PolyCLEAN

A Polyatomic CLEAN-like algorithm for sparse Bayesian imaging

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EPFL

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Radio Interferometry and the CLEAN realm

02
MAP estimation
Optimization problems and numerical challenges

03
PolyCLEAN
Convex optimization solved in an atomic manner

04
Demonstration
Performances and experimental reconstructions

01. Background

Radio Interferometric Imaging

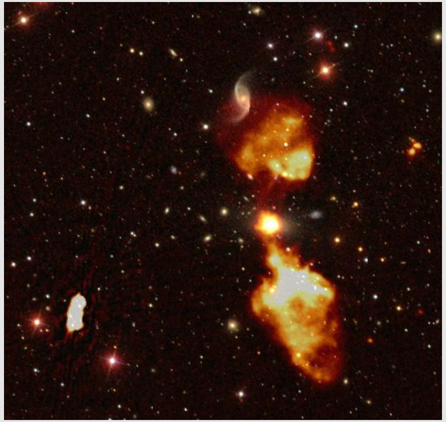
Linear Inverse Problem

$$\boxed{\mathbf{V} = \mathbf{\Phi} \mathbf{I}}$$

Linear Inverse Problem

$$\boxed{\mathbf{V} = \mathbf{\Phi} \mathbf{I}}$$

Observed area of the sky



[Credits: Cyril Tasse and the LOFAR surveys team.]

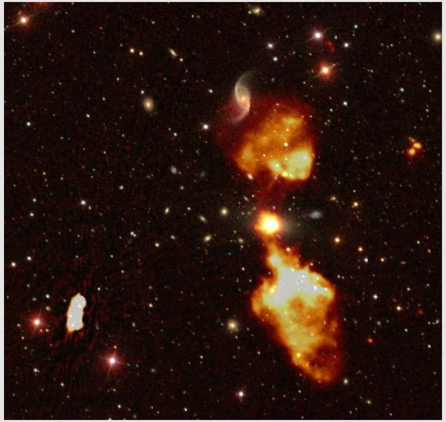
Linear Inverse Problem

Visibility measurements

Spatial frequency information,
Fourier-like measurements,
complex valued

$$\boxed{\mathbf{V} = \mathbf{\Phi I}}$$

Observed area of the sky



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Linear Inverse Problem

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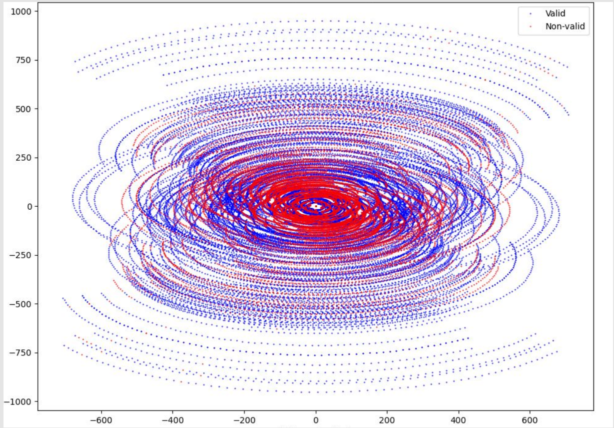
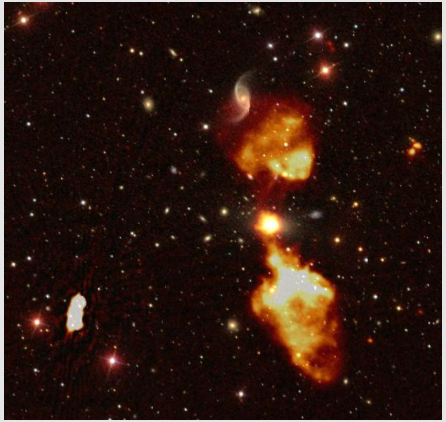


$$\mathbf{V} = \mathbf{\Phi I}$$

Interferometry operator

Depends on the location of the
antennas (*baselines)*
And the observed wavelength

Observed area of the sky



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Measurement Operator

Image

Sky coordinates

$$\boldsymbol{\Phi}(\mathbf{I})_k = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\mathbf{I}[i,j]}{n(l_i, m_j)} e^{-\mathrm{j}2\pi(u_k l_i + v_k m_j)} \mathcal{W}(l_i, m_j; w_k)$$

Baseline coordinates

Challenges of RI

Linear IP

$$\boxed{\mathbf{V} = \mathbf{\Phi} \mathbf{I}}$$

- Noisy measurements

$$\mathbf{V} = \mathbf{\Phi} \mathbf{I} + \varepsilon$$

Challenges of RI

Linear IP

$$\boxed{\mathbf{V} = \mathbf{\Phi I}}$$

- Noisy measurements $\qquad \mathbf{V} = \mathbf{\Phi I} + \varepsilon$

- Ill-posed problem $\qquad \mathrm{Null}(\mathbf{\Phi}) \neq \{0\}$

Linear IP

$$\boxed{\mathbf{V} = \mathbf{\Phi}\mathbf{I}}$$

Challenges of RI

- Noisy measurements

$$\mathbf{V} = \mathbf{\Phi}\mathbf{I} + \varepsilon$$

- Ill-posed problem

$$\mathrm{Null}(\mathbf{\Phi}) \neq \{0\}$$

Use of priors for reconstruction!

Challenges of RI

Linear IP

$$\boxed{\mathbf{V} = \mathbf{\Phi}\mathbf{I}}$$

- Noisy measurements

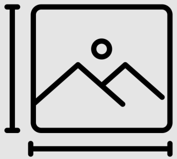
$$\mathbf{V} = \mathbf{\Phi}\mathbf{I} + \varepsilon$$

- Ill-posed problem

$$\mathrm{Null}(\mathbf{\Phi}) \neq \{0\}$$

Use of priors for reconstruction!

- Huge volumes of data



The CLEAN Algorithm

Linear IP

$$\boxed{\mathbf{V} = \mathbf{\Phi}\mathbf{I}}$$

Parametric Shape of the Solutions

$$\mathbf{I}^*[n] = \sum_k \alpha_k g(n - n_k)$$

Matching Pursuit Algorithm

Iterative atomic updates

$$\mathbf{I}^{(k+1)} \leftarrow \mathbf{I}^{(k)} + \alpha \mathbf{g}(\cdot - n_k)$$

Empirical sparsity along iteration

The CLEAN Algorithm

Linear IP

$$\boxed{\mathbf{V} = \mathbf{\Phi}\mathbf{I}}$$

Parametric Shape of the Solutions

$$\mathbf{I}^*[n] = \sum_k \alpha_k \boxed{\delta}(n - n_k)$$

Point sources

Matching Pursuit Algorithm

Iterative atomic updates

$$\mathbf{I}^{(k+1)} \leftarrow \mathbf{I}^{(k)} + \alpha \mathbf{g}(\cdot - n_k)$$

Empirical sparsity along iteration

The Algorithm

Algorithm 1 Högbom CLEAN Algorithm (Major cycles only)

Parameters : $k_{\max}$ (iterations), $\alpha > 0$ (gain)

Initialisation : $\mathbf{I}^{(0)} = \mathbf{0}$, $\mathbf{I}_D = \mathbf{\Phi}^* \mathbf{V}$

for $k = 1, 2, \cdots, k_{\max}$ **do**

1. Compute the dirty residual: $\mathbf{I}_R^{(k)} = \mathbf{I}_D - \mathbf{\Phi}^* \mathbf{\Phi} \mathbf{I}^{(k-1)}$
2. Find the location of the next reconstructed source: $s^{(k)} = \arg\max_{(i,j)} \left| \mathbf{I}_R^{(k)}[i,j] \right|$
3. Update the iterate: $\mathbf{I}^{(k)} = \mathbf{I}^{(k-1)} + \alpha (\max \mathbf{I}_R^{(k)}) \boldsymbol{\delta}_{s^{(k)}}$

end for

Output:

Postprocess $\mathbf{I}^{(k)}$ (convolution with synthetic beam, add residual image)

CLEAN-Like methods

- ✓ Atomic method (scalable)
- ✓ A lot of hacks and tips to make them very fast
- ✓ Developed and maintained by the astronomers
- ✓ Long date expertise
- ✓ Calibration-compliant

CLEAN-Like methods (continued)

- ✓ Atomic method (scalable)
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- ✗ Only denoising = enforcing the prior model
- ✗ Very sensitive to stop
- ✗ Objective function unclear

02. Bayesian MAP Estimation

A principled way to introduce prior information

LASSO as a MAP estimator

$$\arg\min \underbrace{\frac{1}{2}\|\mathbf{V} - \mathbf{\Phi I}\|_2^2}_{\text{Log likelihood}} + \underbrace{\lambda\|\mathbf{I}\|_1}_{\text{Prior distribution}}$$

- Convex optimization methods
- Sparse solutions => Well suited for **Point Sources**

Sparse Dictionary reconstruction

$$\arg\min_{\boldsymbol{\theta}} \frac{1}{2}||\mathbf{V} - \boldsymbol{\Phi}\boxed{\boldsymbol{\Psi}}\boldsymbol{\theta}||_2^2 + \lambda||\boldsymbol{\theta}||_1$$

$\boldsymbol{\Psi} \in \mathbb{R}^{N \times M}$ Dictionary synthesis operator

$\boldsymbol{\theta} \in \mathbb{R}^{M}$ Dictionary coefficients

Optimization methods

- ✓ Denoising (with only one parameter!)
- ✓ Excellent reconstruction quality demonstrated
- ✓ Can handle very complex priors
- ✓ Fast principled algorithms

Optimization methods (continued)

- ✓ Denoising (with only one parameter!)
- ✓ Excellent reconstruction quality demonstrated
- ✓ Can handle very complex priors
- ✓ Fast principled algorithms

- ✗ Completely different implementation paradigm (proximal method)
- ✗ Memory scalability
- ✗ Non calibration-compliant
- ✗ Shrinkage of the reconstructed intensity

03. PolyCLEAN

Convex optimization for RA
enabled with an atomic method

1. Optimization method

Penalty-based prior

2. Atomic behavior

CLEAN-like algorithmic structure
and minor cycles

3. Focus on scalability

Sparsity-informed computations with
Pycsou and NUFFT

1. Optimization method

Penalty-based prior

$$\boxed{\lambda \|\mathbf{I}\|_1 \,,\, \mathbf{I} \geqslant 0}$$

2. Atomic behavior

CLEAN-like algorithmic structure
and minor cycles

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Sparsity-informed computations with
Pycsou and NUFFT

1. Optimization method

Penalty-based prior

$$\boxed{\lambda \|\mathbf{I}\|_1 \,,\ \mathbf{I} \geqslant 0}$$

2. Atomic behavior

CLEAN-like algorithmic structure and minor cycles

$$\boxed{\mathbf{I} = \sum \alpha_k \boldsymbol{\delta}_{i_k}}$$

3. Focus on Scalability

Sparsity-informed computations with Pycsou and NUFFT

1. Optimization method

Penalty-based prior

$$\boxed{\lambda \|\mathbf{I}\|_1 \,,\, \mathbf{I} \geqslant 0}$$

2. Atomic behavior

CLEAN-like algorithmic structure
and minor cycles

$$\boxed{\mathbf{I} = \sum \alpha_k \boldsymbol{\delta}_{i_k}}$$

3. Focus on scalability

$$\boxed{\boldsymbol{\Phi}\left(\mathbf{I}^{(k)}\right)}$$

Sparsity-informed computations with
Pycsou and NUFFT

The Algorithm

Algorithm 2 PolyCLEAN

Initialisation : $\mathbf{I}^{(0)} = \mathbf{0}$, $\mathcal{S}^{(0)} = \mathrm{Supp}(\mathbf{I}^{(0)}) = \emptyset$, $\mathbf{I}_D = \mathbf{\Phi}^* \mathbf{V}$

while `stopping_criterion`$(\mathbf{I}^{(k)})$ not reached **do**

1. Compute the dirty residual: $\mathbf{I}_R^{(k)} = \mathbf{I}_D - \mathbf{\Phi}^* \mathbf{\Phi} \mathbf{I}^{(k-1)}$

2. Place many candidate sources: $s_1^{(k)}, s_2^{(k)}, \cdots =$ `highest_level_set` $(\mathbf{I}_R^{(k)})$
   Update active set : $\mathcal{S}^{(k)} \leftarrow \mathcal{S}^{(k-1)} \cup \{s_1^{(k)}, s_2^{(k)}, \ldots\}$

3. Update the iterate:

$$\mathbf{I}^{(k)} = \underset{\substack{\mathrm{Supp}(\mathbf{I}) \subset \mathcal{S}^{(k)} \\ \mathbf{I} \geqslant 0}}{\arg\min} \frac{1}{2} \|\mathbf{V} - \mathbf{\Phi} \mathbf{I}\|_2^2 + \lambda \|\mathbf{I}\|_1 \tag{R}$$

end while

Focus on the sparse iterates

- Beneficial only if handled correctly
  - → Low memory requirement
  - → Simple model
  - → Fast computation

- NU Fourier Transform: Type II -> Type III

Symbiosis with HVOX

Optimization methods (continued)

- ✓ Denoising (with only one parameter!)
- ✓ Excellent reconstruction quality demonstrated
- ✓ Can handle very complex priors
- ✓ Fast principled algorithms

- ✗ ~~Completely different implementation paradigm (proximal method)~~
- ✗ ~~Memory scalability~~
- ✗ ~~Non calibration-compliant~~
- ✗ Shrinkage of the reconstructed intensity

The Landscape of Methods

CLEAN

Convex Optimization

- L1 penalty and positivity
- Objective function
- Interpretability

- Atomic algorithm
- Scalability

PolyCLEAN

04. Numerical Results

It works, and it is fast

Performance benchmark

1. Pick an interferometer radius and image size

Performance benchmark

1. Pick an interferometer radius and image size
2. Simulate a source sky image

Performance benchmark

1. Pick an interferometer radius and image size
2. Simulate a source sky image
3. Simulate a measurement set

Performance benchmark

1. Pick an interferometer radius and image size
2. Simulate a source sky image
3. Simulate a measurement set
4. Solve with LASSO solvers:
   a. PolyCLEAN
   b. APGD
   c. MonoFW

Performance benchmark

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5. Solve with WS-CLEAN

Performance benchmark

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5. Solve with WS-CLEAN
6. Compare reconstruction time

Performance benchmark

1. Pick an interferometer radius and image size
2. Simulate a source sky image
3. Simulate a measurement set
4. Solve with LASSO solvers:
    a. PolyCLEAN
    b. APGD
    c. MonoFW
5. Solve with WS-CLEAN
6. Compare reconstruction time

