demand, but the model did not account for price dynamics. The Phillips curve filled this gap: for a given period, demand determines output and employment, which in turn determine unemployment. Through the Phillips relationship, unemployment then affects wage growth, and if wage changes pass through to prices, it drives inflation dynamics from one period to the next.

Later, in 1960, Samuelson and Solow extended the Phillips framework using U.S. data from 1900 to 1960. Assuming that wage variations translate into prices through production costs, they reformulated the original relationship between unemployment and wage inflation into one between unemployment and price inflation. This version of the Phillips curve suggested a possible trade-off between inflation and unemployment, with profound implications for economic policy

Actual researches on it

The newest researches go as far as to develop an open-economy Phillips curve, where inflation is influenced by both wage levels and the exchange rate:

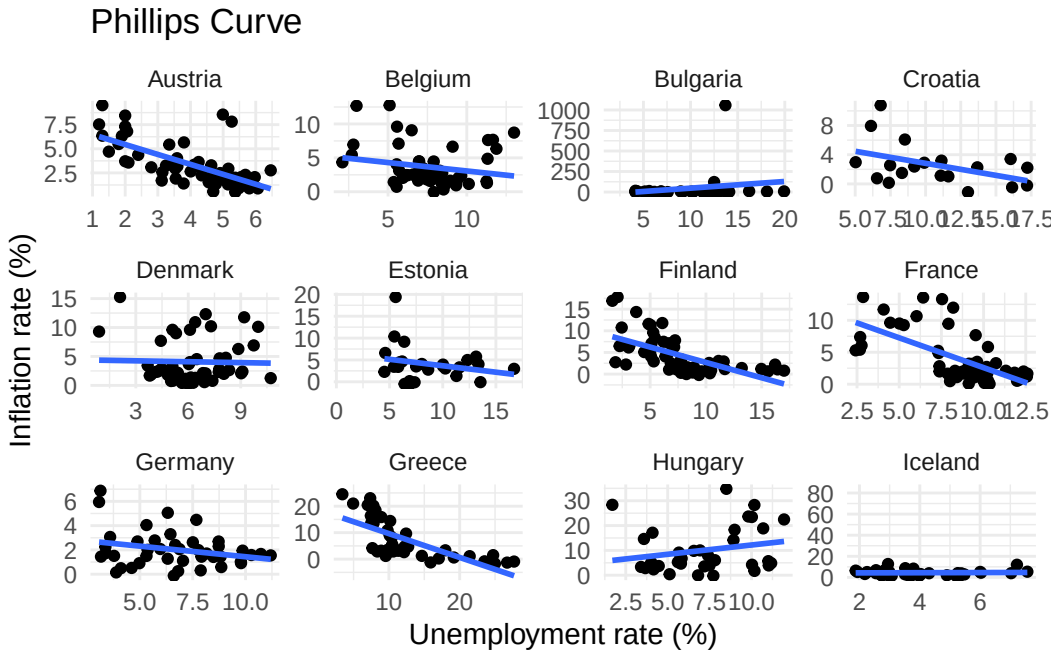
$$\pi_t = \gamma_1 \cdot \mathbb{E}_t[\pi_{t+1}] + \gamma_2 \cdot x_t + \gamma_3 \cdot q_t$$

