Intelligent Decision Support Systems

(Part II - DECISION THEORY)

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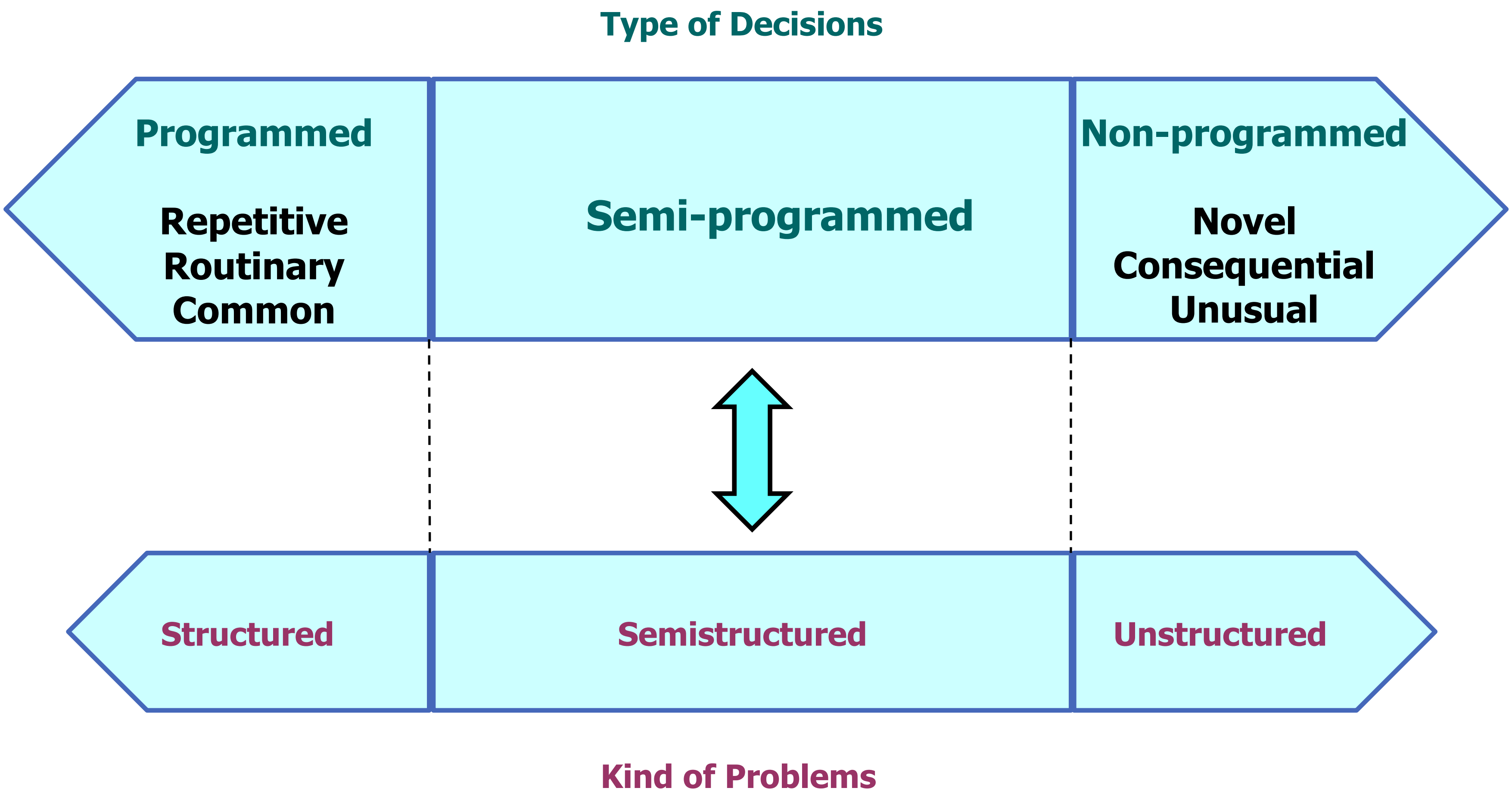
PART 2 – DECISION THEORY

DECISION THEORY

Decisions

Decision Theory

Decision Structure - Kind of problems [Simon, 1960]



Decision Levels in an Organization

[Anthony, 1965]

Decision Typologies (1)

- Negotation-Based Decisions [Delbecq, 1967]. Based on the notion of negotation:
  - Routine decisions
  - Creative decisions
  - Negotiated decisions
- Activity-Based Decisions [Mintzberg, 1973]. Attention focused with the most associated activity to the decision:
  - Entrepreneurial activities
  - Adaptive activities
  - Planning activities

Decision Typologies (2)

- Strategy-Based Decisions [Thompson, 1967]. Primary strategy used in making the final choice:
  - Computational strategies
  - Judgemental strategies
  - Compromise strategies
  - Inspirational strategies

Decision Theory

Types of decision

	Requiring IDSS	
Low uncertainty	Middle/High uncertainty	Very high uncertainty
Stable context	Medium-Stable context	Volatile Context
Commonplace, ordinary	Mostly Commonplace	Atypical, unique
Recurrent	Appear sometimes in a year	Rarely, Discrete times
Programmable	Semi-Programmable	Non-programmable, creative
Easily accessible information	Partially accessible information	Problematic access to information
Decision criterion understood	Partially Decision criterion understood	Decision criterion unclear
Focused decision strategy	Partially Focused decision strategy	Multiple decision strategies
Operational	Tactical/management decisions	Strategic
Routine	Creative	Negotiated
Adaptive activity	Entrepreneurial activity	Planning activity
Computational strategies	Compromise – Judgemental strategy	Inspirational-intuitive strategy
Near-term	Short-term	Long-term
Reactive	Proactive	Proactive & Reactive
Structured Decisions	**Semistructured Decisions**	**Unstructured Decisions**

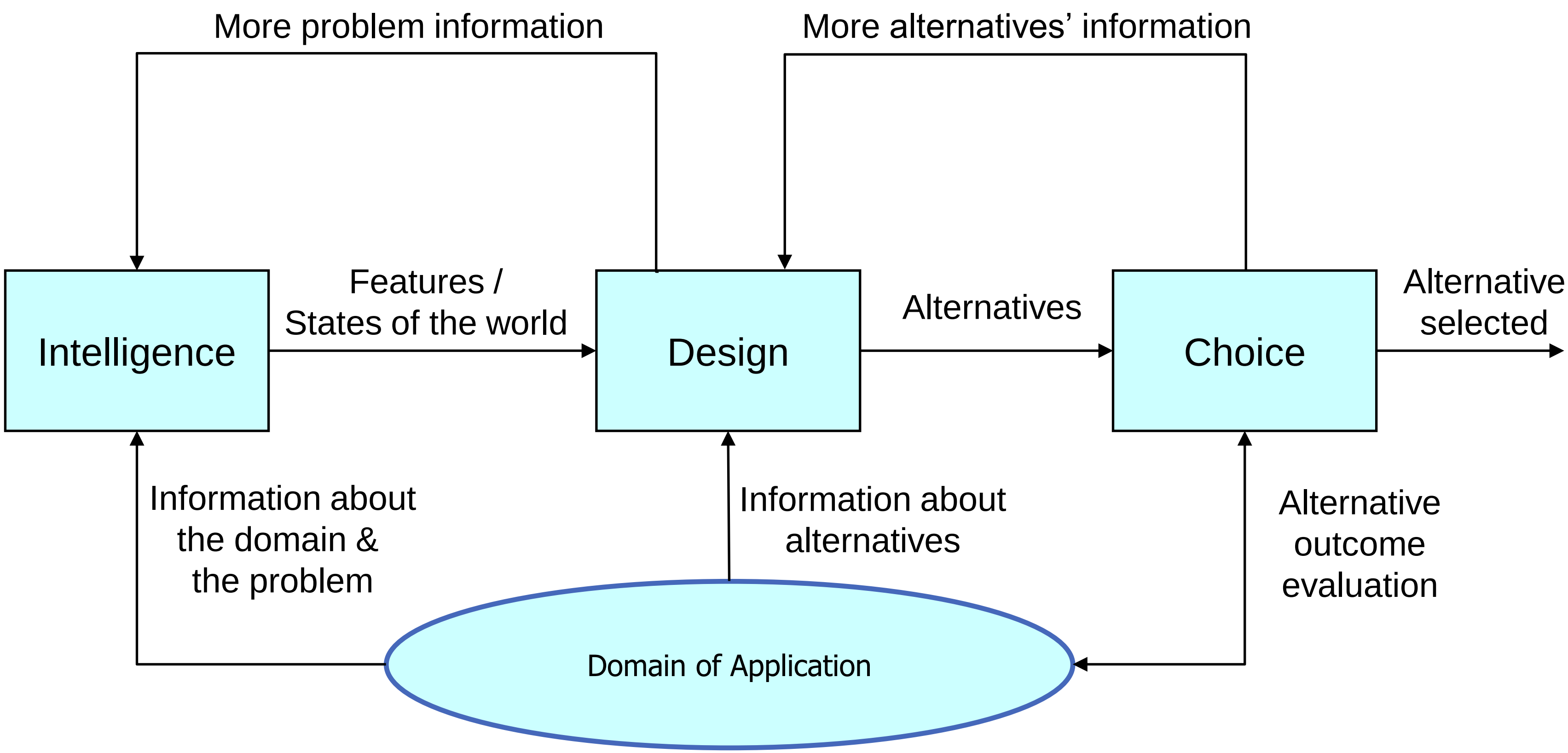
Problem Solving Method (1)

Decision Making Model [Simon, 1960]

- Three-step process:
  - **Intelligence**: Knowledge and information search for ***identifying a problem*** requiring a decision making process
  - **Design**: formation and analysis of the ***possible alternatives*** (strategies) for solving the detected problem
  - **Choice**: the decisor **chooses** one of the possible alternatives analysed and generated in the previous step. The selected alternative satisfies the *rational behaviour criterion*. Thus, it is the best alternative taking into account that provides the decisor with the maximum utility or benefit

Problem Solving Method (2)

Decision Making Model [Simon, 1960]



Decision Making Model: Intelligence phase

- The decision maker should carefully ***analyse the problem*** or domain at hand.
- The decision maker must try to ***identify the decision or decisions*** involved in the problem at hand.
- For each decision, the ***objectives*** or ***issues*** must be clearly stated.
- In addition, the different ***relevant features***, which can influence the decision process, should be searched and elicited.
- These features could be of different nature:
  - *deterministic*
  - *stochastic*
- Usually these features, especially in the case of stochastic nature are known as the ***states of the world***, which reflects the effects of what happens outside the control of the decision maker or what other persons decide.