Largest measured frequency ↗
Image size (in px) ↗
Pixel size ↘

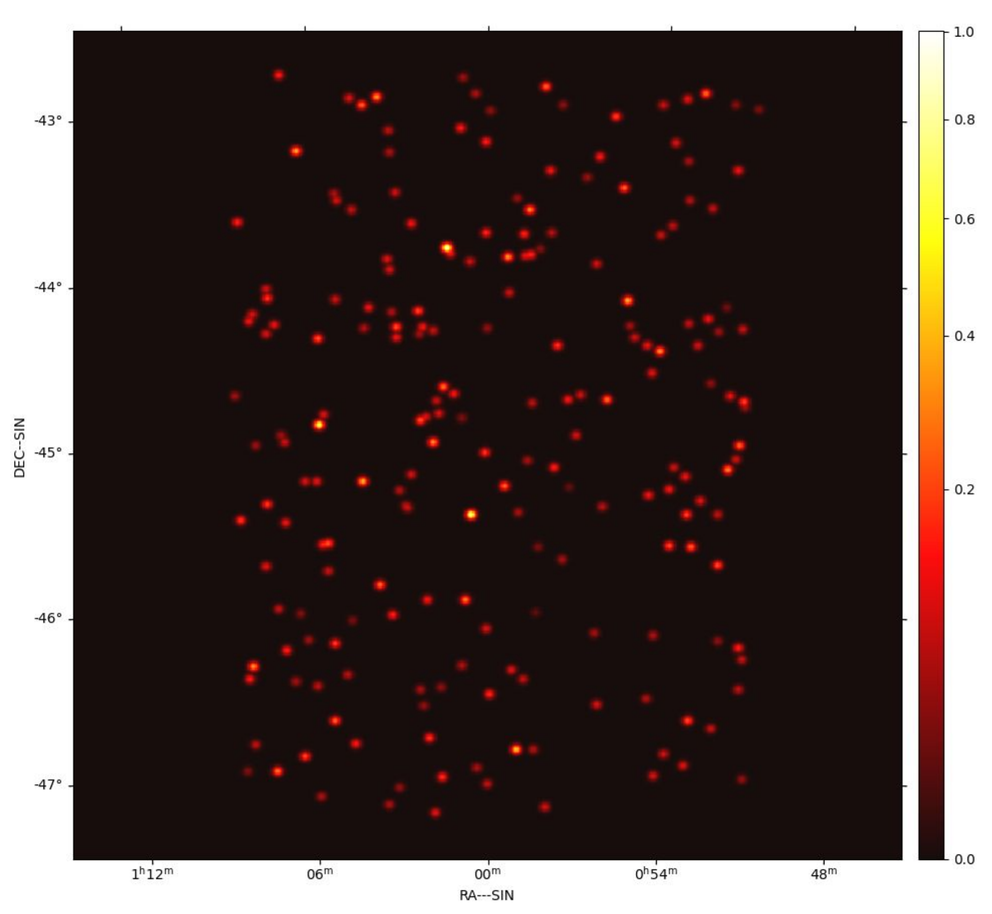
→ Observe scalability

Simulated Source image

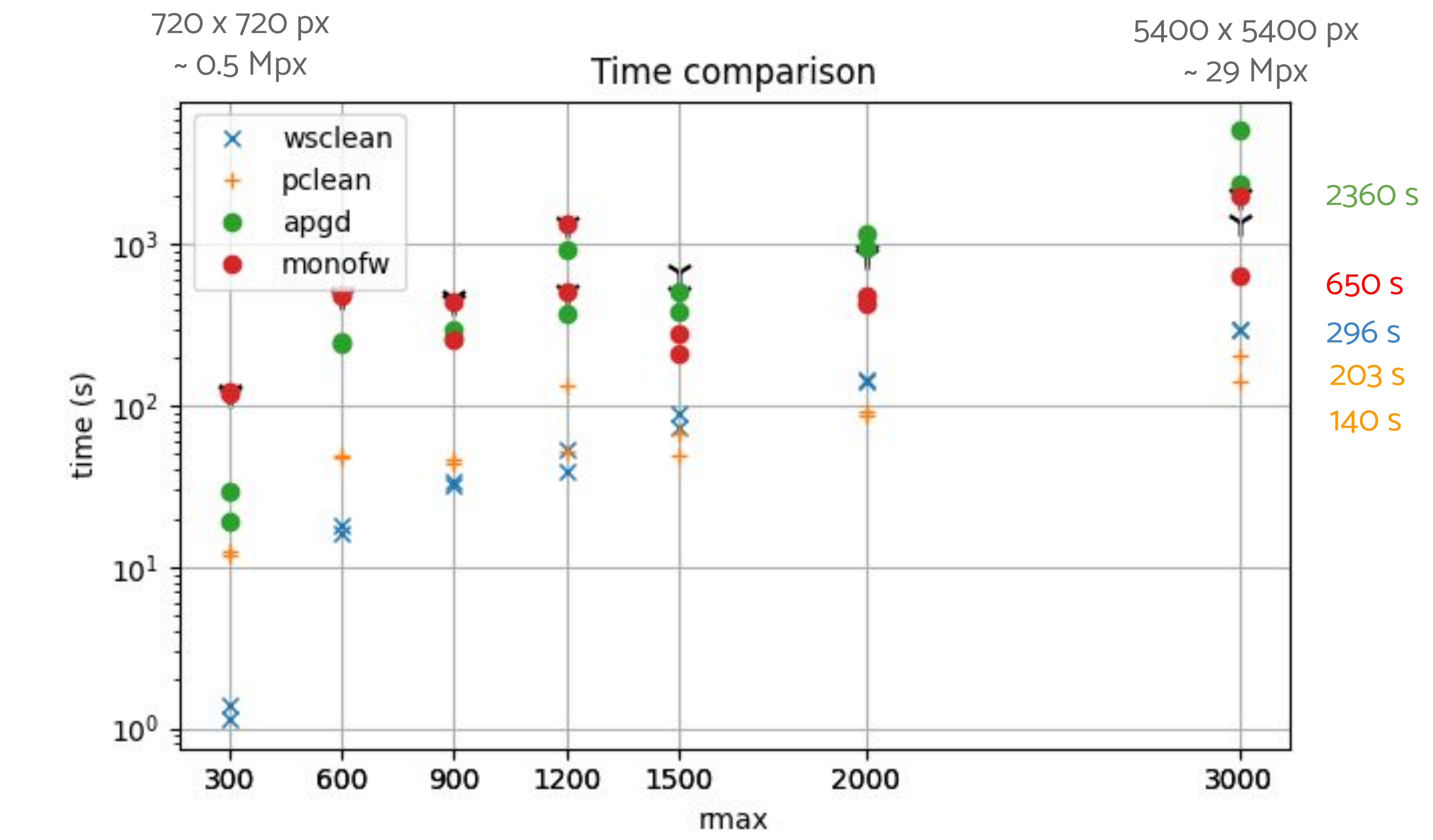
200 point sources

5° x 5° FOV

Image size: 720 -> 5400 pixels



Reconstruction benchmark



Real world measurement

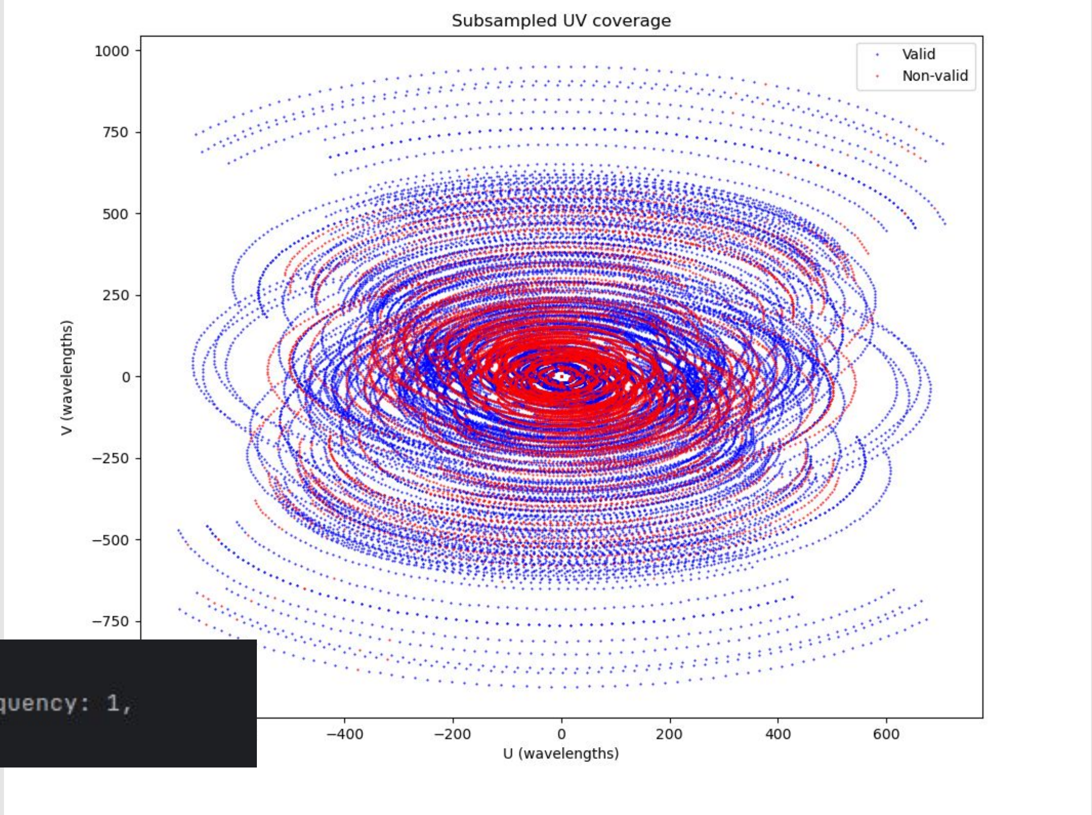
```
<xarray.Visibility>
Dimensions:          (time: 3595, baselines: 1953, frequency: 1,
                      polarisation: 1, spatial: 3)
Coordinates:
  * time             (time) float64 4.914e+09 4.914e+09 ... 4.914e+09
  * baselines        (baselines) object MultiIndex
  * antenna1         (baselines) int64 0 0 0 0 0 0 0 ... 58 59 59 59 60 60 61
  * antenna2         (baselines) int64 0 1 2 3 4 5 6 ... 61 59 60 61 60 61 61
  * frequency        (frequency) float64 1.458e+08
  * polarisation     (polarisation) <U1 'I'
  * spatial          (spatial) <U1 'u' 'v' 'w'
```

~ 400 MB

Selection of antennas

1/50 measurement times

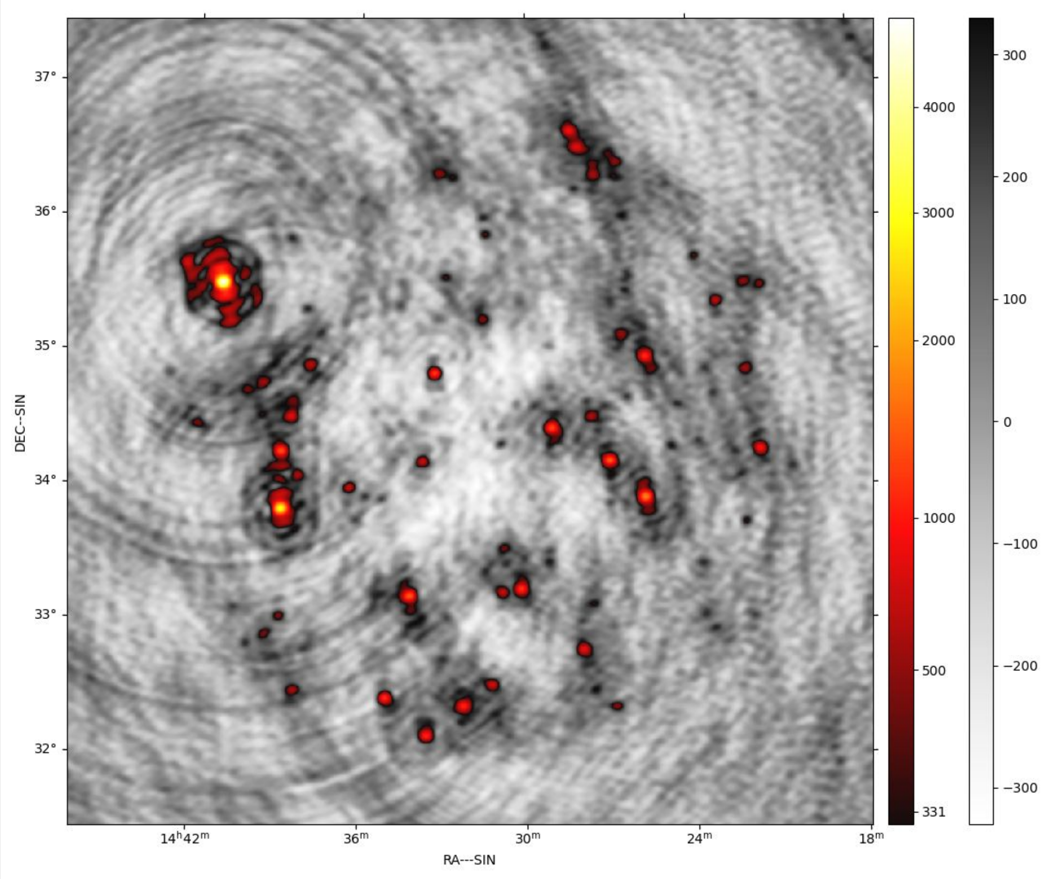
24 antennas




```
<xarray.Visibility>
Dimensions:            (time: 72, baselines: 406, frequency: 1,
                        polarisation: 1, spatial: 3)
```

