ON THE SCALING OF COHERENCE TIMES, BY TH TAMINIAU

This document gives an overview of the values and scaling of coherence times with ^{13}C concentration. The goal is to determine realistic maximum vales for the interpulse delay τ and the total gate time, and minimum vaues for the hyperfine interaction that can still be used.

What sets the limits?

Our gate fidelities are limited in three ways. First, the gates themselves are not perfect because the operation performed is only approximately ideal (e.g. the nuclear spin does not rotatate by $\pi/2$ exactly) and the electron also can entangle with other spins (e.g. other nuclear spins also undergo a conditional evolution. These effects are included in our simulations (electron-nuclear interaction).

Second, nuclear-nuclear interactions cause the nuclear spin bath to evolve so that the electron spin feels a fluctuating magnetic field. We decouple the electron from this field through the pi-pulses, but if the finite time 2τ between the pulses sets a limit to how well this works. Very slow, quasistatic, fluctuations (i.e. $T^*_{2,e}$) are decoupled by a single echo, but faster fluctuations give rise to a $T_{2,e}$ time. These effects are not included in our simulations (nuclear-nuclear interaction) and thus set a manual limit $T_{coh,e}(\tau)$ that also limits τ_{max}.

Third, the nuclear-nuclear interactions dephase the nuclear spins. As we are not doing any echoes on the nuclear spins the nuclear coherence time is set by $T^*_{2,n}$ for the nuclear spins. This effect is not taken into account in our simulations and sets a manual limit for the total maximum gate time.

Our standard concentration $\mu = 1.1\%$

For a concentration of $\mu = 1.1\%$ we typically have $T^*_{2,e} = 3$ μs and $T_{2,e} = 200$ μs. The nuclear $T^*_{2,n}$ can be estimate from the ratio of the gyromagnetic ratios so that:

$$T^*_{2,n} \approx \frac{\gamma_e}{\gamma_n} T^*_{2,e} \approx 8ms \tag{1}$$

with $\gamma_e \approx 2.8$ MHz/G and $\gamma_n \approx 1.07$ kHz/G. $T^*_{2,n}$ limits the total time of the experiment. Assuming we want to be able to perform at least 4 of the longest gates the maximum gate time is $T_{gate,max} = 2$ ms. Realistically, and for high fidelities, we will need more gates, e.g. $T_{gate,max} = 0.5$ ms for 10 gates.

The electron coherence as the number of pulses during dynamical decoupling is given by:

$$S = e^{-(\frac{2\tau}{T_{2,e}})^3}, \tag{2}$$

So that the $1/e$ time $t_{1/e} = 2\tau(\frac{T_{2,e}}{2\tau})^3$. We want this time to be larger than $T^*_{2,n} \approx 8$, which means that $\tau_{max} \leq 16$ μs. For the electron coherence to be 10 times larger than $T^*_{2,n}$, we need $\tau_{max} \leq 5$ μs. There remains the problem that if the gates are much faster (e.g. at low magnetic fields) so that the total experiment is much shorter we might not need the full coherence time and τ can be taken longer.

The minimum hyperfine strength A that can be used is more tricky to determine. An absolute minimum seems to be set by that the total gate time will not be faster than $\sim 1/A$, suggesting that a hyperfine strength of at least 2 kHz is required.

Scaling with concentration

TBD