Decision Making Model: Design phase (1)

- The set of possible ***alternative options*** for the actual decision must be enumerated. This step is a synthetic task where all possible options must be considered. This step is a formalisation of all elements in the scenario:
  - The set of possible alternatives $A = \{a_1, a_2, \ldots, a_n\}$
  - The set of ***deterministic features*** (p features) and/or ***stochastic features*** (m-p features), usually named as the ***states of the world*** $S = \{s_1, \ldots, s_p, s_{p+1}, \ldots, s_m\}$
  - The ***outcomes*** of each state of the world for each alternative (a_i), $O_i = \{o_{i1}, \ldots, o_{ip}, \ldots, o_{im}\}$
- **Outcomes** can be expressed in different ways:
  - In a binary way {high, not-high}
  - In a set of ordered qualitative values like {low, medium, high},
  - In a textual representation
  - In a numerical representation

Decision Making Model: Design phase (2)

- Usually, in decision theory literature, the formalisation has been done using a ***decision matrix***.
- A *decision matrix*, is a matrix like the following one:

	s_1	…	s_p	s_{p+1}	…	s_m
a_1	o_{11}	…	o_{1p}	o_{1p+1}	…	o_{1m}
a_2	o_{21}	…	o_{2p}	o_{2p+1}		o_{2m}
…	…	…	…	…	…	…
a_n	o_{n1}	…	o_{np}	o_{np+1}	…	o_{nm}

Decision Making Model: Design phase (3)

- For example, for the "*which car to buy*" decision problem:

	Price	***Performance***	***no mech. probs & no electr. probs***	***no mech. probs & electr. probs***	***mech. probs & no electr. probs***	***mech. probs & electr. probs***
buy car A	not-high	low	completely satisfied	partially satisfied	partially satisfied	partially unsatisfied
buy car B	high	high	completely satisfied	partially unsatisfied	partially unsatisfied	completely unsatisfied
buy car C	high	medium	completely satisfied	partially unsatisfied	partially unsatisfied	completely unsatisfied
buy car D	not-high	low	completely satisfied	partially satisfied	partially satisfied	partially unsatisfied
buy car E	not-high	medium	completely satisfied	partially satisfied	partially satisfied	partially unsatisfied

Decision Making Model: Choice phase (1)

- *Evaluate* the different alternatives available to the decision-maker
- *Select* the alternative producing the "best outcome".
  - The question is how to evaluate the best outcome for the decision-maker?
  - This best outcome should be evaluated according to the *objectives* of the decision-maker.
- In decision theory, there are some approaches to measure the degree of desirability of one alternative.
  - Based on the use of ***preferences***. The decision-maker makes comparisons between any two alternatives, with the hope that a ***preference ordering*** could be established among all alternatives.
  - Based on the use of a ***numerical representation*** of the values of the alternatives/outcomes. These numbers are commonly named as ***utilities***. An ***utility function*** *(u(o))* assigns the utility value of an outcome.

Decision Making Model: Choice phase (2)

Preferences

- ***Preferences*** are a comparative method, expressing a relation between two alternatives. Some definitions:
  - ***Weak preference* definition**: $A \succcurlyeq B$ defines a *weak preference* relation, which means that A is "at least as good as" B.
    Therefore, $A \succcurlyeq B$ represents that the decision-maker considers option B is not preferred to A.
  - ***Strict/Strong preference* definition**: From the weak preference relation, it can be defined the *strict preference* relation, $A \succ B$, which means that A "is better than" B, as follows:

    $$A \succ B \Leftrightarrow A \succcurlyeq B \text{ and } \neg\,(B \succcurlyeq A)$$

  - ***Indifference* definition**: The *indifference* relation, $\sim$, which means that A and B are equally preferable, is defined as:

    $$A \sim B \Leftrightarrow A \succcurlyeq B \text{ and } B \succcurlyeq A$$

Decision Making Model: Choice phase (3)

Preferences

- It can be stated that the *weakly preference relation* $\succcurlyeq$, *weakly orders* a set *S* of options, *without any cycle of preferences*, whenever it satisfies the following two conditions:
  - **Completeness Axiom:**
    For any $A, B \in S \Rightarrow A \succcurlyeq B$ or $B \succcurlyeq A$ or $A \sim B$
    It means that all options are comparable.
  - **Transitivity Axiom:**
    For any $A, B, C \in S \Rightarrow$ if $A \succcurlyeq B$ and $B \succcurlyeq C$ then $A \succcurlyeq C$
- Therefore, if we have a weakly ordered set of alternatives, then ***a preference ordering*** could be made explicit, and then, the *most preferred alternative,* which is as good as all the others should be the rational choice.

Decision Making Model: Choice phase (4)

Preferences

- Supposing that in our previous example the decision-maker states these set of preferences:

  to buy car E $\succcurlyeq$ *to buy car D*
  to buy car E $\succcurlyeq$ *to buy car C*
  to buy car E $\succcurlyeq$ *to buy car B*
  to buy car E $\succcurlyeq$ *to buy car A*
  to buy car C $\succcurlyeq$ *to buy car D*
  to buy car B $\succcurlyeq$ *to buy car D*
  to buy car A $\sim$ *to buy car D*
  to buy car B $\succcurlyeq$ *to buy car C*
  to buy car C $\succcurlyeq$ *to buy car A*
  to buy car B $\succcurlyeq$ *to buy car A*

- It can be outlined that all pair of preferences can be compared, and that the preferences satisfy the transitivity property. Hence, a *preference ordering* can be drawn:

  to buy car E $\succcurlyeq$ *to buy car B* $\succcurlyeq$ *to buy car C* $\succcurlyeq$ *to buy car A* $\sim$ *to buy car D*

- Therefore, the selected choice, which is the alternative preferred to all the other ones, is the alternative "***to buy car E***".

Decision Making Model: Choice phase (5)

Numerical Representation of outcomes: utilities

- An ***utility function*** *(u(o))* assigns the utility value of an outcome:

$$u: O \rightarrow \mathbb{R}$$
$$o_i \mapsto u(o_i)$$

Where,

O is the set of all outcomes,
o_i is a particular outcome, and
$u(o_i)$ is the utility value of outcome o_i

- For example:

	Price	*Performance*	*no mech. probs & no electr. probs*	*no mech. probs & electr. probs*	*mech. probs & no electr. probs*	*mech. probs & electr. probs*
buy car A	2	3	+50	+25	+25	-25
buy car B	1	10	+50	-25	-25	-50
buy car C	1	7	+50	-25	-25	-50
buy car D	2	3	+50	+25	+25	-25
buy car E	2	7	+50	+25	+25	-25

Decision Making Model: Choice phase (6)

Numerical Representation of outcomes: utilities

- Ordinal utilities (ordinal scale). No way to know the difference between the utilities of two alternatives.
  - Price in the example. Value 1 is the first ranked utility, and value 2 is the second one, but we have no idea of which difference is between any pair of alternatives
- Cardinal utilities (interval scale, ratio scale)
  - All the other attributes (performance, etc.) are expressed in cardinal utilities. Here two alternatives can be compared, and the *difference operator* (interval scale) or the *division operator* (ratio scale) gives this utility difference.
  - Cardinal interval-value scale:

	Grade
Student 1	7
Student 2	9
Student 3	4
Student 4	3
Student 5	8

  - Cardinal ratio scale:

	Weight (ratio)	**Absolute Weight (kg)**
Package 1	5	10
Package 2	1.5	3
Package 3	½	1
Package 4	**1**	**2**
Package 5	2	4
Package 6	¼	0.5
Package 7	10	20

DECISIONS

Decision Process Modelling

Decision Process Modelling (1)

- Single Decision Scenario
  - ***Decision-making under certainty***: *the states of the world* are *deterministic*, and thus, the outcomes of a given alternative are invariably known.
  - ***Decision-making under no-certainty***: *the states of the world* are *stochastic*:
    - ***Decision-making under risk*** happens when each alternative leads to the outcomes, and *each outcome is occurring with a known probability value*.
    - ***Decision-making under uncertainty*** or ***ignorance*** includes two situations:
      - ***Decision-making under classical ignorance****:* the alternatives and outcomes are known, the states are stochastic, and have an *associated probabilistic distribution, which is unknown* by the decision-maker.
      - ***Decision-making under unknown consequences***: *there are unknown states and/or outcomes* by the decision-maker.
- Multiple Decision Scenario

Decision Process Modelling (2)

Single Decision Scenario / Decision under Certainty

- ***Decision-making under certainty***: *the states of the world* are *deterministic*, and thus, the outcomes of a given alternative are invariably known.
  - ***Single-attribute approach***: just one attribute describes the alternatives.

$$a_i \text{ is the best alternative } \Leftrightarrow \forall a_j \in A \;\; u(a_i) \cdot \;\; u(a_j)$$

  - ***Multiple-attribute approach***: Several attributes describe the alternatives
    - ***Combine*** several attributes in a ***unique attribute*** in a ***common scale***

	Price	*Performance*
buy car A	2	3
buy car B	1	10
buy car C	1	7
buy car D	2	3
buy car E	2	7

	Price & Performance	*Price & Performance$_2$*
buy car A	4	11/3
buy car B	5	**20/3**
buy car C	3.5	14/3
buy car D	4	11/3
buy car E	**6**	19/3

Decision Process Modelling (3)

Single Decision Scenario / Decision under Certainty

- ***Aggregation* of the utilities of all attributes**
  - ***Additive approach***: aggregated weighted sum of the utilities

    $$aggu(a_i) = \sum_{j=1}^{m} w_j * u(o_{ij}) \quad where\ 0 \leq w_j \leq 1\ \ and\ \ \sum_{j=1}^{m} w_j = 1$$

    Given that,
    o_{ij} is the outcome for the attribute *j* in the alternative a_i
    w_j is the relevance/weight of the attribute *j*
    m is the number of attributes
    $aggu(a_i)$ is the aggregated utility of the alternative a_i

  - ***Non-additive approaches***: using other aggregation operation like *multiplication of the utility values of each attribute*. Thus, it can be computed as follows:

    $$aggu(a_i) = \prod_{j=1}^{m} u(o_{ij})$$

Decision Process Modelling (4)