Dirty image - Point Sources

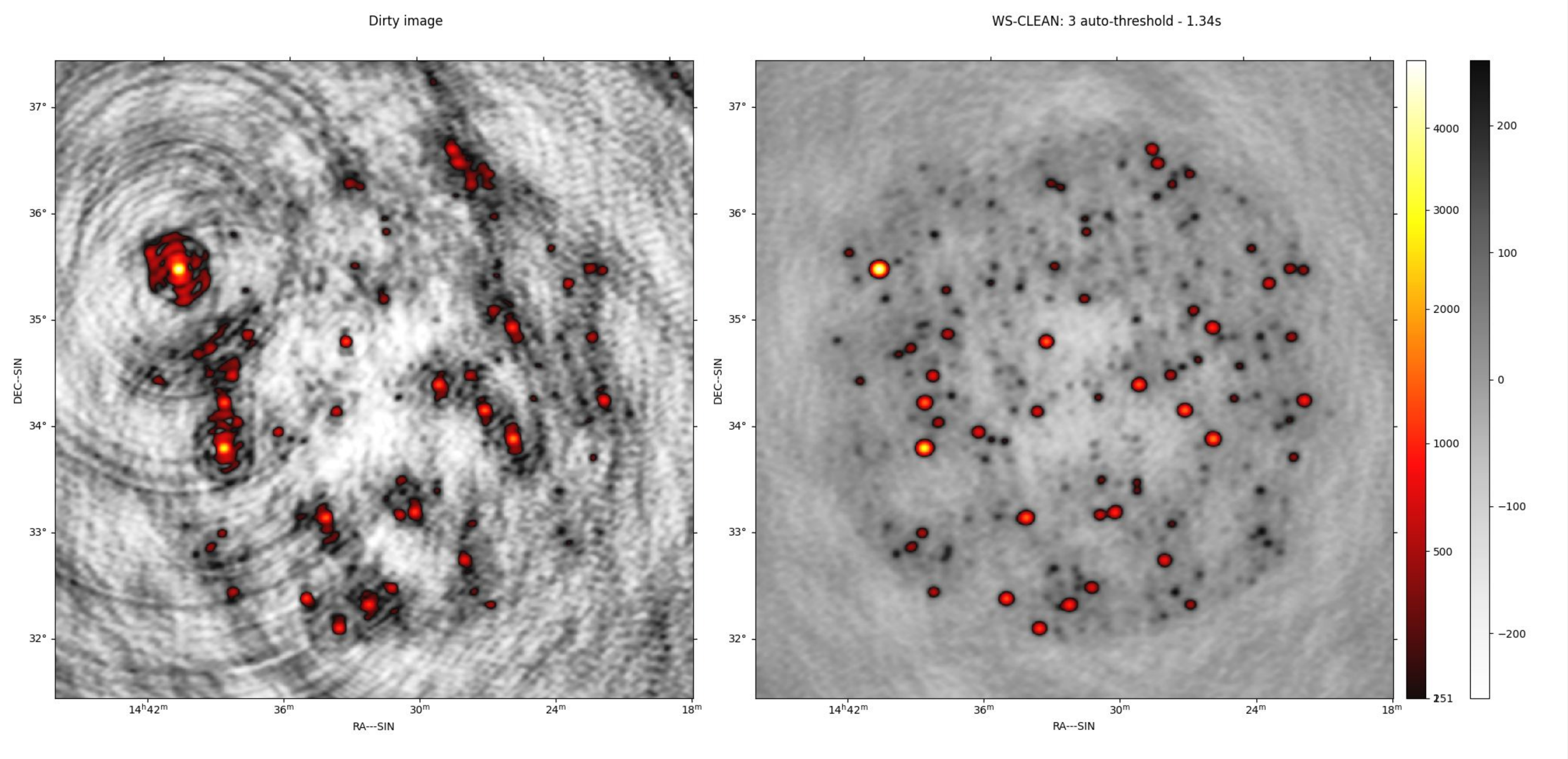
1024 pixels
=
6° FOV



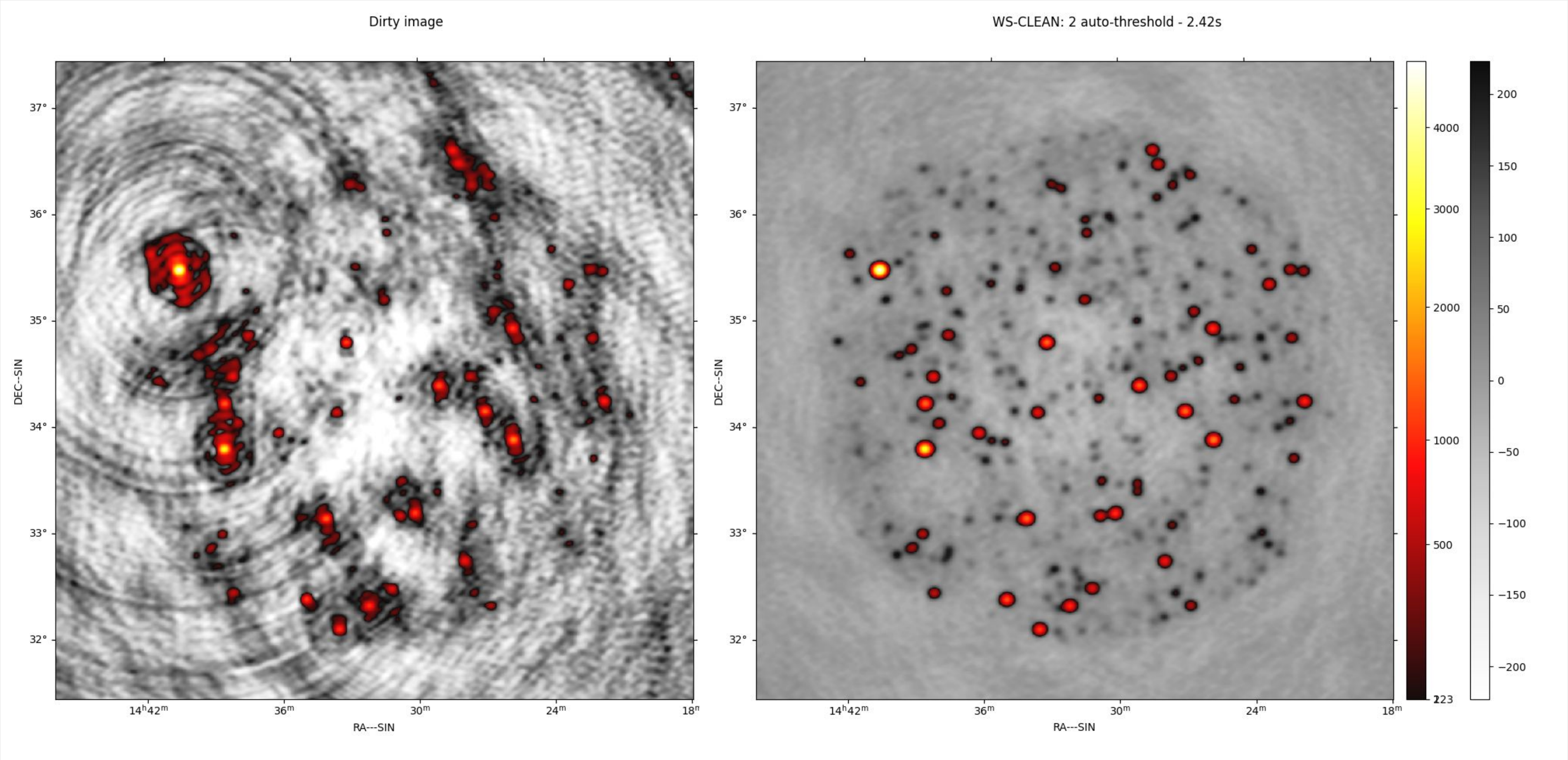
$$\mathbf{I}_D = \mathbf{\Phi}^*\mathbf{V}$$

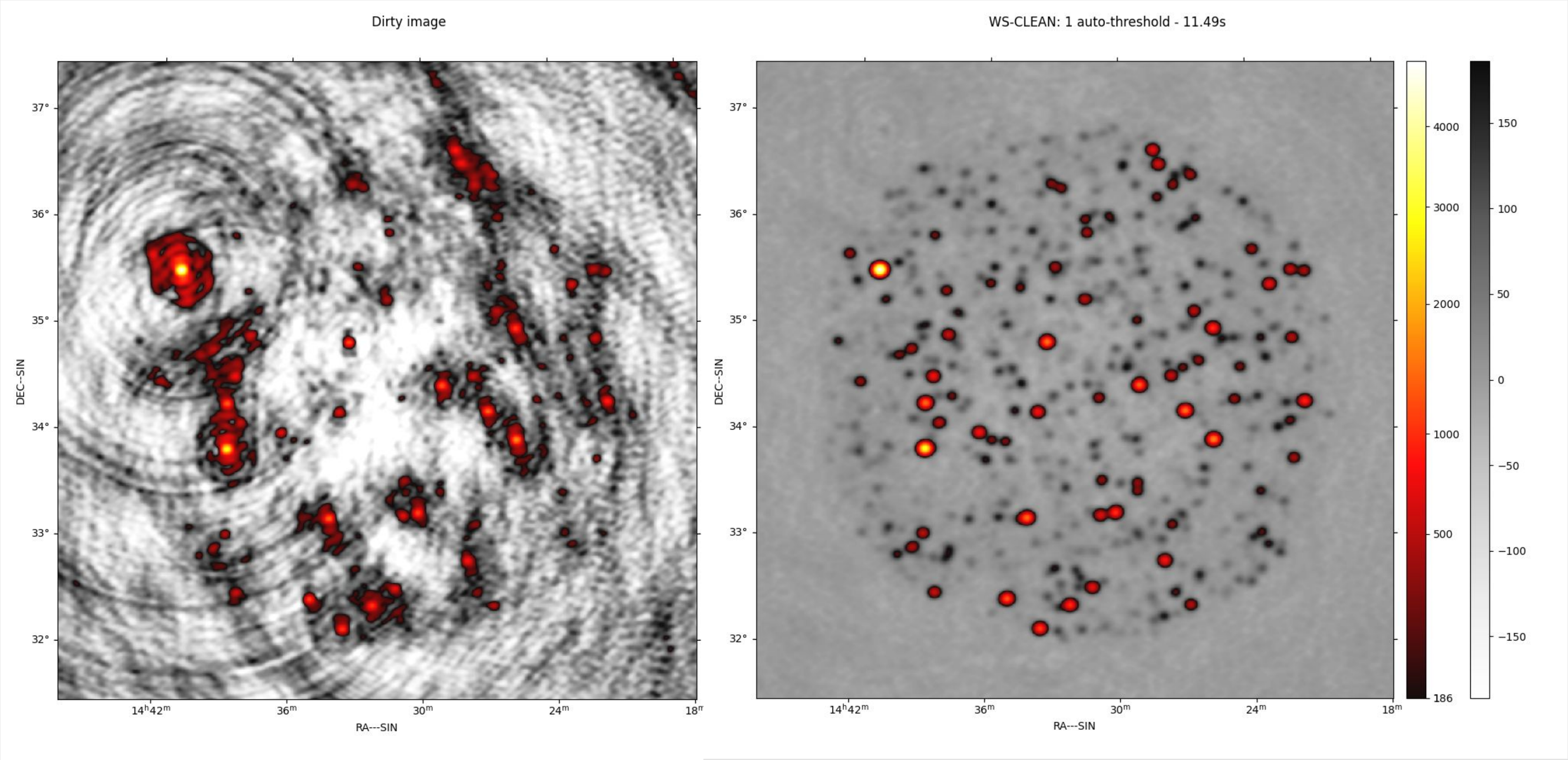
Reconstruction parameters

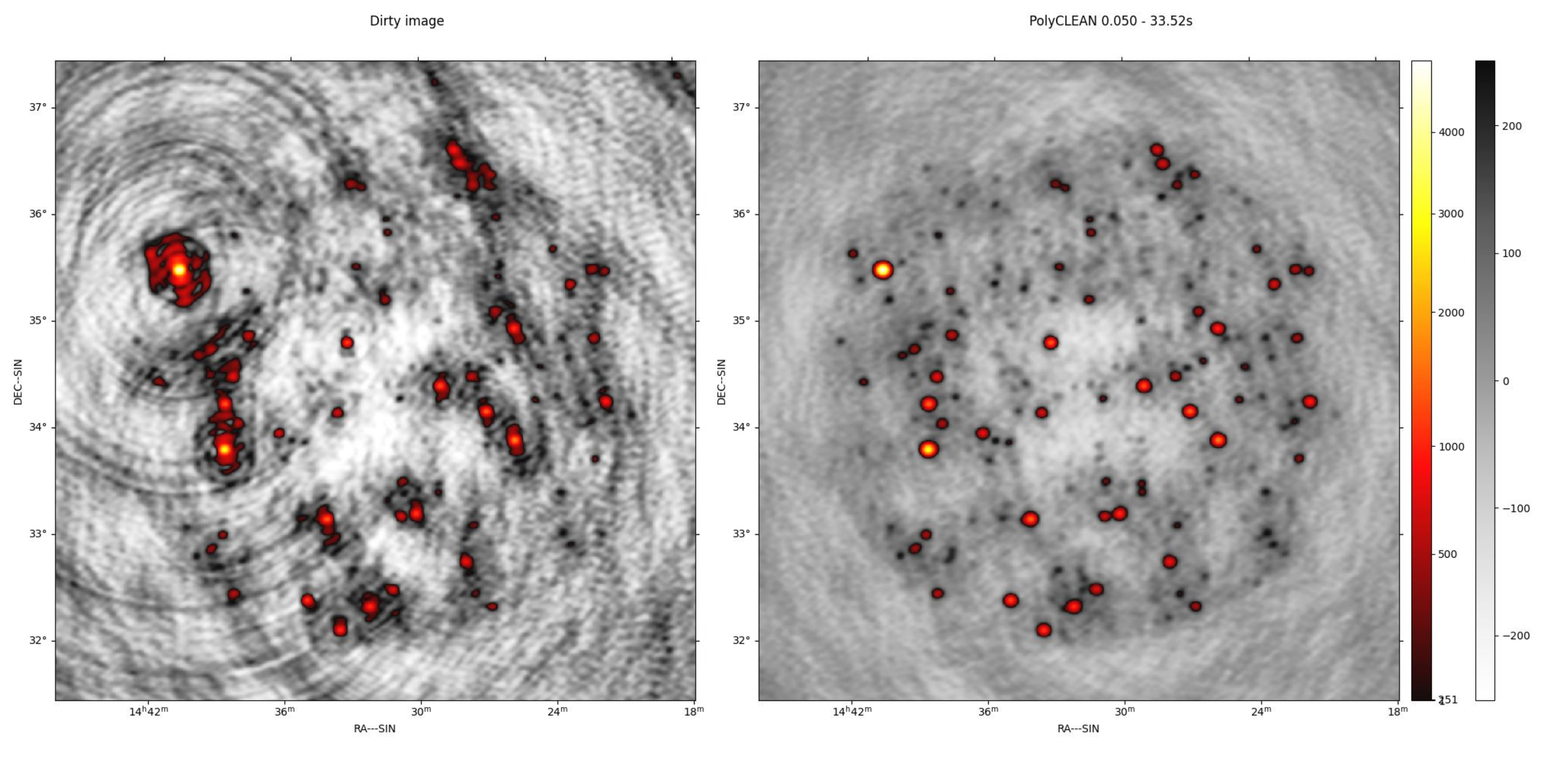
WS-CLEAN Auto-threshold parameter	PolyCLEAN Penalty parameter
3 σ	5%
2 σ	2%
1 σ	0.5%



