Supplemental material

Derivation of Equation (1)

Consider a point mass m and a distance L from the anchor point P of the pendulum to the payload. Let Φ be the swing angle. The conservation of angular momentum H around point P can be written as:

$$\frac{dH}{dt} = M \tag{A1}$$

where M is the net torque. In this case $H = mL^2\dot{\Phi}$. Introducing the gravitational force as $-mg\sin(\Phi)$ to pull on the pendulum via the arm L, we can derive:

$$\begin{aligned} &\frac{d}{dt}\left(mL^2\dot{\Phi}\right) = -mgL\sin(\Phi) \\ \leftrightarrow\ &\frac{d}{dt}\left(L^2\dot{\Phi}\right) = -gL\sin(\Phi) \end{aligned} \tag{A2}$$

Since we know that the length needs to change based on the angle Φ and the velocity $\dot{\Phi}$, we can state $L = L\left(\Phi, \dot{\Phi}\right)$. A solenoid provides the actuation to change the length of the pendulum and has a transition time which is very small. We will consequently represent the length of the pendulum and the magnitudes of the angular velocities with superscripts of $-$ and $+$ to represent the system prior to and posterior to the activation of the solenoid. Assume that the solenoid is actuated when the angular displacement of the pendulum is nearly zero and the transition is completed in a time interval Δt. Representing the angular displacement as $\Phi \approx 0 \to |\Phi| < \epsilon$ and integrating Eq. A2 over $[t_0, t_0 + \Delta t]$ leads to:

$$(L^+)^2\dot{\Phi}^+ - (L^-)^2\dot{\Phi}^- = -\int_{t_0}^{t_0+\Delta t} gL\sin(\Phi)dt \tag{A3}$$

as explained in detail in subsection "Predicting angular velocity and determining angle" in the supplemental material.[1] We assume that $|\sin(\Phi)| \leq \epsilon$ in Eq. A3, over the time interval when the actuator transitions. If $\epsilon \to 0$ we know that:

$$(L^+)^2\dot{\Phi}^+ - (L^-)^2\dot{\Phi}^- \to 0.$$

which reduces to Eq. (1) to:

$$\dot{\Phi}^+ = \left(\frac{L^-}{L^+}\right)^2 \dot{\Phi}^- \tag{A4}$$

which was presented in[1]. To illustrate pumping of a swing, we require $\dot{\Phi}^+ > \dot{\Phi}^-$, which requires $L^+ < L^-$, and to illustrate damping of oscillations of the pendulum, we require $\dot{\Phi}^+ < \dot{\Phi}^-$, which requires $L^+ > L^-$. To permit recurring application of this control logic, there is a need to reset the pendulum length. This change in length is executed when $\dot{\Phi} = 0$, since as can be inferred from Eq. A2, there is no change in the momentum of the pendulum.

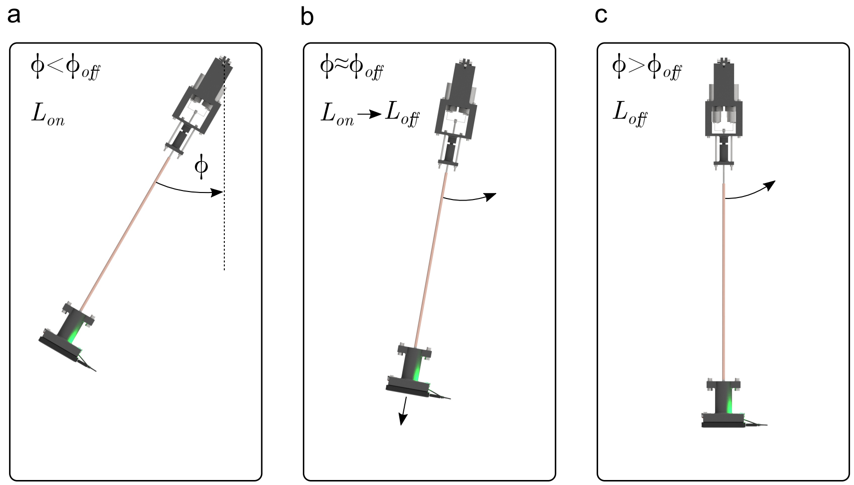

Figure A1. Illustration of the active sway attenuation where: a) The solenoids are activated and pendulum length is L_{on}, b) The solenoids are being deactivated because $\Phi = \Phi_{off}$ and c) The solenoids are deactivated and pendulum length is L_{off}.

Parts list

The following parts and tools are needed for the assembly:

- Solder equipment with solder wire
- 3D printer
- M5 x 0.8 thread cutter (or 10/32 inch)
- Metal saw
- Wood saw
- Wood drill
- Vernier caliper
- Drilling machine with cross screwdriver bit and wood drill and steel drill attachments
- Bending machine
- M5 and M8 wrench
- Pliers for the solenoid nuts
- Cross screwdriver
- 22 gauge wire
- Sticky tape
- Thin sponge material (about $20mm$ thickness)
- Super glue

Table A1 lists all the materials which need to be ordered. The total price of the materials is: $ 496.8 (before tax). Table A2 shows all the 3D printed parts needed.

A Creality Ender 3 V2 3D printer was used to print the 3D printed parts. Figure A2 illustrates the location off all 3D printed parts for the experiment.

Table A1. Purchased mechanical, electrical and miscellaneous material with the price from 12/13/2021.

