Combining SOLA and Deterministic Linear Inferences

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1 Introduction

The Problem

- Poor seismic data coverage leads to non-uniqueness[1].
- Non-uniqueness is typically mitigated by regularisations that impose strong prior constraints.
- Robust uncertainty and resolution quantification is often difficult and absent in inverse methods.

Motivation

- Instead of finding a model, focus on constraining desired properties (Figure 1) of the model using mathematical inferences.
- This allows for a relaxation of prior regularisations (Figure 2).

2 Methods (noise-free)

SOLA[2] (Subtractive Optimally Localized Averages) and DLI[3] are linear inference methods stemming from the work of Backus and Gilbert[1].

Research Question

- How are SOLA and Deterministic Linear Inferences (DLI) related and how can they be combined?

SOLA

- Provides an *approximate property of the true model* without the need for prior model constraints[3].
- Non-uniqueness errors are quantified in the resolving kernels[5] (Figure 4) (approximate mapping).

DLI

- Provides bounds for the *true property of the true model* using deterministic prior constraints (hard prior, Figure 2).
- Non-uniqueness errors are quantified in property bounds.
- Results are interpreted through target kernels.

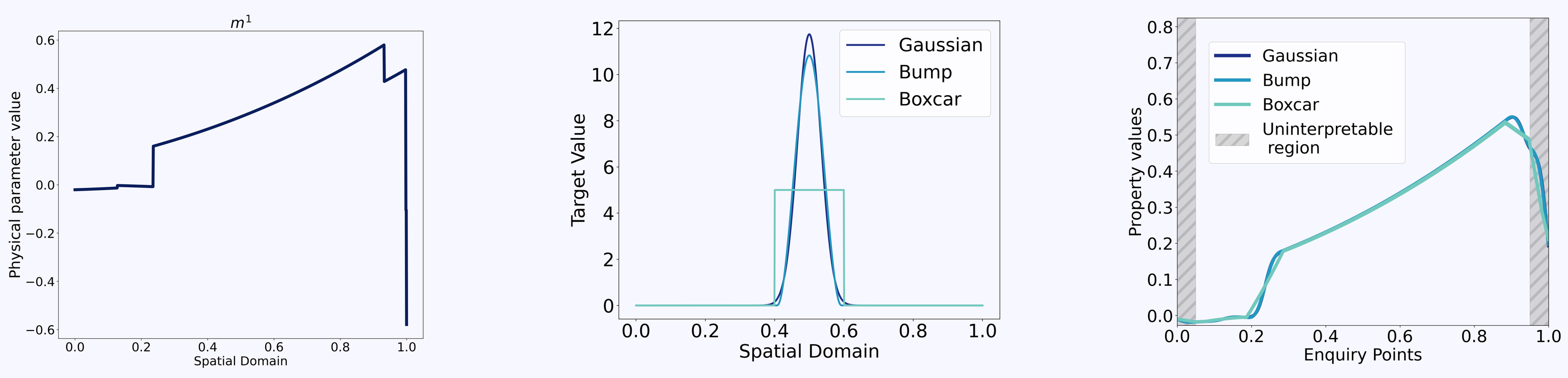
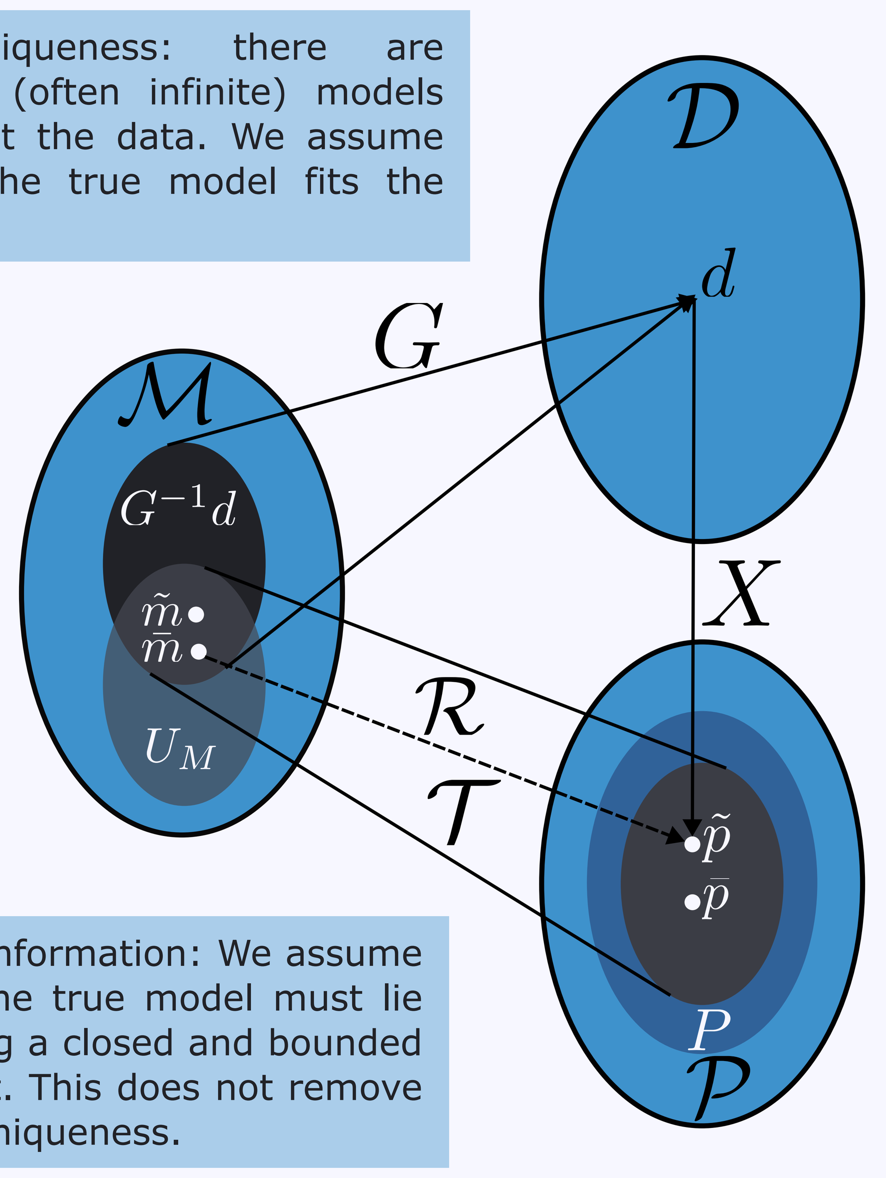