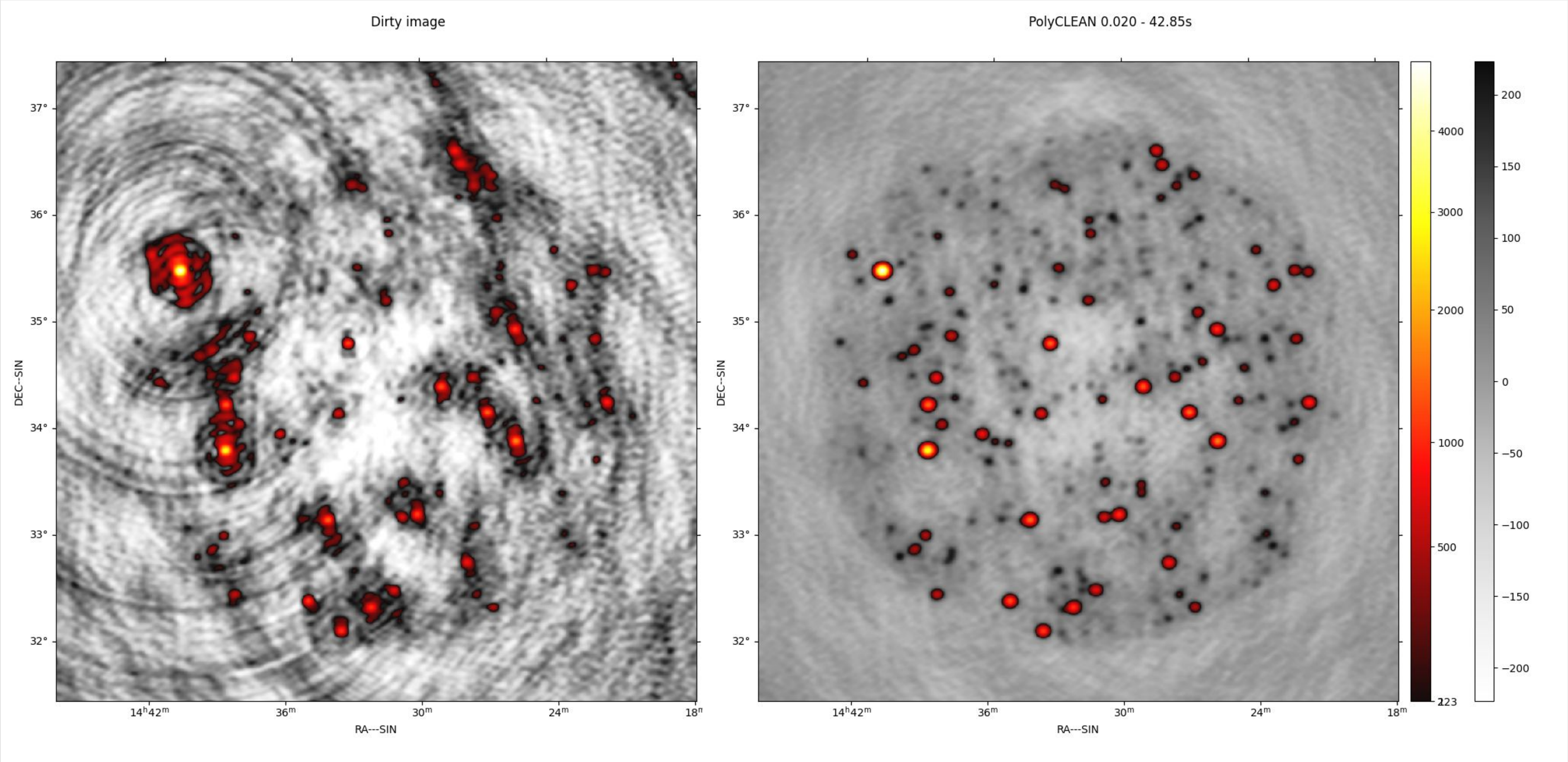
Dirty image
WS-CLEAN: 3 auto-threshold - 1.34s
DEC--SIN
RA---SIN
37°
36°
35°
34°
33°
32°
14h42m
36m
30m
24m
18m
4000
3000
2000
1000
500
251
200
100
0
−100
−200

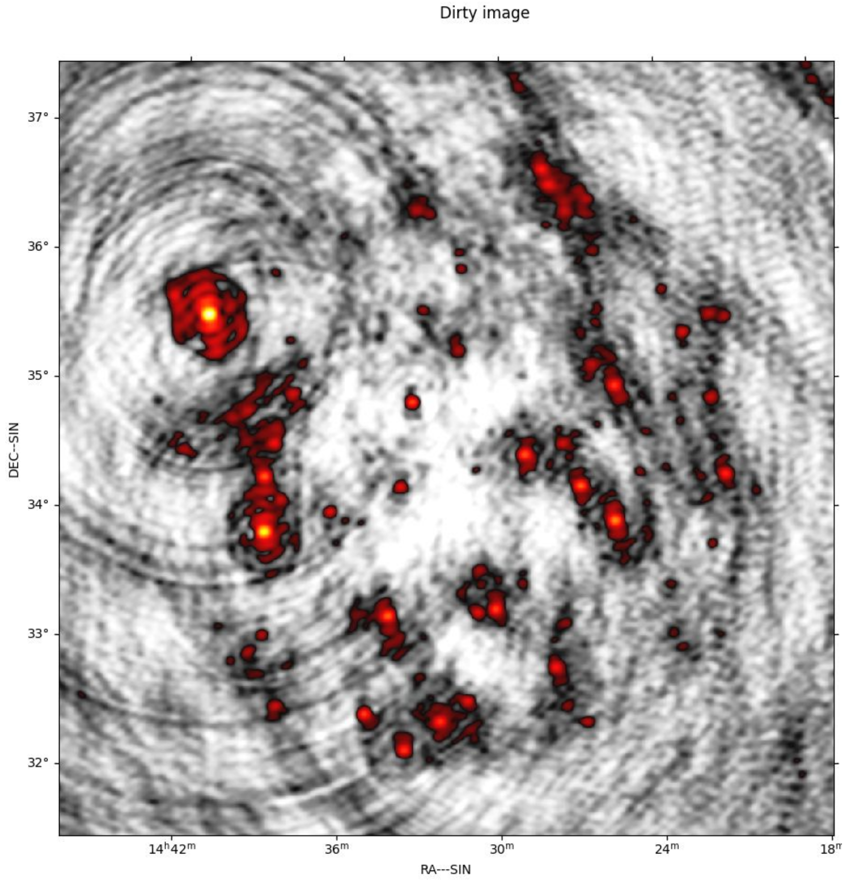

Dirty image
WS-CLEAN: 2 auto-threshold - 2.42s
37°
36°
35°
34°
33°
32°
DEC--SIN
14h42m
36m
30m
24m
18m
RA---SIN
4000
3000
2000
1000
500
223
200
150
100
50
0
−50
−100
−150
−200


Dirty image
WS-CLEAN: 1 auto-threshold - 11.49s
DEC--SIN
RA---SIN
37°
36°
35°
34°
33°
32°
14h42m
36m
30m
24m
18m
4000
3000
2000
1000
500
186
150
100
50
0
−50
−100
−150

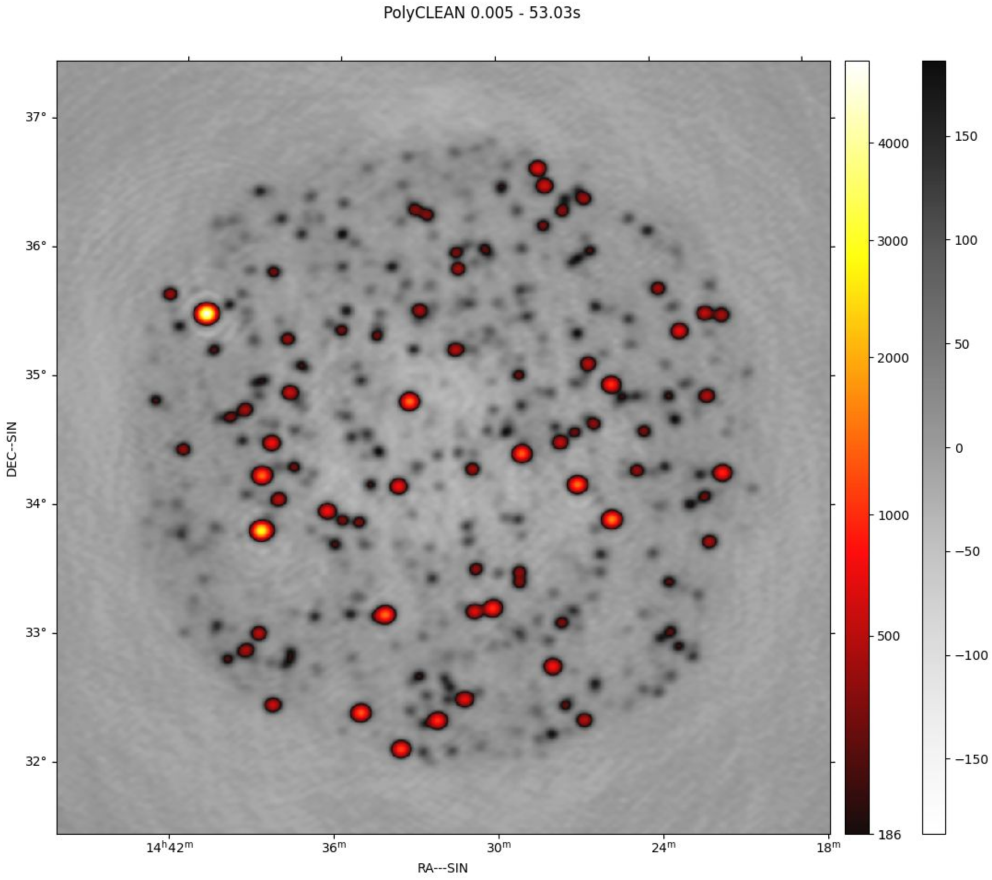


Dirty image
PolyCLEAN 0.050 - 33.52s
DEC--SIN
RA---SIN
37°
36°
35°
34°
33°
32°
14h42m
36m
30m
24m
18m
4000
3000
2000
1000
500
251
200
100
0
−100
−200


Dirty image
PolyCLEAN 0.020 - 42.85s
37°
36°
35°
34°
33°
32°
DEC--SIN
14h42m
36m
30m
24m
18m
RA---SIN
4000
3000
2000
1000
500
223
200
150
100
50
0
−50
−100
−150
−200

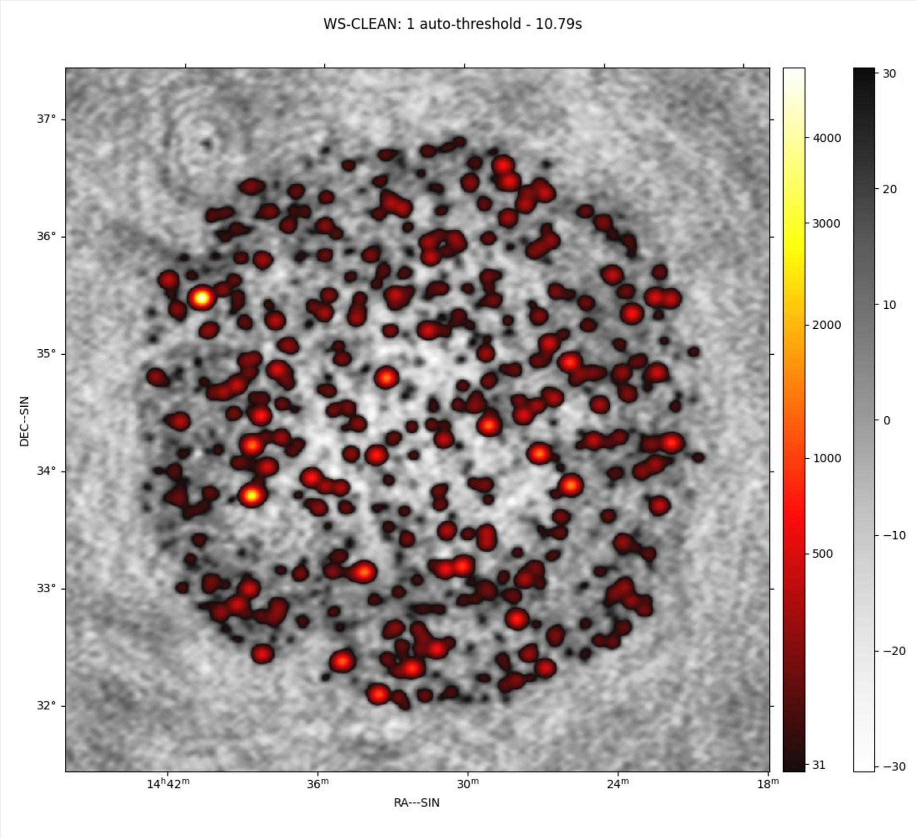


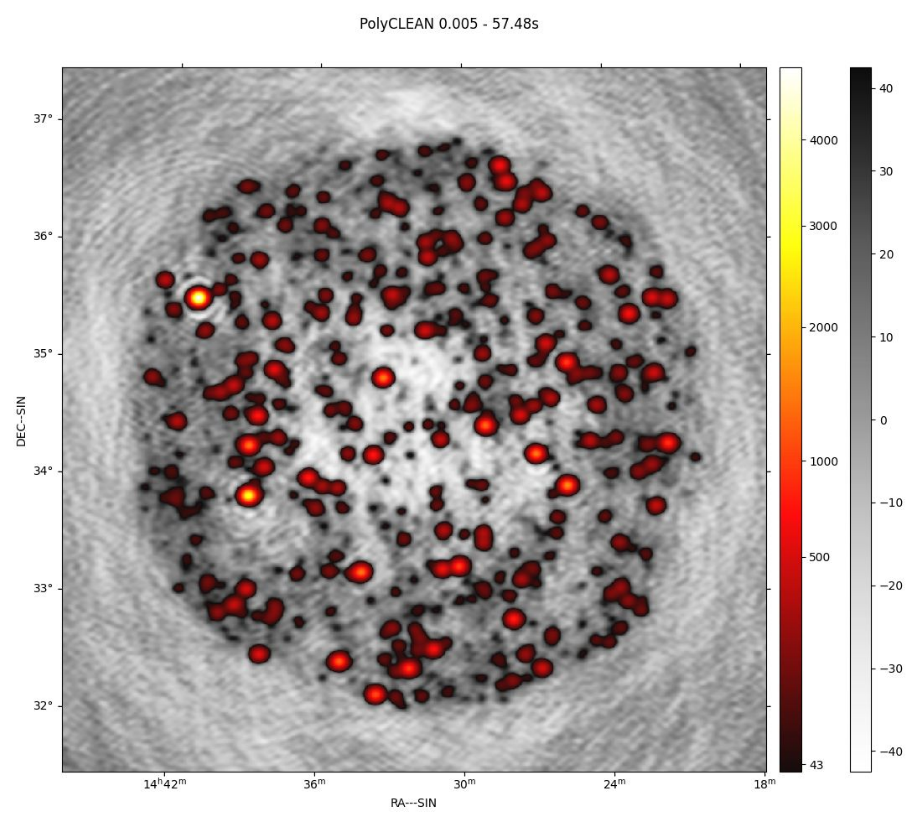
Dirty image
DEC--SIN
RA---SIN
37°
36°
35°
34°
33°
32°
14h42m
36m
30m
24m
18m



PolyCLEAN 0.005 - 53.03s
DEC--SIN
RA---SIN
37°
36°
35°
34°
33°
32°
14h42m
36m
30m
24m
18m
4000
3000
2000
1000
500
186
150
100
50
0
−50
−100
−150

Longest deconvolution example





Dual certificate

Definition:

$$\mu_\lambda = \frac{1}{\lambda}\mathbf{\Phi}^*(\mathbf{V} - \mathbf{\Phi}\mathbf{I}^*)$$