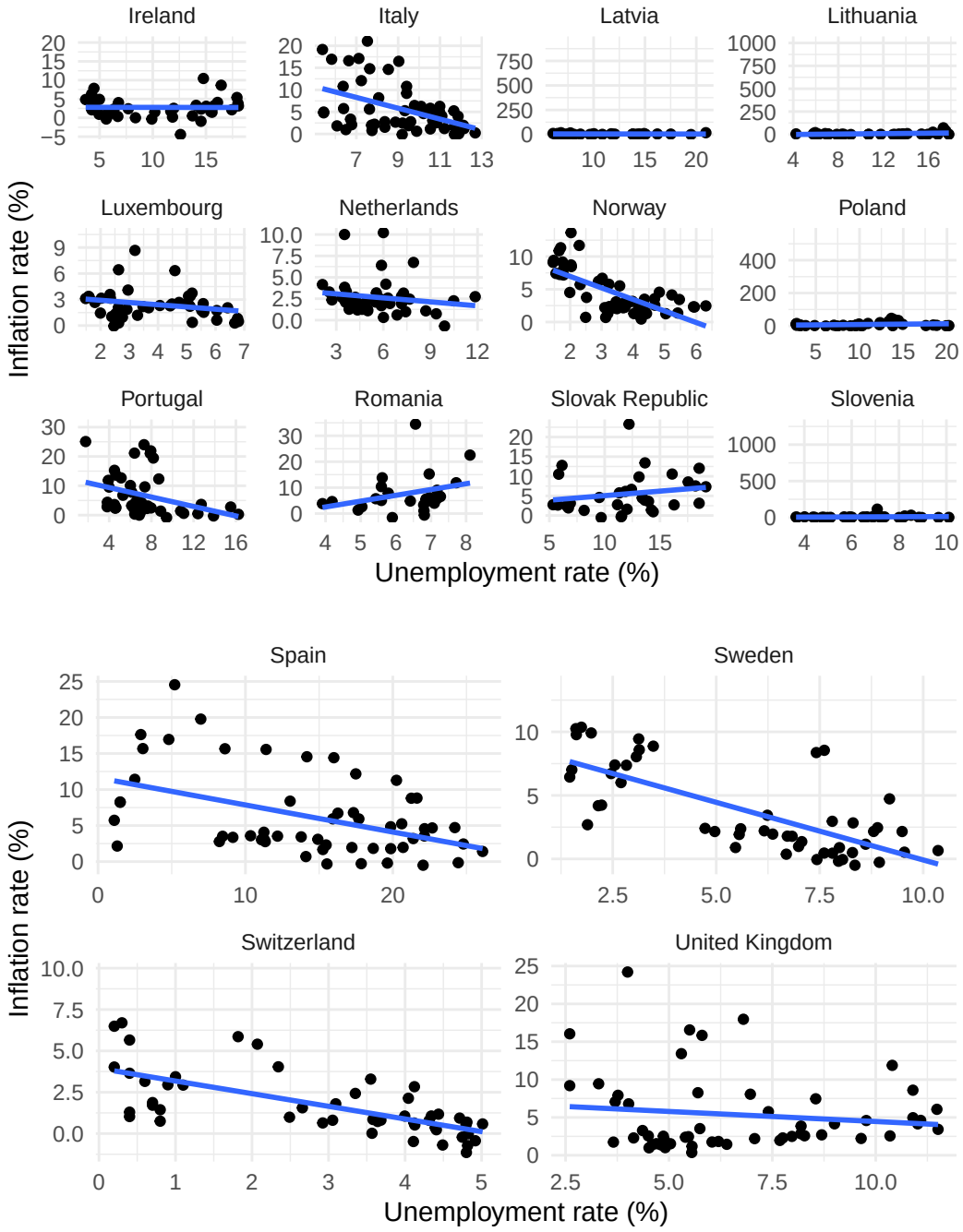
Where γ_1, γ_2 and γ_3 are parameters, π_t represents domestic inflation and x_t is the (real) output gap and q_t the (real) exchange rate. This equation is based on Gali & Monacelli (2005). See Appendix A for full derivation.

