Supplementary material for BSM 2024 poster Examples

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1 Example 1

1.1 Setup

For this example we will use a purely synthetic case. Any model is formed by a tuple $m = (m^1, m^2)$, where m^1, m^2 are the physical parameters. Each physical parameter is assumed to be a piece-wise continuous and bounded function defined over the domain $[0, 1]$ (denoted by $PCb[0, 1]$). The model space $\mathcal{M}$ is formed from the direct sum of two $PCb[0, 1]$ spaces. We use a quasi-random function to generate a true model $\bar{m}$ (see Fig. 3). The true model is linked to the data d via:

$$d_i = G(m) = \int_0^1 K_i^1 m^1 dr + \int_0^1 K_i^2 m^2 dr \qquad (1)$$

where K_i^j are some quasi-randomly generated 1D sensitivity kernels (see Fig. 2). In total, 150 sensitivity kernels have been generated for each physical parameter. The data space is therefore $\mathcal{D} = \mathbb{R}^{150}$.

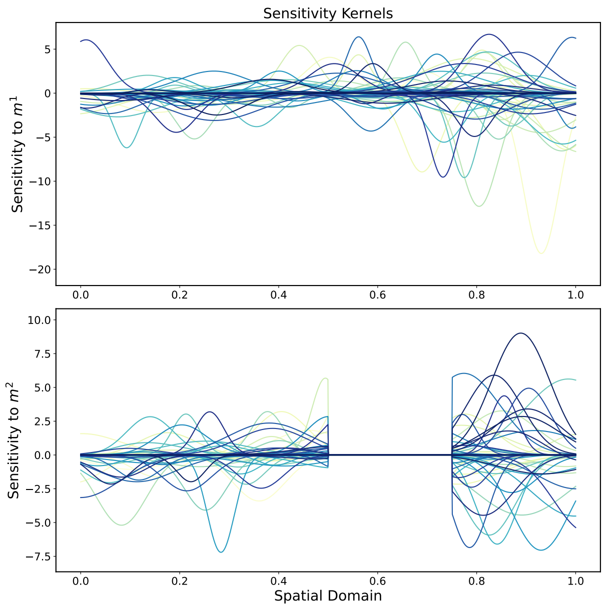

Figure 2: 150 quasi-randomly generated synthetic kernels for each physical parameter.

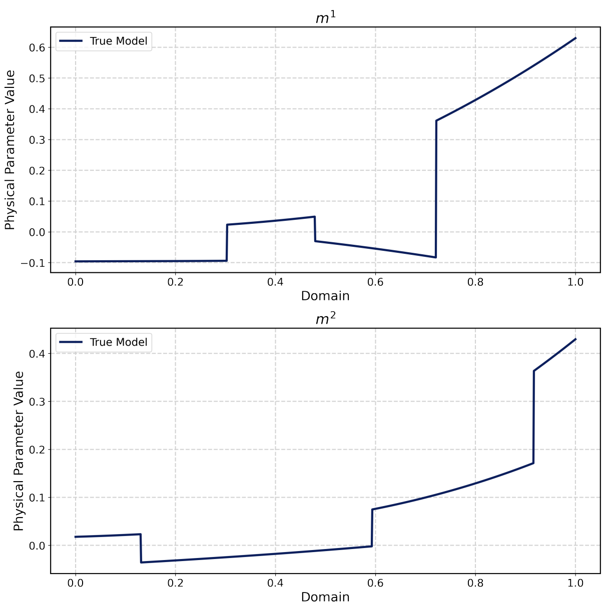

Figure 1: True model $\bar{m}$.

For the prior norm bounds we created some upper bound functions b^i (see Fig. 3). In reality these bounding functions would be obtained from some physical arguments. The model norm bound is obtained from:

$$M = \sqrt{\int_0^1 (b^1)^2 dr} + \sqrt{\int_0^1 (b^2)^2 dr} \qquad (2)$$

and in this case is roughly 0.895 while the true model norm is roughly 0.402.

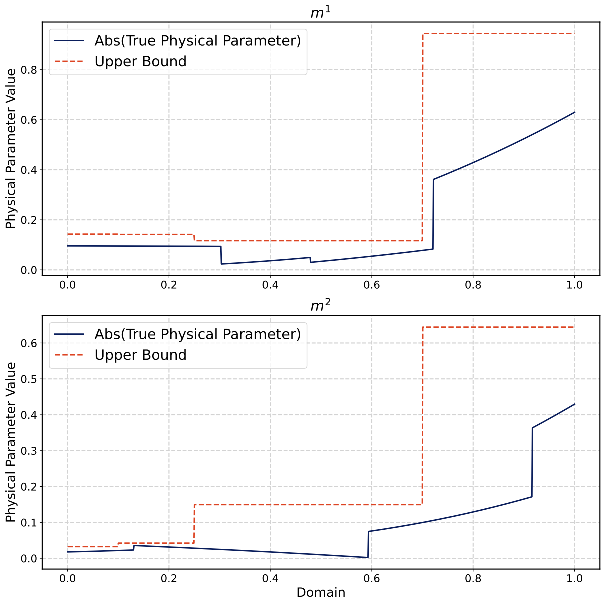

Figure 3: Absolute value of the true model $\bar{m}$ and the prior model bounds used. The red dashed lines represent our prior bounding functions b^i.

With the model space $\mathcal{M}$, data space $\mathcal{D}$, and model-data mapping G defined, and the prior model norm bound cmoputed.

IN WORK