Figure 1: Example of properties. Column 1: True model. Column 2: Three examples of target kernels centered at 0.5 with width 0.2, each associated with a type of local average. Column 3: Each of the three types of local averages evaluated againts the true model at 100 points in the domain of the model

Nonuniqueness: there are many (often infinite) models that fit the data. We assume that the true model fits the data.



- G Forward map
- $\mathcal{T}$ Property map
- $\mathcal{R}$ Approximate map
- X SOLA* inverse
- $\mathcal{M}$ Model space
- $\mathcal{D}$ Data space
- $\mathcal{P}$ Property space
- $\tilde{p}$ Approximate property
- $\bar{p}$ True property
- $\tilde{m}$ Regularized model
- $\bar{m}$ True model
- d Data
- $G^{-1}d$ Possible solutions
- U_M Prior constraint
- P DLI solution

Prior information: We assume that the true model must lie withing a closed and bounded subset. This does not remove non-uniqueness.

Figure 2: Graph showing the relationships between the main spaces used in the noise-free SOLA and DLI methods. Ellipses represent spaces, subspaces and/or subsets. Dots represent elements of the space. Arrows indicate the direction of the relation between spaces, sets, or elements. *SOLA inverse is for the case when unimodularity is not imposed on resolving kernels.

Combined SOLA-DLI method

- Provides bounds on the true property of the true model as solution, but also uses resolving kernels to analyse trade-offs between physical parameters.