	Part	Quantity	Price ($)	Vendor
Mechanic	Ball Bearing ($\varnothing$ $8mm$)	2	9.64	McMasterCarr
	Ball Bearing (LM5UU) ($\varnothing$ $5mm$)	1	11.99	Amazon
	Metal rod ($\varnothing$ $8mm$ and 1' length)	1	4.38	McMasterCarr
	Metal rod ($\varnothing$ $5mm$ and 3' length)	1	5.63	McMasterCarr
	Copper tube ($\varnothing$ $5/16$" and 3' length)	1	10.95	Mc Master Carr
	Solenoid	2	113.28	Digi-Key
Electric	Transceiver set	1	19.99	Amazon
	Gyroscope	1	5.99	Amazon
	Powerbank	1	19.99	Amazon
	Relay Module set	1	6.19	Amazon
	5 A Fuse	1	4.99	Amazon
	Power Supply (30 V; 10 A)	1	89.98	Amazon
Miscellaneous	M8x1.25 screw (80 mm length)	1	11.82	McMasterCarr
	M8x1.25 screw (30 mm length)	1	12.22	McMasterCarr
	M5x0.8 screw (110 mm length)	1	11.03	McMasterCarr
	M2.5x0.45 screw (20 mm length)	1	6.43	McMasterCarr
	M8 washer	1	7.14	McMasterCarr
	M5 washer	1	3.14	McMasterCarr
	M2.5 washer	1	1.67	McMasterCarr
	M8x1.25 nut	1	6.14	McMasterCarr
	M5x0.8 nut	1	1.76	McMasterCarr
	M2.5x0.45 nut	1	1.94	McMasterCarr
	Wood (2" x 6" x 10')	1	≈ 10	Home Depot
	Wood (2" x 4" x 8')	1	≈ 4	Home Depot
	Corner Braces (2" x 1-1/2" x 2-3/4")	4	3.36	Home Depot
	Screws #9 x 1"	1	6.84	McMasterCarr
	Screws #9 x 3"	1	8.56	McMasterCarr
	Wood (20" x 20" x 1")	1	50.59	Amazon
	Steel sheet (12" x 12" x 0.035")	1	23.25	Amazon
	#9 O-ring	1	2.92	Home depot
	Filament	1	20.99	Amazon

Assembly

- The $\varnothing$ 8 mm metal rod needs to be cut to a length of 200 mm.
- The $\varnothing$ 5 metal rod needs to be cut to a length of 700 mm. A M5x0.8 thread of 20 mm length needs to be cut on both ends of the metal rod. (Hint: In case there is no M5x0.8 thread cutter available, a 10/32 inch thread cutter can be used too).
- Cut the steel sheet into a rectangular shape of 20 mm x 115 mm and drill holes at the places shown in the Figure A3. Finally the steel sheet needs to be bent into a U-shape profile (each angle is 90°).
- The copper tube needs to be cut to a length of 520 mm.

Table A2. 3D printed parts for the experiment.

Part number	Part name	Quantity
Bearing bracket	1	2
Housing solenoid	2	1
Adapter spacer	3	4
Mid base	4	1
Spacer Bearing	5	1
Lower base	6	1
Spacer solenoid	7	2
Payload chassis cover	8	1
Payload chassis	9	1
Copper tube spacer	10	2

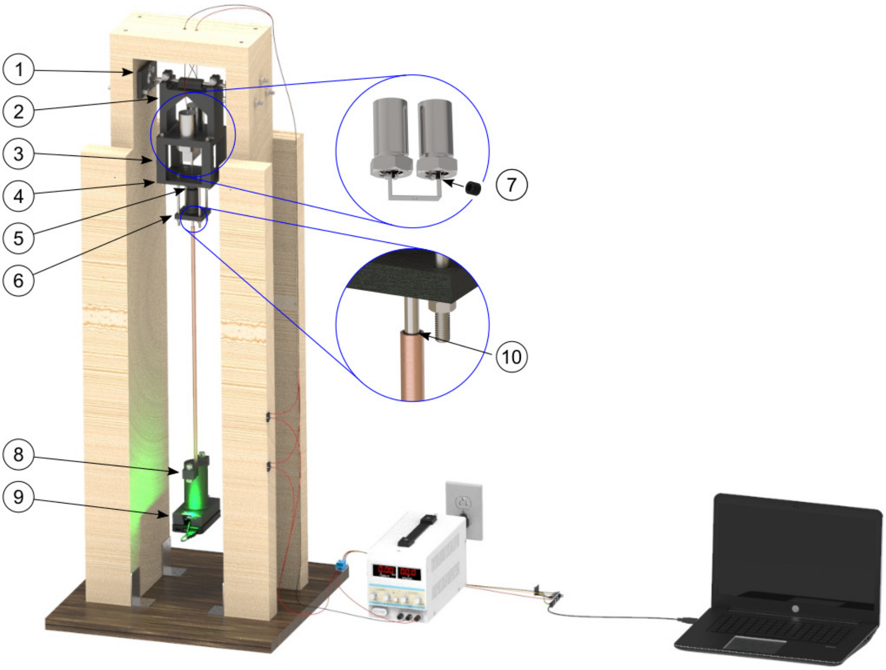

Figure A2. General setup of the pendulum with highlighting all the 3D printed parts

- The 6” x 2” x 10’ wood needs to be cut in two 1200 mm long pieces (wood 1) and one 306 mm long piece (wood 2). The holes illustrated in Figure A4 need to be pre-drilled.

