

# **Titel der Arbeit Extra Lang und noch Länger und noch noch Extra Lang und noch Länger Extra Lang und noch Länger**

Dissertation  
zur Erlangung des akademischen Grades  
"‘doctor rerum naturalium’"  
(Dr. rer. nat.)  
in der Wissenschaftsdisziplin Informatik

eingereicht an der  
Mathematisch-Naturwissenschaftlichen Fakultät  
der Universität Potsdam  
von

**Foo Bar**

Potsdam, Germany,  
10. Juli 2012



# Inhaltsverzeichnis

<b>Vorwort</b>	<b>v</b>
<b>1 Einleitung</b>	<b>1</b>
1.1 General Related Work . . . . .	1
1.2 Contributions . . . . .	2
1.3 Application Areas . . . . .	3
<b>2 Erstes Hauptteil Kapitel</b>	<b>5</b>
2.1 Aat accusam aliquyam diam . . . . .	5
2.1.1 Stet clita kasd gubergren . . . . .	6
2.1.2 Stet clita kasd gubergren . . . . .	7
2.1.3 At vero eos et accusam . . . . .	8
2.2 At accusam aliquyam . . . . .	8
<b>3 Sample File from SIAM L<sup>A</sup>T<sub>E</sub>X Book Macro Package</b>	<b>17</b>
3.1 Introduction and Examples . . . . .	17
3.1.1 Sample text . . . . .	17
3.1.2 Some list environments . . . . .	18
3.1.3 Some displayed equations . . . . .	19
3.2 Main Results . . . . .	20
3.2.1 Numerical experiments . . . . .	22
<b>Literaturverzeichnis</b>	<b>25</b>



# Vorwort

Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim Alvarez et al. (1992) ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi.

Test: Ä, ä, ß



## Kapitel 1

# Einleitung

Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet.

Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consetetur adipiscing elit, sed diam nonumy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat.

Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi.

## 1.1 General Related Work

Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet.

Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat,

vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat.

Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi.

Nam liber tempor cum soluta nobis eleifend option congue nihil imperdiet doming id quod mazim placerat facer possim assum. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat.

Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis.

At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur.

## 1.2 Contributions

Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet.

Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat.

Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi.

Nam liber tempor cum soluta nobis eleifend option congue nihil imperdiet doming id quod mazim placerat facer possim assum. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam



erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat.

Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis.

At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur

## 1.3 Application Areas

Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet.

Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consetetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat.

Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi.

Nam liber tempor cum soluta nobis eleifend option congue nihil imperdiet doming id quod mazim placerat facer possim assum. Lorem ipsum dolor sit amet, consetetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat.



## Kapitel 2

# Erstes Hauptteil Kapitel

Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et (Aubert & Kornprobst, 2006) dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet.

Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consetetur adipiscing elit, sed diam nonumy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Nam liber tempor cum soluta nobis eleifend option congue nihil imperdiet doming id quod mazim placerat facer possim assum. Lorem ipsum dolor sit amet, consetetur adipiscing elit, sed diam nonumy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat.

## 2.1 Aat accusam aliquyam diam

Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus

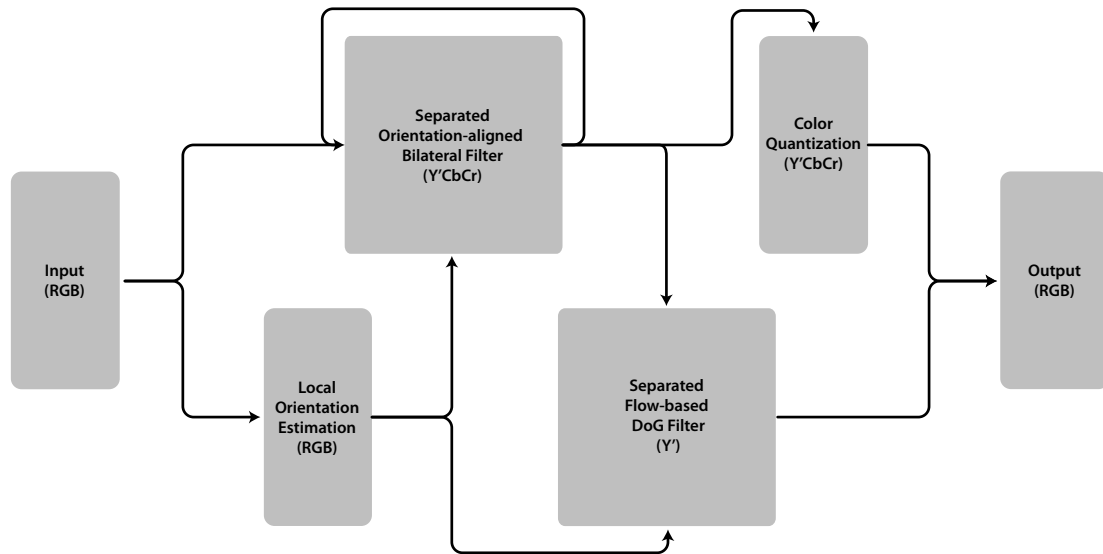
est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, At accusam aliquyam diam diam dolore dolores duo eirmod eos erat, et nonumy sed tempor et et invidunt justo labore Stet clita ea et gubergren, kasd magna no rebum. sanctus sea sed takimata ut vero voluptua. est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat.