Improvements there could be (according to us)

Data visualization

Let us represent as an "entrée" a basic scatter plot for the Phillips curve for 28 European countries over the period and we are using WDI data because it is more complete:





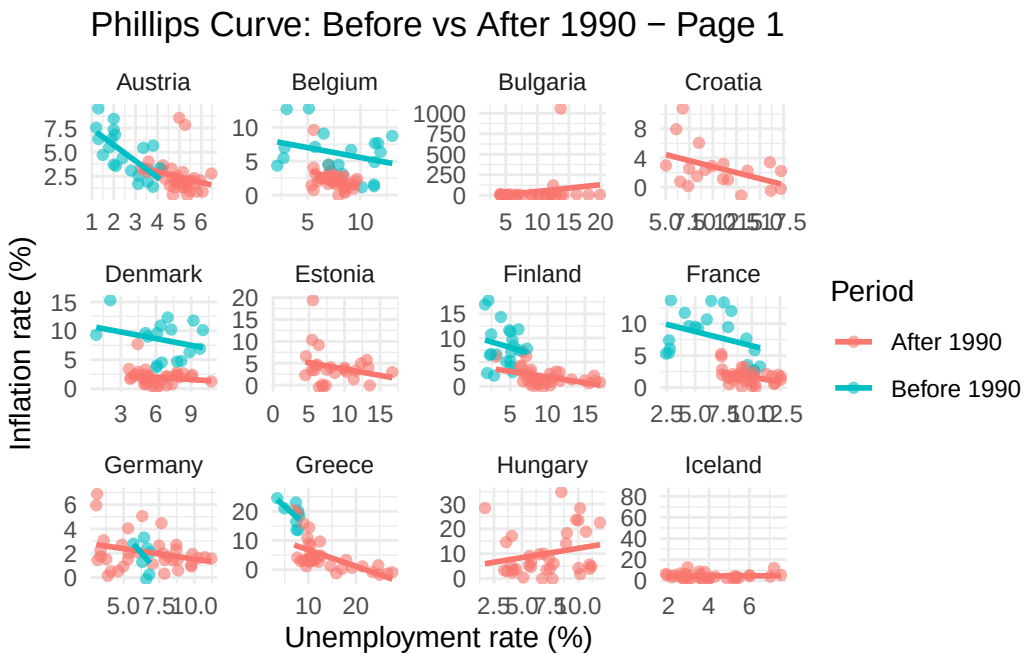
A simple visualization reassures us on theory. We can indeed see that over the complete span 1960-2026, the Phillips curve seems, for most countries, to obey the theory and display a downward slopping tendency. There are though some surprising results, as for Bulgaria, Romania, Hungary or Slovak Republic, with upward slopping curves, or Latvia, Lithuania,

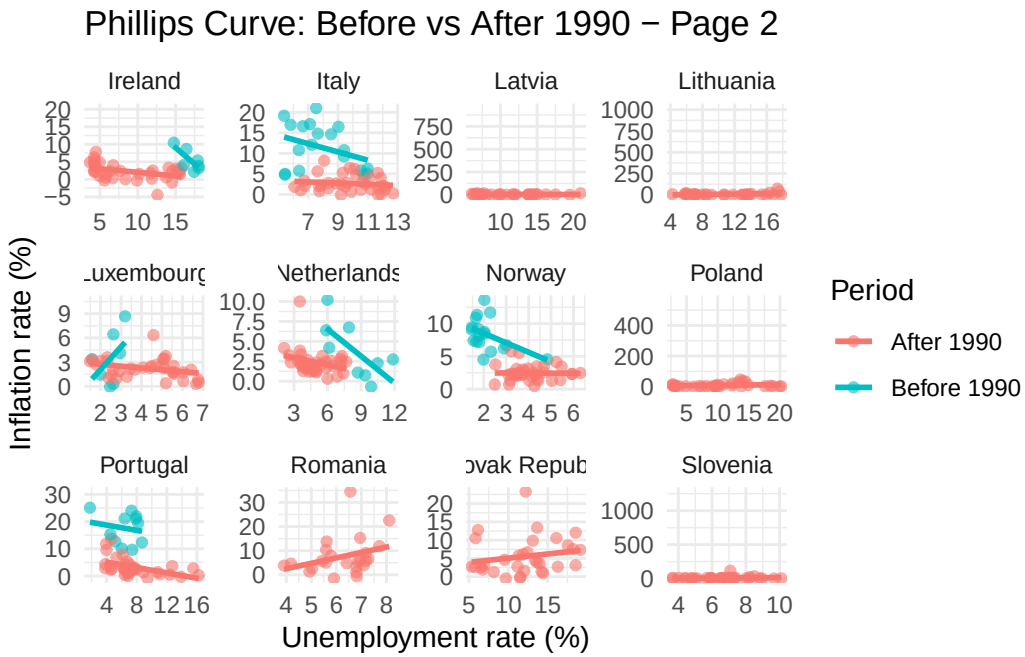
Poland, Slovania, or Iceland with hardly interpretable results.
Whether these results are due to true economic tendencies or simply a lack of data or data mismatch will be determined with the analysis.
The question of the curvature of the Phillips curve, however, requires the differentiation between periods, say decades or key dates, which will be our next observation.

