

Abstract

Contents

1	Introduction	5
1.1	Quantum Computing	5
1.2	Quantum Error Correction	5
1.3	Weakly coupled carbons; a naturally occurring register	5
2	Electronic spins in Diamond	7
2.1	Spin Control	7
3	Addressing Weakly-coupled Carbon Spins	9
3.1	Coupling to the Environment	9
3.1.1	Coupling regimes	9
3.1.2	The Hyperfine Interaction	10
3.1.3	Strongly-coupled spins	11
3.1.4	Weakly-coupled carbon spins	11
3.1.5	Extending electron coherence	12
3.1.6	Dynamical decoupling spectroscopy	12
3.2	Addressing Weakly-coupled Carbons through Dynamical Decoupling	12
3.2.1	The effect of dynamical decoupling	13
3.2.2	Response of an isolated carbon to dynamical decoupling spectroscopy	14
3.2.3	Characterizing the Nuclear-spin environment	14
3.2.4	Effect of the magnetic field	15
3.2.5	Identifying Individual Carbon-spins	15
4	Controlling Weakly-coupled Carbon Spins	17
4.1	Carbon control (more of a theory chapter, change title)	17
4.1.1	Carbon Ramsey experiment	17
4.1.2	Measuring Precession Frequencies/Carbon Ramsey experiment	17
4.2	Controlling weakly coupled carbons trough the electronic spin	17
4.2.1	Initialized Carbon Ramsey	18
4.3	Carbon Initialization & Readout	18
5	Deterministic Parity Measurements	19
5.1	Entanglement	19
5.2	Verification of Entanglement	19
6	Outlook: towards Quantum Error Correction	21
A	Fingerprintdata	25
B	State Initialization	27
C	Bell State Tomography	29

D	Entanglement witness	31
E	Simulations and Calculations for Number of Addressable Carbons	33
E.1	Scaling of number of resonances with magnetic field	33
F	Constants and Experimental values	35

1.1 Quantum Computing

The idea of using a quantum mechanical system to simulate physics was first explored by Feynman[6]. Because the Hilbert space(/state space?) of a quantum mechanical system scales exponentially with its size one would need an exponentially large classical computer to simulate its behavior. By manipulating a quantum mechanical system directly this scaling problem can be circumvented.

It was quantum simulation that eventually led to the idea of exploiting quantum effects to perform more efficient calculations but it wasn't until Shor's discovery of a remarkably efficient quantum algorithm for prime factorization in 1994[13] that quantum information science really took off.

Shor's algorithm was the first example where a quantum computer can provide an exponential speedup over a classical computer. Shor's and other quantum algorithms allow solving classes of problems that were previously unsolvable, a well known example being the breaking of classical encryption codes.

By now Shor's algorithm has been shown to work on a range of different small scale quantum computers [19] [Needs reference to Shor in different systems or basic algorithms in range of systems] but making a scalable quantum computer that can take full advantage of the exponential speedup proves elusive.

1.2 Quantum Error Correction

1.3 Weakly coupled carbons; a naturally occurring register

The Nitrogen Vacancy centre in diamond is a well investigated system[5] and a promising candidate for quantum computation[2]. In order to implement three qubit measurement based QEC we need three qubits plus ancillae that we can initialise, measure and conditionally perform operations on. These extra qubits are found in Carbon-13 atoms, which are normally a source of decoherence. These atoms can be addressed using a resonant decoupling sequence[15].

Electronic spins in Diamond

It has been shown that the nuclear- and electron- spin-state of the NV- center can be initialized, controlled and read-out using microwave- and laser- pulses[12]. In these experiments two lasers that are resonant with transitions in the NV- center are used to initialize the electronic spin state. One of these two lasers is used to read out the electronic spin state and an off-resonant laser is used to reset the system. Microwaves are used to drive transitions between the different nuclear and electronic spin states.

Strongly coupled nuclear spins can be initialized by conditionally rotating the electronic state to a state that is read out only if the Carbon is in the desired state, when the electronic state readout has a positive result the system is projected into the desired state. We call this Measurement Based Initialization (MBI). Our experiments are build around the same basic tools. Each experiment starts with a Charge-Resonance check that verifies if the lasers are still on resonance. After that the Nitrogen spin state is initialized using MBI. Once the system is initialized the actual experiment is performed. An experiment consists of one or multiple blocks of microwave pulses and optical readouts.

All experiments were performed on a custom-build cryostat setup operating at liquid helium temperatures described in detail in Bernien [1, chap. 3]. The setup was additionally outfitted with a movable neodymium magnet that applied a magnetic field of 300G to the sample.

2.1 Spin Control

The electronic ground state Hamiltonian can be written as[11]:

$$H_{GS} = \Delta S_z^2 + \gamma_e \mathbf{B} \cdot \mathbf{S} \quad (2.1)$$

With zero field splitting $\Delta \approx 2.88\text{GHz}$ and gyro-magnetic ratio $\gamma_e = 2.802 \text{ MHz/G}$. In this expression the interactions with the nitrogen nucleus and the carbon spin bath are not included. By applying a magnetic field B_z along the NV-axis the degeneracy of the $m_s = \pm 1$ states is lifted by the Zeeman effect. We define our electronic qubit by the two level system with $m_s = 0 := |0\rangle$ and $m_s = +1 := |1\rangle$.

