Overview on Functions
MAT091 & MAT092

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1 Function

1.1 What is function

Before going into details about "Function", let's talk about a simple machine first. If it is asked, "What does a simple machine do?" the very first thing should come to your mind is, "It takes an input, process it and gives an output". If it is represented through a diagram, then it should look like,

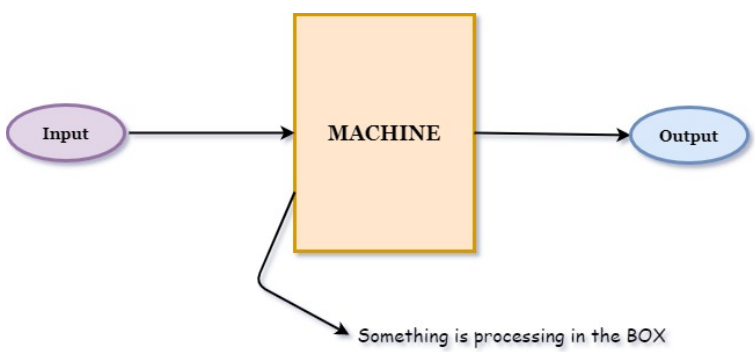

Figure 1: How a Simple Machine works

So, now let's get back to the topic "Function". "What does a Function do?" The answer is, "A **Function** exactly works like a **Simple Machine**". You can get a clear idea when you see the below diagram.

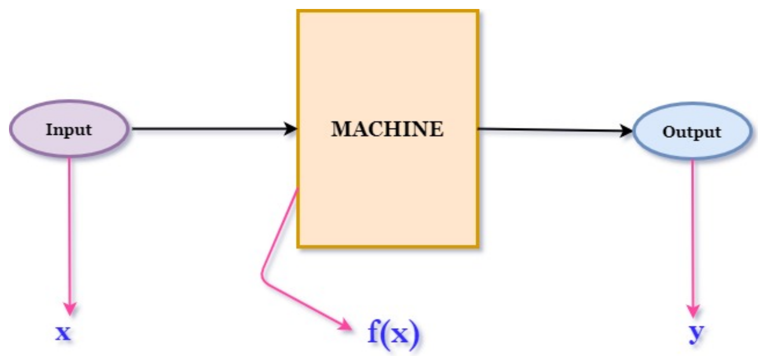

Figure 2: How a Function works

1.2 Domain & Range

In very simple words, “All the possible inputs for a function” is called the Domain of that particular function. Domain represents all the values of input or x.
On the other hand, “All possible outputs due to given input” is called the Range for a particular function.
Now, it can be asked “What is the relationship between Domain and Range?” Well, the answer is, Range is the corresponding output for a particular Domain/Input which is processed by the given function f(x). The given diagrams will make the idea clearer.

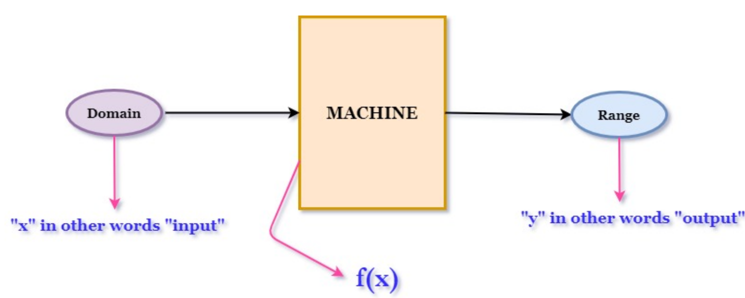

Figure 3: Relationship between Domain & Range

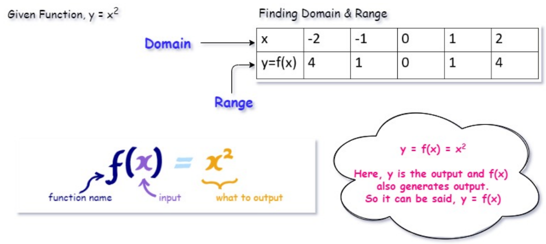

Figure 4: Example of Domain & Range

1.3 Graphing of Functions

It is familiar to almost everyone "How to Graph a Function?" However, I will give a short overview about it. You can see it in the below figure.