Single Decision Scenario / Decision under Certainty

- Examples
  - Aggregated utility values using an ***additive approach*** for all the alternatives, assuming an equal relevance situation (weights equal to ½) and a situation where the importance of *performance* is higher (2/3):

Equal relevance	**Higher relevance of performance**
$aggu(a_1)$ = ½ * 2 + ½ * 3 = 5/2	$aggu(a_1)$ = 1/3 * 2 + 2/3 * 3 = 8/3
$aggu(\boldsymbol{a_2})$ = ½ * 1 + ½ * 10 = **11/2**	$aggu(\boldsymbol{a_2})$ = 1/3 * 1 + 2/3 * 10 = **21/3**
$aggu(a_3)$ = ½ * 1 + ½ * 7 = 8/2	$aggu(a_3)$ = 1/3 * 1 + 2/3 * 7 = 15/3
$aggu(a_4)$ = ½ * 2 + ½ * 3 = 5/2	$aggu(a_4)$ = 1/3 * 2 + 2/3 * 3 = 8/3
$aggu(a_5)$ = ½ * 2 + ½ * 7 = 9/2	$aggu(a_5)$ = 1/3 * 2 + 2/3 * 7 = 16/3

  - Using a ***multiplicative approach***:

$aggu(a_1)$ = 2 * 3 = 6
$aggu(a_2)$ = 1 * 10 = 10
$aggu(a_3)$ = 1 * 7 = 7
$aggu(a_4)$ = 2 * 3 = 6
$aggu(\boldsymbol{a_5})$ = 2 * 7 = **14**

Decision Process Modelling (5)

Single Decision Scenario / Decision under risk

- ***Decision-making under risk*** happens when each alternative leads to the outcomes, and *each outcome is occurring with a known probability value*. The *states of the world are stochastic*, and they have an *associated probability distribution, which is known* by the decision-maker.
- Based on the **Expected Utility** (***EU***) as defined by von Neumann and Morgensten:

$$EU(a_i) = \sum_{k=1}^{m} p_{ik} * u(O_{ik})$$

- The best alternative can be selected:

a_i is the best alternative $\Leftrightarrow$

$$\operatorname*{argmax}_i \{EU(a_i) = \sum_{k=1}^{m} p_{ik} * u(O_{ik})\}$$

Decision Process Modelling (6)

Single Decision Scenario / Decision under risk

- Example:

	no mech. probs & no electr. probs	*no mech. probs & electr. probs*	*mech. probs & no electr. probs*	*mech. probs & electr. probs*
buy car A	+50	+25	+25	-25
buy car B	+50	-25	-25	-50
buy car C	+50	-25	-25	-50
buy car D	+50	+25	+25	-25
buy car E	+50	+25	+25	-25

- Assuming that probability of having electrical problems is 0.5 and mechanical problems is 0.5, and they are independent, the ***expected utility*** of all alternatives is:

$EU(\boldsymbol{a_1})$ = 0.25 * (+50) + 0.25 * (+25) + 0.25 * (+25) + 0.25 * (-25) =**18.75**
$EU(a_2)$ = 0.25 * (+50) + 0.25 * (-25) + 0.25 * (-25) + 0.25 * (-50) = -12.5
$EU(a_3)$ = 0.25 * (+50) + 0.25 * (-25) + 0.25 * (-25) + 0.25 * (-50) = -12.5
$EU(\boldsymbol{a_4})$ = 0.25 * (+50) + 0.25 * (+25) + 0.25 * (+25) + 0.25 * (-25) =**18.75**
$EU(\boldsymbol{a_5})$ = 0.25 * (+50) + 0.25 * (+25) + 0.25 * (+25) + 0.25 * (-25) =**18.75**

- Thus, "*buy car A*", "*buy car D*" and "*buy car E*" are the alternatives maximizing the expected utility

Decision Process Modelling (7)

Single Decision Scenario / Decision under Uncertainty or Ignorance

- ***Decision-making under classical ignorance:*** the alternatives and outcomes are known, the states are stochastic, and have an *associated probabilistic distribution, which is unknown* by the decision-maker.

- In the Decision Theory literature, there are several decision criteria for choosing the best alternative:
  - ***Maximin*** decision rule
  - ***Leximin*** (lexicographic maximin) decision rule
  - ***Maximax*** decision rule
  - ***Optimism-pessimism*** decision rule or ***alpha-index*** rule
  - ***Minimax*** **regret** rule
  - ***The principle of insufficient reason*** decision rule

Decision Process Modelling (8)

Single Decision Scenario / Decision under Classical Ignorance

- **Maximin decision rule** (*Wald's criterion*): This decision rule is based on the idea of maximizing the minimal utility of each alternative. It is a *pessimistic criterion*. von Neumann proposed this criterion in adversarial game theory, but was popularized by Wald [Wald, 1950]. It is commonly named as *Wald's criterion*.
- Maximin decision rule:

  a_i is the best alternative $\Leftrightarrow$ $\forall a_j \in A \;\; min(a_i) \cdot \;\; min(a_j)$

  Given that,

  $A = \{a_1, a_2, \ldots, a_n\}$ is the set of possible alternatives

  $min(a_j)$ is the minimal utility/value of the alternative a_j
- Example:

	s_1	s_2	s_3	s_4
a_1	6	4	*3*	9
a_2	10	*2*	5	6
$\boldsymbol{a_3}$	***4***	8	7	12
a_4	5	4	9	*3*
a_5	8	11	5	*2*

Decision Process Modelling (9)

Single Decision Scenario / Decision under Classical Ignorance

- **Leximin (lexicographic maximin) decision rule**: According to *leximin rule*, when there is a tie among the worst value in some alternatives, the second worst value of those alternatives must be compared. In the case that a new tie is produced, then the third-worst values should be compared and so on. Finally, the maximum value is the one, which identifies the best alternative.
- Leximin decision rule:

$$a_i \text{ is the best alternative } \Leftrightarrow$$

$$\forall a_j \in A, \exists n \in \mathbb{Z}, n > 0, \; min^n(a_i) \cdot min^n(a_j) \text{ and } \forall m \in \mathbb{Z}, 0 < m < n, \; min^m(a_i) = min^m(a_j)$$

Given that,

$A = \{a_1, a_2, ..., a_n\}$ is the set of possible alternatives

$min^k(a_j)$ is the k^{th} worst utility/value of the alternative a_j

- Example:

	s_1	s_2	s_3	s_4
a_1	6	*4*	*3*	10
a_2	10	*2*	5	6
a_3	*3*	8	*4*	12
$\boldsymbol{a_4}$	11	*4*	***9***	*3*
a_5	8	11	5	*2*

Decision Process Modelling (10)

Single Decision Scenario / Decision under Classical Ignorance

- **Maximax decision rule**: This decision rule proposes to choose the best option as the alternative that maximizes the maximum utility values of all alternatives. Thus, this criterion is radically different from *maximin* and *leximin* criteria. It is completely optimistic, as it is focussing on the best possible outcomes of the alternatives, and selects the maximum one among the bests.
- Maximax decision rule:

$$a_i \text{ is the best alternative } \Leftrightarrow \forall a_j \in A \;\; max(a_i) \geq max(a_j)$$

Given that,

$A = \{a_1, a_2, \ldots, a_n\}$ is the set of possible alternatives

$max(a_j)$ *is the maximal utility/value of the alternative* a_j

- Example:

	s_1	s_2	s_3	s_4
a_1	6	4	3	9
a_2	10	2	5	6
a_3	4	8	7	**12**
a_4	5	4	9	3
a_5	8	11	5	2

Decision Process Modelling (11)

Single Decision Scenario / Decision under Classical Ignorance

- **Optimism-pessimism decision rule** or **alpha-index rule** (*Hurwicz's criterion*): It was proposed by Hurwicz [Hurwicz, 1951]. This proposal considers both the *worst outcome* and the *best outcome* of each alternative, and according to the degree of optimism and pessimism of the decision maker, the best alternative is selected. The evaluation of each alternative a_i is done through a weighted formula of the best and worst values:

$$▪ * max(a_i) + (1-▪) * min(a_i)$$

  Where $▪ \in \mathbb{R}$, $0 < ▪ < 1$, represents the degree of optimism of the decision-maker

- <u>Alpha-index rule</u>:

$$a_i \text{ is the best alternative } \Leftrightarrow$$
$$\forall a_j \in A,\quad ▪ * max(a_i) + (1-▪) * min(a_i)\ ▪\ ▪ * max(a_j) + (1-▪) * min(a_j)$$

- Example:

	s_1	s_2	s_3	s_4	$\alpha = 0.8$	$\alpha = 0.2$
a_1	6	4	3	10	8.6	4.4
a_2	10	2	5	6	8.4	3.6
$\mathbf{a_3}$	2	8	4	12	**10**	4
$\mathbf{a_4}$	11	4	9	3	9.4	**4.6**
a_5	8	11	5	2	9.2	3.8

Decision Process Modelling (12)

Single Decision Scenario / Decision under Classical Ignorance

- **Minimax regret decision rule**: Savage [Savage, 1951] proposed this decision rule. The rationality of this criteria is related with the human feeling of *regret* that some people experiments when after having made an action, for instance a choice, and given new information available, starts to regret the action (choice) done. The idea of the *minimax regret criterion* is that the best alternative is the one minimizing the maximum amount of regret of the alternatives.
- The usual procedure for computing which is the best alternative is generating a *regret matrix*. The values of the regret matrix are the result of subtracting the value of each outcome from the value of the best outcome of each state. This way the *regret values* (distance of the outcome to the best outcome of each state) are computed.
- Minimax regret rule:

$$a_i \text{ is the best alternative } \Leftrightarrow$$
$$\forall a_j \in A \; min\{o_{i1} - max(s_1), \ldots, o_{im} - max(s_m)\} \cdot min\{o_{j1} - max(s_1), \ldots, o_{jm} - max(s_m)\}$$

Given that,

$A = \{a_1, a_2, \ldots, a_n\}$ is the set of possible alternatives
o_{ip} is the outcome of alternative a_i for the state of the world s_p
$max(s_p)$ is the maximal value of the state of the world s_p across all alternative

Decision Process Modelling (13)

Single Decision Scenario / Decision under Classical Ignorance

- Example:

	s_1	s_2	s_3	s_4
a_1	6	4	3	10
a_2	10	2	5	6
a_3	2	8	4	**12**
a_4	**11**	4	**9**	3
a_5	8	**11**	5	2