On the Bloch-sphere the state vector rotates around the quantization-axis with a frequency depending on the energy splitting between the two states; the Larmor frequency. For the NV-electronic spin transition used the Larmor frequency is given by eq. (2.2).

$$\omega_L = \Delta + \gamma_e B_z \quad (2.2)$$

By applying an external field a term is effectively added to the Hamiltonian, changing the quantization-axis and thereby its evolution. By applying microwaves with the right frequency this can be used to selectively drive the transition from the $|0\rangle$ state to the $|1\rangle$ state[10].

Addressing Weakly-coupled Carbon Spins

The electronic-spin of the NV-center does not live in a vacuum but in an environment full of nuclear spins. To some of these spins the NV-center couples strongly, these spins can be controlled and can serve as qubits. To others it couples weakly these spins are a source of decoherence and cannot be controlled directly.

In this chapter I will first explain what strong and weakly coupled spins are, how this relates to coherence and how coherence can be extended through dynamical decoupling. In the second part of this chapter I will explain how dynamical decoupling can be used to identify and control some of these weakly coupled spins, transforming them from a source of decoherence to a resource for qubits.

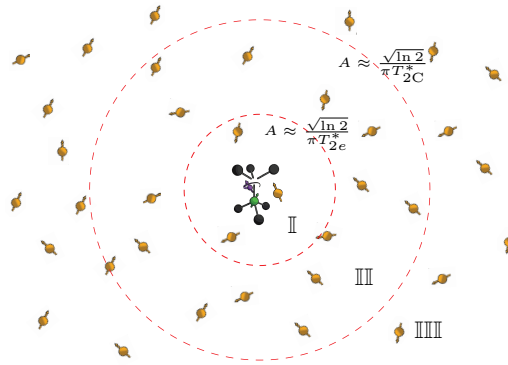


Figure 3.1 – Schematic representation of different coupling regimes. In the strong coupling regime (region I) carbon-spins are coupled to the NV-center stronger than the coupling of the NV-center to the spin-bath. These carbons can be addressed directly. In the weak coupling regime (region II) carbon-spins are coupled more strongly to the NV-center than to the spin-bath but not strong enough to be addressed directly. In the very-weak coupling regime (region III) the coupling to the spin-bath is stronger than the coupling to the NV-center. These spins cannot be addressed.

3.1 Coupling to the Environment

3.1.1 Coupling regimes

When addressing a spin qubit we usually drive transitions between energy levels. When the electronic-spin couples to a nuclear-spin these energy levels are shifted depending on the state of the nucleus. In this way each spin has an individual back-action on the electronic-spin.

Most of these spins only shift the spin by a tiny amount making it hard to distinguish them from each other. Because the states of these spins fluctuate the transitions of the NV-center are continuously shifting around, effectively causing a broadening of the transitions. We call these very-weakly-coupled spins the spin-bath.

If the coupling between a spin and the NV-center is strong enough it is possible to resolve and address its transitions. Transitions between energy levels can be resolved if the difference between

them is larger than the width of transition.

The width of the transitions is related to the rate at which the transitions shift out of resonance. The time before a transition shifts out of resonance, or a signal decoheres, is usually expressed by T_2^* and is measured by a Ramsey experiment, see fig. 3.2.

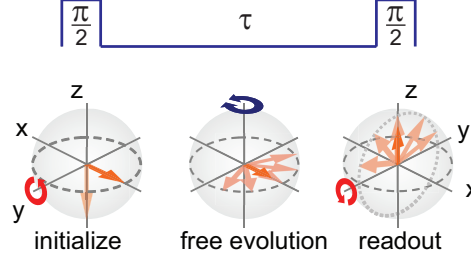


Figure 3.2 – In a Ramsey experiment a qubit is initialized along the z-axis before being subjected to a $\pi/2$ -pulse that moves it into the xy plane of the Bloch-sphere. It freely evolves for a time τ before being subjected to a final $\pi/2$ pulse and read out along the z-direction. The final pulse will rotate the spin towards the poles depending on the phase picked up during free evolution. This manifests itself as an oscillation. This oscillation will slowly decay because the final pulse will be more out of resonance with the transition the longer the free evolution time. Figure from [3].

The decay of the amplitude K in an electron Ramsey experiment is given by eq. (3.1).

$$K(t) = e^{-\left(\frac{\tau}{T_{2e}^*}\right)^2} \quad (3.1)$$

We define T_{2e}^* as the $1/e$ value of the Gaussian decay. The dark Electron-Spin-Resonance (ESR)¹ frequency spectrum for negligible power broadening is given by the Fourier transform of eq. (3.1). Where $\omega = 2\pi \cdot f$ ².

$$\mathcal{F}\{K(\tau)\} = T_{2e}^* \sqrt{\pi} e^{-\frac{(2\pi \cdot f)^2 \cdot T_{2e}^{*2}}{4}} \quad (3.2)$$

Two identical Gaussians can be readily resolved if the separation between their maxima is larger than their full-width-half-maximum (FWHM). The FWHM of the dark ESR is given by eq. (3.3).

$$2\pi \cdot \text{FWHM} = 2\pi \cdot \frac{2\sqrt{\ln 2}}{\pi T_{2e}^*} \quad (3.3)$$

We define a spin to be strongly coupled when it is possible to readily resolve its transition. For a carbon-spin that is when twice the interaction strength A [NOTE: $|A|$? A par? shift due to A?] is larger than the FWHM of the transition:

$$2\pi \cdot A > 2\pi \cdot \frac{\sqrt{\ln 2}}{\pi T_{2e}^*} \quad (3.4)$$

NV-centers have a typical electron $T_{2e}^* \approx 2\mu\text{s}$ ³ that depends on the exact configuration of the spin-environment. Increasing the carbon-13 concentration generally reduces T_{2e}^* . For a typical NV-center this means that the coupling between the carbon and the NV-center must be larger than $2\pi \cdot 130\text{kHz}$ for the carbon to be directly addressable.