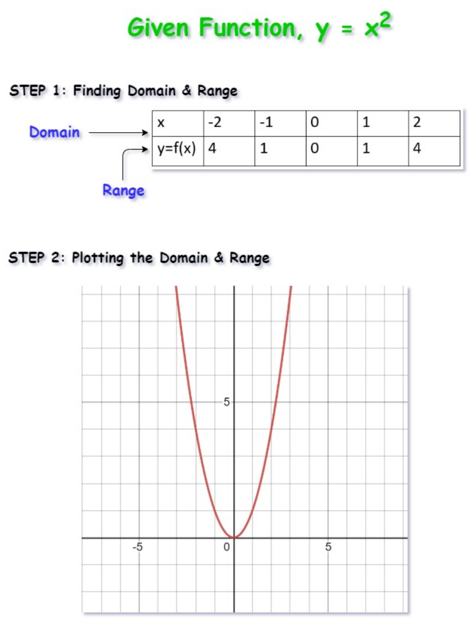

Figure 5: Graphing of a Function (step by step)

1.3.1 Squeezing & Stretching

When a graph Squeezes: Whenever, a function is being multiplied by **whole number** it squeezes. With the increment of the 'Number', the 'Graph Squeezes More' gradually.

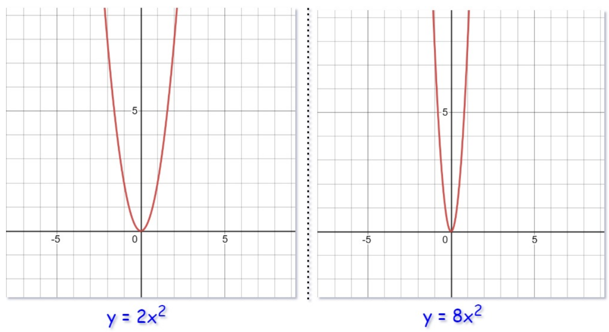

Figure 6: Squeezing of a graph

When a graph Stretches: Whenever, a function is being multiplied by **fractional number** it stretches. With the increment of the 'Fractional Number', the 'Graph Stretches More' gradually.

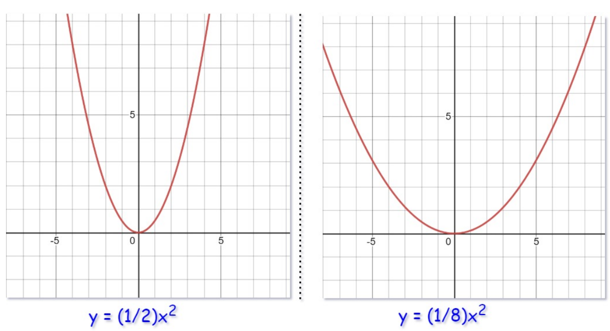

Figure 7: Stretching of a graph

1.3.2 Shifting

Vertical Shifting: When an additional number (it can be whole or fractional) is being **added** to the function the graph shifts **upward**.

Horizontal Shifting: When an additional number (it can be whole or

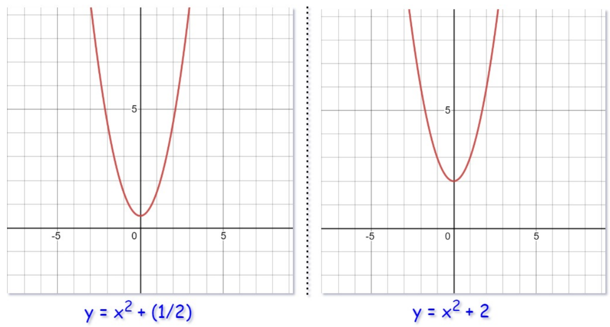

Figure 8: Vertical Shifting of a graph

fractional) is being **subtracted** from the function the graph shifts **downward**.

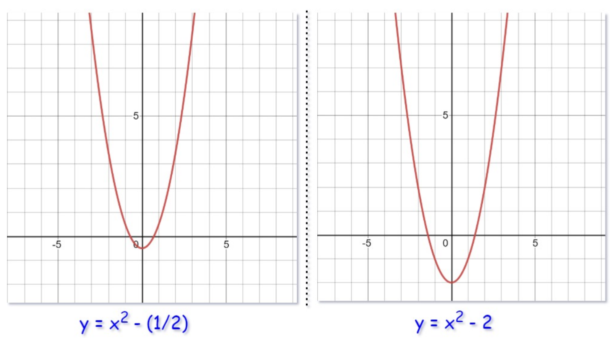

Figure 9: Horizontal Shifting of a graph

1.3.3 Change the Origin

By adding two units to the x axis, it shifts the curve to the right by two units. On the other hand, by subtracting two units to the x axis, it shifts the curve to the left by two units.

