Supplementary material for BSM 2024 poster Examples

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1 Example 1

1.1 Setup

For this example we will use a purely synthetic case. Any model is formed by a tuple $m = (m^1, m^2)$, where m^1, m^2 are the physical parameters. Each physical parameter is assumed to be a piece-wise continuous and bounded function defined over the domain $[0, 1]$ (denoted by $PCb[0, 1]$). The model space $\mathcal{M}$ is formed from the direct sum of two $PCb[0, 1]$ spaces. We use a quasi-random function to generate a true model $\bar{m}$ (see Fig. 3). The true model is linked to the data d via:

$$d_i = G(m) = \int_0^1 K_i^1 m^1 dr + \int_0^1 K_i^2 m^2 dr \qquad (1)$$

where K_i^j are some quasi-randomly generated 1D sensitivity kernels (see Fig. 2). In total, 150 sensitivity kernels have been generated for each physical parameter. The data space is therefore $\mathcal{D} = \mathbb{R}^{150}$.

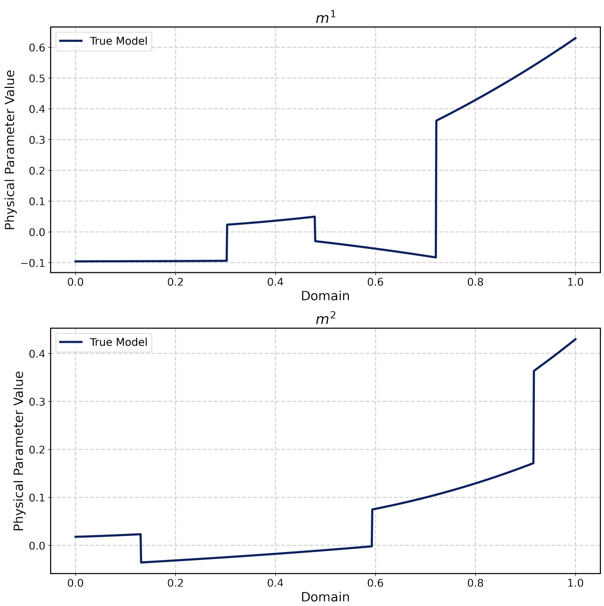

Figure 1: True model $\bar{m}$.

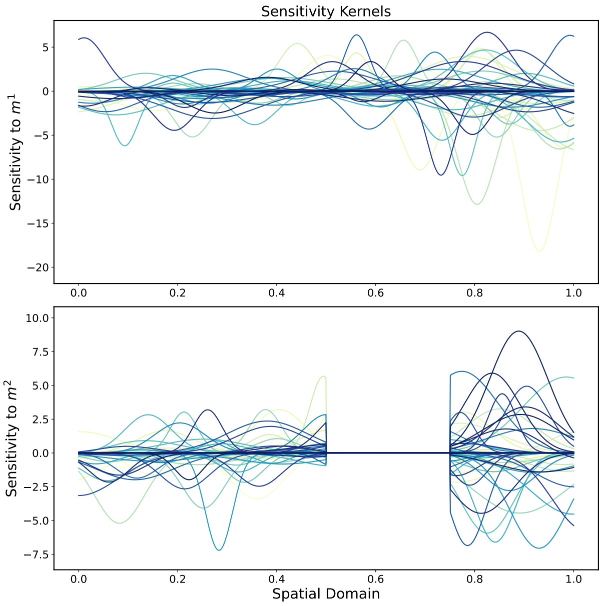

Figure 2: 150 quasi-randomly generated synthetic kernels for each physical parameter.

For the prior norm bounds we created some upper bound functions b^i (see Fig. 3). In reality these bounding functions would be obtained from some physical arguments. The model norm bound is obtained from:

$$M = \sqrt{\int_0^1 (b^1)^2 dr} + \sqrt{\int_0^1 (b^2)^2 dr} \qquad (2)$$

and in this case is roughly 0.895 while the true model norm is roughly 0.402.

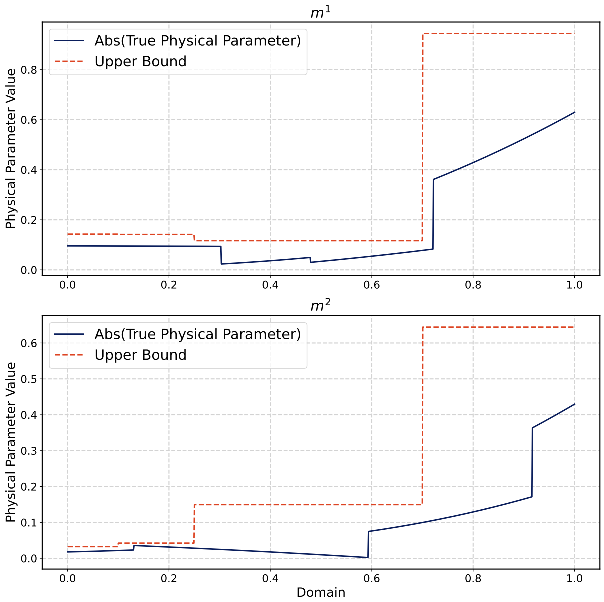

Figure 3: Absolute value of the true model $\bar{m}$ and the prior model bounds used. The red dashed lines represent our prior bounding functions b^i.

With the model space $\mathcal{M}$, data space $\mathcal{D}$, and model-data mapping G defined, and the prior model norm bound computed, we can now defined various properties and constrain them.

1.2 Local Averages

We will first look at uniform local averages obtained from boxcar target functions. 100 evenly distributed boxcar functions with a width of 0.2 are used.

IN WORK