3.1.2 The Hyperfine Interaction

The coupling between the NV-centers electronic spin and a nuclear-spin is given by the hyperfine-interaction. The hyperfine-interaction is a spin dependent interaction that is not present for spin-0 particles such as carbon-12.

For nuclear spins the Hamiltonian depends on the electronic spin-state of the NV-center. For a magnetic field (B_z) in the z-direction the Hamiltonian is given by eq. (3.5) when the electronic-spin is in the $m_s = 0$ state, and by eq. (3.6) when in the $m_s = +1$ state[17]. Where γ_n is the gyro-magnetic ratio of the nucleus.

$$H_0 = \gamma_n B_z I_z \quad (3.5)$$

¹See fig. 3.3 for an example of a dark ESR.

²For clarity the factor of 2π is stated explicitly throughout this thesis to distinguish real and angular frequency.

³At a natural concentration of 1.1% carbon-13.

$$H_1 = \gamma_n B_z I_z + H_{\text{HF}} \quad (3.6)$$

The Larmor frequency for a nucleus is given by eq. (3.7).

$$\omega_L = \gamma_n B_z \cdot \hat{\mathbf{z}} \quad (3.7)$$

The hyperfine (H_{HF}) term consists of a contact term and a dipole term. The contact term results from an overlap between the electronic- and nuclear- wave-functions making it negligible for all but the nuclear-spins closest to the NV-center.

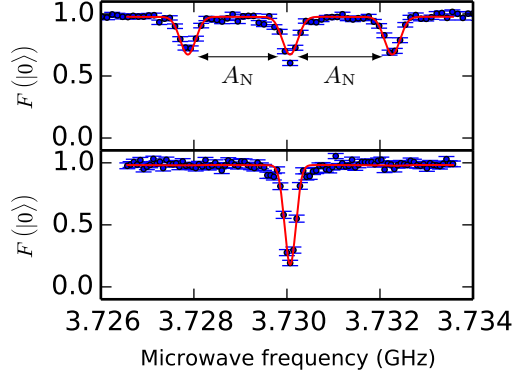


Figure 3.3 – dark Electron Spin Resonance (ESR) for uninitialized (top) and initialized nitrogen spin (bottom) of the $m_s = 0 \rightarrow m_s = +1$ transition. In a dark ESR the spin is prepared in $|0\rangle$, a microwave pulse is applied and then the $|0\rangle$ state is read-out again. This is done for a range of frequencies. When the microwave is on resonance with a transition the spin will be rotated and a dip will be visible in the signal. In the top figure the transition is split due to the interaction with the nearby nitrogen atom. In the lower figure the nitrogen state is initialized and the splitting disappears.

3.1.3 Strongly-coupled spins

An example of a strongly coupled spin is the NV-nitrogen spin. The strength of the coupling between the nitrogen and the electronic spin is $A_N = 2\pi \cdot 2.186$ MHz[1] and it acts along the NV-axis. Figure 3.3 clearly shows the $m_s = 0 \rightarrow m_s = +1$ transitions being split due to hyperfine-coupling to the nitrogen. This splitting can be used to control the nitrogen. By first initializing in the $m_s = +1$ -state and then applying a slow pulse that turns only one of the three nitrogen spin-states back to $m_s = 0$ and reading out the nitrogen-spin can be initialized. By only continuing on a positive outcome the spin is initialized in the state that was rotated back to $m_s = 0$. We call this measurement-based-initialization (MBI). The lower panel of fig. 3.3 shows a dark Electron-Spin-Resonance (ESR) after nitrogen-MBI.

In a similar way strongly-coupled carbon-13 atoms can be controlled[12]. For most strongly-coupled carbons the contact term in the hyperfine is not negligible. For these carbons hyperfine couplings have been measured[14] and calculated[7, 8].

3.1.4 Weakly-coupled carbon spins

For carbons where the contact term is negligible the hyperfine-term is equal to the dipole term and is given by eq. (3.8)[3]. With \mathbf{n}_{HF} is a unit vector pointing from the NV-center to the nucleus and r the distance between them. \mathbf{S} and \mathbf{I} are the spin operators for the NV-spin and the nucleus, γ_e the gyromagnetic ratio of the electron, γ_n the gyromagnetic ratio of the nucleus, and μ_0 the magnetic permeability. For weakly-coupled carbons the contact-term of the hyperfine is generally⁴ negligible.

$$H_{\text{dip}} = \frac{\mu_0 \gamma_e \gamma_n \hbar^2}{4\pi r^3} [\mathbf{S} \cdot \mathbf{I} - 3(\mathbf{S} \cdot \hat{\mathbf{n}}_{\text{hf}})(\mathbf{I} \cdot \hat{\mathbf{n}}_{\text{hf}})] \quad (3.8)$$

⁴Only under exceptionally high carbon-13 concentrations is T_{2e}^* short enough for close by carbon-13 atoms to fall under the definition of weakly coupled.