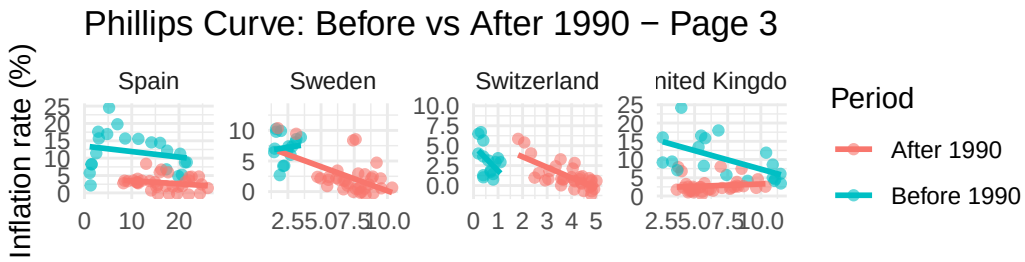
Before/after 1990

We focus on 1990 as the dividing year because it marks a significant structural shift in the European and global economy. Around this time, Europe experienced major political and economic transformations, including the fall of the Berlin Wall, the reunification of Germany, and the liberalization of formerly planned economies in Eastern Europe. These events altered labor markets, inflation dynamics, and macroeconomic policies across the continent. Additionally, the early 1990s coincide with the widespread adoption of modern central banking practices, with many European countries emphasizing stricter inflation targeting. Using 1990 as a breakpoint allows us to examine how the Phillips curve relationship between unemployment and inflation may have changed before and after these structural shifts.

remarque pr le MAZERE : les pays sans données bleues (cad avant 90) on les tej ? et j'ai choisi 90 mais on peut prendre aussi 2008 idk what u prefer







The Phillips Curve graphs compare inflation and unemployment rates before and after 1990 across various countries. Before 1990, many nations, like France and Spain, show a clearer inverse relationship, aligning with the traditional Phillips Curve theory. However, post-1990, this relationship often weakens or disappears, as seen in Germany and the UK, suggesting

shifts in economic dynamics. Furthermore, some countries, such as Lithuania and Latvia, lack pre-1990 data.

Focus on one country : France ?

Data analysis

Numerical model

Conclusion

Appendix

Appendix A: Derivation of the open economy New Keynesian Phillips Curve (NKPC)

In the classical New Keynesian model, the demand side of the economy yields the CES consumption function:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} \cdot C_t$$

For $i \in [0,1]$ firms. Market clears at $Y_t(i) = C_t(i)$. Firms produce according to a linear production function $Y_t(i) = A_t N_t(i)$ where $a_t \equiv \log A_t$ follows an AR(1) process with MA(∞) representation $a_t = \sum_{k=0}^{+\infty} \rho_a^k \epsilon_{t-k}$. Moreover, firms price *a la Calvo* (1983), where $1-\theta$ (oligopolist) firms can reset prices at level $P_t^*(i)$ in each period, which implies maximizing:

$$\max_{P_t(i)} \mathbb{E}_t \left[\sum_{k=0}^{+\infty} (\beta\theta)^k \Lambda_{t,t+k} Y_{t+k|t} \left(P_t^*(i) - MC_{t+k} P_{t+k}\right) \right]$$