### 2.1.1 Stet clita kasd gubergren

Consetetur (Goshtasby & Satter, 2008) sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus.

- Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et
- Et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit.
  - Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et.
  - Et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit.
  - Amet, consetetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat.
- Amet, consetetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat.

Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel



**Abbildung 2.1:** *Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr.*

illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consetetur adipiscing elit, sed diam nonumy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi.

### 2.1.2 Stet clita kasd gubergren

Nam liber tempor cum soluta nobis eleifend option congue nihil imperdiet doming id quod mazim placerat facer possim assum. Lorem ipsum dolor sit amet, consetetur adipiscing elit, sed diam nonumy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, At accusam aliquyam diam diam dolore dolores duo eirmod eos erat, et nonumy sed tempor et et invidunt justo labore Stet clita ea et gubergren, kasd magna no rebum. sanctus sea sed takimata ut vero voluptua. est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat.

Consetetur sadipscing elitr, (Yang et al., 1996) sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus.

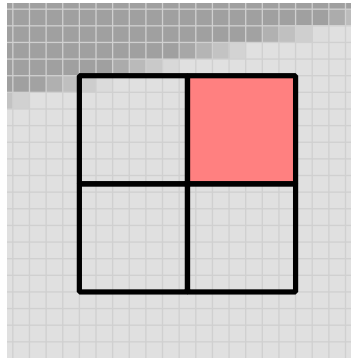
### **2.1.3 At vero eos et accusam**

Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consetetur adipiscing elit, sed diam nonumy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat.

Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat (Guichard & Morel, 2003) nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Nam liber tempor cum soluta nobis eleifend option congue nihil imperdiet doming id quod mazim placerat facer possim assum. Lorem ipsum dolor sit amet, consetetur adipiscing elit, sed diam nonumy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis.

## **2.2 At accusam aliquyam**

At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur

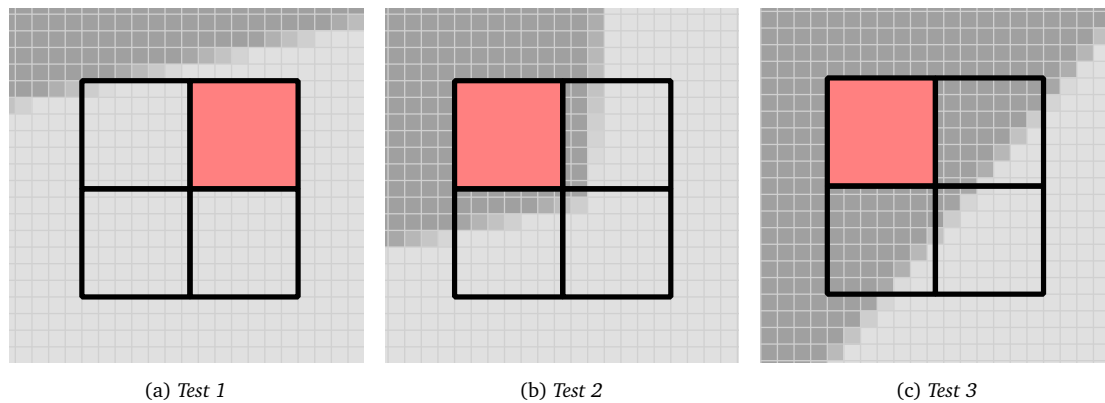


**Abbildung 2.2:** *Stet clita kasd gubergren, no sea takimata sanctus est.*

sadipscing elit, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, At accusam aliquyam diam diam dolore dolores duo eirmod eos erat, et nonumy sed tempor et et invidunt justo labore Stet clita ea et gubergren, kasd magna no rebum. sanctus sea sed takimata ut vero voluptua. est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat.

- Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et
- Et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit.
- Amet, consetetur adipiscing elit, sed diam nonumy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat.

Consetetur sadipscing elitr, (Barash & Comaniciu, 2004) sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus



**Abbildung 2.3:** *Stet clita kasd gubergren, no sea takimata*

est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consetetur adipiscing elit, sed diam nonumy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat.

**Ut wisi enim** ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Nam liber tempor cum soluta nobis eleifend option congue nihil imperdiet doming id quod mazim placerat facer possim assum. Lorem ipsum dolor sit amet, consetetur adipiscing elit, sed diam nonumy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis.

At vero eos et accusam et (Black & Rangarajan, 1996) justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, At accusam aliquyam diam diam dolore dolores duo eirmod eos erat, et nonumy sed tempor et et invidunt justo labore Stet clita ea et gubergren, kasd magna no rebum. sanctus sea sed takimata ut vero voluptua. est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat.



Referenzieren von Abbildungen: Abbildung 2.3, Abbildung 2.3(a), Abbildung 2.3(a)-(c).

Consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus.

Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue dui dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue dui dolore te feugait nulla facilisi. Nam liber tempor cum soluta nobis eleifend option congue nihil imperdiet doming id quod mazim placerat facer possim assum. Lorem ipsum dolor sit amet, consetetur adipiscing elit, sed diam nonumy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis.

At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, At accusam aliquyam diam diam dolore dolores duo eirmod eos erat, et nonumy sed tempor et et invidunt justo labore Stet

```
1  uniform sampler2D img;
2  uniform sampler2D tfm;
3  uniform int pass;
4  uniform float sigma_d;
5  uniform float sigma_r;
6
7  void main (void) {
8      float twoSigmaD2 = 2.0 * sigma_d * sigma_d;
9      float twoSigmaR2 = 2.0 * sigma_r * sigma_r;
10     vec2 uv = gl_TexCoord[0].xy;
11
12     vec2 t = texture2D(tfm, uv).xy;
13     vec2 dir = (pass == 0)? vec2(t.y, -t.x) : t;
14     vec2 dabs = abs(dir);
15     float ds = 1.0 / ((dabs.x > dabs.y)? dabs.x : dabs.y);
16     dir /= textureSize2D(img, 0);
17
18     vec3 center = texture2D(img, uv).rgb;
19     vec3 sum = center;
20     float norm = 1.0;
21
22     float halfWidth = 2.0 * sigma_d;
23     for (float d = ds; d <= halfWidth; d += ds) {
24         vec3 c0 = texture2D(img, uv + d * dir).rgb;
25         vec3 c1 = texture2D(img, uv - d * dir).rgb;
26         float e0 = length(c0 - center);
27         float e1 = length(c1 - center);
28
29         float kerneld = exp( - d *d / twoSigmaD2 );
30         float kernele0 = exp( - e0 *e0 / twoSigmaR2 );
31         float kernele1 = exp( - e1 *e1 / twoSigmaR2 );
32         norm += kerneld * kernele0;
33         norm += kerneld * kernele1;
34
35         sum += kerneld * kernele0 * c0;
36         sum += kerneld * kernele1 * c1;
37     }
38     sum /= norm;
39     gl_FragColor = vec4(sum, 1.0);
40 }
```