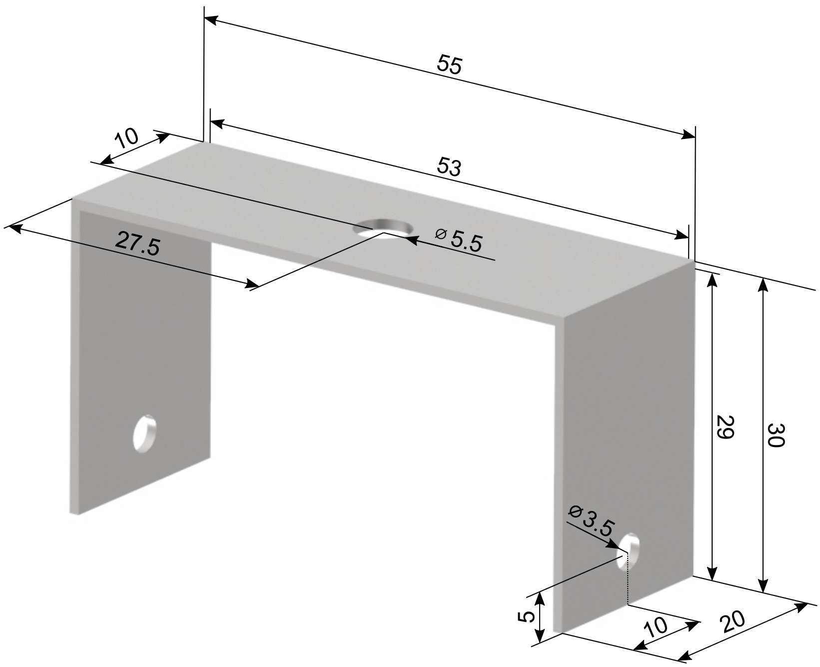

Figure A3. Steel sheet bent to a U-shape profile with bore placement instructions. All dimensions are in mm.

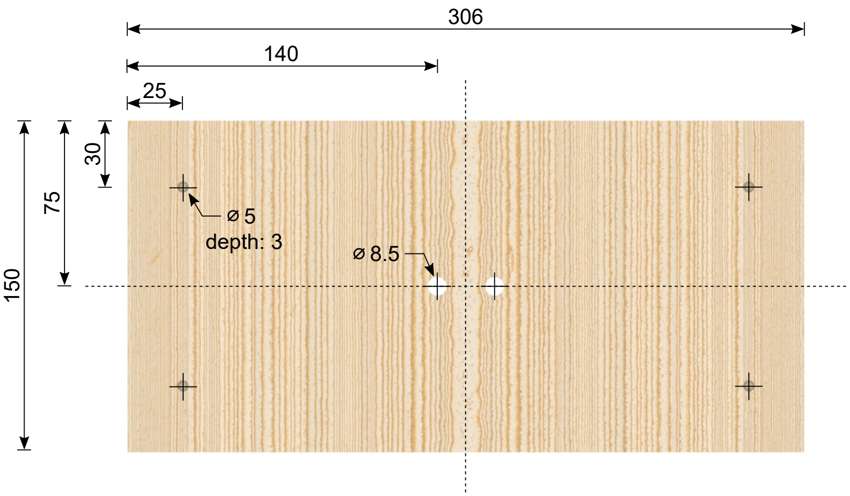

Figure A4. Drill pattern of wood 2 (top). The depth of the $\varnothing\ 5$ mm holes is 3 mm. All dimensions are in mm.

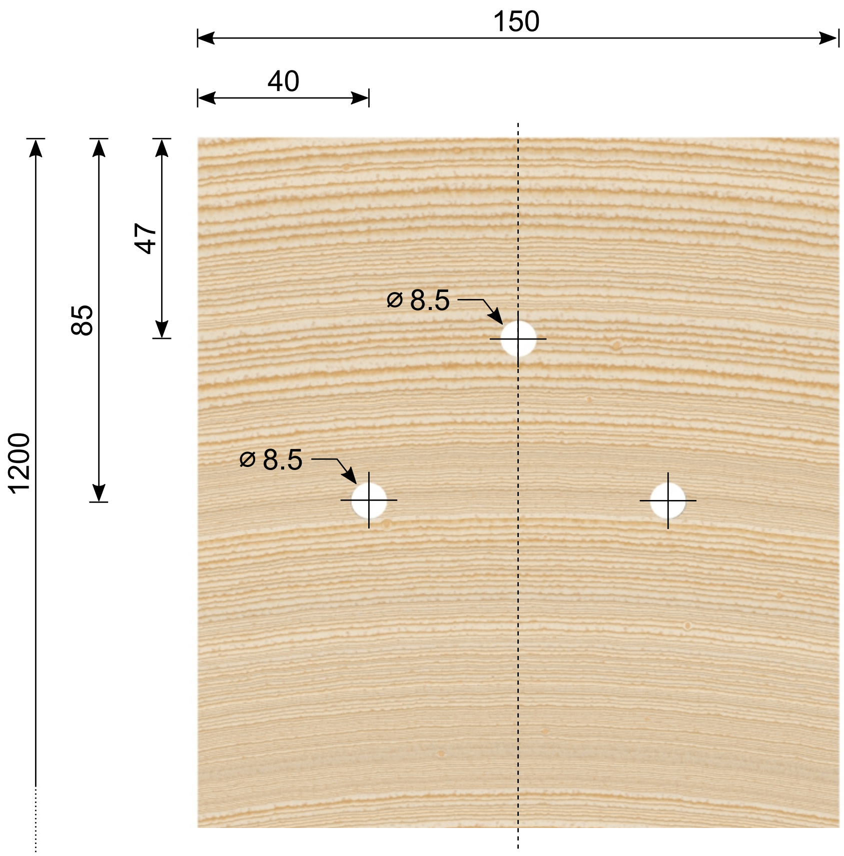

Figure A5. Drill pattern of wood 1, where the bearing will be attached to. All dimensions are in mm.