Regret matrix

	s_1	s_2	s_3	s_4
$\boldsymbol{a_1}$	-5	**-7**	-6	-2
a_2	-1	**-9**	-4	-6
a_3	**-9**	-3	-5	0
a_4	0	-7	0	**-9**
a_5	-3	0	-4	**-10**

Decision Process Modelling (14)

Single Decision Scenario / Decision under Classical Ignorance

- **The principle of insufficient reason decision rule**: Jacques Bernoulli (1654-1705) formulated this criterion. This principle states that if the decision-maker has no reason to believe that one state of the world is more probable to occur than the other states, then equal probabilities should be assigned to all the states. In general, if there are *m* states, the probability assigned to each state will be $1/m$.
- The principle of insufficient reason decision rule: As the most common decision strategy in decision under risk is the use of the Expected Utility theory, the rule can be formalised as follows:

$$a_i \text{ is the best alternative } \Leftrightarrow$$

$$\forall a_j \in A \ \sum_{k=1}^{m} \frac{1}{m} * u(o_{ik}) \ ▪ \ \sum_{k=1}^{m} \frac{1}{m} * u(o_{jk})$$

Given that,

$A = \{a_1, a_2, \ldots, a_n\}$ is the set of possible alternatives

$u(o_{ip})$ is the utility of the outcome of alternative a_i for the state of the world s_p

Decision Process Modelling (15)

Single Decision Scenario / Decision under Classical Ignorance

- Example:

	s_1	s_2	s_3	s_4
a_1	6	4	3	10
a_2	10	2	5	6
a_3	2	8	4	**12**
a_4	**11**	4	**9**	3
a_5	8	**11**	5	2

- Assuming that the probability of each state is ¼. The Expected Utilities of the alternatives are:

$$EU(a_1) = 0.25 * 6 + 0.25 * 4 + 0.25 * 3 + 0.25 * 10 = 23/4$$
$$EU(a_2) = 0.25 * 10 + 0.25 * 2 + 0.25 * 5 + 0.25 * 6 = 23/4$$
$$EU(a_3) = 0.25 * 2 + 0.25 * 8 + 0.25 * 4 + 0.25 * 12 = 26/4$$
$$EU(\mathbf{a_4}) = 0.25 * 11 + 0.25 * 4 + 0.25 * 9 + 0.25 * 3 = \mathbf{27/4}$$
$$EU(a_5) = 0.25 * 8 + 0.25 * 11 + 0.25 * 5 + 0.25 * 2 = 26/4$$

Decision Process Modelling (16)

Single Decision Scenario / Decision under Uncertainty or Ignorance

- ***Decision-making under unknown consequences***: *there are unknown states and/or outcomes* by the decision-maker.
- This scenario implies *a higher degree of uncertainty or ignorance*. The decision-maker ignore what the possible consequences and the corresponding outcomes are. That means that some consequence of a certain alternative could not be known by the decision-maker when making the choice.
- This is really a *complex and extremely difficult scenario*. The scenario can be even worse when some unknown consequences could lead to *catastrophic outcomes*.
- With this higher degree of uncertainty and ignorance, *there are not decision criteria available* to cope with this scenario. What is suggested from a rational point of view is to *make a rational analysis of these possible uncertain consequences, and decide whether they could be unconsidered or not*.
- **A rational rule**: to avoid the alternatives most related with higher degrees of ignorance or uncertainty.
- These complex decision scenarios are **the focus of Intelligent Decision Support Systems (IDSS)**

Decision Process Modelling (17)

Multiple Decision Scenario

- Multiple sequential decisions scenario
  - Real-world problems are even more complex because usually there is more than one decision that must be coped with.
  - These decisions are processed ***sequentially***, one after another, because some decisions can depend on the previous decisions made as well as other random or stochastic events than can happen.
  - To model all decisions in a concrete problem, some formalism is needed to model and visualize sequentially all the decision process.
  - Most commonly used tools are:
    - *Decision trees*
    - *Influence diagrams*

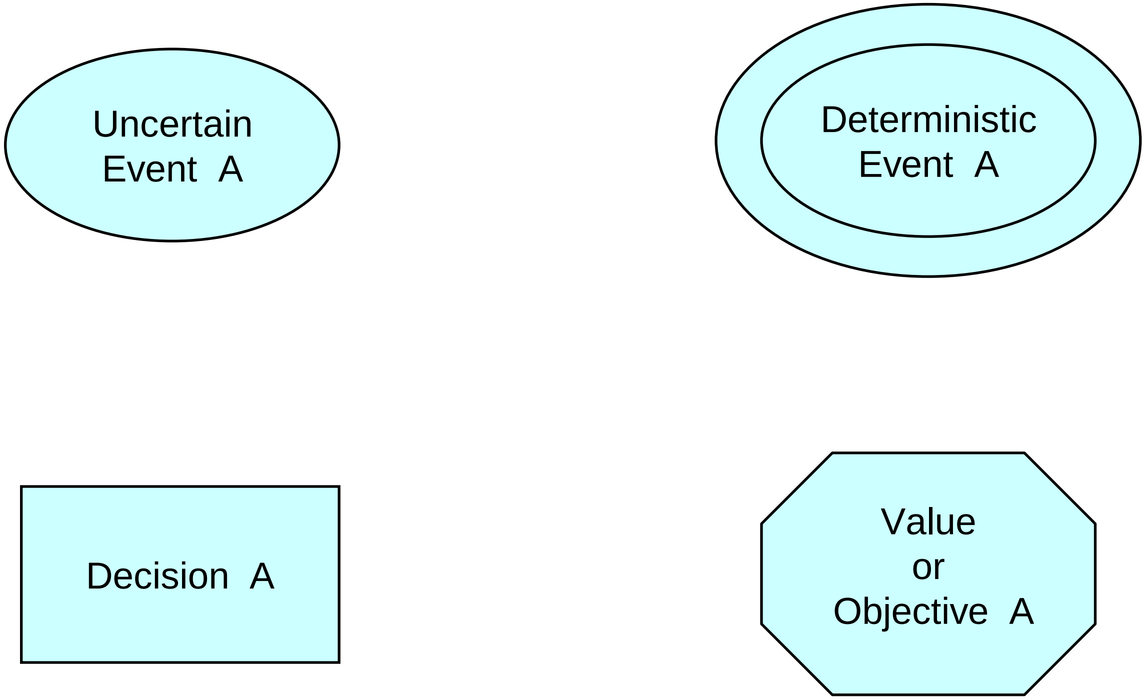
Decision Process Modelling (18)

Multiple Decision Scenario/Influence Diagrams [Howard & Matheson, 1984]

- Influence Diagram / Relevance Diagram / Decision Diagram / Decision Network
- They are diagrams to graphically model a decision making process
  - Directed Acyclic Graphs (DAGs)
  - Generalization of Bayesian Networks
- The **nodes** represent ***decisions***, ***uncertain events*** *or* ***deterministic events***, ***objectives/values***
  - A **decision node** is drawn as **a rectangle**. It represents variables under the control of the decision maker
  - An **uncertain node** is drawn as **a circle or oval**. It is a random variable representing uncertain quantities that are relevant to the decision problem.
  - A **deterministic node** is drawn as **a double circle or double oval**. It represent constant values or algebraically determined values from the states of their parents
  - A **value/objective node** is drawn as **an octagon or diamond**. It represents utility, i.e., a measure of desirability/satisfaction of the outcomes of the decision process

Decision Process Modelling (19)

Multiple Decision Scenario / Influence Diagrams



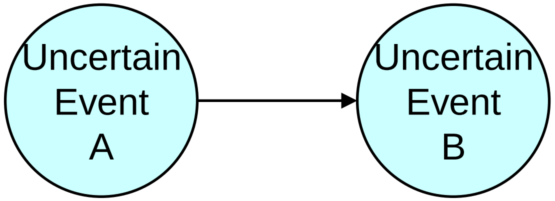
Decision Process Modelling (20)

Multiple Decision Scenario / Influence Diagrams

- Influence Diagrams
  - The edges connecting the nodes could be of three different types:
    - ***Conditional arcs*** (solid edges): indicate that the preceding node is **relevant** for the assessment of the value of the following component. Always are directed to *events*
    - ***Informational arcs*** (dashed edges): indicate that a decision has been made **knowing the result of the preceding node**. Always are directed to *decision nodes*
    - ***Functional arcs*** (solid edges): indicate that one of the components of additively separable utility function **is a function of all nodes at their tails**. Always end in a *value/objective node*.

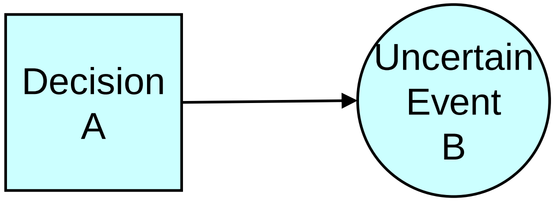
Decision Process Modelling (21)

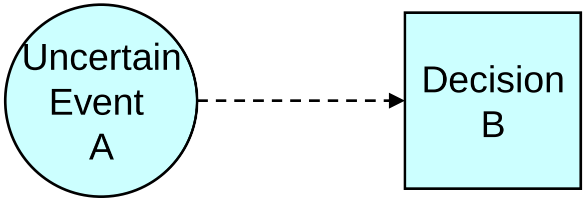
Multiple Decision Scenario / Influence Diagrams



RELEVANCE / DEPENDENCE
CONDITIONAL INFLUENCE

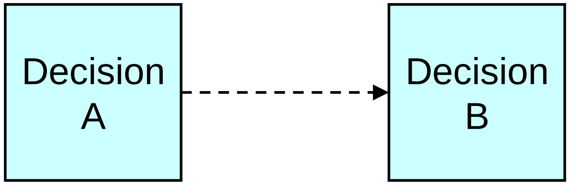
The probability of event B **depends** on the decision or random variable A





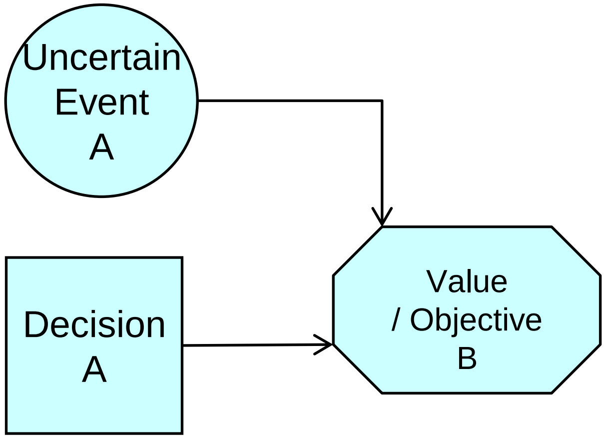
PRECEDENCE
INFORMATIONAL INFLUENCE

The result of the event A or the decision A is known **before** making decision B



