Kevin Pacheco

15. $\int \frac{1}{x\sqrt{16-x^2}}\,dx$ $\qquad u = \sqrt{16-x^2} = \frac{1}{2\sqrt{16-x^2}}$

$16-x^2 = -2x$

$= \frac{1}{2\sqrt{16-x^2}}(-2x) = -\frac{x}{\sqrt{16-x^2}}$

$$= \int \frac{1}{xu}\left(-\frac{u}{x}\right)$$

$$= -\frac{1\,u}{xux} = -\frac{1}{xx} = \frac{1}{x^2}$$

$$\int -\frac{1}{u^2+16} \quad \frac{du}{dv} = 4$$

$$= \int \frac{1}{-(4v)^2+16}\cdot 4 \Rightarrow \frac{1}{4(v^2+1)}$$

$$= -\frac{1}{4}\cdot\int\frac{1}{v^2+1}$$

$$= -\frac{1}{4}\left|\frac{\ln|v+1)}{2} - \frac{\ln|v-1|}{2}\right|$$

$$= -\frac{1}{4}\left|\frac{\ln\left|\frac{\sqrt{16-x^2}}{4}+1\right|}{2} - \frac{\ln\left|\frac{\sqrt{16-x^2}}{4}-1\right|}{2}\right|$$

$$= -\frac{\left(\frac{\ln\left|\frac{\sqrt{16-x^2}}{4}+1\right|}{2} - \frac{\ln\left|\frac{\sqrt{16-x^2}}{4}-1\right|}{2}\right)}{4}$$

$$= -\frac{\frac{\ln\left|\frac{\sqrt{-x^2+16}}{4}+1\right| - \ln\left|\frac{\sqrt{-x^2+16}}{4}-1\right|}{2}}{4}$$

$$= -\frac{\ln\left|\frac{\sqrt{-x^2+16}}{4}+1\right| - \ln\left|\frac{\sqrt{-x^2+16}}{4}-1\right|}{8}$$

$$= -\frac{1}{8}\left(\ln\frac{\sqrt{16-x^2}}{4}+1\right) - \ln\left|\frac{\sqrt{16-x^2}}{4}-1\right| + C$$