From eq. (3.8) the parallel and orthogonal components of the Hyperfine interaction, with respect to the NV-axis along the z-direction, can be derived to be:

$$A_{\parallel} = -\frac{\mu_0\gamma_e\gamma_C\hbar^2}{4\pi r^3} \left(3 \cdot \frac{z^2}{r^2} - 1 \right) \quad (3.9)$$

$$A_{\perp} = -\frac{\mu_0\gamma_e\gamma_C\hbar^2}{4\pi r^3} \left(3 \cdot \frac{\sqrt{x^2 + y^2} \cdot z}{r^2} \right) \quad (3.10)$$

Where $H_{\text{HF}} = A_{\parallel} I_z + A_{\perp} I_x$.

At first sight it seems impossible to control weakly coupled spins as their transitions are obscured by the spin-bath. It is however possible to increase the coherence time of the electron, stabilizing the transitions, allowing more spins to be resolved.

3.1.5 Extending electron coherence

In a spin-echo experiment a Ramsey experiment is performed with a small difference. An additional π pulse is applied in the middle of the experiment exactly between the $\pi/2$ pulses. This pulse effectively turns the nuclear-spin environment upside-down halfway through the experiment, making the quasi-static-part of the dephasing during the first and the second part of the free evolution cancel each other out, substantially increasing coherence on short-timescales ($\tau \ll T_2$)⁵. On longer timescales the difference between the initial part of the evolution and the final part of the evolution becomes larger making the initial and final part no longer cancel each other out.

A natural way of extending the short timescale behavior of the spin-echo to longer timescales is by applying more π -pulses. This procedure known as dynamical-decoupling can dramatically improve coherence times by decoupling the central-spin from the environment[4].

Although dynamical-decoupling improves the coherence of the central spin by decoupling from the environment, the central spin is also decoupled from other spins preventing direct two-qubit gates. It was demonstrated by van der Sar et al. [18] how to incorporate dynamical decoupling in a universal gate design by implementing Grover's algorithm. Using this technique Taminiau et al. [16] used the extended coherence to detect and control weakly-coupled carbon spins, before implementing three-qubit quantum-error-correction (QEC) [17].

As these experiments were performed with NV-centers at Room temperature they lack the option to do single-shot readout required to act on a measurement outcome⁶. An essential feature for the parity measurements that form the basis of measurement-based QEC and surface codes.

3.1.6 Dynamical decoupling spectroscopy

As we cannot perform an ESR experiment while decoupling a different technique must be used to resolve and address additional spins. In order to resolve additional spins we perform a dynamical decoupling spectroscopy, resulting in a fingerprint of the nuclear-spin environment[16].

In a dynamical decoupling spectroscopy experiment the electron is prepared in the $|X\rangle = |0\rangle + |1\rangle$ state. It is subjected to a decoupling sequence consisting of $N/2$ blocks of the form $\tau - \pi - 2\tau - \pi - \tau$, and concluded by measuring $\langle X \rangle$. The fingerprint is the result of many repetitions for a range of inter-pulse delays 2τ . π is a π -pulse.

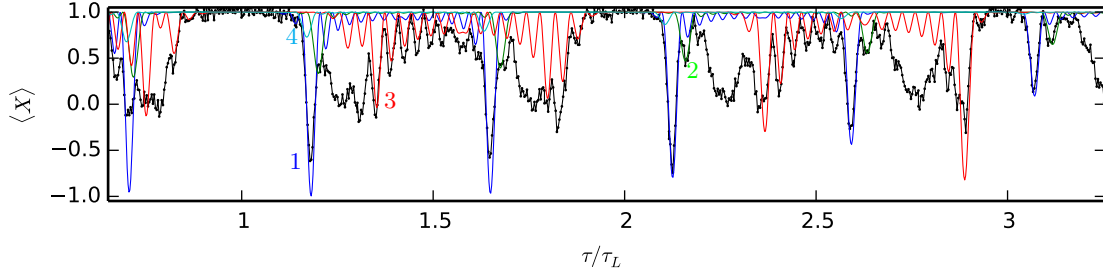
Part of a dynamical decoupling spectroscopy can be seen in fig. 3.4. The spectroscopy was performed for $N = 8, 16, 32$ and 64 pulses. For $N = 8, 16$ and 32 pulses this was done between $\tau = 2\mu\text{s}$ and $72\mu\text{s}$ and for $N = 64$ this was done up to $\tau = 52\mu\text{s}$. A reference to the full spectroscopy can be found in appendix A.

