Applying Backus-Gilbert SOLA-DLI to normal modes to infer Earth model properties

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1 Introduction

The Problem

- Poor seismic data coverage leads to non-uniqueness[1], which manifests in a large number of models for the Deep Earth.
- Non-uniqueness is typically mitigated by regularisations, but these impose strong prior constraints.
- Robust uncertainty and resolution quantification is often difficult and typically absent in inverse methods.

Motivation

- Instead of finding a model, we focus on constraining desired properties (Fig. 1) of the model using mathematical inferences.
- This allows for a relaxation of prior regularisations (Fig. 2).

2 Methods (noise-free)

SOLA[2] (Subtractive Optimally Localized Averages) and DLI[3] (Deterministic Linear Inferences) are linear inference methods stemming from the work of Backus and Gilbert[1].

Research Question

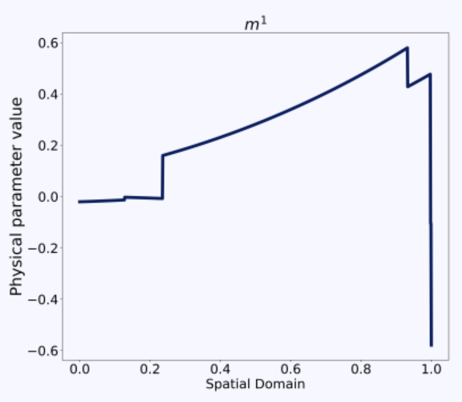
- How can SOLA and DLI be combined to constrain a range of Earth properties?

SOLA

- Provides an *approximate property of the true model* without the need for prior model constraints[3].
- Non-uniqueness errors are quantified in the resolving kernels[5].

DLI

- Provides bounds on the *true property of the true model* using deterministic prior constraints (hard prior, Fig. 2).
- Non-uniqueness errors are quantified in property bounds.
- Results are interpreted through target kernels.



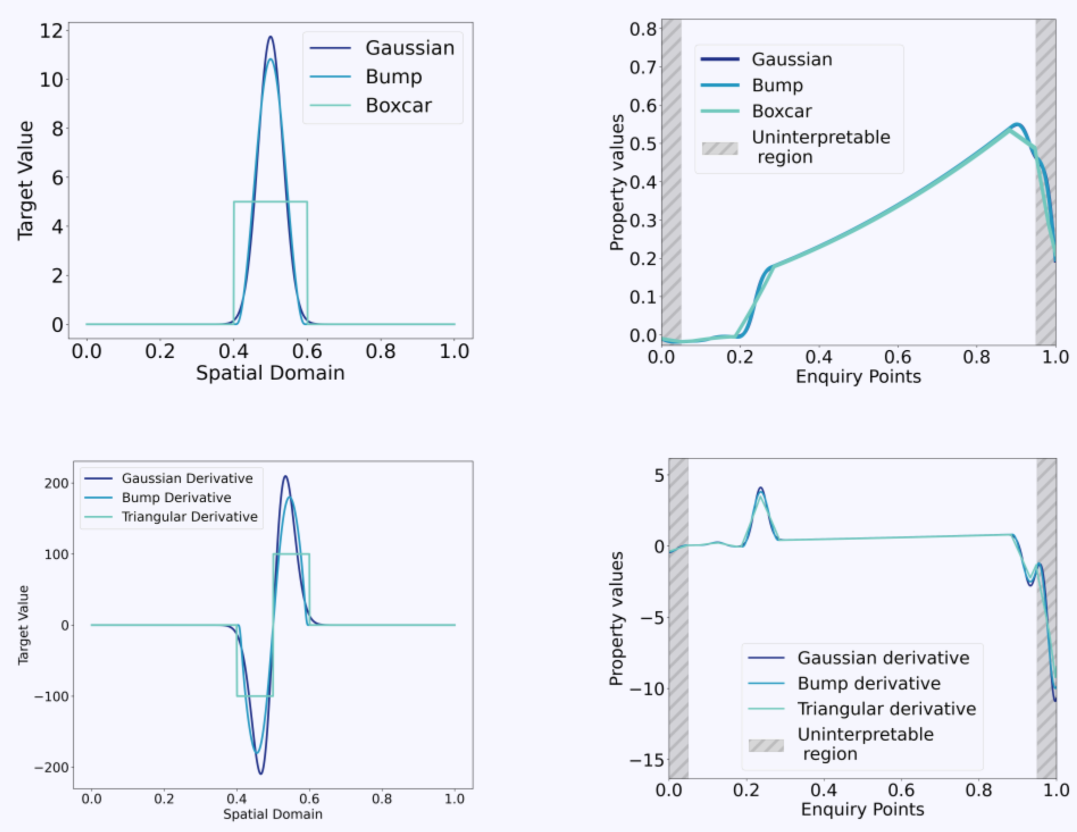


Fig. 1: Example of properties. Column 1: True model. Column 2: Three examples of target kernels centered at 0.5 with width 0.2, each associated with a type of local average. Column 3: Each of the types of local averages evaluated againts the true model at 100 points in the domain of the model