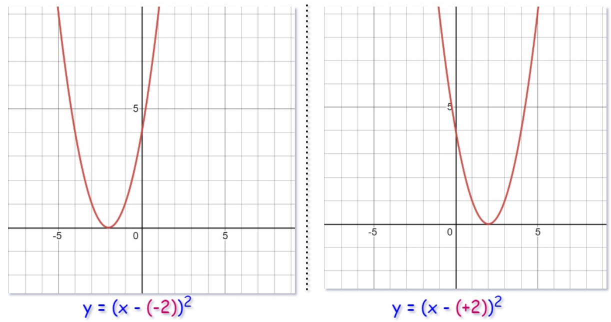

Figure 10: Changing the origin of a graph

1.4 Function Composition

Function Composition is a convenient way for building up **complex functions** by combining **simple functions**. It is done in a shape of a chain design. Intuitively, the output of the previous function is fed as input of the current function and this process continues upto **n** number of times or the number of times it is needed.

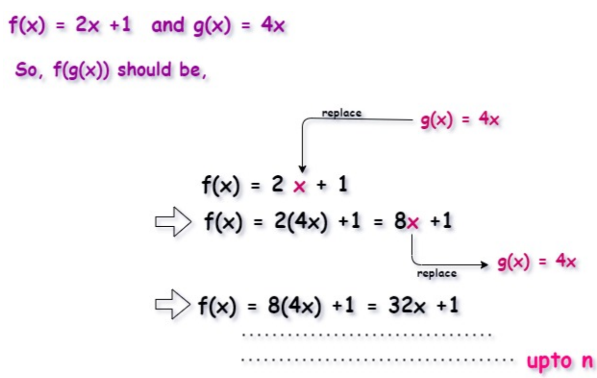

Figure 11: An Example of Function Composition

Now let's see a complex example.
Problem: Given,

$$f(x) = 3x$$
$$g(x) = \sqrt{3x+2}$$
$$h(x) = \frac{2x}{5x-1}$$

find out,

$$f(h(g(x)))$$
$$g(\frac{h(x)}{f(x)})$$

Solution:

f(h(g(x)))

Let's breakdown the complex function, at first we solve, h(g(x)) then we will put this into f(x).

So,

$$h(g(x)) = \frac{2(\sqrt{3x+2})}{5(\sqrt{3x+2})-1}$$

Now,

$$f(h(g(x))) = 3(\frac{2(\sqrt{3x+2})}{5(\sqrt{3x+2})-1})$$

$$g(\frac{h(x)}{f(x)})$$

Similarly like previous one, breaking down the inner part first.

$$\frac{h(x)}{f(x)} = \frac{\frac{2x}{5x-1}}{3x}$$
$$\frac{h(x)}{f(x)} = \frac{2x}{5x-1} * \frac{1}{3x}$$
$$\frac{h(x)}{f(x)} = \frac{2x}{3x(5x-1)}$$

Now,

$$g(\frac{h(x)}{f(x)}) = \sqrt{3(\frac{2x}{3x(5x-1)})} + 2$$
$$g(\frac{h(x)}{f(x)}) = \sqrt{(\frac{2x}{x(5x-1)})} + 2$$

1.5 Inverse Function

The inverse of a function tells you how to get back to the original value. We do this a lot in everyday life, without really thinking about it. For example, you are a new student in your university. You are looking for one of your batch mate but you do not know any basic contact information (Name, Contact Number) of that person. The only thing you know about him is "He is enrolled in the same section as yours in MAT092 course". So if you think mathematically, f(x) = "basic contact information" & $f^{-1}(x)$ = "He is enrolled in the same section as yours in MAT092 course"

Now you can easily find out that person from $f^{-1}(x)$.

Example: Given,

$$f(x) = 2x - 3$$

Find out $f^{-1}(y)$
Solution: We know, f(x) = y
Now,

$$\begin{aligned} f(x) &= 2x - 3 \\ y &= 2x - 3 \\ 3y &= 2x \\ x &= \frac{3y}{2} \end{aligned}$$

Since, $f^{-1}(y)$ = x
So,

$$f^{-1}(y) = \frac{3y}{2}$$