3.2 Addressing Weakly-coupled Carbons through Dynamical Decoupling

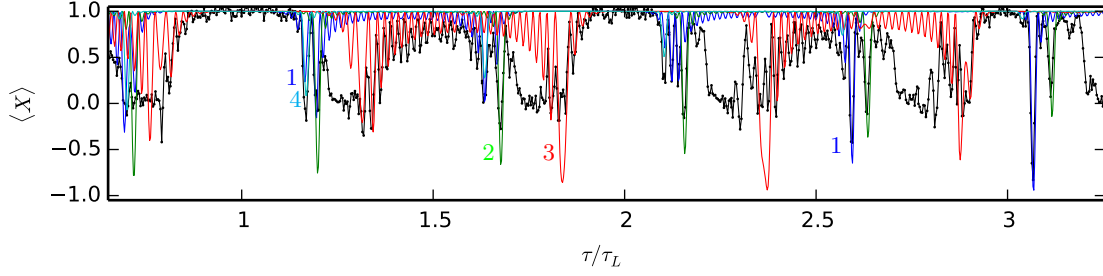
In order to understand how the features in the fingerprint from fig. 3.4 relate to the different spins and how this knowledge can be used to control these carbons it is necessary to understand what dynamical decoupling does to these atoms.

⁵ T_2 is a measure for decoherence. It is similar to T_2^* but does not include variations between experiments. T_2 is defined as the $1/e$ time of the decay of a spin-echo experiment.

⁶@Tim, I think this can be worded more concisely. Do you have any ideas?



(a) Fingerprint for $N=16$ pulses.



(b) Fingerprint for $N=32$ pulses.

Figure 3.4 – Part of a fingerprint resulting from a dynamical-decoupling-spectroscopy experiment performed at 304.12G. A reference to the full spectroscopy can be found in appendix A. Colored lines represent computed responses of carbon spins. Responses were calculated using eq. (3.14) with hyperfine parameters from table 3.1.

3.2.1 The effect of dynamical decoupling

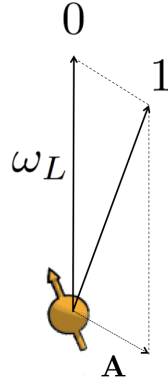


Figure 3.5 – Flipping the electron spin from the $m_s = 0$ to the $m_s = +1$ state changes the quantization axis of nuclear spins. For $m_s = 0$ all nuclear spins precess about ω_L . For $m_s = +1$ each spin precesses about a distinct axis $\tilde{\omega} = \omega_L + \mathbf{A}$.

When the electron is in the $m_s = 0$ state each nuclear spin precesses about ω_L with the Larmor frequency. When the electron is in the $m_s = +1$ state nuclear spins precess about a distinct axis $\tilde{\omega} = \omega_L + \mathbf{A}$ [16]. The hyperfine interaction \mathbf{A} depends on the position of that particular nuclear spin relative to the NV- center.

When applying a decoupling sequence with $N/2$ decoupling units of the form $\tau - \pi - 2\tau - \pi - \tau$, the nuclear spin alternately rotates around the ω_L and the $\tilde{\omega}$ axis. The net result of one such decoupling sequence is a rotation around an axis $\hat{\mathbf{n}}_i$ by an angle ϕ . Where $\hat{\mathbf{n}}_i$ depends on the initial state of the electron: $\hat{\mathbf{n}}_0$ when the electron starts in $m_s = 0$ and $\hat{\mathbf{n}}_1$ when the electron starts in $m_s = +1$ [16].

To understand how a carbon-13 atom can be controlled it is useful to consider three situations. In the first situation the ω_L and \mathbf{A} point in the same direction. In the second situation ω_L and \mathbf{A}_\perp are of comparable magnitude, resulting in a large angle between the quantization axes. In the last situation $|\mathbf{A}|$ is small compared to $|\omega_L|$ resulting in a small angle between the quantization axes.



Figure 3.6 – Figure 3.6a When the net rotation axes $\hat{\mathbf{n}}_0$ and $\hat{\mathbf{n}}_1$ point in the same direction the carbon experiences an unconditional rotation and cannot be controlled. Figure 3.6b When the net rotation axes $\hat{\mathbf{n}}_0$ and $\hat{\mathbf{n}}_1$ are anti-parallel the carbon experiences a conditional rotation, either around $+x$ or $-x$, and can be controlled.

When ω_L and \mathbf{A} point in the same direction, the net rotation axis is independent of the initial electron-state making it impossible to use the electron to control the carbon-13 atom using this decoupling sequence.

In the case where ω_L and \mathbf{A}_\perp are of comparable magnitude the net rotation axes $\hat{\mathbf{n}}_i$ are strongly dependent on the initial electron-state for almost any τ . This creates entanglement between the electron and this carbon for a wide range of inter pulse-delays τ . For future reference we say that these weakly-coupled carbons are in the *complex regime*.

When considering the case where the hyperfine interaction is much smaller than the Larmor frequency ($\omega_L \gg |\mathbf{A}|$), the net rotation axes $\hat{\mathbf{n}}_0$ and $\hat{\mathbf{n}}_1$ are practically parallel and the nuclear spin undergoes an unconditional evolution. Only when the inter-pulse delay is precisely resonant with the spin dynamics the axes are anti-parallel leading to a conditional rotation[16]. The resonant condition is given by eq. (3.11), where k is an integer and the FWHM of the Lorentzian-shaped resonance is given by eq. (3.12). To distinguish these carbons from those in the complex regime we say that these weakly-coupled carbons are in the *basic regime*.

$$\tau = \frac{(2k + 1)\pi}{2\omega_L + A_\parallel} \quad (3.11)$$

$$\Delta = \frac{A_\perp}{2\omega_L^2} \quad (3.12)$$

If $\hat{\mathbf{n}}_0$ and $\hat{\mathbf{n}}_1$ are not parallel, the resulting conditional rotation of the nuclear spin generally entangles the electron and nuclear spins.