- The wood base (wood 3) needs to be pre-drilled as follows
- The two 4” x 2” x 8’ wood pieces need to be cut to 4 x 1000 mm long wood pieces. Two of them (wood 4) and the other two (wood 5) get pre-drilled as follows

The whole instruction is explained on YouTube. These steps provide an instruction for the assembly:

Step 1

At the beginning of the assembly process, the solenoids are placed in the upper part of the 3D printed part (2) using 1 1/8”-7 nuts. The ⌀ 8 mm metal rod gets pressed symmetric about the center through the guidance of part (2) and is secured with 2 x M8x1.25 screws (30 mm length), 4 x M8 washers and 2 x M8 nuts. 4 x M5x0.8 screw (110 mm length) and 4 x M5 washers need to be put in place in part (4) for the next step’s assembly with part (6). Then, part (4) can be attached

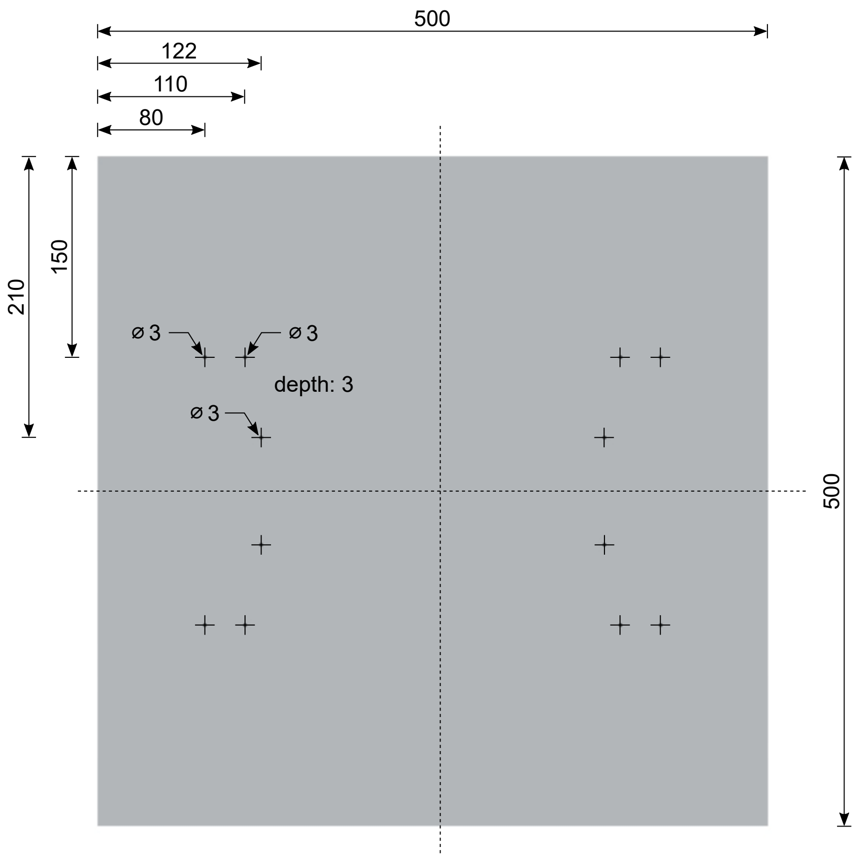

Figure A6. Drill pattern of wood 3 (bottom). The depth of the ⌀ 3 mm holes is 3 mm. Note: The color is grey to show the drill pattern better. All dimensions are in mm.

to (2) with 4 x (3), and fastened with 4 x M5x0.8 screws (110 mm length), 8 x M5 washers and 4 x M5 nuts.

Step 2

One ⌀ 5 mm linear ball bearing is pressed in the (4). (5) is pressed in (4) against the linear bearing. Simultaneously another ⌀ 5 mm linear ball bearing is pressed in (6). (6) gets now pressed against (4), where (5) and the 4 pre-assembled M5x0.8 screws (110 mm length) function as guidance. 4 x M5 washers and 4 x M5 nuts are used to fasten the setup.

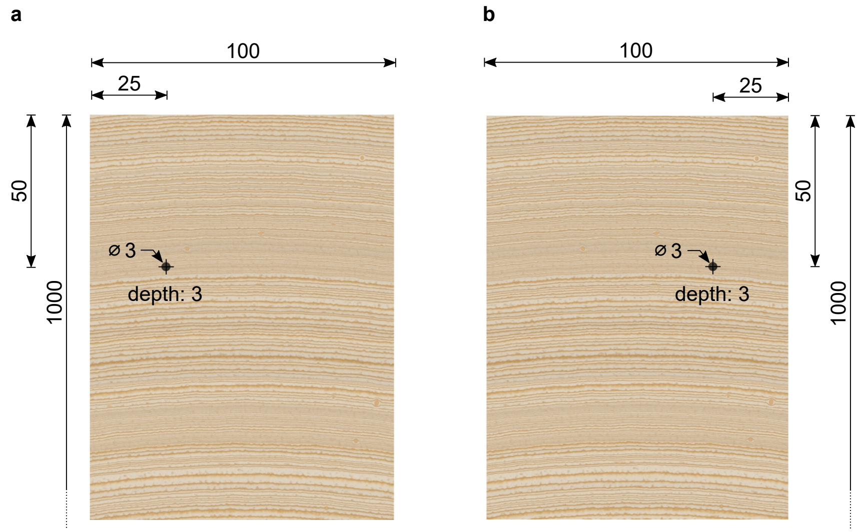

Figure A7. Drill pattern of wood 4 and 5. The depth of the ⌀ 3 mm holes is 3 mm. a) wood 4. b) wood 5. 2 copies of each wood are needed. All dimensions are in mm.