**Listing 2.1:** *Consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna.*

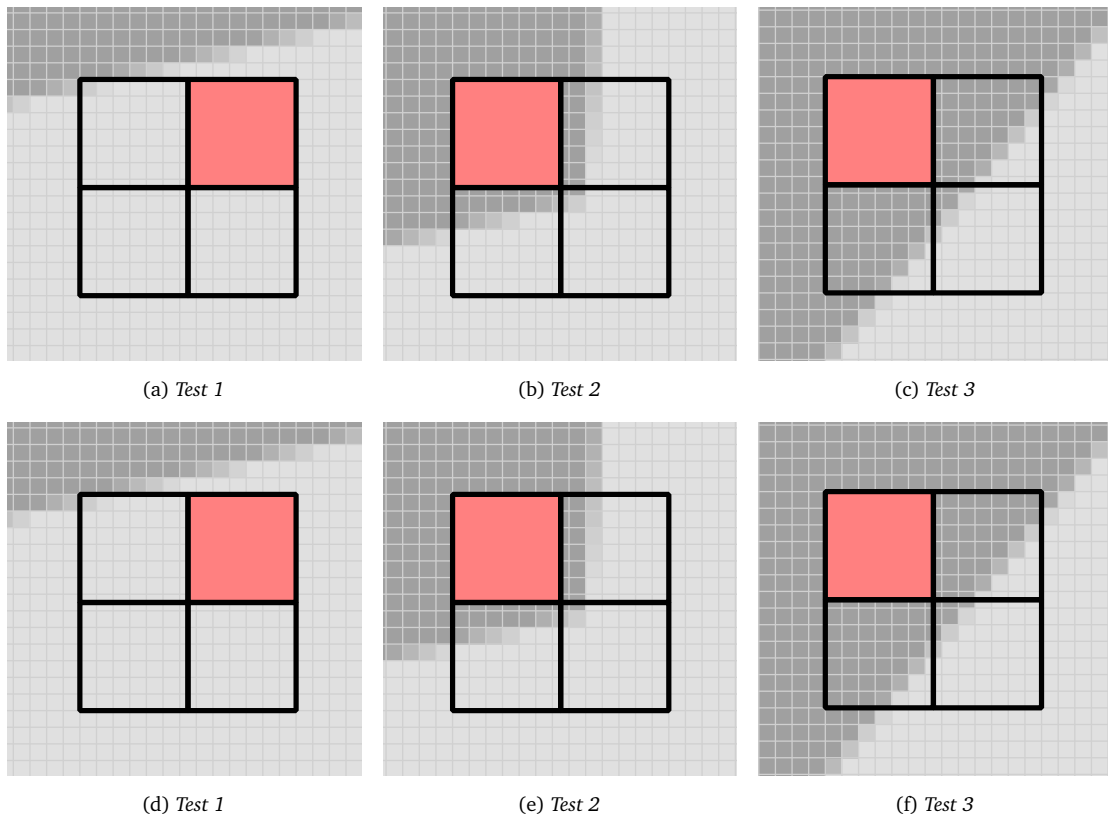
clita ea et gubergren, kasd magna no rebum. sanctus sea sed takimata ut vero voluptua. est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat.

Consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus.

Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consetetur adipiscing elit, sed diam nonumy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat.

Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Nam liber tempor cum soluta nobis eleifend option congue nihil imperdiet doming id quod mazim placerat facer possim assum. Lorem ipsum dolor sit amet, consetetur adipiscing elit, sed diam nonumy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis.

At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor



**Abbildung 2.4:** *Stet clita kasd gubergren, no sea takimata sanctus est. Stet clita kasd gubergren, no sea takimata sanctus est.*

sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, At accusam aliquyam diam diam dolore dolores duo eirmod eos erat, et nonumy sed tempor et et invidunt justo labore Stet clita ea et gubergren, kasd magna no rebum. sanctus sea sed takimata ut vero voluptua. est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat. Consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum



## Kapitel 3

# Sample File from SIAM L<sup>A</sup>T<sub>E</sub>X Book Macro Package

### 3.1 Introduction and Examples

This paper presents a sample file for the use of SIAM's L<sup>A</sup>T<sub>E</sub>X macro package. It illustrates the features of the<sup>1</sup> macro package, using actual examples culled from various papers published in SIAM's journals. It is to be expected that this sample will provide examples of how to use the macros to generate standard elements of journal papers, e.g., theorems, definitions, or figures. This paper also serves as an example of SIAM's stylistic preferences for the formatting of such elements as bibliographic references, displayed equations, and equation arrays, among others. Some special circumstances are not dealt with in this sample file; for such information one should see the included documentation file.

*Note:* This paper is not to be read in any form for content. The conglomeration of equations, lemmas, and other text elements were put together solely for typographic illustrative purposes and don't make any sense as lemmas, equations, etc.