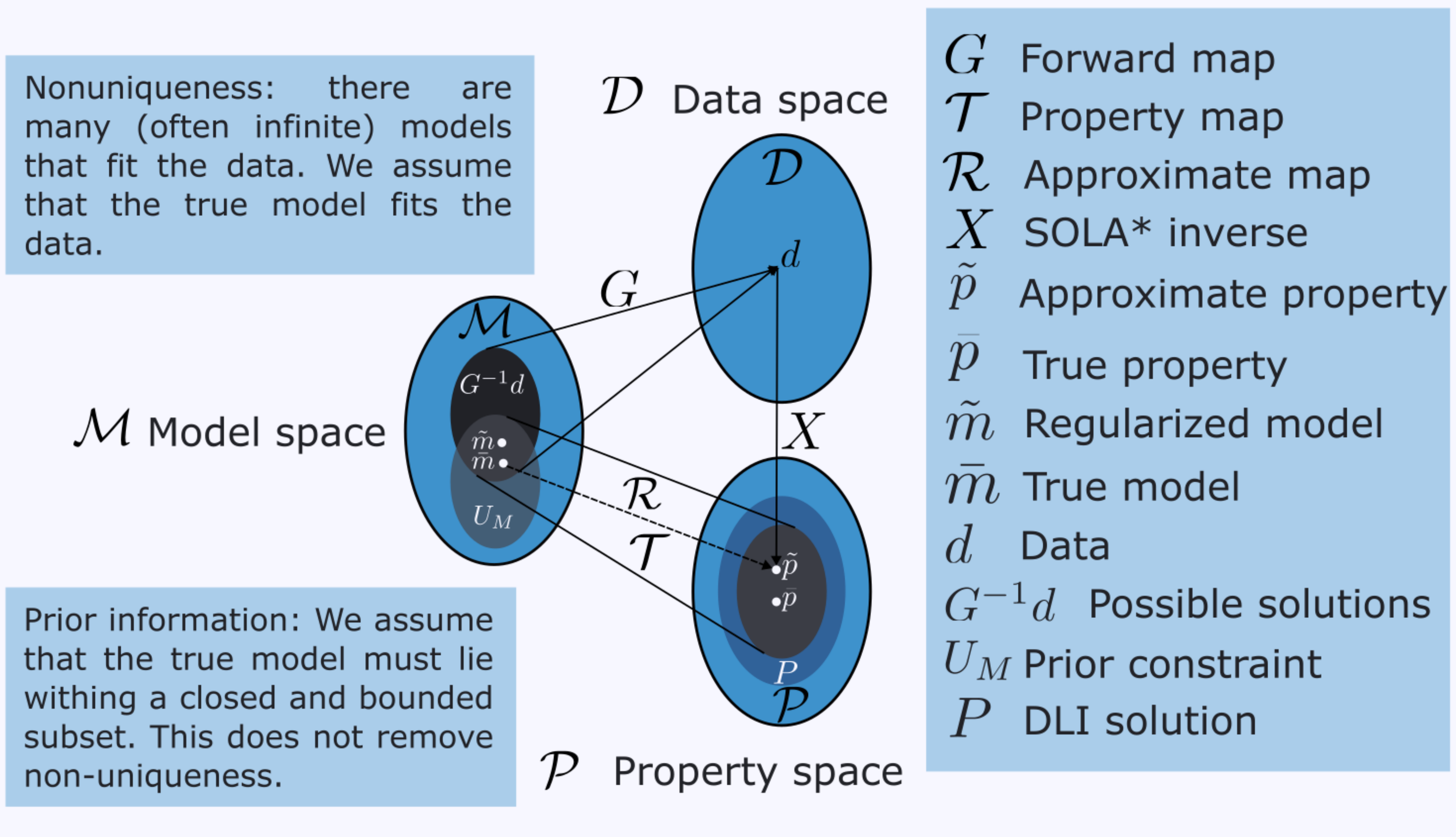


Fig. 2: Graph showing the relationships between the main spaces used in the noise-free SOLA and DLI methods. Ellipses represent spaces, subspaces and/or subsets. Dots represent elements of the space. Arrows indicate the direction of the relation between spaces, sets, or elements. *SOLA inverse is for the case when unimodularity is not imposed on resolving kernels.