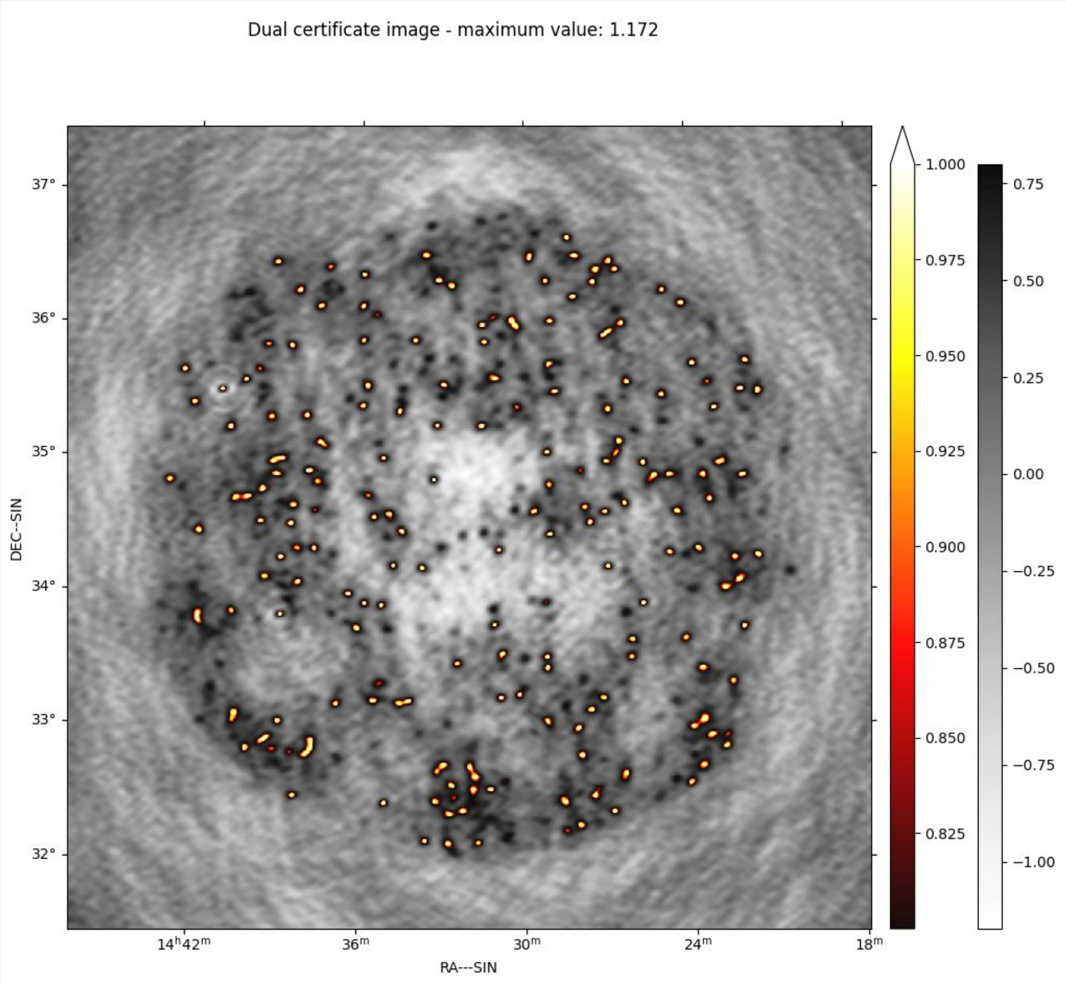
Properties:

$$\|\mu_\lambda\|_\infty \leq 1$$

$$\langle \mu_\lambda, \mathbf{I}^* \rangle = \|\mathbf{I}^*\|_1$$

Usages:

- Convergence
- Saturation set

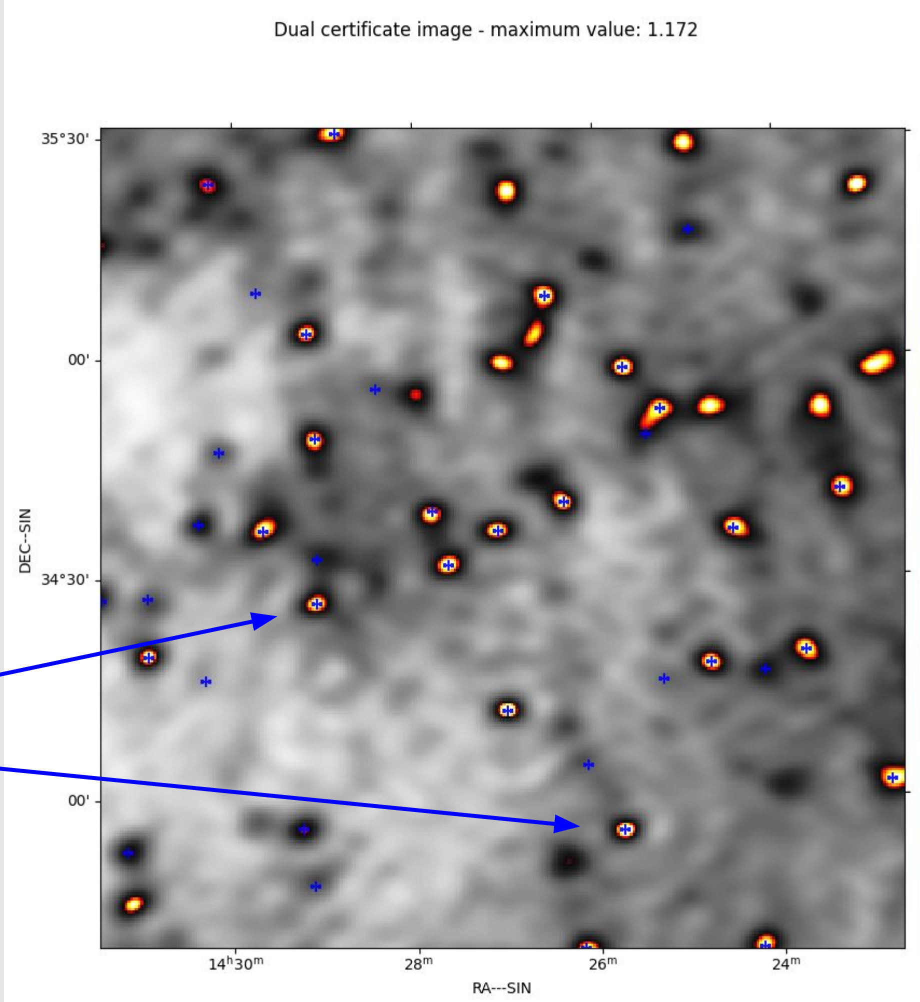


Dual certificate

Rate of information explained

$$\|\mu_\lambda\|_\infty = \frac{\|\mathbf{\Phi}^*(\mathbf{V} - \mathbf{\Phi}\mathbf{I}^*)\|_\infty}{0.02 \ \|\mathbf{\Phi}^*\mathbf{V}\|_\infty} \leq 1$$

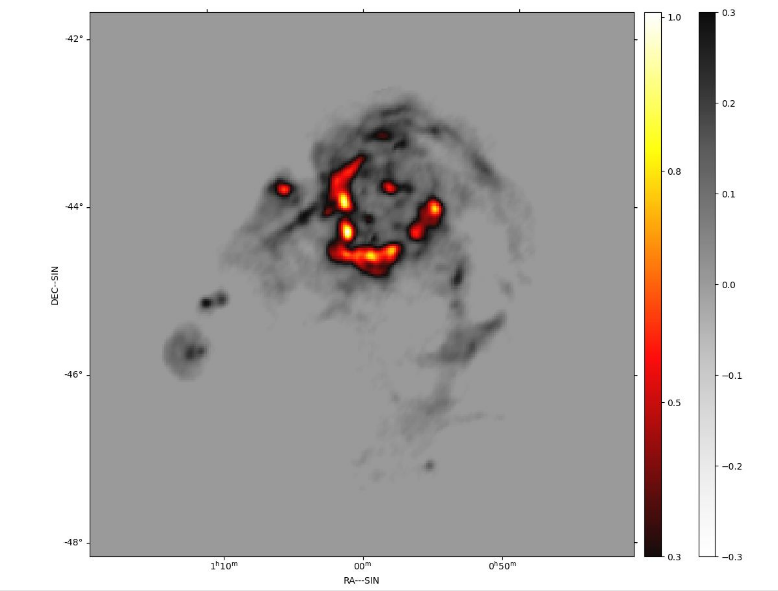
Interpretation of the saturation set



Extended Sources: Simulations

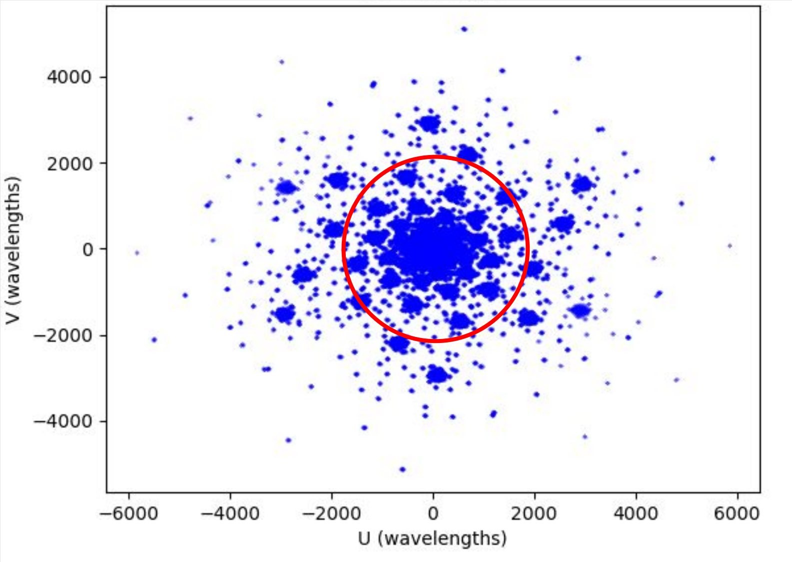
M31 image:

6.5 degrees -> 256 pixels / side



Baselines:

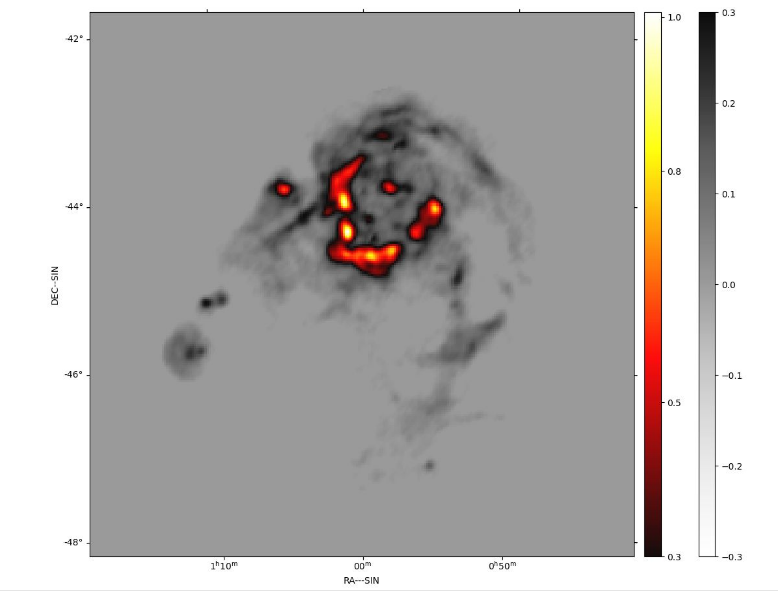
rmax = 1000m -> 31500 baselines



Extended Sources: Simulations

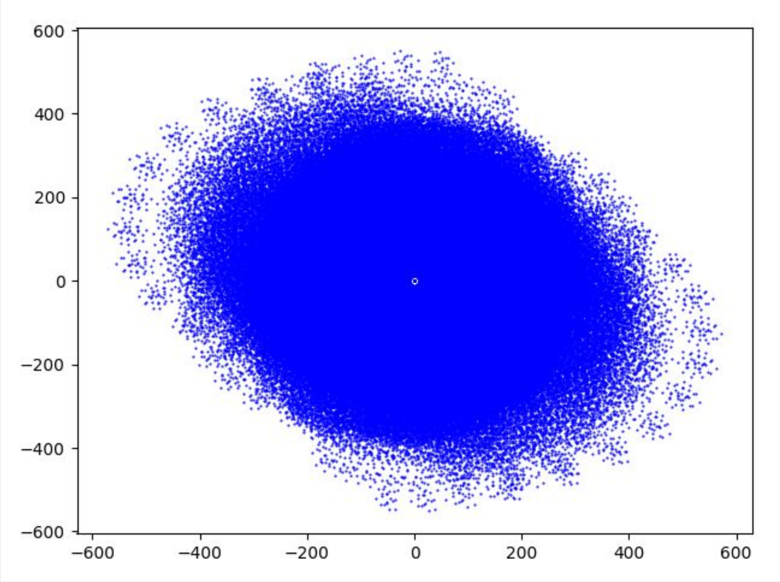
M31 image:

6.5 degrees -> 256 pixels / side



Baselines:

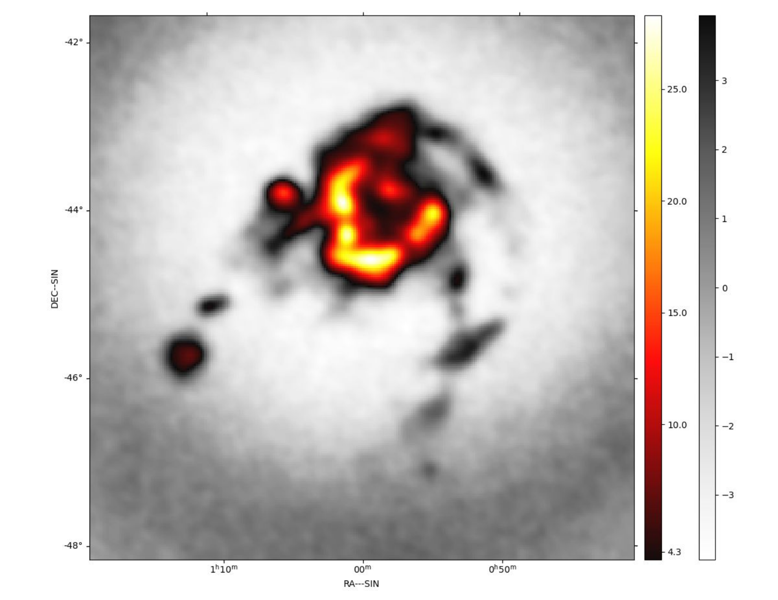
rmax = 1000m -> 31500 baselines,
11 measurement times



Extended Sources: Simulations

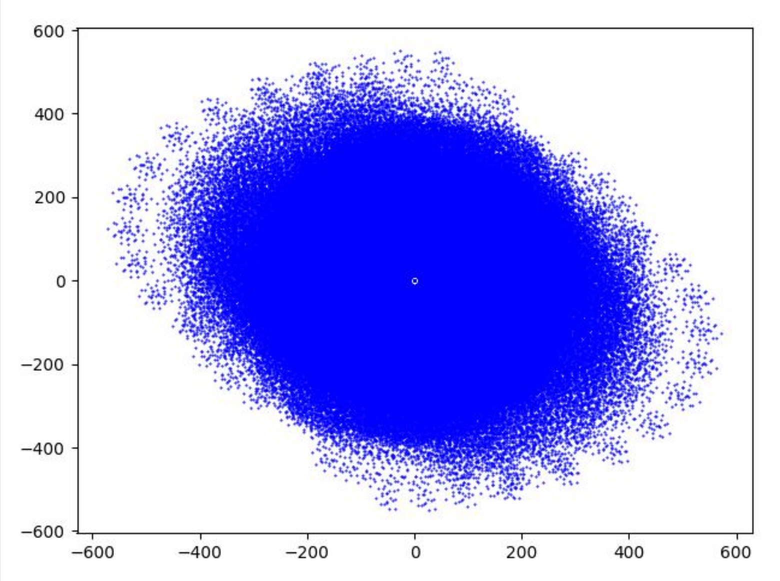
M31 image:

6.5 degrees -> 256 pixels / side

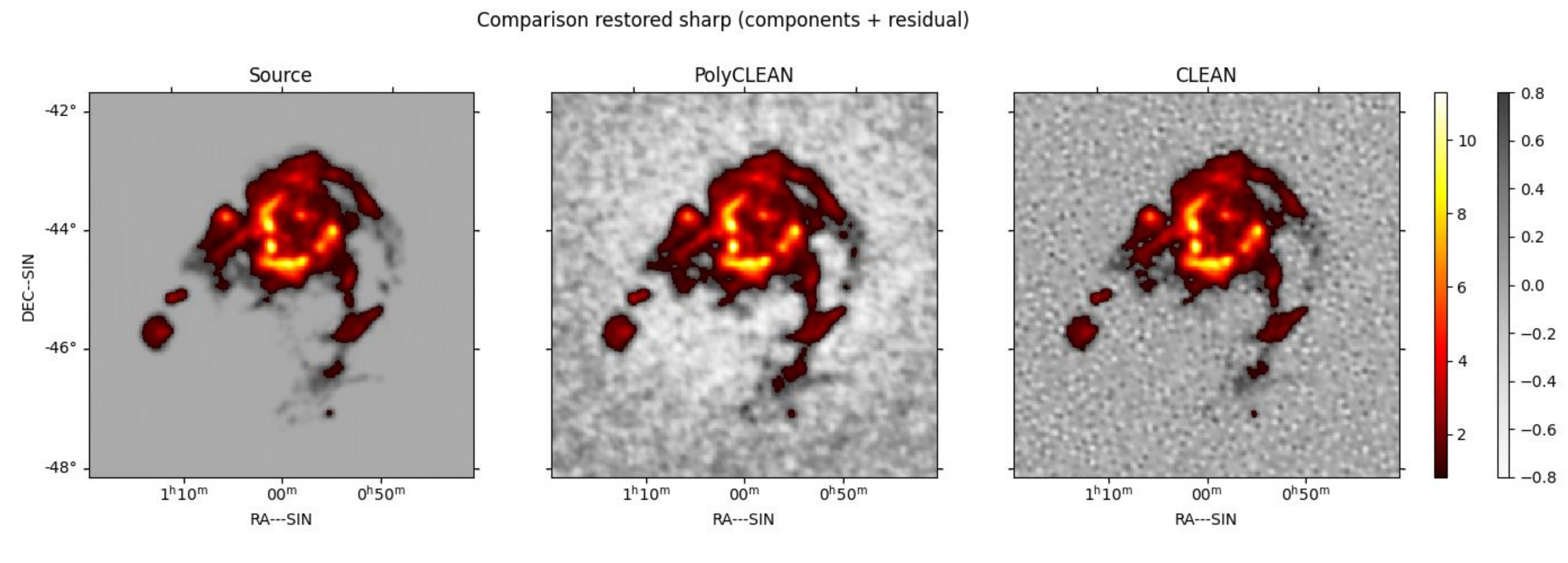


Baselines:

rmax = 1000m -> 31500 baselines,
11 measurement times

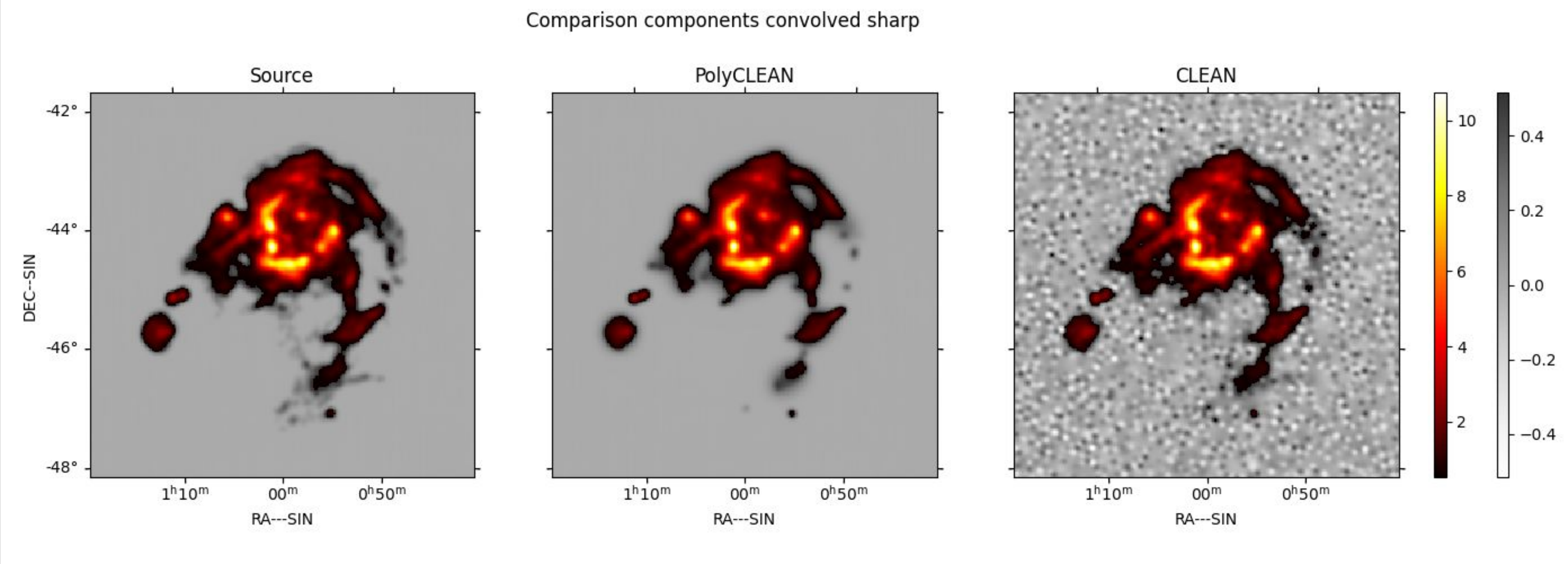


Reconstructions



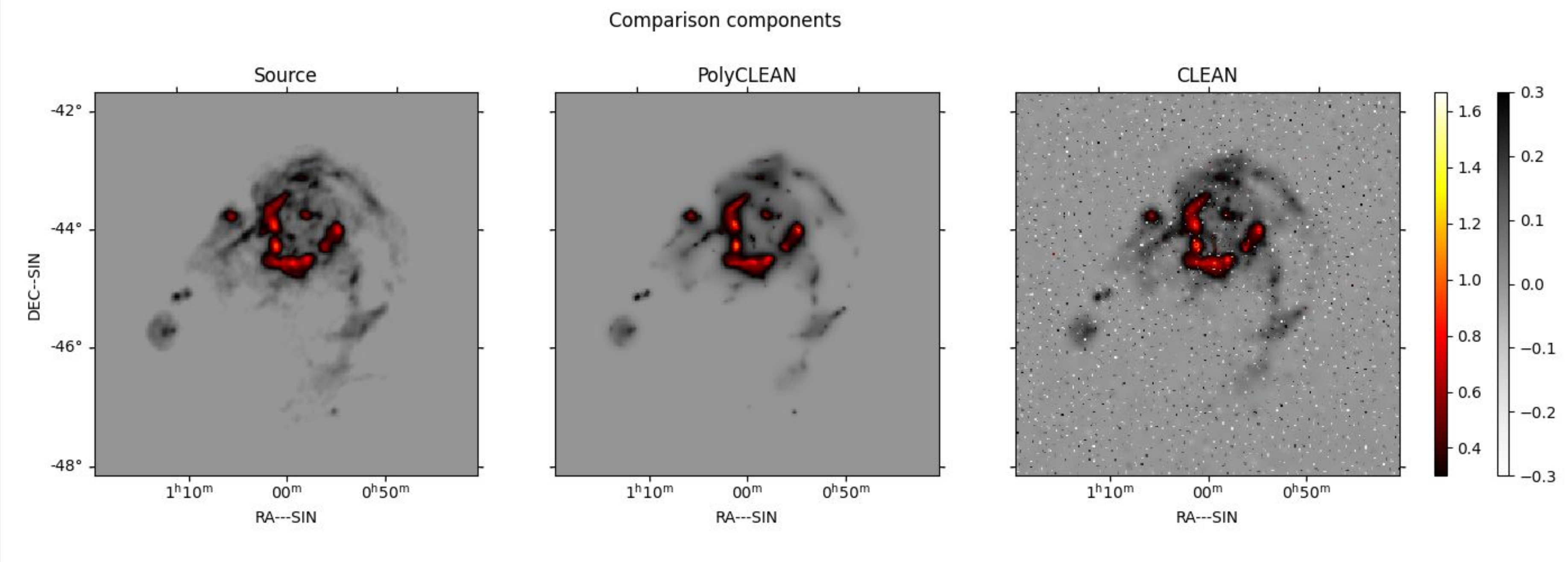
	PolyCLEAN	CLEAN
MSE:	5.5 x 10^-2	1.4 x 10^-2
MAD:	1.9 x 10^-1	8.9 x 10^-2

Reconstructions (without res.)



MSE:	1.2 x 10^-2	MSE:	1.0 x 10^-2
MAD:	4.6 x 10^-2	MAD:	7.5 x 10^-2

Reconstructions (model)



	PolyCLEAN	CLEAN
MSE:	1.5 x 10^-4	1.2 x 10^-3
MAD:	4.3 x 10^-3	1.2 x 10^-2

Summary

1. Numerical performance

- Scalability achievement
- Optimization method up to speed with atomic method
- Sparsity-based method

2. Versatility

- Adaptable:
  - Tune the parameters
  - Fine control
- Point and extended sources

3. Ongoing research work

- **Dual certificate**: promising new scientific tool for RI image reconstruction
- Room for improvement in the code as well as in the algorithm
- Bayes estimation of the parameters

Thanks!

	CLEAN	MAP Estimation	PolyCLEAN
Sparse iterates	✅	❌	✅
Flexible priors	~	✅	~
Fast solvers	✅	~	✅
Calibration compliant	✅	❌	✅
Interpretable obj. function	❌	✅	✅

Multi-Scales dictionaries

Redundant dictionary:

$$\boxed{\mathbf{\Psi} = [\mathbf{\Psi}_1 \ldots \mathbf{\Psi}_D]} \in \mathbb{R}^{N \times DN}$$

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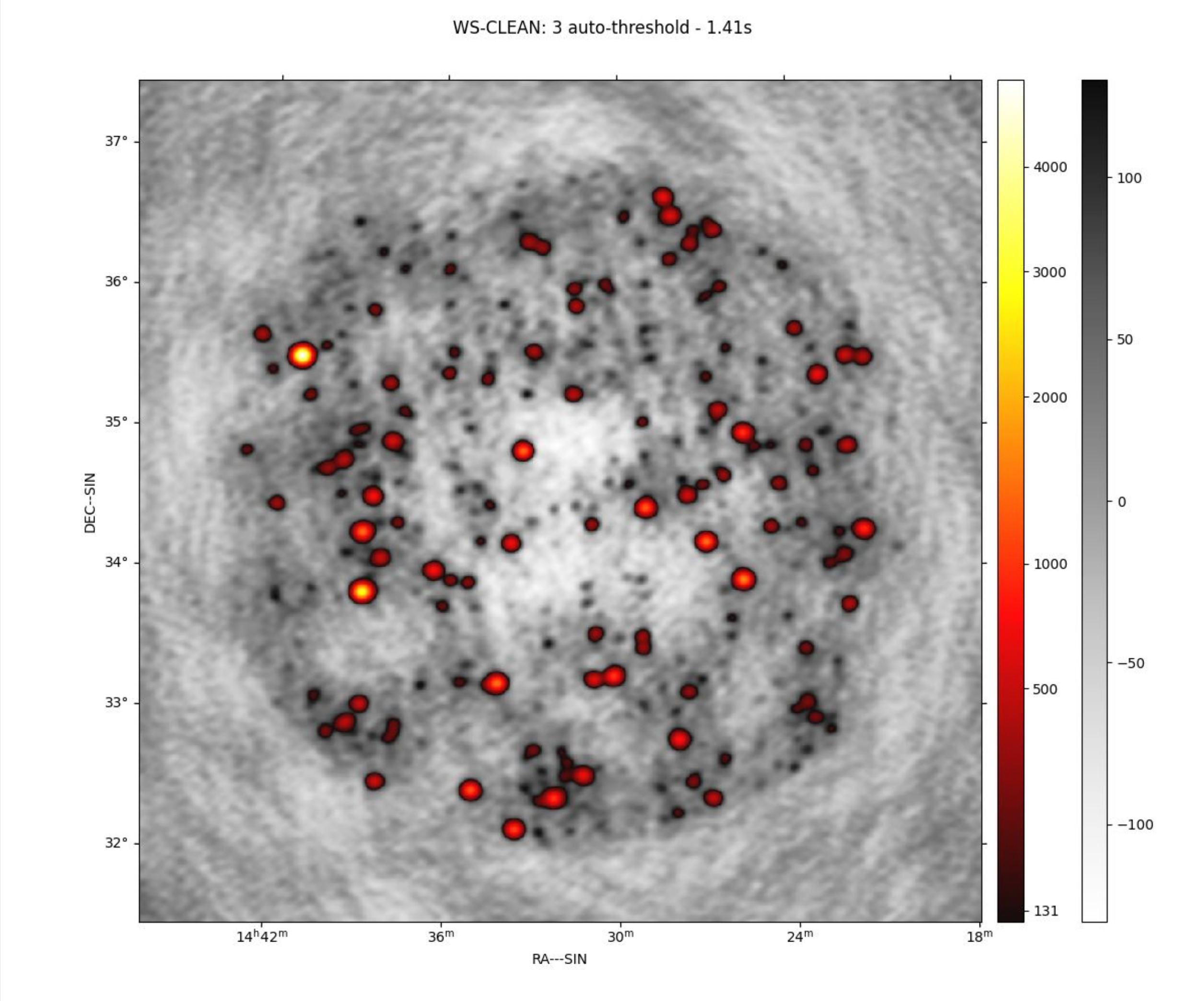
Redundant dictionary:

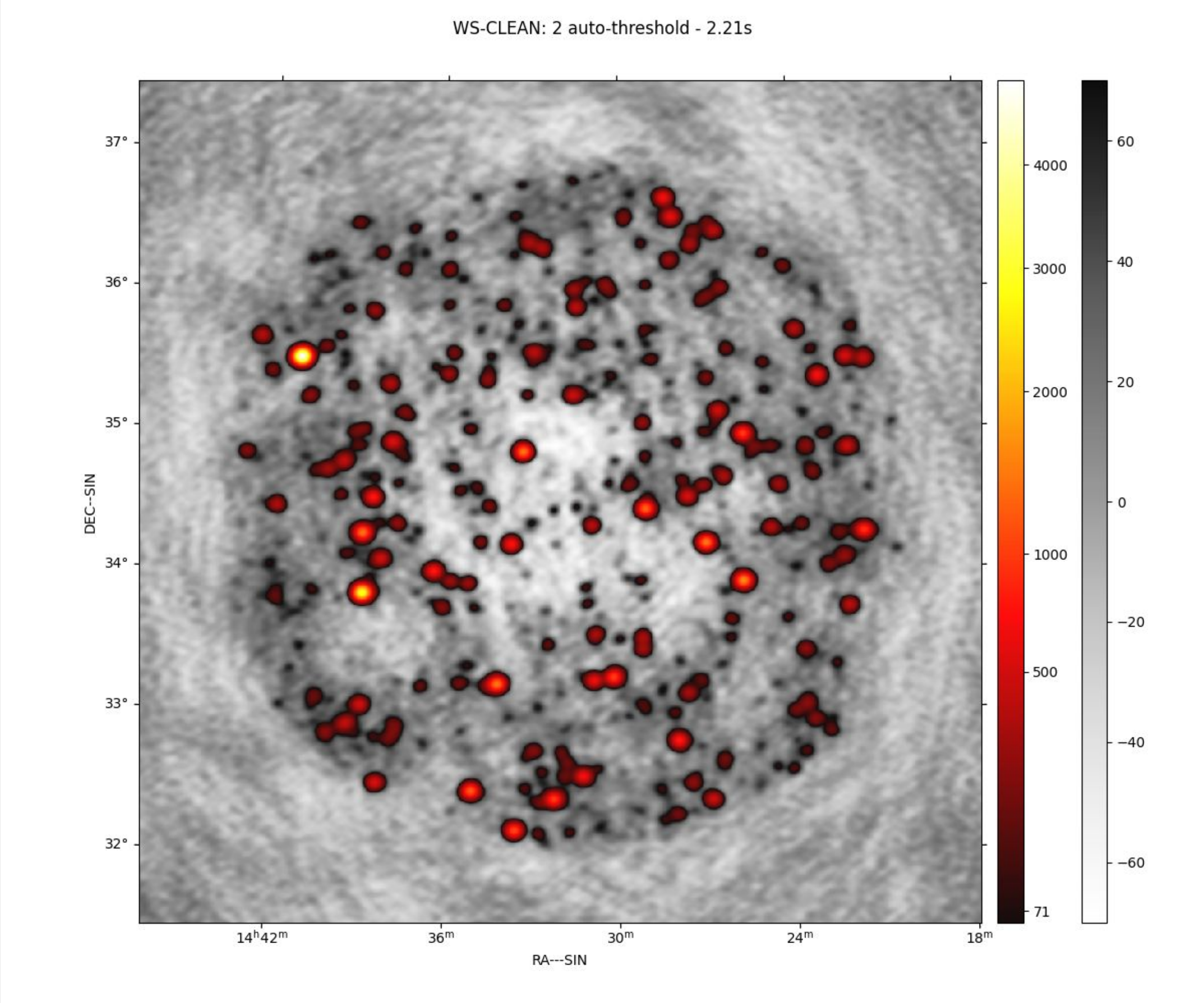
$$\boxed{\boldsymbol{\Psi} = [\boldsymbol{\Psi}_1 \ldots \boldsymbol{\Psi}_D]} \quad \in \mathbb{R}^{N \times DN}$$

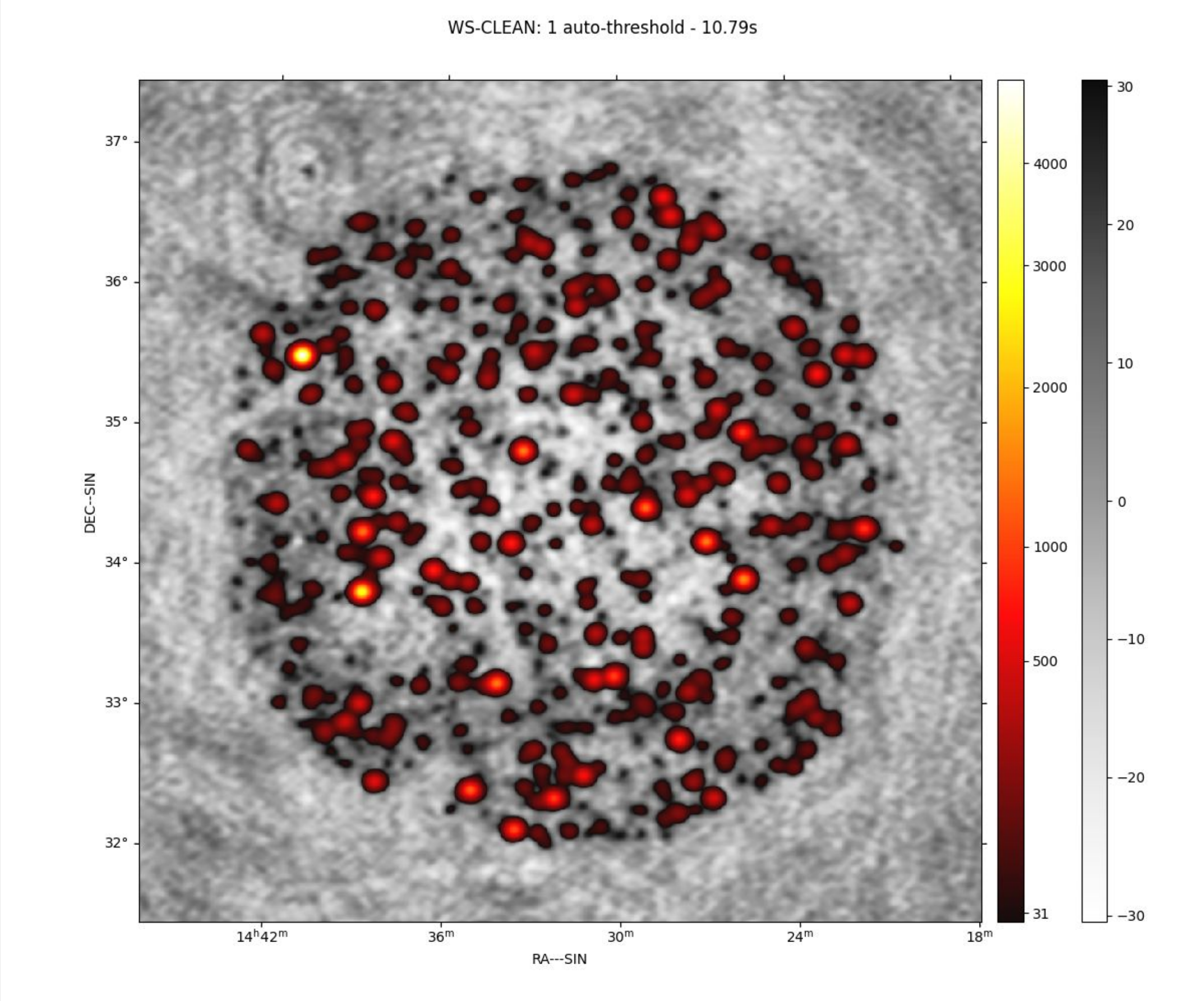
- Wavelets: $$\boldsymbol{\Psi}_d = \mathbf{W}_d \qquad \in \mathbb{R}^{N \times N}$$
- Gaussian kernels: $$\boldsymbol{\Psi}_d \boldsymbol{\theta}_d = \boldsymbol{\theta}_d * \mathbf{g}_{\sigma_d} \qquad \in \mathbb{R}^{N \times N}$$

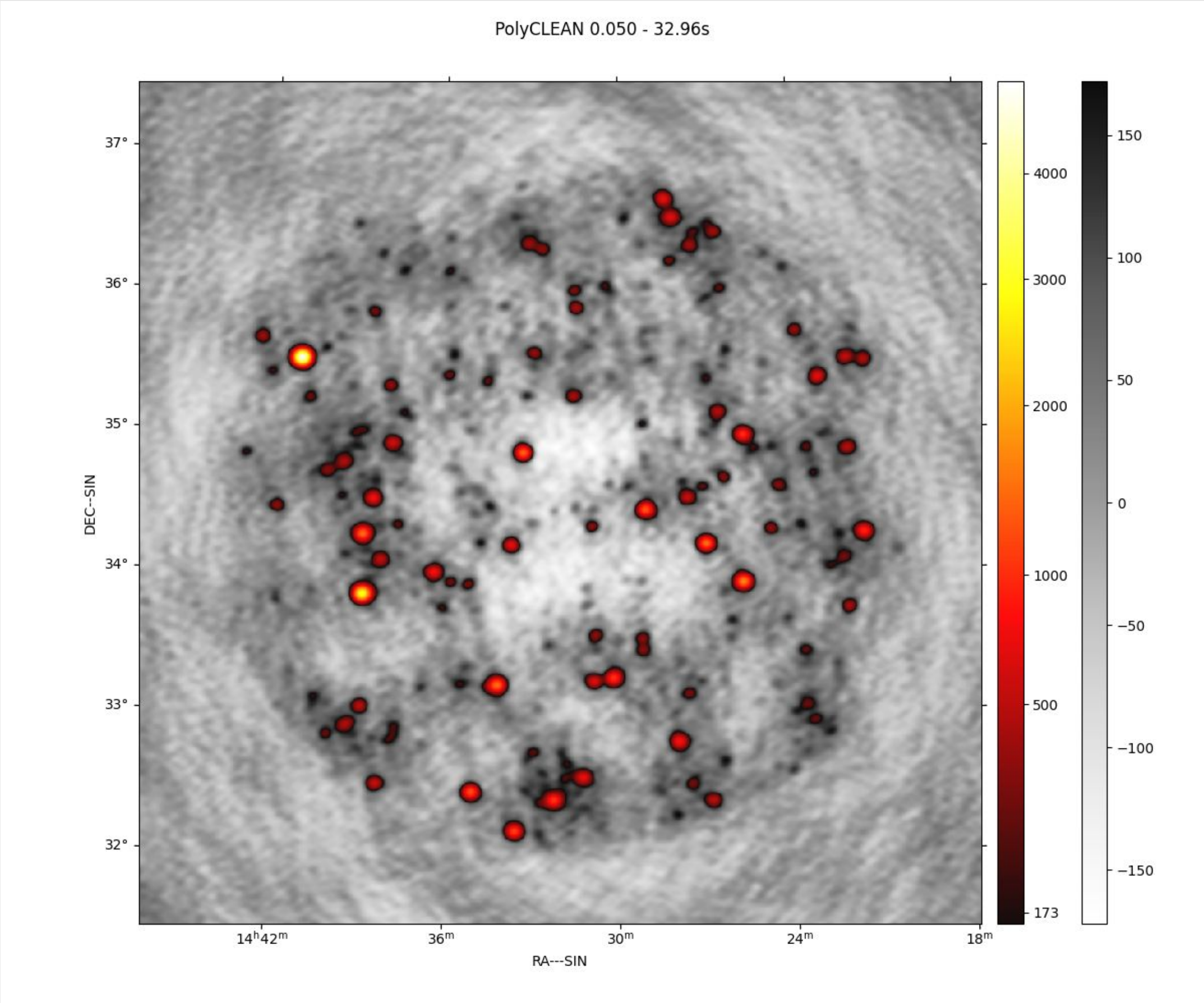
A Frank-Wolfe Solver

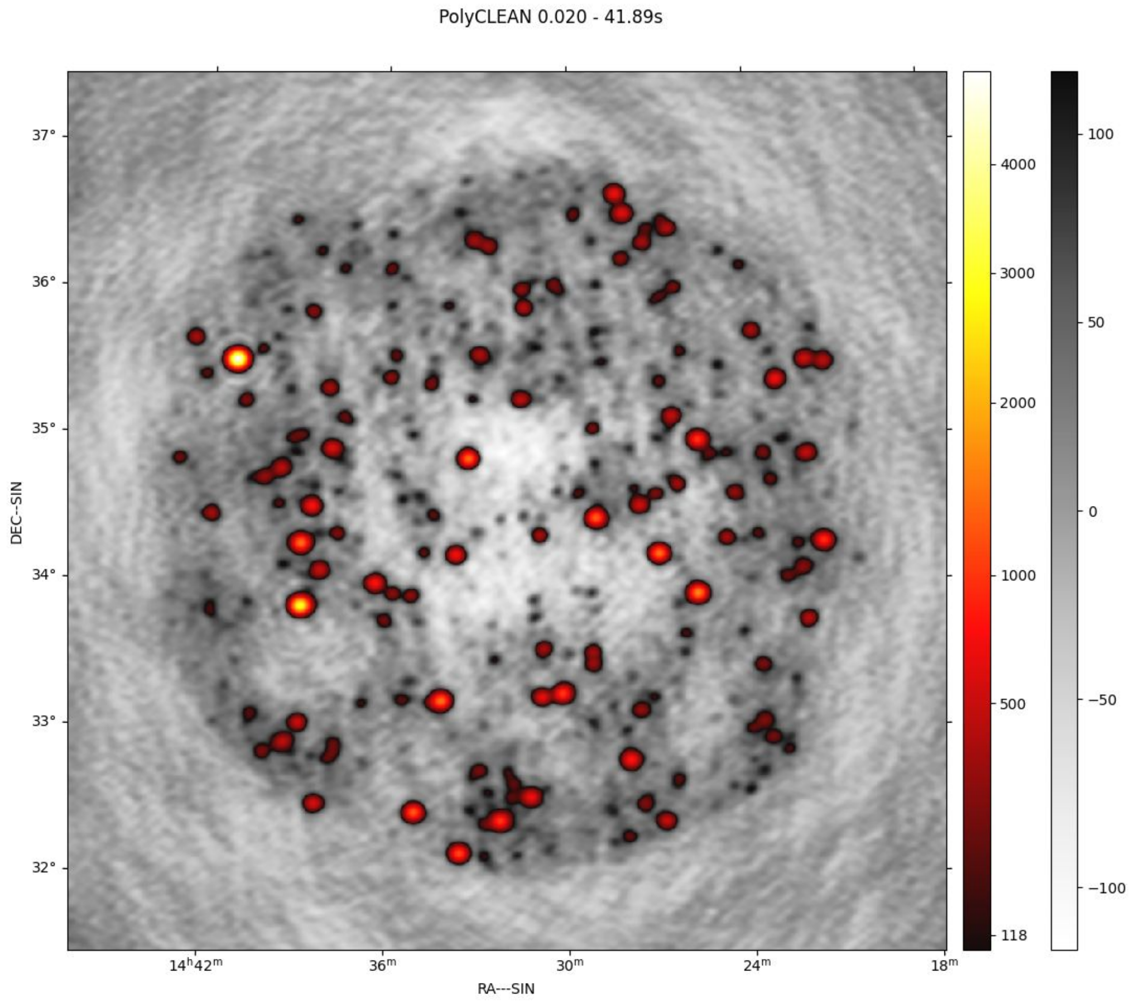
- Convex optimization method:
  - → Convergence guarantee
- Frank-Wolfe for atomic norm:
  - → Atomic behavior
  - → Sparse iterates
- Polyatomic variation:
  - → Fast solver