3.2.2 Response of an isolated carbon to dynamical decoupling spectroscopy

As a result the electronic spin, starting out in $|X\rangle$, entangles with the nuclear spin for specific values of τ during a dynamical decoupling spectroscopy. When reading out the electronic spin along the X-axis this creates a dip in the signal. The probability that the initial state is preserved is given by eq. (3.13). Where the contrast M_j for a single nuclear spin is given by eq. (3.14)[16].

$$P_x = (M + 1)/2 \quad (3.13)$$

$$M_j = 1 - (1 - \hat{\mathbf{n}}_0 \cdot \hat{\mathbf{n}}_1) \sin^2 \frac{N\phi}{2} \quad (3.14)$$

$$1 - \hat{\mathbf{n}}_0 \cdot \hat{\mathbf{n}}_1 = \frac{A_\perp^2}{\tilde{\omega}^2} \frac{(1 - \cos(\tilde{\omega}\tau))(1 - \cos(\omega_L\tau))}{1 + \cos(\tilde{\omega}\tau) \cos(\omega_L\tau) - \left(\frac{A_\parallel + \omega_L}{\tilde{\omega}}\right) \sin(\tilde{\omega}\tau) \sin(\omega_L\tau)} \quad (3.15)$$

$$\phi = \cos^{-1} \left(\cos(\tilde{\omega}\tau) \cos(\omega_L\tau) - \left(\frac{A_\parallel + \omega_L}{\tilde{\omega}} \right) \sin(\tilde{\omega}\tau) \sin(\omega_L\tau) \right) \quad (3.16)$$

3.2.3 Characterizing the Nuclear-spin environment

In reality the electron is not interacting with a single carbon but with a bath of carbon atoms. When the electron interacts with multiple carbons at the same time the contrast M is given by the product of all individual values M_j for each individual spin j (eq. (3.17)). In order to selectively control one carbon the electron should not entangle with any other carbon when addressing it.

$$M = \prod_j M_j \quad (3.17)$$

When entanglement is created with multiple carbons at the same time coherence is quickly lost and contrast drops to 0. By sweeping the number of pulses π -pulses the response of an individual carbon can be distinguished from the response of multiple spins. Only when an individual carbon is being addressed is it possible to sweep the contrast to -1.

Most spins are relatively far away from the NV-center and have similar hyperfine couplings causing their resonances to overlap. This causes a broad feature with low coherence known as the spin-bath collapse. This feature is clearly visible in the fingerprint (fig. 3.4) at $\tau/(4\tau_L) = m$ for odd m , where τ_L is the Larmor period ($\tau_L = \frac{2\pi}{\omega_L}$).

Spins that have a stronger than average hyperfine-interaction show up outside or at the edge of the spin-bath collapse. Spins that are in the basic regime show up as a narrow dip. Going to larger τ separates these dips further as the order of the resonance k increases. By looking at larger τ it is possible to resolve and address more resonances. Several spins in the basic regime have been identified 3 of these are visible as colored lines in fig. 3.4. As computations are fundamentally limited by the coherence time there is a limit to the resonance-order that can be used to address carbons, making it impossible to resolve all weakly coupled spins.

Besides the carbons in the basic regime there are also weakly-coupled carbons that are more strongly coupled. When a carbon in the complex regime is present in the NV-center this manifests itself as a resonance with strong oscillations on the side. Such a feature is also clearly visible in fig. 3.4. We have identified the oscillations in the fingerprint as belong to a single spin which is denoted by the red line.

When a weakly coupled carbon in the complex regime is present a significant part of the fingerprint spectrum is inaccessible for controlling other carbons making them an undesired feature when attempting to control weakly coupled carbon spins.

3.2.4 Effect of the magnetic field

There are significant advantages to increasing the magnetic field when attempting to address weakly coupled carbons. By increasing the magnetic field the Larmor frequency can be increased, reducing the number of carbons that are in the complex regime. This causes the broad oscillating resonances to disappear allowing more carbons to be addressed.

Although increasing the magnetic field can improve the situation it is not always possible or desired. When the magnetic field becomes too strong too strong the resonances become narrower than the resolution of the Arbitrary Waveform Generator used to generate the pulses that address the resonances, making it impossible to address these resonances effectively. Simulations were performed (see appendix E) that indicate that for a natural carbon-13 concentration there is a range between 400G and 1400G where the magnetic field is optimal for controlling weakly coupled spins.

Besides the spin environment there are other factors affecting the choice for magnetic field. Because the optical transitions used for readout and initialization depend on strain and magnetic field field[9], care must be taken when measuring that states do not mix in the excited state. This combined with the fact that few experiments have been performed at high magnetic field and low temperature make it more practical to settle for a more moderate magnetic field of 300G.

3.2.5 Identifying Individual Carbon-spins

We identify distinct features in the fingerprint, of which fig. 3.4 shows a small part, and try to assign different hyperfine-couplings to them. We then compute the response for these hyperfine couplings using eq. (3.14). The parameters are varied such that the computed response agrees with the data as well as possible. Using this method 13 distinct carbon spins were identified.