3 Results

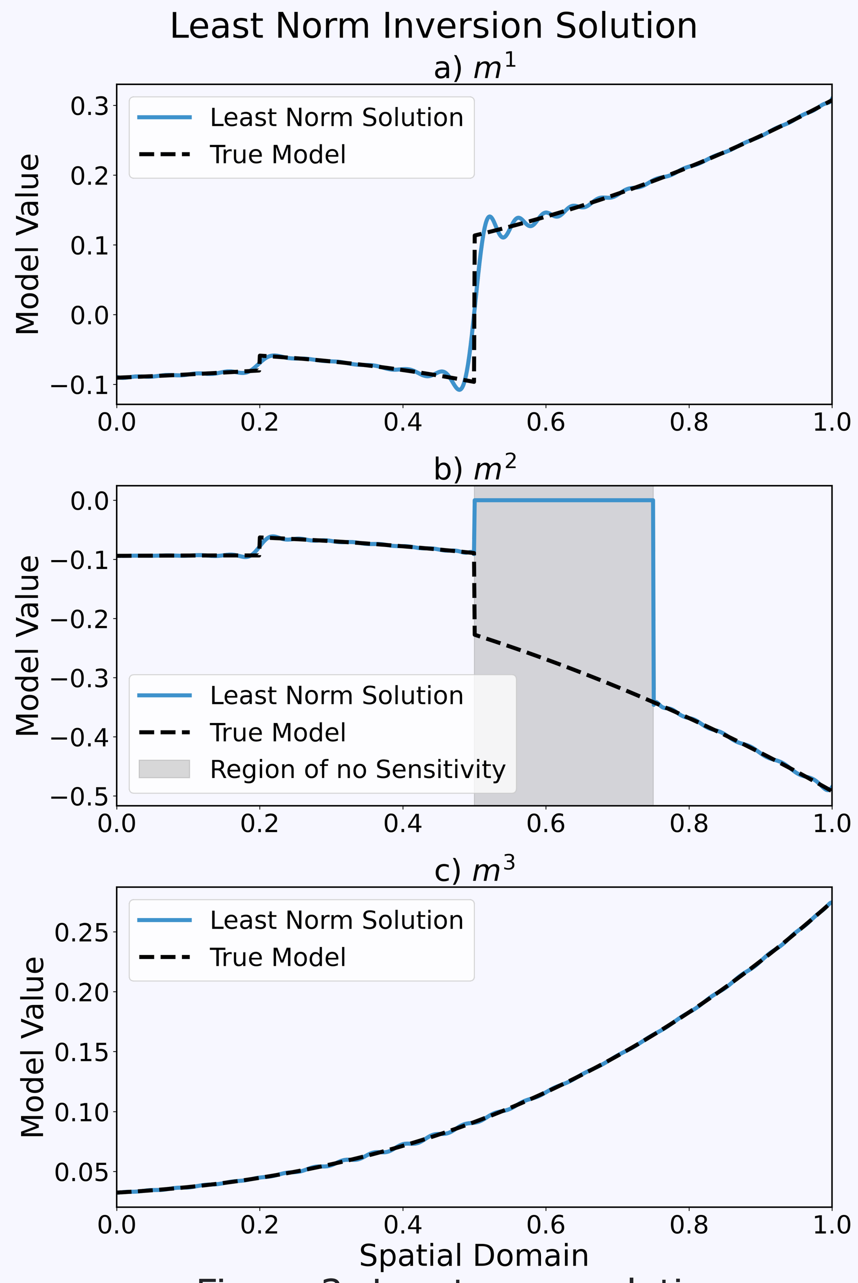


Figure 3: Least norm solution

- Least norm solution provides unique answer due to strong regularizations (Figure 3), while inferences give bounds thanks to more relaxed regularization (Figure 4).
- Inferences take into account lack of data sensitivity by construction and provide broad bounds in such cases (Figure 4).
- Some properties are better constrained than others (Figure 4).
- We can extract more than just local averages[6] (for eg. gradients and coefficients of basis functions) (Figure 4).
- Inference methods work naturally on undiscretised model spaces.