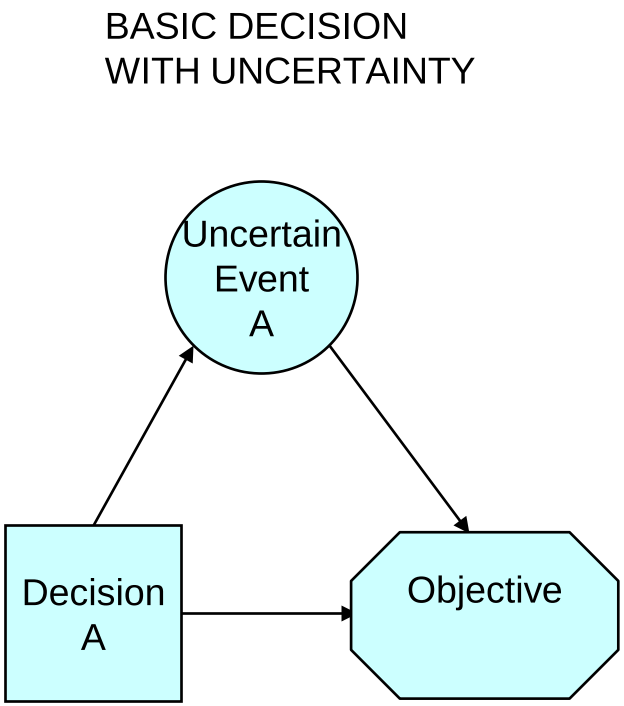
FUNCTIONAL DEPENDENCE

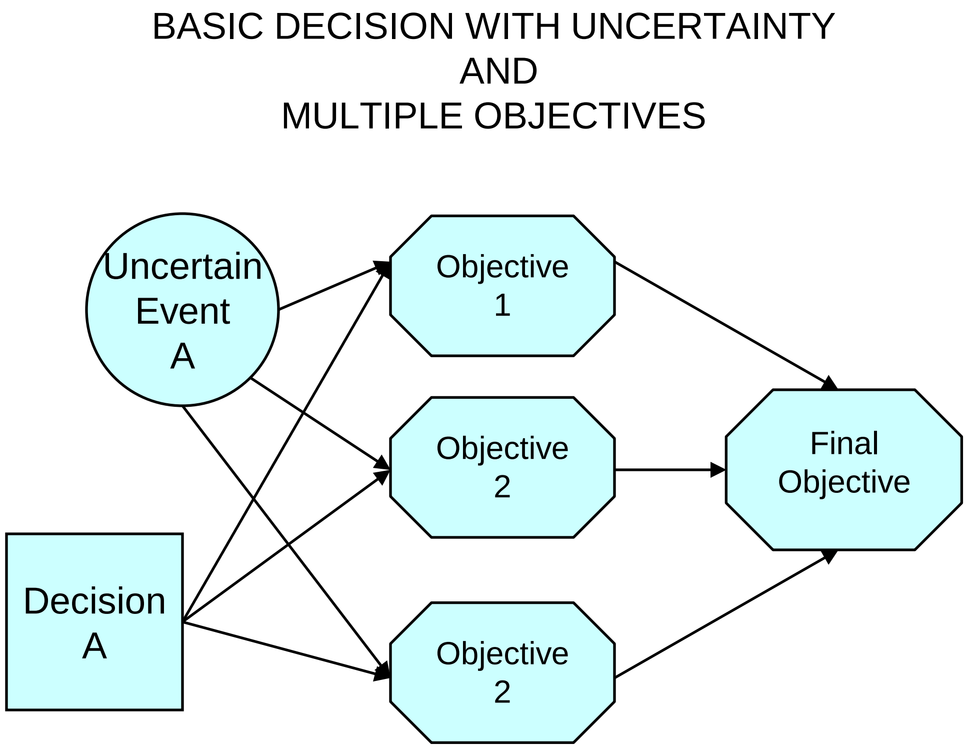
The measure of satisfaction of Value B is **the utility function** of Event A and Decision A



Decision Process Modelling (22)

Multiple Decision Scenario / Influence Diagrams

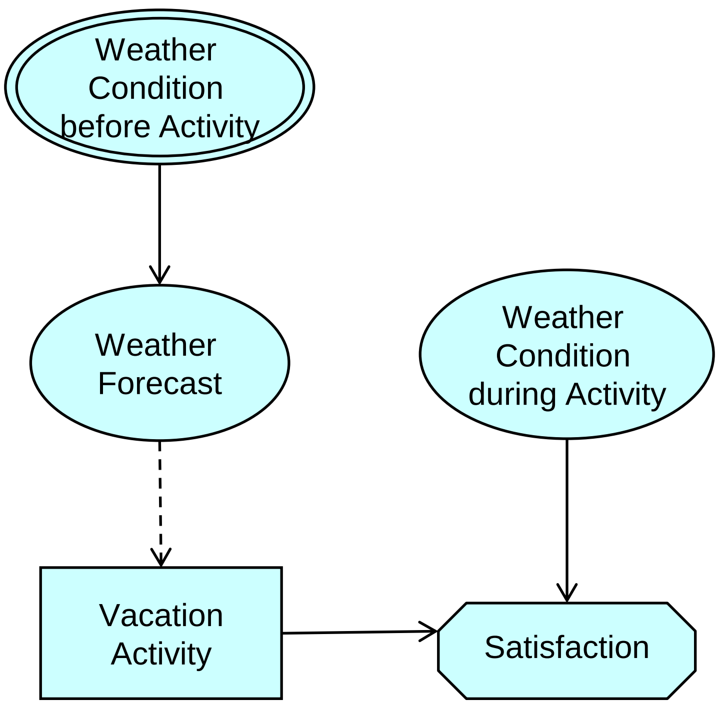




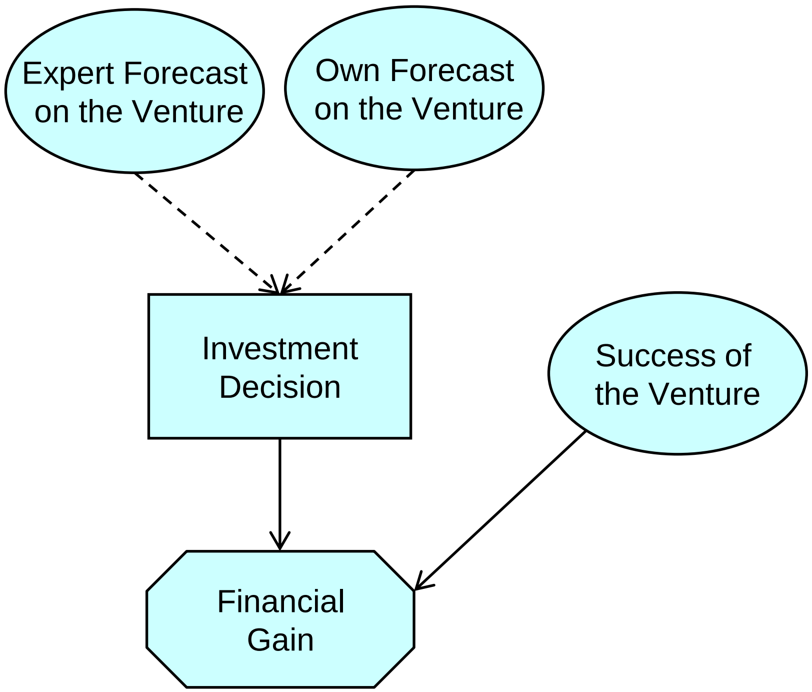
Decision Process Modelling (23)

Multiple Decision Scenario / Influence Diagrams

MAKING DECISION
ABOUT VACATION ACTIVITY



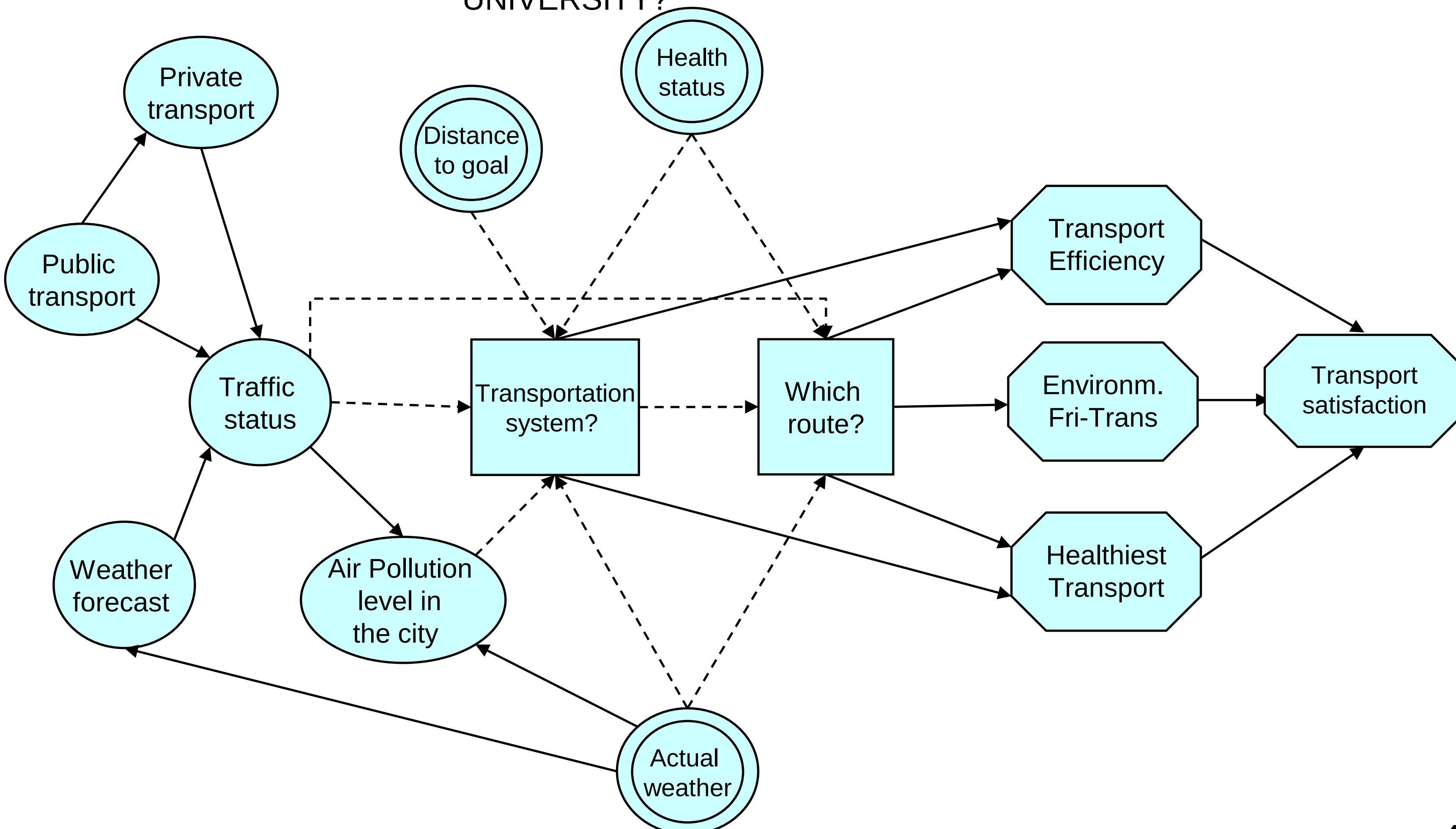
INVESTMENT IN A RISKY VENTURE



Decision Process Modelling (24)