Under the demand constraint $Y_{t+k|t} = \left(\frac{P_t^*(i)}{P_t}\right)^{-\varepsilon} \cdot Y_{t+k}$ and the marginal cost of the firm defined as:

$$MC_t = (1-\chi)\frac{W_t}{P_t} + \chi S_t P_t^f$$

Where S_t is the nominal exchange rate and P_t^f is the foreign price index. χ represents home bias preference, and therefore is a natural index for openness to trade The optimal consumer price index (CPI) inflation condition, where inflation is defined as $\pi_t \equiv p_t - p_{t-1}$, is given by:

$$\pi_t = \beta \cdot \mathbb{E}_t[\pi_{t+1}] + \kappa \cdot \widehat{mc}_t$$

Where $\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$. Real marginal cost $\widehat{mc}_t$ can be found by log-linearizing the firms' nominal marginal cost, which holds for every firm due to the linearity of the production function:

$$\widehat{mc}_t = (1-\chi) \cdot (\hat{w}_t - \hat{p}_t - \hat{a}_t) + \chi \cdot (\hat{s}_t + \hat{p}_t^f - \hat{p}_t) \Leftrightarrow \widehat{mc}_t^* = (\hat{w}_t - \hat{p}_t - \hat{a}_t) + \alpha \cdot \hat{q}_t$$

Where $\hat{s}_t + \hat{p}_t^f - \hat{p}_t \equiv \hat{q}_t$ is exactly the formula for the real exchange rate, $\widehat{mc}_t^* \equiv \frac{\widehat{mc}_t}{1-\chi}$ and $\alpha \equiv \frac{\chi}{1-\chi}$. This allows to derive the final New Keynesian Phillips Curve (NKPC):

$$\pi_t = \beta \cdot \mathbb{E}_t[\pi_{t+1}] + \kappa \cdot (\hat{w}_t - \hat{p}_t - \hat{a}_t) + \kappa\alpha \cdot \hat{q}_t$$

Which allow us to derive three different possible NKPC:

$$\text{NKPC (1):} \quad \pi_t = \gamma_1 \cdot \mathbb{E}_t[\pi_{t+1}] + \gamma_2 \cdot (\hat{w}_t - \hat{p}_t - \hat{a}_t) + \gamma_3 \cdot \hat{q}_t$$

With parameters $\gamma_1 \equiv \beta$, $\gamma_2 \equiv \kappa$ and $\gamma_3 \equiv \kappa\alpha$.

$$\text{NKPC (2):} \quad \pi_t = \gamma_1 \cdot \mathbb{E}_t[\pi_{t+1}] + \gamma_2 \cdot (\hat{y}_t - \hat{y}_t^n) + \gamma_3 \cdot \hat{q}_t$$

Using the fact that under imperfect competition, the real unit labor cost is equal to the output gap such that $\hat{w}_t - \hat{p}_t - \hat{a}_t = \hat{y}_t - \hat{y}_t^n$, and with parameters $\gamma_1 \equiv \beta$, $\gamma_2 \equiv \kappa\varphi$ and $\gamma_3 \equiv \kappa\alpha$.

$$\text{NKPC (3):} \quad \pi_t = \gamma_1 \cdot \mathbb{E}_t[\pi_{t+1}] + \gamma_2 \cdot (\hat{u}_t - \hat{u}_t^n) + \gamma_3 \cdot \hat{q}_t$$

Using Okun's law, which can be expressed as $-\delta(\hat{y}_t - \hat{y}_t^n) = \hat{u}_t - \hat{u}_t^n$, and with parameters $\gamma_1 \equiv \beta$, $\gamma_2 \equiv -\frac{\kappa}{\delta}$ and $\gamma_3 \equiv \kappa\alpha$.