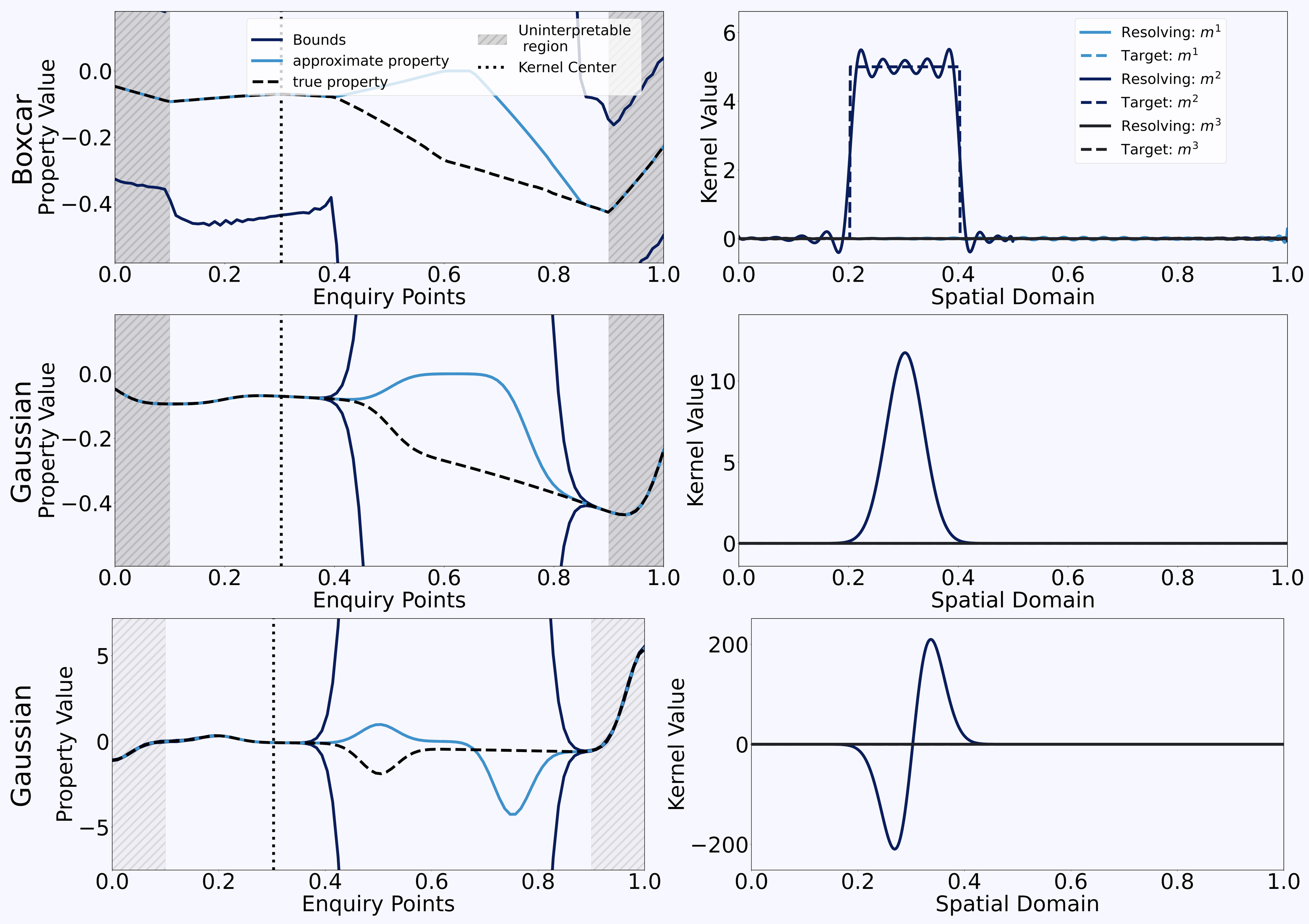


Figure 4: SOLA-DLI solutions for two different types of local average and a local gradient. First column: property bounds for the second physical parameter evaluated at 100 evenly spaced enquiry points. Second column: target and resolving kernels for each type of property

4 Conclusions and Discussion

- Combining SOLA and DLI we can place the result interpretation on the target kernels, which are well known and consistent.
- Smart choices of target kernels can improve the precision of the results and their interpretability.
- Resolving kernels can provide additional information about spatial trade-offs and trade-offs between physical parameters.
- Currently we are working on introducing data noise and probabilistic prior information.
- These methods are best suited for obtaining rigorous uncertainty and resolution quantification on a linear problem or a linearazible problem that has small amounts of data and would otherwise rely on strong regularizations.

References and Acknowledgements

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