#### 3.1.1 Sample text

Let  $S = [s_{ij}]$  ( $1 \leq i, j \leq n$ ) be a  $(0, 1, -1)$ -matrix of order  $n$ . Then  $S$  is a *sign-nonsingular matrix* (SNS-matrix) provided that each real matrix with the same sign pattern as  $S$  is nonsingular. There has been considerable recent interest in constructing and characterizing SNS-matrices Healey et al. (2004), Dai et al. (2007). There has also been interest in strong forms of sign-nonsingularity Balikai et al. (2008). In this paper we give a new generalization of SNS-matrices and investigate some of their basic properties.

Let  $S = [s_{ij}]$  be a  $(0, 1, -1)$ -matrix of order  $n$  and let  $C = [c_{ij}]$  be a real matrix of order  $n$ . The pair  $(S, C)$  is called a *matrix pair of order  $n$* . Throughout,  $X = [x_{ij}]$  denotes a matrix of order  $n$  whose entries are algebraically independent indeterminates over the real field. Let  $S \circ X$  denote the Hadamard product (entrywise product) of  $S$  and  $X$ . We say that the pair  $(S, C)$  is a *sign-nonsingular matrix pair of order  $n$* , abbreviated *SNS-matrix pair of order  $n$* , provided that the matrix

$$A = S \circ X + C$$

---

<sup>1</sup>This is a sample footnote. This is a sample footnote. This is a sample footnote. This is a sample footnote. This is a sample footnote. This is a sample footnote.

is nonsingular for all positive real values of the  $x_{ij}$ . If  $C = O$  then the pair  $(S, O)$  is a SNS-matrix pair if and only if  $S$  is a SNS-matrix. If  $S = O$  then the pair  $(O, C)$  is a SNS-matrix pair if and only if  $C$  is nonsingular. Thus SNS-matrix pairs include both nonsingular matrices and sign-nonsingular matrices as special cases.

The pairs  $(S, C)$  with

$$S = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

and

$$S = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

are examples of SNS-matrix pairs.

### 3.1.2 Some list environments

In this paper we consider the evaluation of integrals of the following forms:

$$\int_a^b \left( \sum_i E_i B_{i,k,x}(t) \right) \left( \sum_j F_j B_{j,l,y}(t) \right) dt, \quad (3.1)$$

$$\int_a^b f(t) \left( \sum_i E_i B_{i,k,x}(t) \right) dt, \quad (3.2)$$

where  $B_{i,k,x}$  is the  $i$ th B-spline of order  $k$  defined over the knots  $x_i, x_{i+1}, \dots, x_{i+k}$ . We will consider B-splines normalized so that their integral is one. The splines may be of different orders and defined on different knot sequences  $x$  and  $y$ . Often the limits of integration will be the entire real line,  $-\infty$  to  $+\infty$ . Note that (3.1) is a special case of (3.2) where  $f(t)$  is a spline.

There are five different methods for calculating (3.1) that will be considered; here is the remunerate list:

1. Use Gauss quadrature on each interval.
2. Convert the integral to a linear combination of integrals of products of B-splines and provide a recurrence for integrating the product of a pair of B-splines.
3. Convert the sums of B-splines to piecewise Bézier format and integrate segment by segment using the properties of the Bernstein polynomials.
4. Express the product of a pair of B-splines as a linear combination of B-splines. Use this to reformulate the integrand as a linear combination of B-splines, and integrate term by term.
5. Integrate by parts.



Of these five, only methods 1 and 5 are suitable for calculating (3.2). The first four methods will be touched on and the last will be discussed at length.

Here is the bullet list:

- Use Gauss quadrature on each interval.
- Convert the integral to a linear combination of integrals of products of B-splines and provide a recurrence for integrating the product of a pair of B-splines.
- Convert the sums of B-splines to piecewise Bézier format and integrate segment by segment using the properties of the Bernstein polynomials.
- Express the product of a pair of B-splines as a linear combination of B-splines. Use this to reformulate the integrand as a linear combination of B-splines, and integrate term by term.
- Integrate by parts.

and, finally, the romannum list:

1. Use Gauss quadrature on each interval.
2. Convert the integral to a linear combination of integrals of products of B-splines and provide a recurrence for integrating the product of a pair of B-splines.
3. Convert the sums of B-splines to piecewise Bézier format and integrate segment by segment using the properties of the Bernstein polynomials.
4. Express the product of a pair of B-splines as a linear combination of B-splines. Use this to reformulate the integrand as a linear combination of B-splines, and integrate term by term.
5. Integrate by parts.