Multiple Decision Scenario / Influence Diagrams

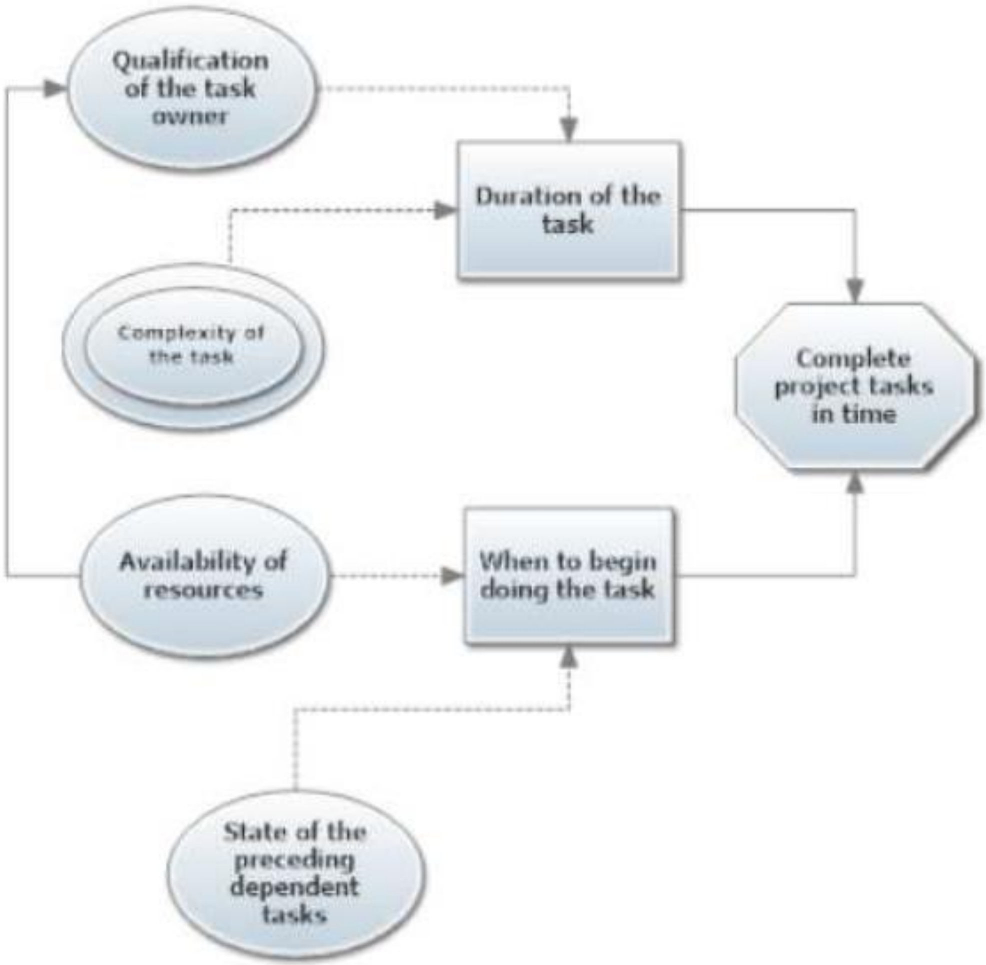
HOW TO ARRIVE AT UNIVERSITY?



Decision Process Modelling (25)

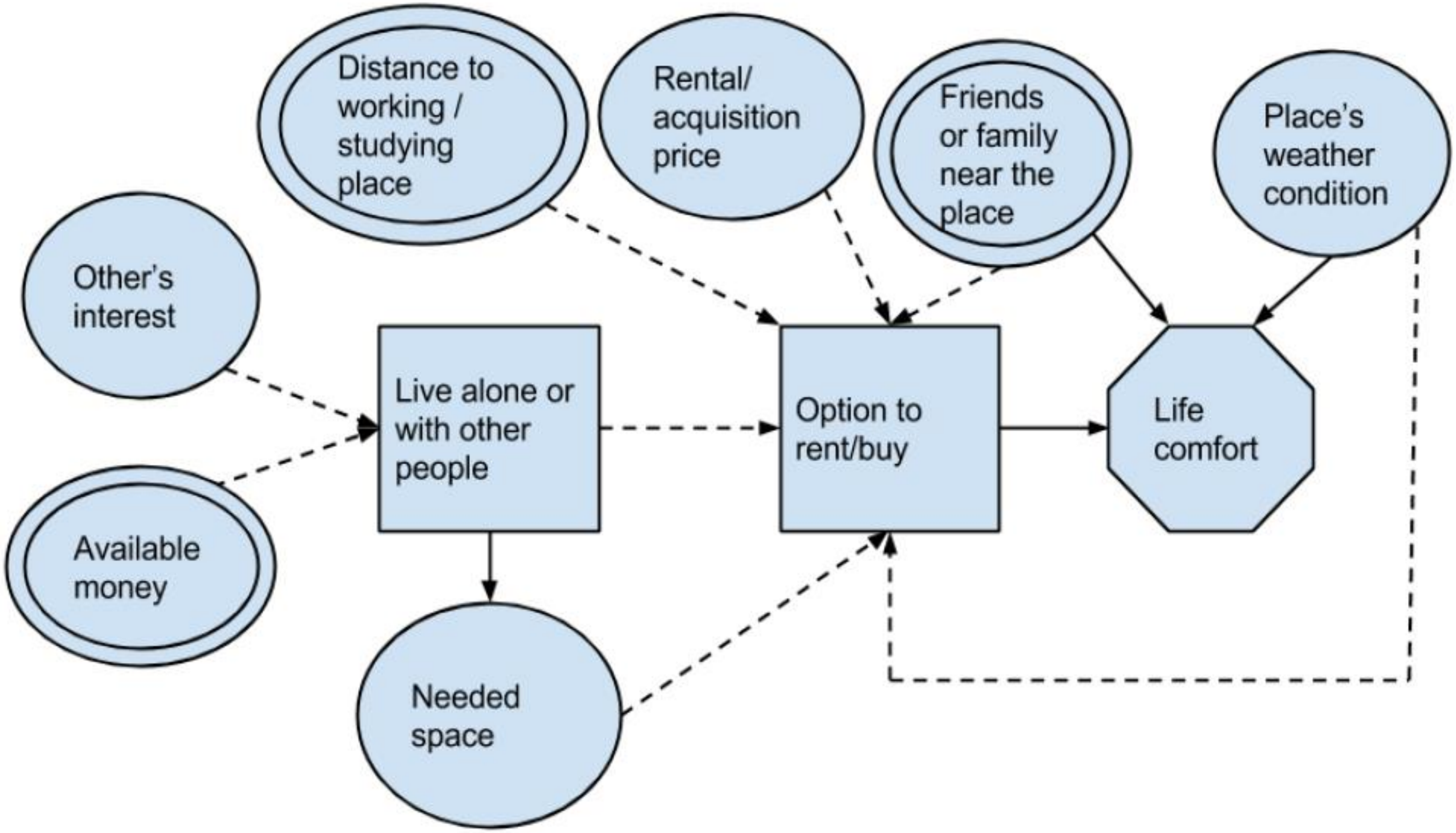
Multiple Decision Scenario / Influence Diagrams