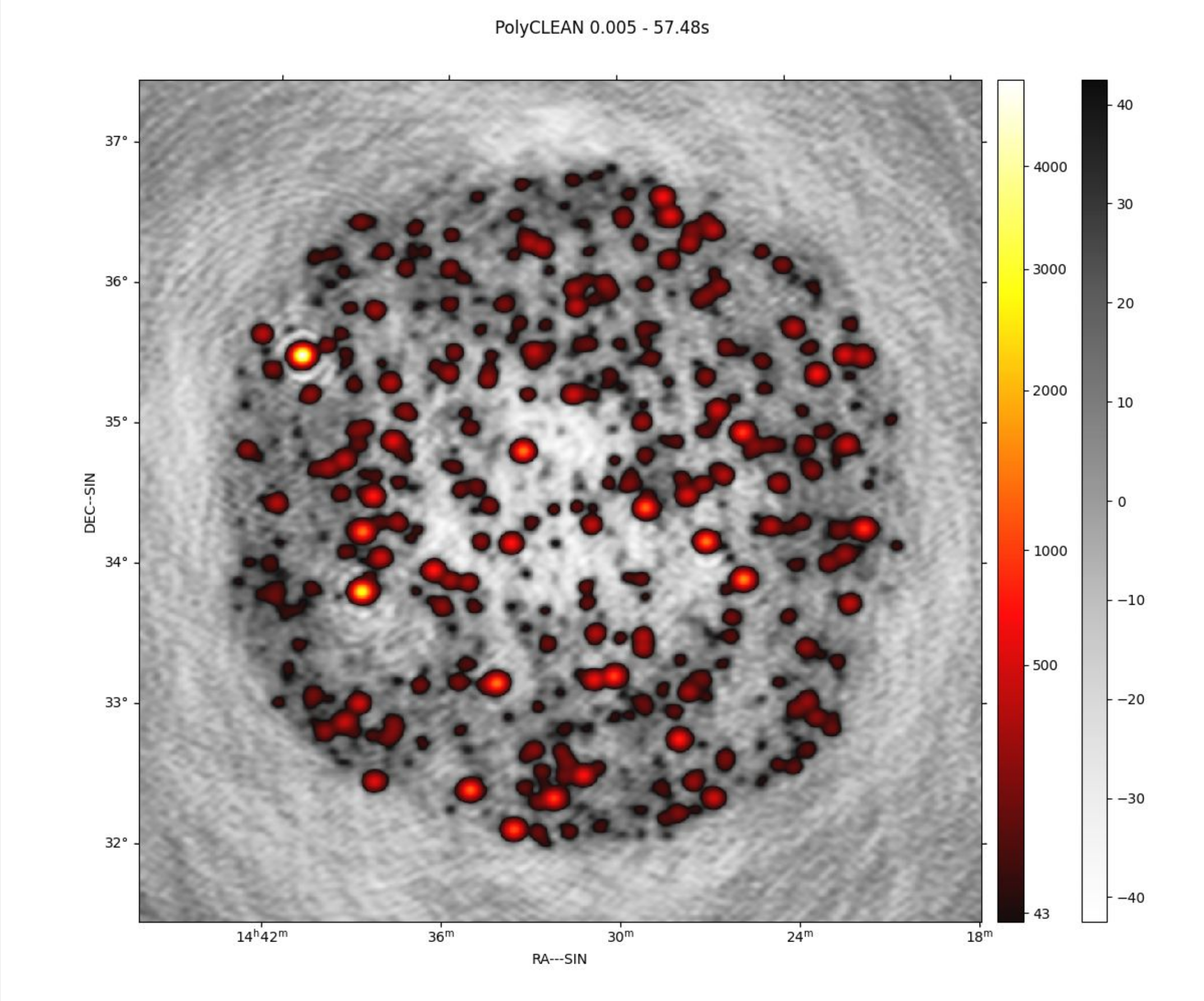

WS-CLEAN: 3 auto-threshold - 1.41s
DEC--SIN
RA---SIN
37°
36°
35°
34°
33°
32°
14h42m
36m
30m
24m
18m
4000
3000
2000
1000
500
131
100
50
0
−50
−100

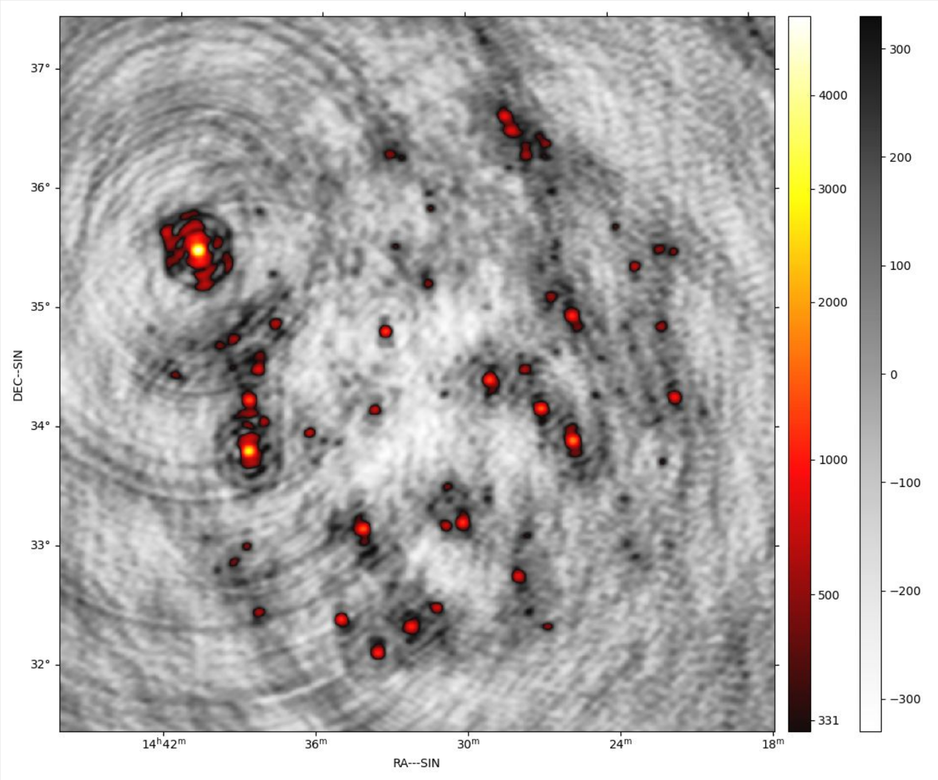

WS-CLEAN: 2 auto-threshold - 2.21s
37°
36°
35°
34°
33°
32°
DEC--SIN
14h42m
36m
30m
24m
18m
RA---SIN
4000
3000
2000
1000
500
71
60
40
20
0
−20
−40
−60

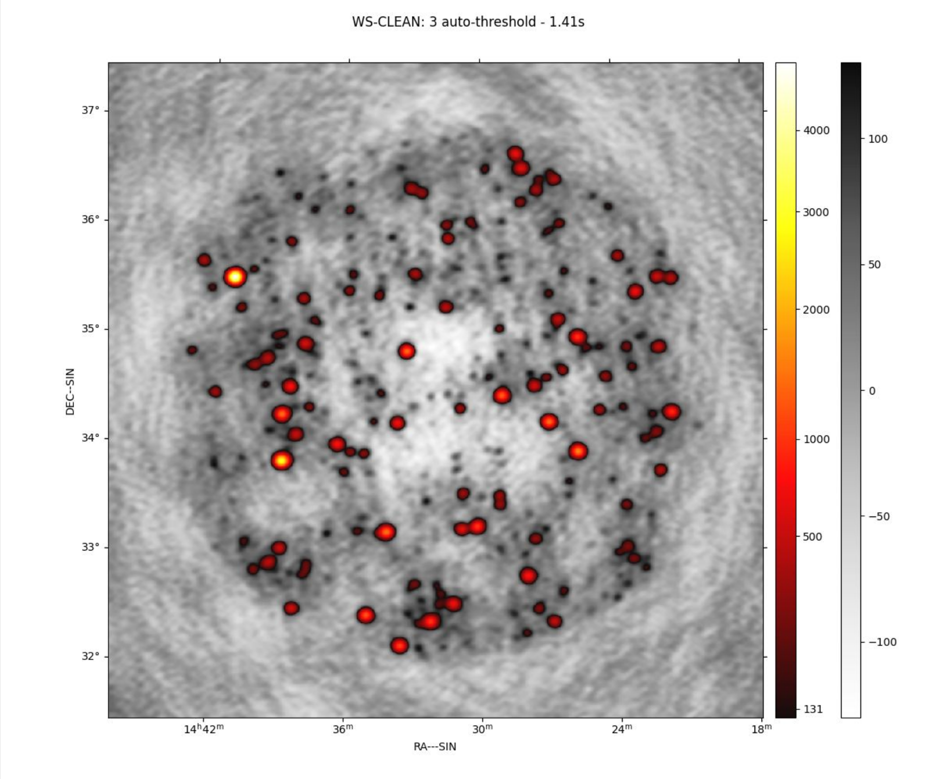

WS-CLEAN: 1 auto-threshold - 10.79s
DEC--SIN
RA---SIN
37°
36°
35°
34°
33°
32°
14h42m
36m
30m
24m
18m
4000
3000
2000
1000
500
31
30
20
10
0
−10
−20
−30


PolyCLEAN 0.050 - 32.96s
DEC--SIN
37°
36°
35°
34°
33°
32°
RA---SIN
14h42m
36m
30m
24m
18m
4000
3000
2000
1000
500
173
150
100
50
0
−50
−100
−150

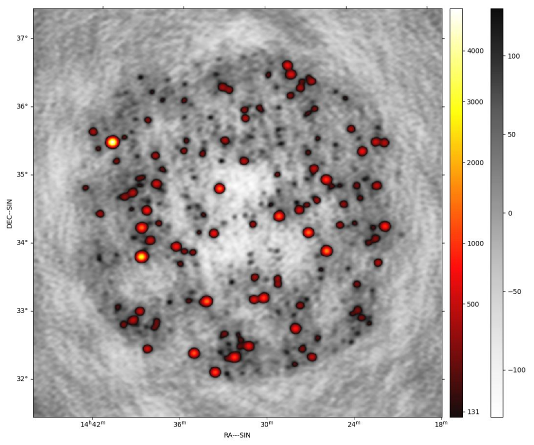

PolyCLEAN 0.020 - 41.89s
37°
36°
35°
34°
33°
32°
DEC--SIN
14h42m
36m
30m
24m
18m
RA---SIN
4000
3000
2000
1000
500
118
100
50
0
−50
−100


PolyCLEAN 0.005 - 57.48s
37°
36°
35°
34°
33°
32°
DEC--SIN
14h42m
36m
30m
24m
18m
RA---SIN
4000
3000
2000
1000
500
43
40
30
20
10
0
−10
−20
−30
−40

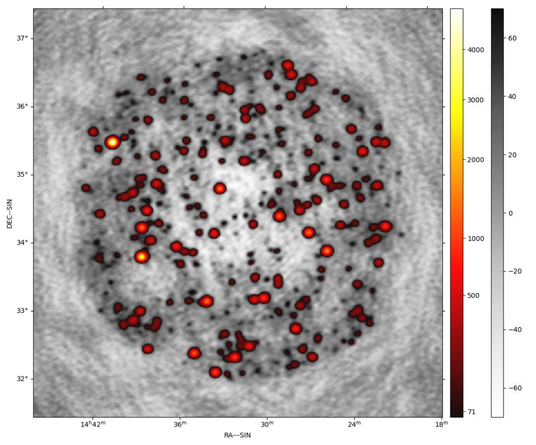

37°
36°
35°
34°
33°
32°
DEC--SIN
14h42m
36m
30m
24m
18m
RA---SIN
4000
3000
2000
1000
500
331
300
200
100
0
−100
−200
−300


WS-CLEAN: 3 auto-threshold - 1.41s
37°
36°
35°
34°
33°
32°
DEC--SIN
14h42m
36m
30m
24m
18m
RA---SIN
4000
3000
2000
1000
500
131
100
50
0
−50
−100

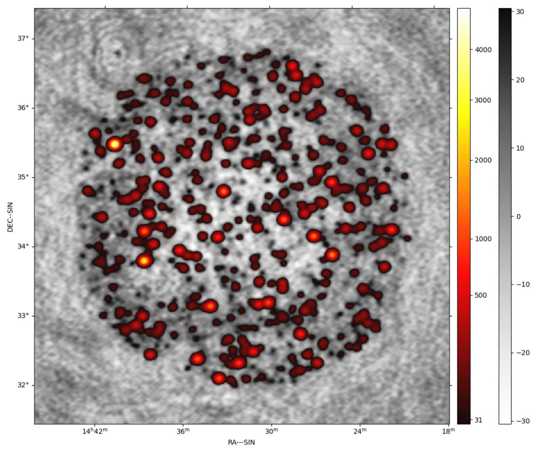
WS-CLEAN: 3 auto-threshold - 1.41s



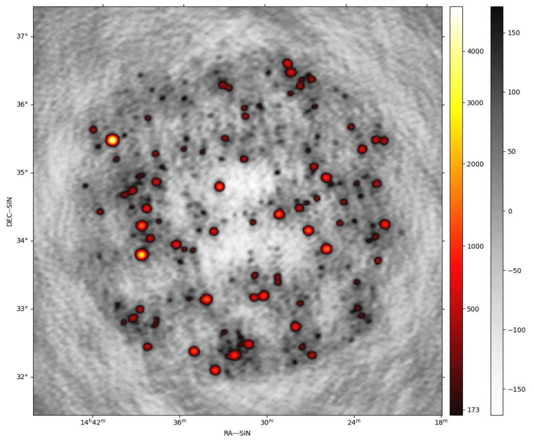
WS-CLEAN: 2 auto-threshold - 2.21s



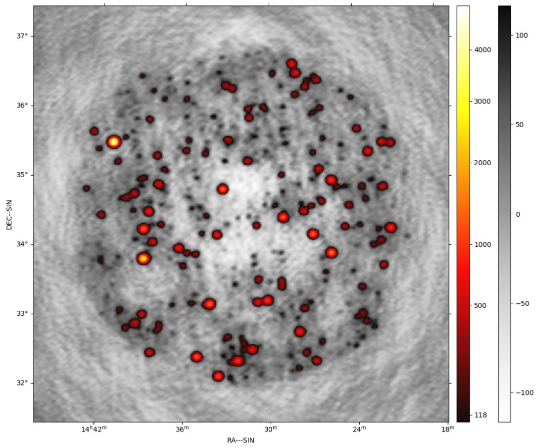
WS-CLEAN: 1 auto-threshold - 10.79s



PolyCLEAN 0.050 - 32.96s



PolyCLEAN 0.020 - 41.89s



PolyCLEAN 0.005 - 57.48s