### 3.1.3 Some displayed equations

By introducing the product topology on  $\mathbb{R}^{m \times m} \times \mathbb{R}^{n \times n}$  with the induced inner product

$$\langle (A_1, B_1), (A_2, B_2) \rangle := \langle A_1, A_2 \rangle + \langle B_1, B_2 \rangle, \quad (3.3)$$

we calculate the Fréchet derivative of  $F$  as follows:

$$\begin{aligned} F'(U, V)(H, K) &= \langle R(U, V), H\Sigma V^T + U\Sigma K^T - P(H\Sigma V^T + U\Sigma K^T) \rangle \\ &= \langle R(U, V), H\Sigma V^T + U\Sigma K^T \rangle \\ &= \langle R(U, V)V\Sigma^T, H \rangle + \langle \Sigma^T U^T R(U, V), K^T \rangle. \end{aligned} \quad (3.4)$$

In the middle line of (3.4) we have used the fact that the range of  $R$  is always perpendicular to the range of  $P$ . The gradient  $\nabla F$  of  $F$ , therefore, may be interpreted as the pair of matrices:

$$\nabla F(U, V) = (R(U, V)V\Sigma^T, R(U, V)^T U\Sigma) \in \mathbb{R}^{m \times m} \times \mathbb{R}^{n \times n}. \quad (3.5)$$

Because of the product topology, we know

$$\mathcal{T}_{(U,V)}(\mathcal{O}(m) \times \mathcal{O}(n)) = \mathcal{T}_U \mathcal{O}(m) \times \mathcal{T}_V \mathcal{O}(n), \quad (3.6)$$

where  $\mathcal{T}_{(U,V)}(\mathcal{O}(m) \times \mathcal{O}(n))$  stands for the tangent space to the manifold  $\mathcal{O}(m) \times \mathcal{O}(n)$  at  $(U, V) \in \mathcal{O}(m) \times \mathcal{O}(n)$  and so on. The projection of  $\nabla F(U, V)$  onto  $\mathcal{T}_{(U,V)}(\mathcal{O}(m) \times \mathcal{O}(n))$ , therefore, is the product of the projection of the first component of  $\nabla F(U, V)$  onto  $\mathcal{T}_U \mathcal{O}(m)$  and the projection of the second component of  $\nabla F(U, V)$  onto  $\mathcal{T}_V \mathcal{O}(n)$ . In particular, we claim that the projection  $g(U, V)$  of the gradient  $\nabla F(U, V)$  onto  $\mathcal{T}_{(U,V)}(\mathcal{O}(m) \times \mathcal{O}(n))$  is given by the pair of matrices:

$$g(U, V) = \left( \frac{R(U, V)V\Sigma^T U^T - U\Sigma V^T R(U, V)^T}{2} U, \right. \\ \left. \frac{R(U, V)^T U\Sigma V^T - V\Sigma^T U^T R(U, V)}{2} V \right). \quad (3.7)$$

Thus, the vector field

$$\frac{d(U, V)}{dt} = -g(U, V) \quad (3.8)$$

defines a steepest descent flow on the manifold  $\mathcal{O}(m) \times \mathcal{O}(n)$  for the objective function  $F(U, V)$ .

## 3.2 Main Results

Let  $(S, C)$  be a matrix pair of order  $n$ . The determinant

$$\det(S \circ X + C)$$

is a polynomial in the indeterminates of  $X$  of degree at most  $n$  over the real field. We call this polynomial the *indicator polynomial* of the matrix pair  $(S, C)$  because of the following proposition.