TASKS MANAGEMENT IN IT PROJECT MANAGEMENT – Agne Grinciunaite



Decision Process Modelling (26)
Multiple Decision Scenario / Influence Diagrams

WHERE TO LIVE? – Gerard Canal



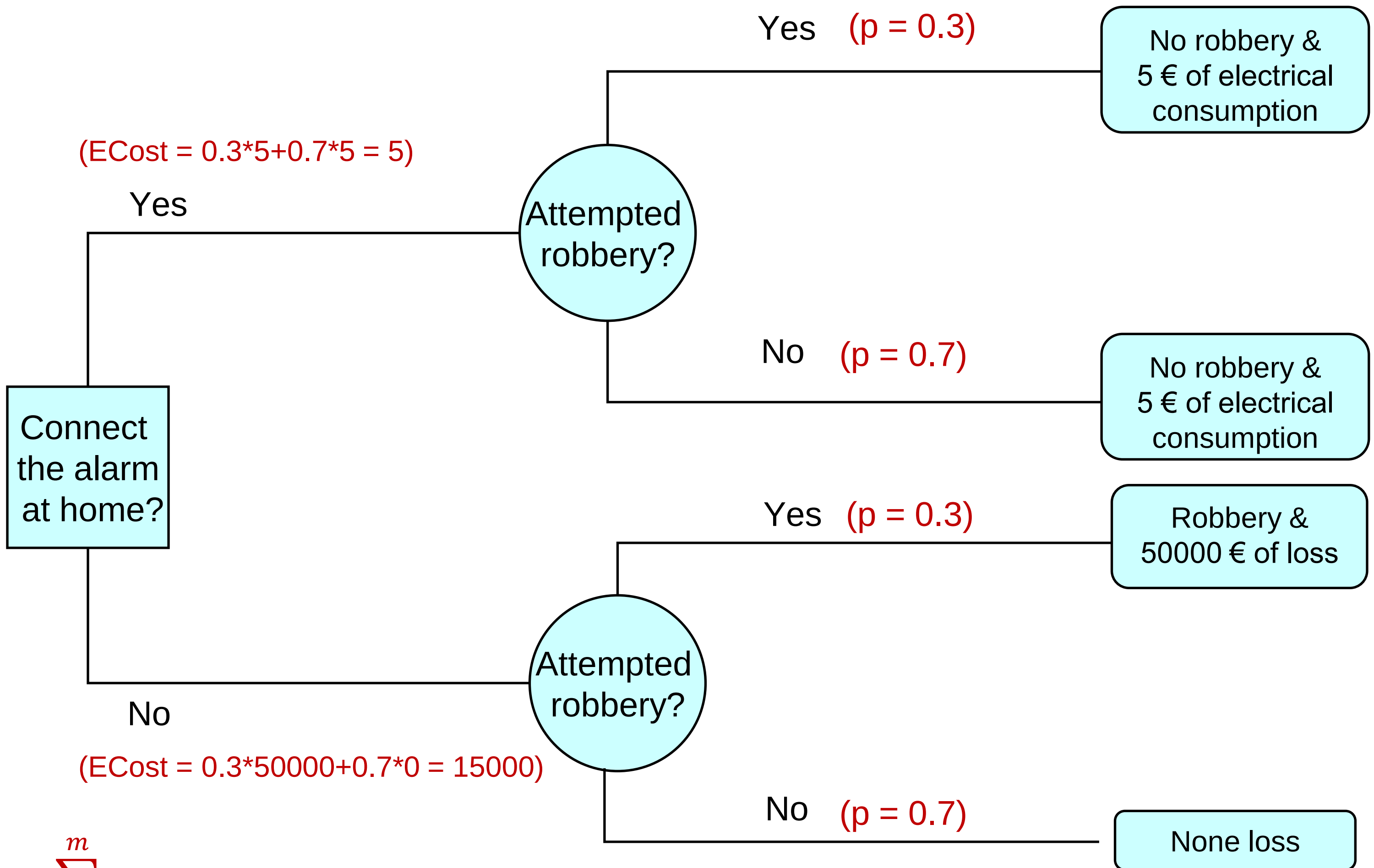
Decision Process Modelling (27)

Multiple Decision Scenario / Decision Trees

- Decision Trees
  - They are trees which graphically model a decision making process
  - The **nodes** represent ***decisions*** or ***uncertain events***
    - ***Decision nodes*** are represented by ***rectangles***
    - ***Uncertain Events*** are represented by ***circles***
  - The **edges** connecting the nodes are named as ***branches***:
    - The ***branches leaving a decision node*** are the ***set of available alternatives***
    - The ***branches leaving an uncertain event node*** are ***the possible results of the event***. Probability values can be associated to the branches.
  - The ***final leaves of the tree*** are the ***outcomes*** of the decision path. They are represented as ***round-shaped rectangles***
  - It is a mechanism allowing to make some estimations of the results (utility/benefit) based on **probabilities**

Decision Process Models (28)

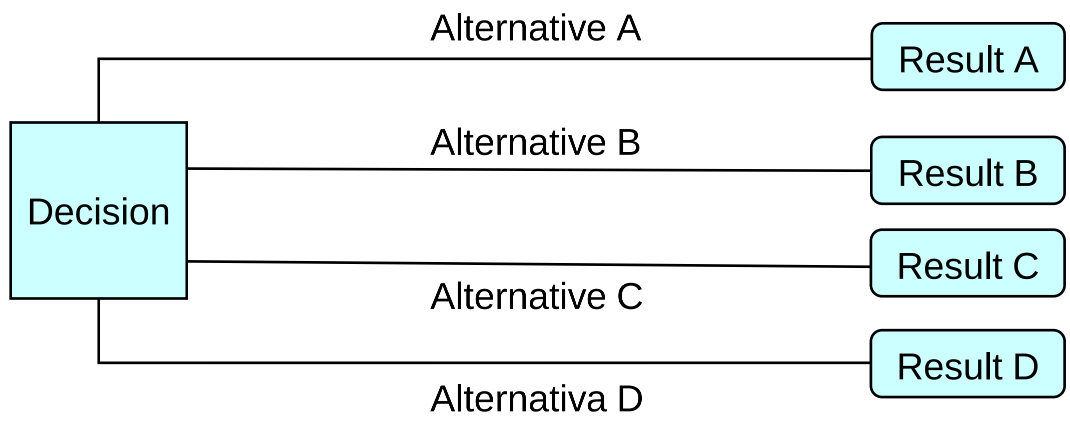
Multiple Decision Scenario / Decision Trees

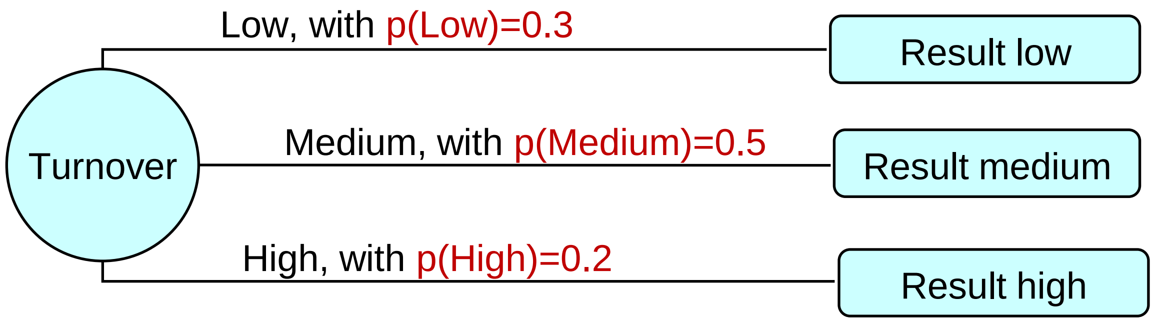


$$EU(a_i) = \sum_{k=1}^{m} p_{ik} * u(O_{ik})$$

Decision Process Models (29)