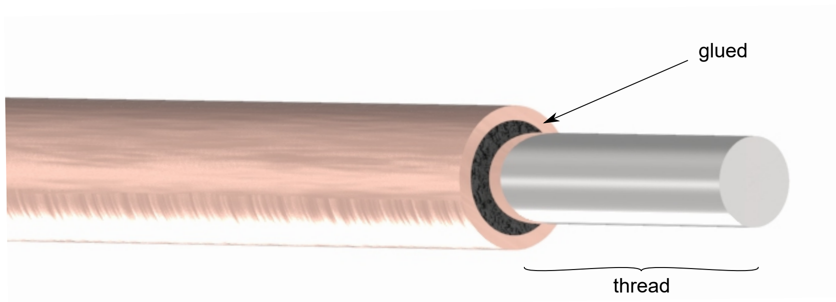

Figure A8. Copper tube spacer. The spacer (black) is 3D printed.

Step 3

1 x 10 gets slid over the ⌀ 5 mm metal rod until the end of the thread from either side. The copper tube is slid over 10 until the ends of 10 and the copper tube match (as illustrated in Figure A8). Super glue is used to create a stiff connection between the metal rod, 10 and the copper tube as shown in Figure A8. From the other end of the rod, another 10 is slid over the metal rod and guided into the copper tube until it is completely in between the metal rod and the copper tube. Super glue is used again to create a stiff connection. 8 is attached to the ⌀ 5 mm metal rod by 2 x M5x0.8 nuts and 2 x M5 washers from each end of 8 respectively. 8 is connected to 9 by 2 x

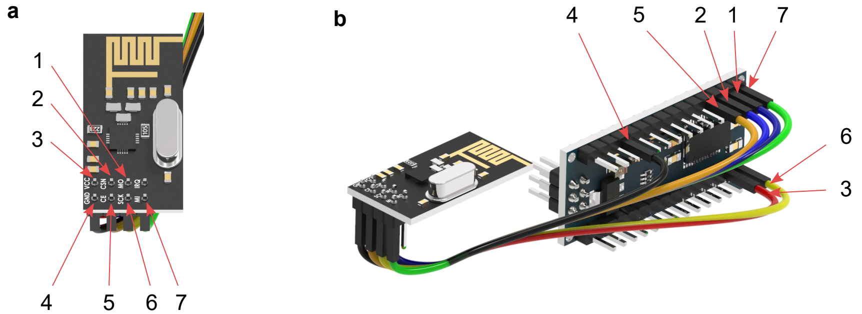

Figure A9. Wiring Transmitter and Arduino Nano. **Transmitter**: 1 - MOSI, 2 - CSN, 3 - VCC, 4 - GND, 5 - CE, 6 - SCK, 7 - MISO; **Arduino Nano**: 1 - D11, 2 - D10, 3 - 3V3, 4 - GND, 5 - D9, 6 - D13, 7 - D12.

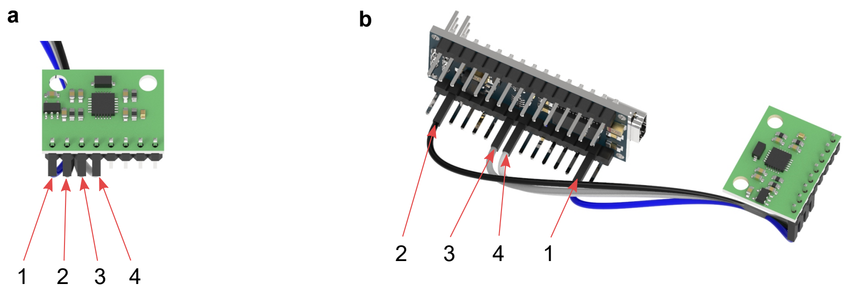

Figure A10. Wiring Gyroscope and Arduino Nano. **Gyroscope**: 1 - VCC, 2 - GND, 3 - SCL, 4 - SDA; **Arduino Nano**: 1 - 3V3, 2 - GND, 3 - A5, 4 - A4.

M8x1.25 screws (30 mm length), 4 x M8 washers and 2 x M8 nuts. 9 houses an Arduino Nano, and holds a MPU6050 gyroscope and a transmitter. A powerbank supplies the components with power. The gyroscope and the transmitter are attached to the 3D printed body with sticky tape and a spongy material is used between the sensors and 9. A mini USB cable connects the Arduino Nano to the powerbank. The wiring between the Arduino Nano, gyroscope and the transmitter can be found in Figure A9 and Figure A10. Note: Cable 3 of Figure A9 and cable 1 of Figure A10 need to be connected by soldering and then plugged to the Arduino Nano. Figure A9 shows how the final sensor part looks.