**Theorem 1.** *The matrix pair  $(S, C)$  is a SNS-matrix pair if and only if all the nonzero coefficients in its indicator polynomial have the same sign and there is at least one nonzero coefficient.*

*Beweis.* Assume that  $(S, C)$  is a SNS-matrix pair. Clearly the indicator polynomial has a nonzero coefficient. Consider a monomial

$$b_{i_1, \dots, i_k; j_1, \dots, j_k} x_{i_1 j_1} \cdots x_{i_k j_k} \quad (3.9)$$

occurring in the indicator polynomial with a nonzero coefficient. By taking the  $x_{ij}$  that occur in (3.9) large and all others small, we see that any monomial that occurs in the indicator polynomial with a nonzero coefficient can be made to dominate all others. Hence all the nonzero coefficients have the same sign. The converse is immediate.  $\square$

For SNS-matrix pairs  $(S, C)$  with  $C = O$  the indicator polynomial is a homogeneous polynomial of degree  $n$ . In this case Theorem 1 is a standard fact about SNS-matrices.

**Lemma 1** (Stability). *Given  $T > 0$ , suppose that  $\|\epsilon(t)\|_{1,2} \leq h^{q-2}$  for  $0 \leq t \leq T$  and  $q \geq 6$ . Then there exists a positive number  $B$  that depends on  $T$  and the exact solution  $\psi$  only such that for all  $0 \leq t \leq T$ ,*

$$\frac{d}{dt} \|\epsilon(t)\|_{1,2} \leq B(h^{q-3/2} + \|\epsilon(t)\|_{1,2}). \quad (3.10)$$

*The function  $B(T)$  can be chosen to be nondecreasing in time.*

**Theorem 2.** *The maximum number of nonzero entries in a SNS-matrix  $S$  of order  $n$  equals*

$$\frac{n^2 + 3n - 2}{2}$$

*with equality if and only if there exist permutation matrices such that  $P|S|Q = T_n$  where*

$$T_n = \begin{bmatrix} 1 & 1 & \cdots & 1 & 1 & 1 \\ 1 & 1 & \cdots & 1 & 1 & 1 \\ 0 & 1 & \cdots & 1 & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 1 & 1 \\ 0 & 0 & \cdots & 0 & 1 & 1 \end{bmatrix}. \quad (3.11)$$

We note for later use that each submatrix of  $T_n$  of order  $n - 1$  has all 1s on its main diagonal.

We now obtain a bound on the number of nonzero entries of  $S$  in a SNS-matrix pair  $(S, C)$  in terms of the degree of the indicator polynomial. We denote the strictly upper triangular  $(0,1)$ -matrix of order  $m$  with all 1s above the main diagonal by  $U_m$ . The all 1s matrix of size  $m$  by  $p$  is denoted by  $J_{m,p}$ .

**Proposition 1** (Convolution theorem). *Let*

$$a * u(t) = \int_0^t a(t - \tau)u(\tau)d\tau, \quad t \in (0, \infty).$$

*Then*

$$\widehat{a * u}(s) = \widehat{a}(s)\widehat{u}(s).$$

**Lemma 2.** *For  $s_0 > 0$ , if*

$$\int_0^\infty e^{-2s_0 t} v^{(1)}(t)v(t)dt \leq 0,$$

*then*

$$\int_0^\infty e^{-2s_0 t} v^2(t)dt \leq \frac{1}{2s_0} v^2(0).$$

*Proof.* Applying integration by parts, we obtain

$$\begin{aligned} \int_0^\infty e^{-2s_0 t} [v^2(t) - v^2(0)]dt &= \lim_{t \rightarrow \infty} \left( -\frac{1}{2s_0} e^{-2s_0 t} v^2(t) \right) + \frac{1}{s_0} \int_0^\infty e^{-2s_0 t} v^{(1)}(t)v(t)dt \\ &\leq \frac{1}{s_0} \int_0^\infty e^{-2s_0 t} v^{(1)}(t)v(t)dt \leq 0. \end{aligned}$$

**Abbildung 3.1:**  $\log_{10}$  of the residual norm versus the number of GMRES( $m$ ) iterations for the finite difference methods.

**Corollary 1.** Let  $E$  satisfy (5)–(6) and suppose  $E^h$  satisfies (7) and (8) with a general  $G$ . Let  $G = \nabla \times \Phi + \nabla p$ ,  $p \in H_0^1(\Omega)$ . Suppose that  $\nabla p$  and  $\nabla \times \Phi$  satisfy all the assumptions of Theorems 4.1 and 4.2, respectively. In addition suppose all the regularity assumptions of Theorems 4.1–4.2 are satisfied. Then for  $0 \leq t \leq T$  and  $0 < \epsilon \leq \epsilon_0$  there exists a constant  $C = C(\epsilon, T)$  such that

$$\|(E - E^h)(t)\|_0 \leq Ch^{k+1-\epsilon},$$

where  $C$  also depends on the constants given in Theorems 4.1 and 4.2.