Multiple Decision Scenario / Decision Trees

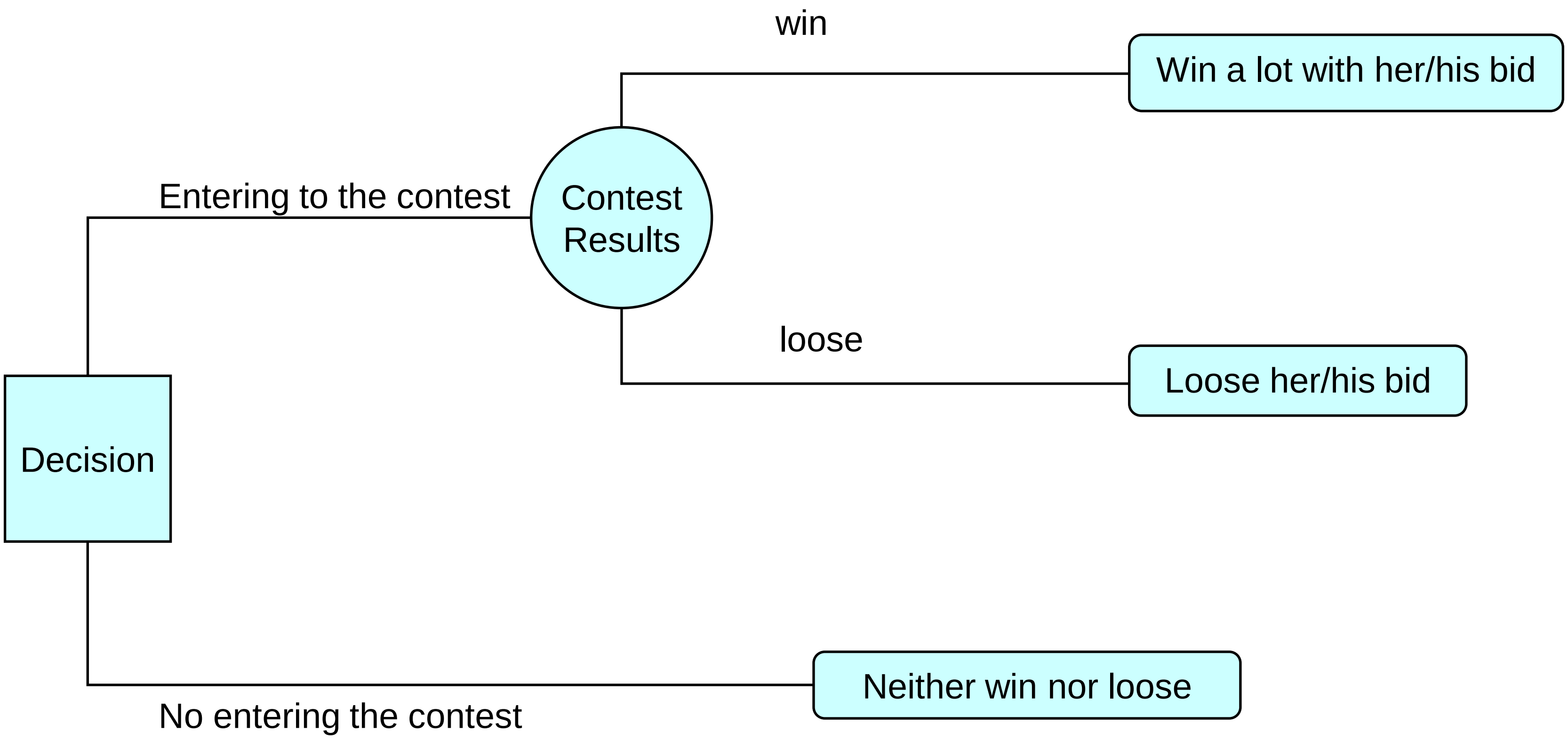




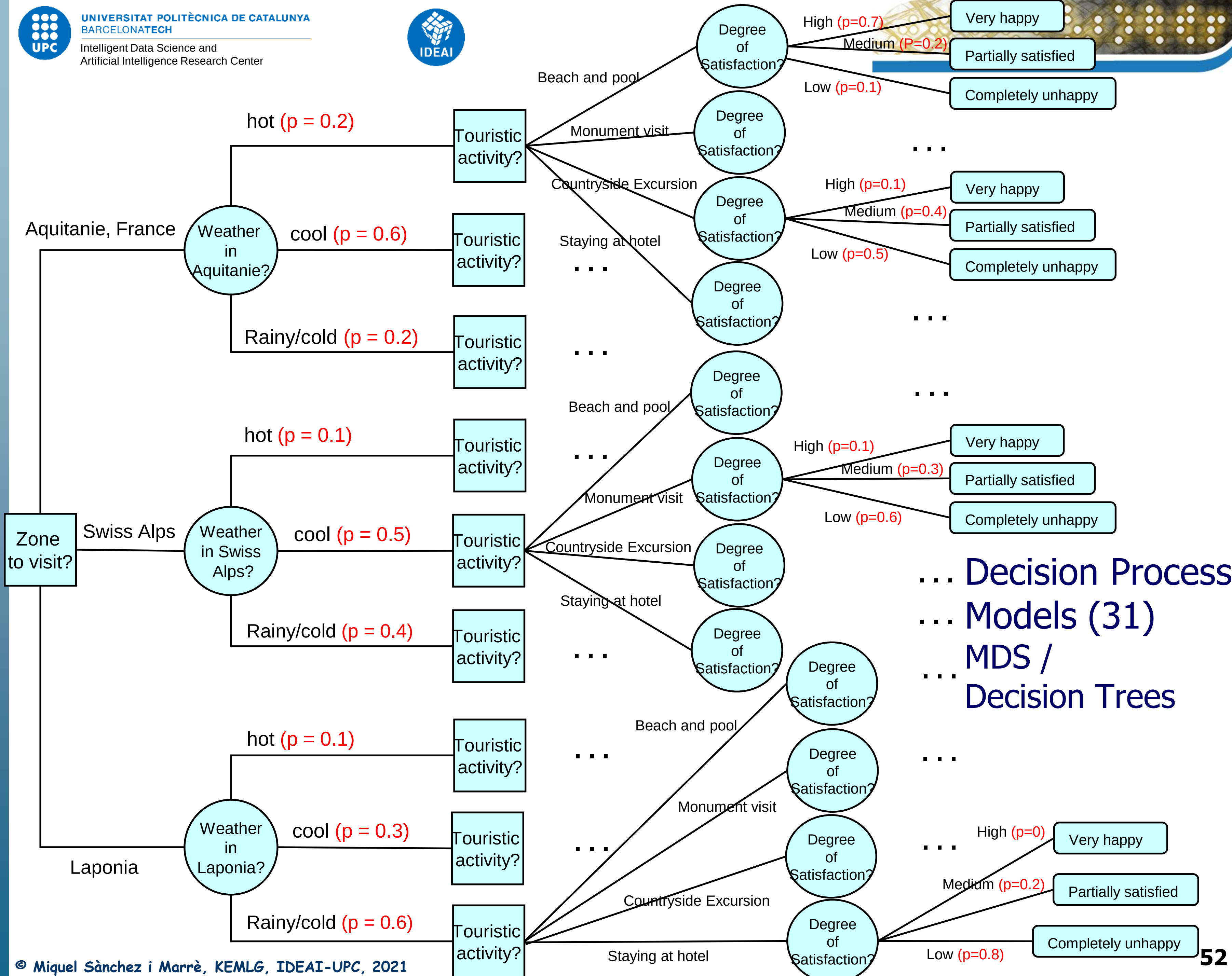
EU = 0.3*2000+0.5*15000+0.2*45000 = 17100

Decision Process Models (30)

Multiple Decision Scenario / Decision Trees

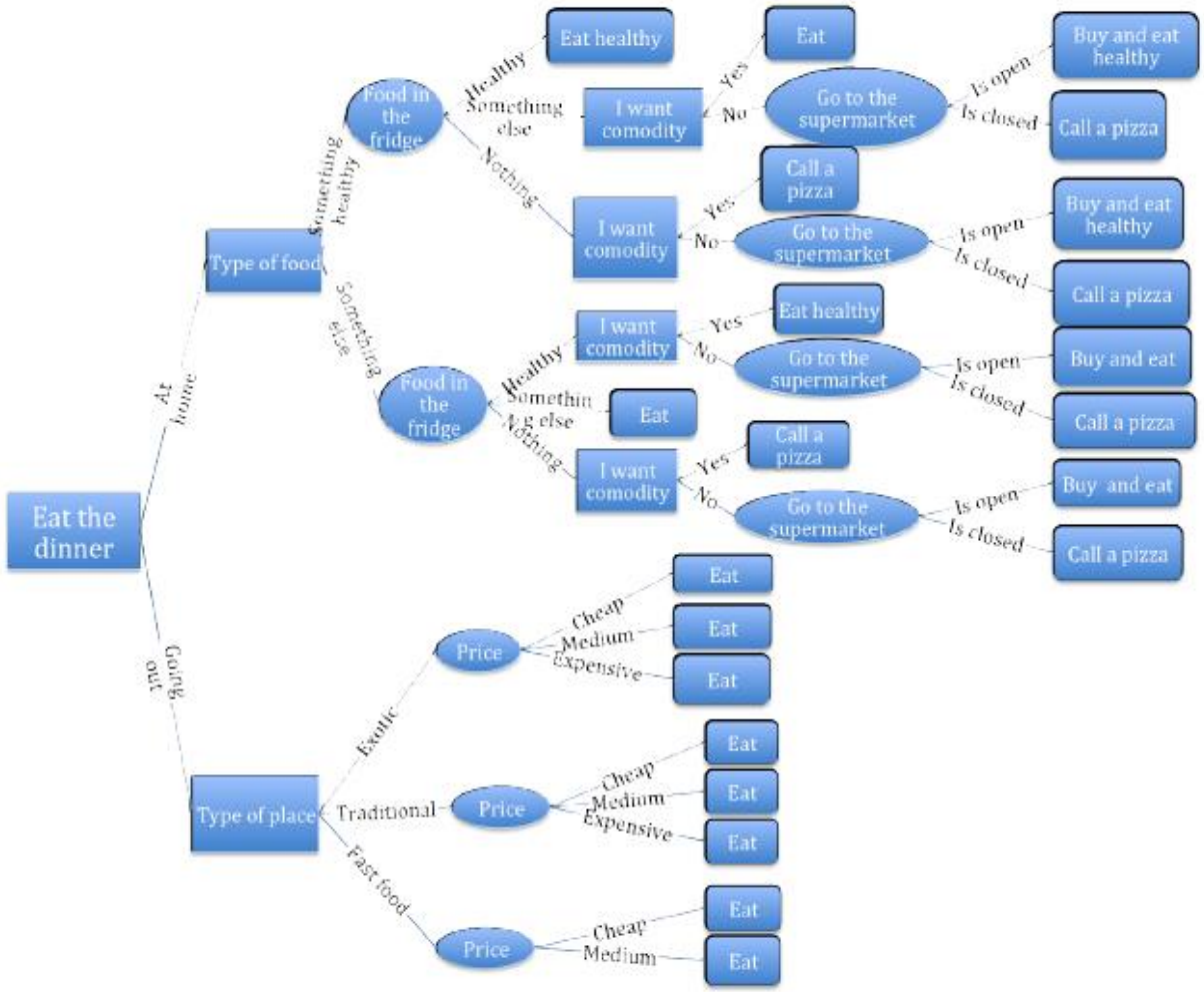


Decision Process Models (31)
MDS / Decision Trees



Decision Process Models (32)
Multiple Decision Scenario / Decision Trees

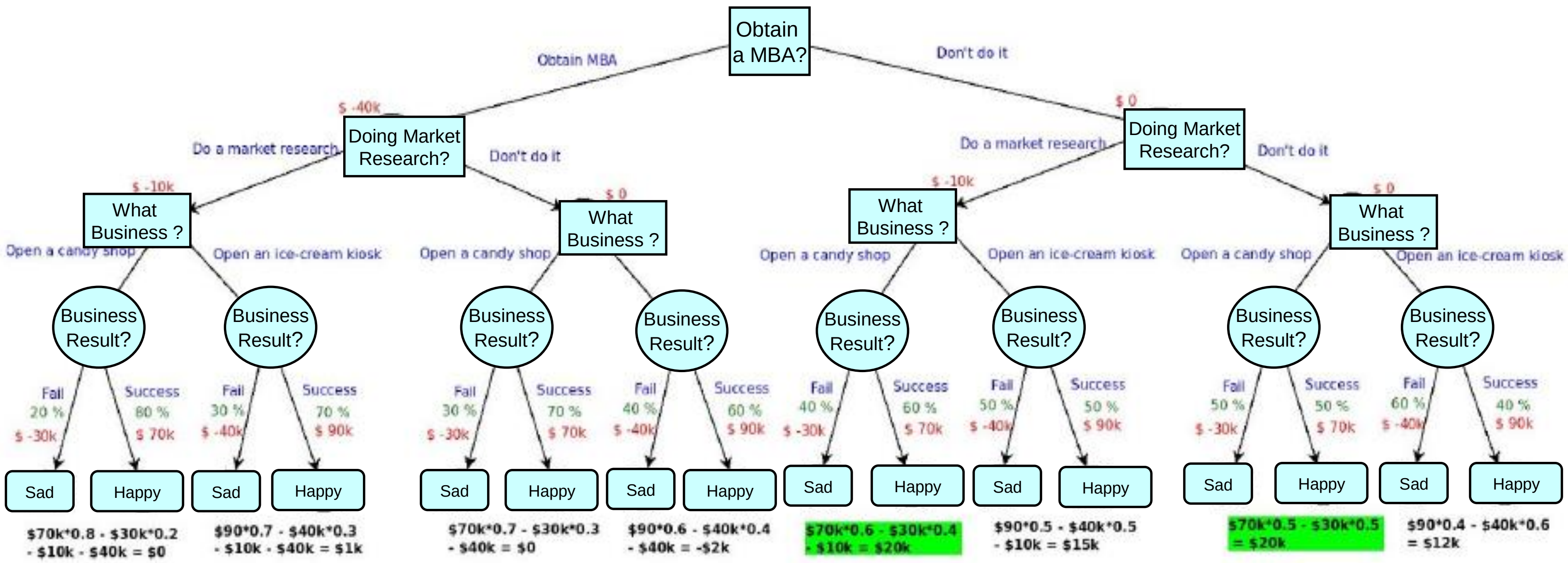
GET SOMETHING TO DINNER – Anna Guitart



Decision Process Models (33)

Multiple Decision Scenario / Decision Trees

BEST PLANNING FOR A YOUNG BUSINESSMAN – Aleksandr Beliaev



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