Combined SOLA-DLI method

Provides bounds on the true property of the true model, but also uses resolving kernels to analyse trade-offs between physical parameters.

3 Applications

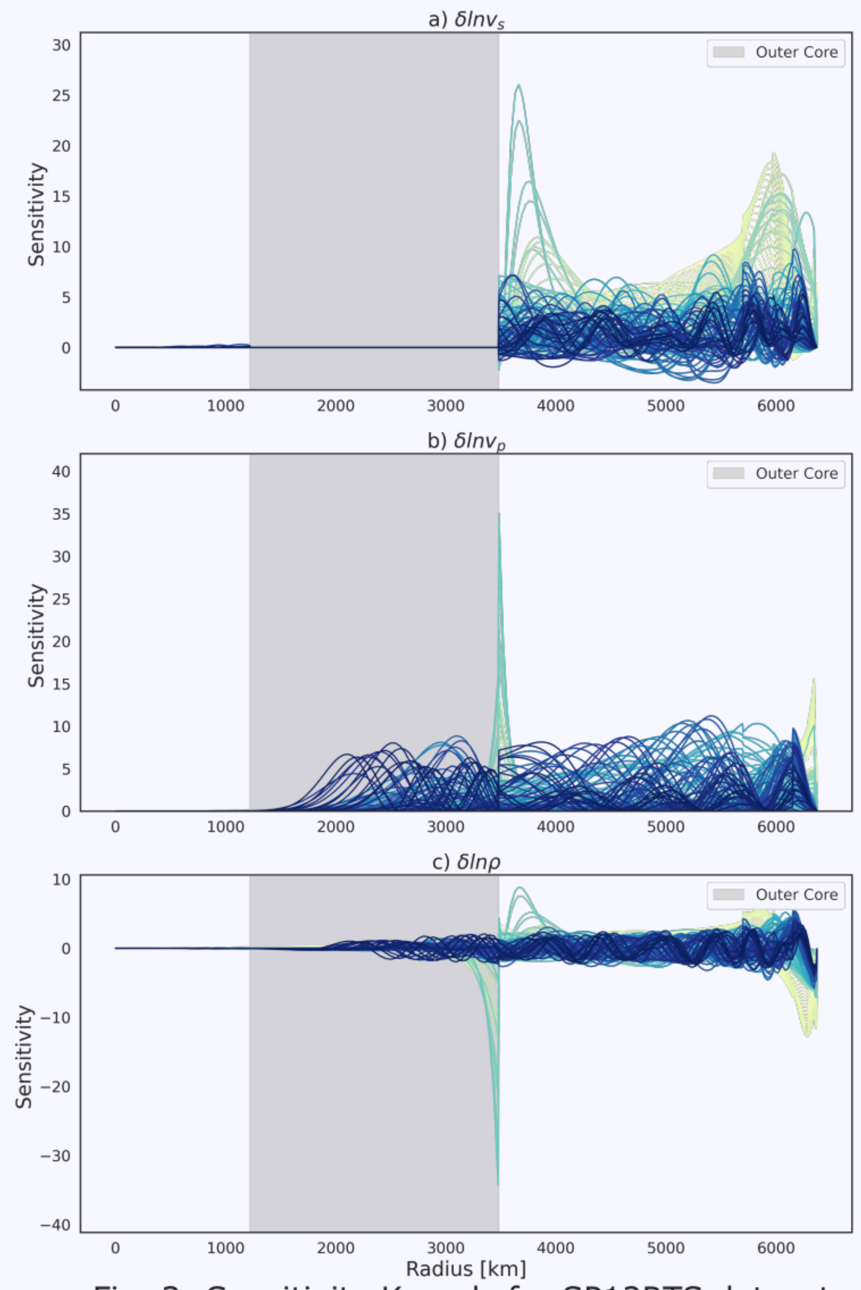


Fig. 3: Sensitivity Kernels for SP12RTS dataset

- We apply SOLA-DLI to the normal-mode data set of SP12RTS (see Fig. 3 for the sensitivity kernels) and investigate the ability to constrain properties in the mantle
- Some properties are better constrained than others (Fig. 4).
- Trade-offs between physical parameters can be visualized. Naturally, regions with large trade-offs result in large resolution misfits and large uncertainties in the solution.
- We can extract more than just local averages[6] (for eg. gradients and coefficients of basis functions) (Fig. 4).