Step 4

1 x #9 O-ring needs to be placed on each solenoid pin. The U-shaped profile gets plugged into the solenoids moving pins. 2 x part 7 need to be placed between the U-profile and the solenoid pins. 2 x M2.5x0.45 (20 mm length) screws, 4 x M2.5 washers and 4 x M2.5 nuts are used to fasten the U-shaped profile to the solenoid pins. The $\varnothing$ 5 mm metal rod is slid through 6, 4 and both linear bearings. On the metal rod thread a M5x0.8 nut and a M5 washer need to be placed on the rod to hold it back, before the rod slides through the U-shaped profile and gets fastened by another

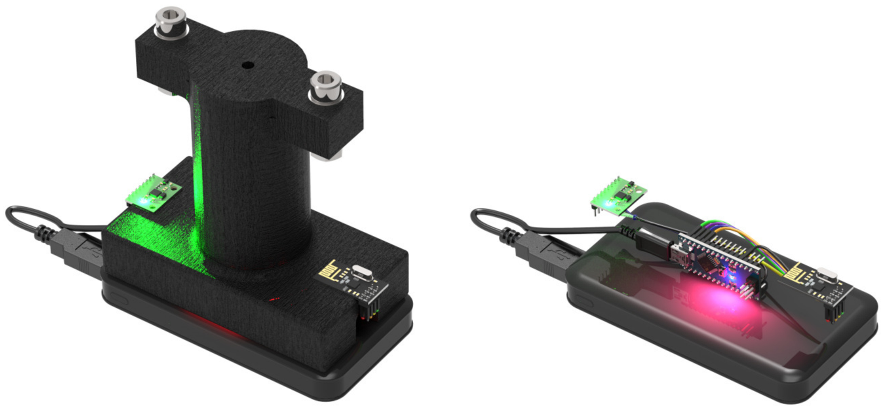

Figure A11. Assembled payload chassis cover and payload chassis with transmitter, Arduino Nano and Gyroscope

M5x0.8 nut and a M5 washer. A spongy material needs to be placed between 9 and the U-shaped profile to dampen the impact of the solenoid's movement.

Step 5

Both wood 2 are being put on the wood 3 so that the assembled pendulum is placed as centered. 3 x M8x1.25 screws (80 mm length), 6 x M8 washers and 3 x M8 nuts are used to attach 1 to each wood 2. 2 x $\varnothing$ 8 mm Ball Bearings are pressed into 1. Then both wood 2 can be put together with the assembled pendulum from both sides, where the bearing function as a guidance. Wood 1 needs to be put on the top and fastened with 4 x #9x3 screws in total using the pre-drilled holes as guidance. 2 x wood 4 and 2 x wood 5 get put against the assembled structure, so that the inner parts align. 4 x #9x3 screws are used in total to fasten wood 4 and wood 5 to the assembled structure by using the pre-drilled holes as guidance. 4 x corner braces with each 8 x #9x1 screws are used to stabilize wood 4 and wood 5 by being placed on the inner side. Finally, 12 x #9x3 screws are screwed from the bottom side of wood 3 by using the pre-drilled whole as orientation points to stabilize the whole structure.

Step 6

Connect the power cord of the Power Supply to a socket and place the relay module on wood 3. Insert the mini USB cable in the computer and connect it to the Arduino Nano which is used to receive the data. Wire the receiving Arduino Nano to the receiver module as prescribed in Figure A12 Use 2 x 5 A fuses and place them on wood 4. Fix them with sticky tape. Wire the receiving Arduino Nano to the relay module as prescribed in Figure A13. Solder 22 gauge wire to each black wire coming out of the solenoid to extend them. Use the black crocodile clip which came with the power supply and grab both extended black cables from the solenoids and plug the other end in the power supply. Connect the red crocodile clip to the power supply and connect it to

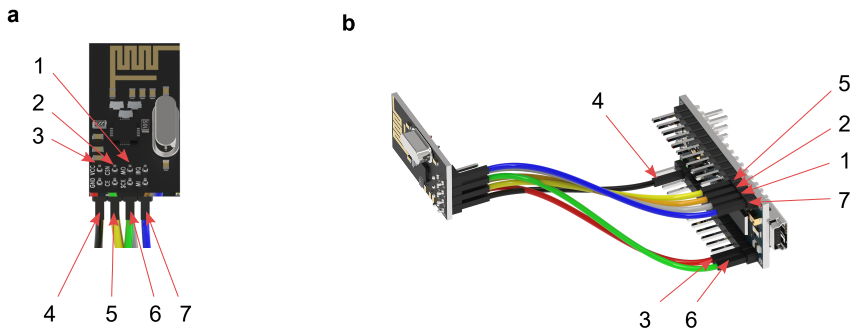

Figure A12. Wiring Receiver and Arduino Nano. **Receiver**: 1 - MOSI, 2 - CSN, 3 - VCC, 4 - GND, 5 - CE, 6 - SCK, 7 - MISO; **Arduino Nano**: 1 - D11, 2 - D10, 3 - 3V3, 4 - GND, 5 - D9, 6 - D13, 7 - D12.