The parameters of the 4 strongest coupled carbons are listed in table 3.1 and their computed responses are visible as colored lines in fig. 3.4. All estimated hyperfine parameters and a link to the full fingerprint measurements can be found in appendix A.

Carbon	A_{\parallel}	A_{\perp}
1	$2\pi \cdot 30.0$ kHz	$2\pi \cdot 80.0$ kHz
2	$2\pi \cdot 27.0$ kHz	$2\pi \cdot 28.5$ kHz
3	$2\pi \cdot -51.0$ kHz	$2\pi \cdot 105.0$ kHz
4	$2\pi \cdot 45.1$ kHz	$2\pi \cdot 20.0$ kHz

Table 3.1 – Estimated hyperfine parameters for spins 1 to 4 in fig. 3.4.

Controlling Weakly-coupled Carbon Spins

4.1 Carbon control (more of a theory chapter, change title)

When on resonance (eq. (3.11)) the carbon rotates around on of two distinct anti-parallel axes based on the state the electron is in. [Need some statement that puts the axis in the equator when on resonance. Look in appendix again] We define the

Figure of Bloch sphere whoing n_0 and n_1 axis. A state starting of in 0 being rotated to $+y$ and $-y$ ($\pm x$ operation). Different arrow
adf

4.1.1 Carbon Ramsey experiment

4.1.2 Measuring Precession Frequencies/Carbon Ramsey experiment

Idea explain init with swap init. And from there go back to X-init. Refer to appendix for more rigorous derivation.

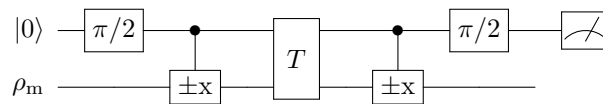


Figure 4.1 – Carbon Ramsey experiment.

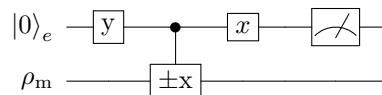


Figure 4.2 – MBI-based initialization into $\pm|X\rangle$. Initializes the carbon into $|X\rangle_C$ when $|0\rangle_e$ is measured and into $|-X\rangle_C$ when $|1\rangle_e$ is measured for the electron.

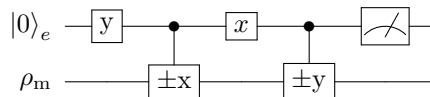


Figure 4.3 – MBI-swap initialization into $\pm|0\rangle$. Initializes the carbon into $|0\rangle_C$ when $|0\rangle_e$ is measured and into $|1\rangle_C$ when $|1\rangle_e$ is measured for the electron.

4.2 Controlling weakly coupled carbons trough the electronic spin

Explain how carbon control works in theory. Explain how a conditional and unconditional gate can be performed. Explain initialization on gate level, refer to appendix for calculations. Explain Readout.

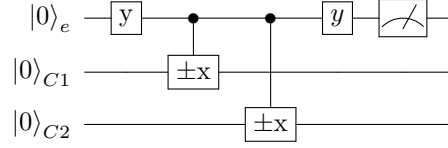


Figure 4.4 – XX-parity-measurement

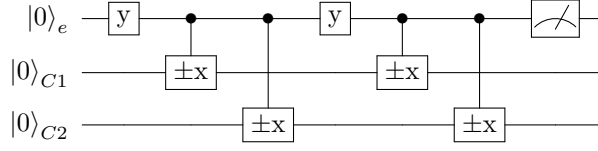
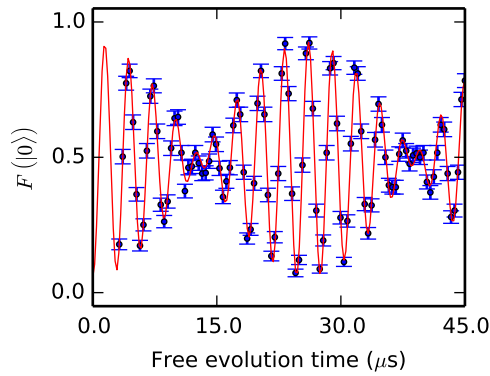


Figure 4.5 – General parity RO

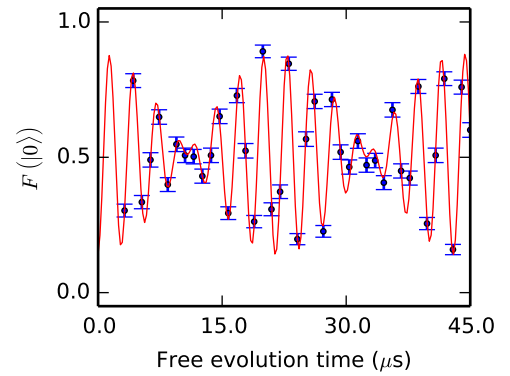
4.2.1 Initialized Carbon Ramsey

4.3 Carbon Initialization & Readout

Show results that demonstrate carbon control.



(a) Nuclear Ramsey of Carbon 1



(b) Nuclear Ramsey of Carbon 4

Figure 4.6 – Nuclear Ramsey experiment wit

Deterministic Parity Measurements

5.1 Entanglement

5.2 Verification of Entanglement

Outlook: towards Quantum Error Correction

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Fingerprintdata

The estimated hyperfine paramters of all 13 identified spins can be found in table A.1. Due to the size of the fingerprint analysis it is not possible to include with this thesis. A pdf file containing the fingerprint analysis can be found here: <https://www.dropbox.com/s/gieji9e86bfvsf1/fingerprinting.pdf>.