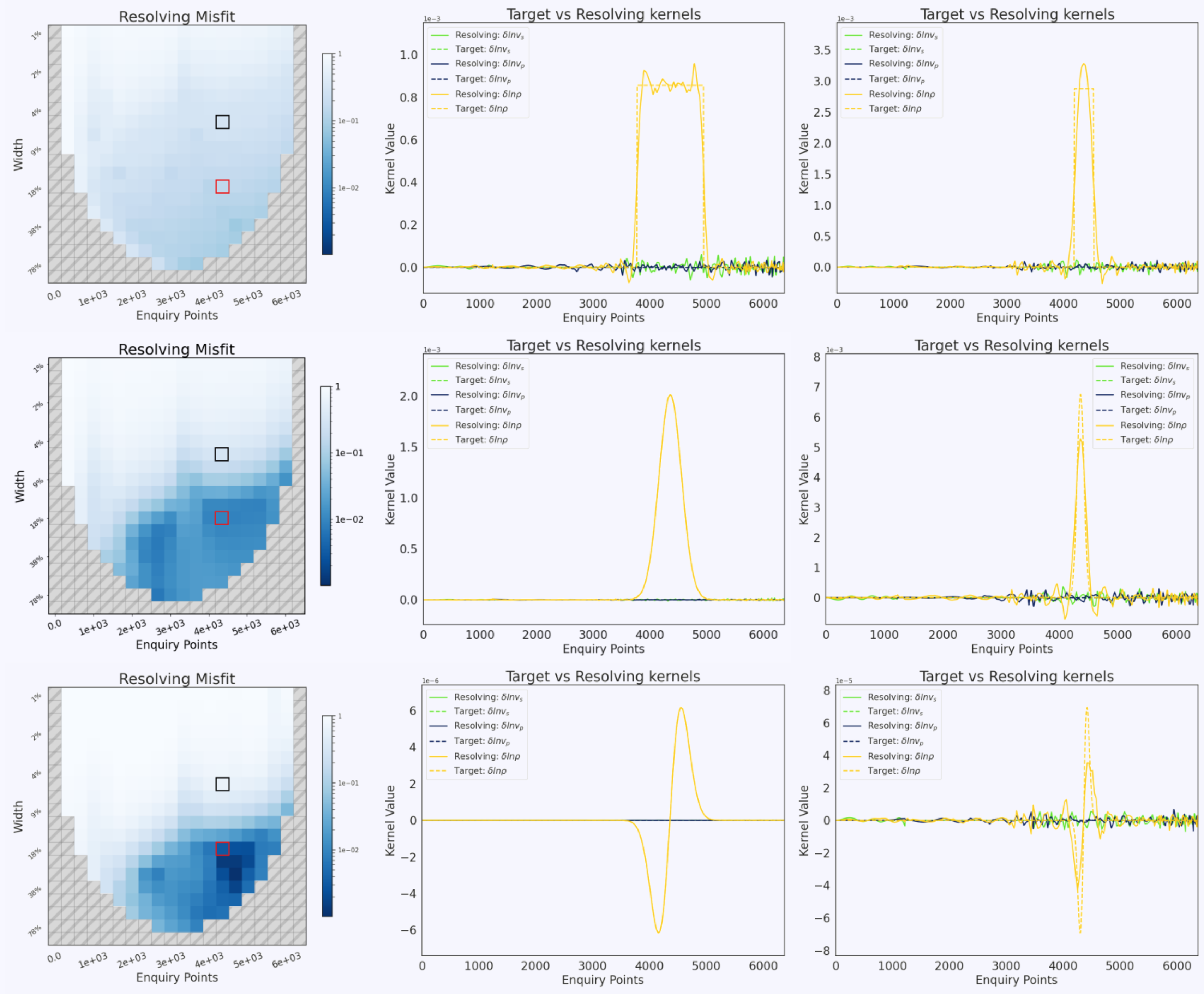


Fig. 4: SOLA-DLI resolution analysis. First column shows the resolution misfit on a logarithmic scale from 0 to 1 (1 means perfect resolution recovery) for 20 different target widths (shown as percentage of Earth's radius on the y axis) and 20 different locations. Second and third columns: examples of resolving and contaminant kernels against their corresponding target kernels (column 1 for red square, and column 2 for black square).

4 Conclusions and Discussion

- Combining SOLA and DLI, we can place the result interpretation on the target kernels, which are well known.
- Smart choices of target kernels can improve the precision of the results and their interpretability.
- Resolving kernels can provide additional information about spatial trade-offs and trade-offs between physical parameters.
- We are working on introducing data noise and probabilistic prior information.
- These methods are best suited for obtaining rigorous uncertainty and resolution quantification in linear or linearazible problems, with sparse data such as typicay is the case in deep Earth applications.

References and Acknowledgements

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