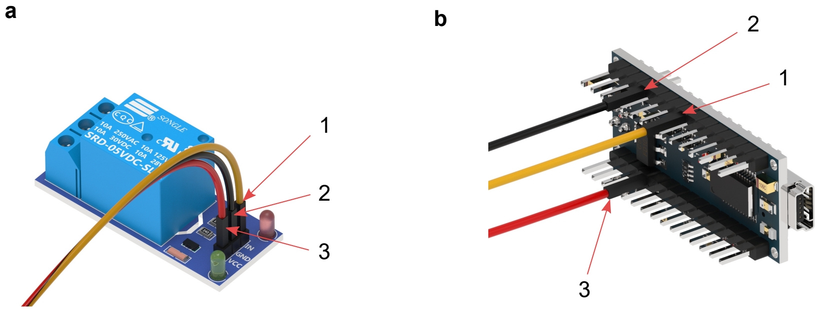

Figure A13. Wiring Relay and Arduino Nano. **Relay module**: 1 - Signal input, 2 - DC-, 3 - DC+; **Arduino Nano**: 1 - D5, 2 - GND, 3 - 5 V.

a 22 gauge wire. Connect this wire to the "COM" output of the relay. Take two 22 gauge wires and insert both of them into the "NO" (Normally open) output of the relay module. The other ends of both wires need to be soldered on the each fuse respectively. Take again 22 gauge wires and solder each of them on the remaining spot of the fuses. The other end of the wires need to be soldered to the red cables of solenoid.

Data acquisition

We decided to use the Arduino IDE software to control the Arduino Nanos. Two codes need to be written for the transmitter and receiver respectively which can be downloaded with the following Github link. Download the "MPU6050_light.h" library which is used to get the data from the gyroscope and then download the "nRF24L01.h" library which is used to send and receive data with the nRF24L01 modules (transmitter & receiver) and embed them in the Arduino IDE environment. For the experiment there are several requirements which need to be taken into account

- 1. Wireless transmission of the angular velocity $\dot{\Phi}$ with a reasonable sampling frequency

- 2. Robustness to the magnetic field caused by the solenoids
- 3. Robustness to the hard impacts caused by the solenoids
- 4. Remote calibration of the gyroscope

1. The difficulty is that 4. is causing a trade-off with 1. If the transmitter can be calibrated at any time instant, then the sampling frequency drops because it needs to be enabled to "listen" during every void loop. We will describe our algorithm on how we achieved a reasonable sampling frequency.

2. Instead of a nRF24L01 receiver module with an antenna, a nRF24L01 receiver without an antenna is used. During conducted experiments the authors found out that an antenna actually gets influenced by the magnetic field and was harming the wireless transmission.

3. A low-pass filter for the angular velocity is implemented because the impacts caused vibrations and influence the angular velocity. A detailed explanation is provided in subsection Low-pass filter.

4. After the transmitter code is uploaded the transmitter is just "listening". The time (in ms) and the angular velocity (in deg/s) are sent with a frequency of ≈ 5 Hz and the low sampling frequency shows that the receiver needs to be calibrated. After a successful calibration the sampling frequency is ≈ 116 Hz. After 100 s have passed the the sampling frequency drops again to ≈ 5 Hz, which shows the user that another calibration is needed and the transmitter is enabled to "listen" again in order to receive the calibration command.

Note: The transmitter code needs to uploaded just once to the Arduino Nano by using the computer. Based on the received data, the receiving Arduino decides if the solenoids are activated or deactivated.

Predicting angular velocity and determining angle

We need to account for the dead time which is the delay from the instant when $\dot{\Phi} = 0$ to the activation of the solenoid. We estimated for our setup that the dead time was $t_{dead,on} = 0.09$ s. Since one cannot exactly measure the time instant when $\dot{\Phi} = 0$, and to account for the dead time, we use linear extrapolation to estimate the zero crossing time for $\dot{\Phi}$ using the equation:

$$\dot{\Phi}_{n+1} = \frac{\dot{\Phi}_n - \dot{\Phi}_{n-1}}{t_n - t_{n-1}} t_{dead,on} + \dot{\Phi}_n \tag{A5}$$

where $\dot{\Phi}_{n+1}$ is the predicted angular velocity, $\dot{\Phi}_n$ and $\dot{\Phi}_{n-1}$ are the angular velocities at times t_n and t_{n-1} respectively. It is obvious that time instants where $\dot{\Phi}_{n+1} = 0$ °/s are difficult to catch. Therefore a lower and upper bound needs to be applied around 0 °/s and we consider:

$$\dot{\Phi}_l < \dot{\Phi}_{n+1} < \dot{\Phi}_u \tag{A6}$$

holds. $\dot{\Phi}_l$ is the lower and $\dot{\Phi}_u$ the upper bound respectively. However, since we are interested in an attenuation of the angle, one can conceive of a scenario where $\dot{\Phi}_l < \dot{\Phi}_{n+1} < \dot{\Phi}_u$, i.e., the angular velocity lies within the bounds used to forecast the time for the zero crossing of Φ. To

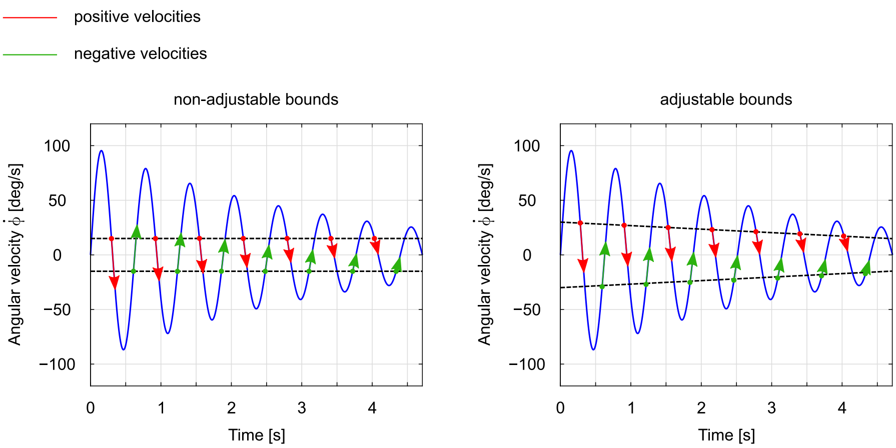

Figure A14. Difference between non-adjustable (left figure) and adjustable (right figure) bounds to capture $\dot{\Phi} = 0$ °/s. Over time the bounds are shrinking as illustrated in the right figure. This is an example of adjustable bounds for an attenuation experiment.