Carbon	A_{\parallel}	A_{\perp}
1	$2\pi \cdot 30.0 \text{ kHz}$	$2\pi \cdot 80.0 \text{ kHz}$
2	$2\pi \cdot 27.0 \text{ kHz}$	$2\pi \cdot 28.5 \text{ kHz}$
3	$2\pi \cdot -51.0 \text{ kHz}$	$2\pi \cdot 105.0 \text{ kHz}$
4	$2\pi \cdot 45.1 \text{ kHz}$	$2\pi \cdot 20.0 \text{ kHz}$
5	$2\pi \cdot 17.0 \text{ kHz}$	$2\pi \cdot 10.0 \text{ kHz}$
6	$2\pi \cdot -15.0 \text{ kHz}$	$2\pi \cdot 12.0 \text{ kHz}$
7	$2\pi \cdot -23.0 \text{ kHz}$	$2\pi \cdot 12.0 \text{ kHz}$
8	$2\pi \cdot 10.0 \text{ kHz}$	$2\pi \cdot 8.0 \text{ kHz}$
9	$2\pi \cdot 8.0 \text{ kHz}$	$2\pi \cdot 12.0 \text{ kHz}$
10	$2\pi \cdot -9.3 \text{ kHz}$	$2\pi \cdot 13.0 \text{ kHz}$
11	$2\pi \cdot -10.0 \text{ kHz}$	$2\pi \cdot 5.0 \text{ kHz}$
12	$2\pi \cdot -30.0 \text{ kHz}$	$2\pi \cdot 35.0 \text{ kHz}$
13	$2\pi \cdot -32.0 \text{ kHz}$	$2\pi \cdot 20.0 \text{ kHz}$

Table A.1 – Estimated hyperfine parameters for spins 1 to 13.

B

State Initialization

C

Bell State Tomography

Derivation, what would a tomography of the Ψ^+ state look like?

D

Entanglement witness

Simulations and Calculations for Number of Addressable Carbons

E.1 Scaling of number of resonances with magnetic field

Starting from eq. (3.11)

$$\tau = \frac{(2k+1)\pi}{2\omega_L + A_{\parallel}} \quad (\text{E.1})$$

In the limit where $\omega_L \gg A$ the number of resonances K of a single carbon between $\tau = 0$ and $\tau = T_{\text{DD},e}$ scales linear with the magnetic field: $N_k \propto \omega_L$.

At the same time the width of these resonances decreases quadratically with magnetic field (eq. (3.12)).

$$\Delta = \frac{A_{\perp}}{2\omega_L^2} \quad (\text{E.2})$$

Combining these two we find that the number of resonances N that fit between two orders to increase linearly with magnetic field.

$$N_k - N_{k-1} = \frac{\tau_{k+1} - \tau_k}{\Delta} \quad (\text{E.3})$$

$$N_k - N_{k-1} = \frac{2\pi}{2\omega_L + A_{\parallel}} \cdot \frac{2\omega_L^2}{A_{\perp}} \quad (\text{E.4})$$

$$N_k - N_{k-1} = \frac{2\pi\omega_L}{A_{\perp}} \quad (\text{E.5})$$

Meanwhile the time it takes to implement a $\pi/2$ -gate is given by eq. (E.6) where $N_{\pi/2}$ is the number of pulses required for a $\pi/2$ -pulse.

$$T_{\pi/2} = N_{\pi/2}\tau \quad (\text{E.6})$$

Using eq. (3.14), and noting that \hat{n}_0 and \hat{n}_1 are anti-parallel at the resonance condition, we can find N to be:

$$\frac{\pi}{4} = \frac{N_{\pi/2}\phi}{2} \quad (\text{E.7})$$

$$N_{\pi/2} = \frac{\pi}{2\phi} \quad (\text{E.8})$$

Where ϕ is given by eq. (3.16).

$$\phi = \cos^{-1} \left(\cos(\tilde{\omega}\tau) \cos(\omega_L\tau) - \left(\frac{A_{\parallel} + \omega_L}{\tilde{\omega}} \right) \sin(\tilde{\omega}\tau) \sin(\omega_L\tau) \right) \quad (\text{E.9})$$

In the limit where $\omega_L \gg A$, $\omega_L \approx \tilde{\omega}$ simplifying eq. (E.9) to:

$$\phi = \cos^{-1} (\cos^2(\omega_L\tau) - \sin^2(\omega_L\tau)) \quad (\text{E.10})$$

$$\phi = \cos^{-1} (2\cos(\omega_L\tau)) \quad (\text{E.11})$$

Constants and Experimental values

Gyromagnetic Ratios

γ_e	$2\pi \cdot 2.8025$	MHz/G
γ_N	$2\pi \cdot 0.3077$	kHz/G
γ_C	$2\pi \cdot 1.0705$	kHz/G

Interaction Strengths

Δ	$2\pi \cdot 2.878$	GHz
Q	$2\pi \cdot 4.946$	MHz
A_N	$2\pi \cdot 2.186$	MHz

Acknowledgements