prevent such a scenario we tighten the bounds over time as illustrated in Figure A14. For our attenuating experiment we start with $\dot{\Phi}_l = -50$ °/s and $\dot{\Phi}_u = 50$ °/s. The final time is 20 s and the bounds are $\dot{\Phi}_l = -45$ °/s and $\dot{\Phi}_u = 45$ °/s. For any time instant in between the start and final time we linearly interpolate the bounds. For catching $\Phi = 0°$ precisely we cannot rely on the angular velocity measurement because even when using a low-pass filtered signal the impact caused by the solenoids create unpredictable peaks which can't be used for an integration over time. As mentioned in section Introduction the natural frequency is changing when the solenoids are activated compared to a deactivated case. Furthermore, the natural frequency is a function of the magnitude of angular displacements and decreases with an increase in the magnitude of displacement. After letting the payload swing the instant where $\dot{\Phi} = 0$ °/s, would provide half a time period of a mixed system (activated & deactivated solenoids). However the natural frequencies are reasonably close, so half of this time period provides a satisfying estimate of the time when the solenoids need to be deactivated. Figure A15 illustrates the described strategy. The dead time to deactivate the solenoids is $t_{dead,off} = 0.1$ s, where gravity is the only active force. It should be noted that the release dynamics of the solenoid on the active pendulum setup is not only a function of gravity but the centrifugal force as well, which reduces the deactivation dead time. Assuming t_1 and t_2 are two consecutive time instants when the angular velocity is zero. The time to deactivate the solenoid is given by the equation:

$$t_{off} = \frac{t_2 - t_1}{2} + t_2 - t_{dead,off}. \tag{A7}$$

We found that if $t_{dead,off}$ in Eq. (A7) is set to 0 s we observed experimental results match the simulation results. This can be explained by the fact that the centrifugal force is the greatest when the solenoid is deactivated and reduces the transition time. However, for the pumping up the swing experiment, the solenoid is deactivated when the centrifugal force is zero and the impact of the gravity force progressively decreases, mandating the inclusion of the transition time of the

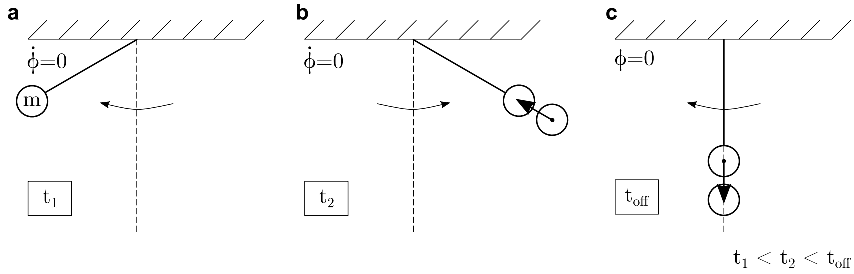

Figure A15. Strategy to capture the angular displacement $\Phi = 0\ ^\circ$ by purely measuring the angular velocity $\dot{\Phi}$. a) $\dot{\Phi}_{n+1}$ got captured at t_1. b) $\dot{\Phi}_{n+1}$ got captured at t_2. c) $1/4$ of the time period has passed and the solenoid is being deactivated.

solenoid. Note that for the next iteration $t_{1,new} = t_2$ and t_2 will be the new measured time instant. For the pumping the swing experiment, the lower bounds for determining $\dot{\Phi}$ were $\dot{\Phi}_l = -10\ ^\circ$ and $\dot{\Phi}_u = 10\ ^\circ$ at the initial time. At the final time of $t = 40$ s, the bounds are $\dot{\Phi}_l = -15\ ^\circ$ and $\dot{\Phi}_u = 15\ ^\circ$. The delay for contracting the solenoids is $t_{dead,on} = 0.1$ s, so the equation is:

$$t_{on} = \frac{t_1 + t_2}{2} + t_1 - t_{dead,on} \tag{A8}$$

Procedure of an experiment

The following steps should help the reader understand how an experiment is conducted. Turn on the power module and set it to 24V.

1. Connect the computer to the transmitting Arduino and upload the transmitter code. Disconnect the computer from the Arduino.
2. Connect the computer to the receiving Arduino and upload the receiver code. Open its Serial Monitor.
3. Place the pendulum in a vertical down position. The user is asked to choose between “1” for calibration or “2” for starting an experimental testing. Concurrently, the sampling rate is visualized. During calibration, the pendulum should be undisturbed. If the sampling is not above 100 Hz, you need to calibrate the device and send “1”. The process takes about 10 s. If the calibration is successful the user gets notified and can choose Start with the “2” option. If the calibration wasn’t successful please try to calibrate again and end a “1”.
4. A successful calibration for our setup a resulted in a sampling frequency in between 110 and 125 Hz. You are then asked to displace the pendulum in a $-90\ ^\circ$ position and hold it.
5. A countdown will run up to 100% and a “Let go!” message will be displayed. This is the moment you need to release the pendulum.