**Definition 1.** Let  $S$  be an isolated invariant set with isolating neighborhood  $N$ . An index pair for  $S$  is a pair of compact sets  $(N_1, N_0)$  with  $N_0 \subset N_1 \subset N$  such that:

1.  $cl(N_1 \setminus N_0)$  is an isolating neighborhood for  $S$ .
2.  $N_i$  is positively invariant relative to  $N$  for  $i = 0, 1$ , i.e., given  $x \in N_i$  and  $x \cdot [0, t] \subset N$ , then  $x \cdot [0, t] \subset N_i$ .
3.  $N_0$  is an exit set for  $N_1$ , i.e. if  $x \in N_1$ ,  $x \cdot [0, \infty) \not\subset N_1$ , then there is a  $T \geq 0$  such that  $x \cdot [0, T] \subset N_1$  and  $x \cdot T \in N_0$ .

### 3.2.1 Numerical experiments

We conducted numerical experiments in computing inexact Newton steps for discretizations of a modified Bratu problem, given by

$$\begin{aligned} \Delta w + ce^w + d \frac{\partial w}{\partial x} &= f \quad \text{in } D, \\ w &= 0 \quad \text{on } \partial D, \end{aligned} \tag{3.12}$$

where  $c$  and  $d$  are constants. The actual Bratu problem has  $d = 0$  and  $f \equiv 0$ . It provides a simplified model of nonlinear diffusion phenomena, e.g., in combustion and semiconductors, and has been considered by Glowinski, Keller, and Rheinhardt Paris (2008), as well as by a number of other investigators; see Brachmann et al. (2007) and the references therein. See also problem 3 by Glowinski and Keller and problem 7 by Mittelman in the collection of nonlinear model problems assembled by Moré Yang et al. (2007). The modified problem (3.12) has been used as a test problem for inexact Newton methods by Brown and Saad Smith et al. (2008).

In our experiments, we took  $D = [0, 1] \times [0, 1]$ ,  $f \equiv 0$ ,  $c = d = 10$ , and discretized (3.12) using the usual second-order centered differences over a  $100 \times 100$  mesh of equally spaced points in  $D$ . In GMRES( $m$ ), we took  $m = 10$  and used fast Poisson right preconditioning as in the experiments in §2. The computing environment was as described in §2. All computing was done in double precision.

In the first set of experiments, we allowed each method to run for 40 GMRES( $m$ ) iterations, starting with zero as the initial approximate solution, after which the limit of residual norm reduction had been reached. The results are shown in Fig. 3.1.

**Tabelle 3.1:** Statistics over 20 trials of GMRES( $m$ ) iteration numbers,  $F$ -evaluations, and run times required to reduce the residual norm by a factor of  $\epsilon$ . For each method, the number of GMRES( $m$ ) iterations and  $F$ -evaluations was the same in every trial.

Method	$\epsilon$	Number of Iterations	Number of $F$ -Evaluations	Mean Run Time (Seconds)	Standard Deviation
EHA2	$10^{-10}$	26	32	47.12	.1048
FD2	$10^{-10}$	26	58	53.79	.1829
EHA4	$10^{-12}$	30	42	56.76	.1855
FD4	$10^{-12}$	30	132	81.35	.3730
EHA6	$10^{-12}$	30	48	58.56	.1952
FD6	$10^{-12}$	30	198	100.6	.3278

In Fig. 3.1, the top curve was produced by method FD1. The second curve from the top is actually a superposition of the curves produced by methods EHA2 and FD2; the two curves are visually indistinguishable. Similarly, the third curve from the top is a superposition of the curves produced by methods EHA4 and FD4, and the fourth curve from the top, which lies barely above the bottom curve, is a superposition of the curves produced by methods EHA6 and FD6. The bottom curve was produced by method A.

In the second set of experiments, our purpose was to assess the relative amount of computational work required by the methods which use higher-order differencing to reach comparable levels of residual norm reduction. We compared pairs of methods EHA2 and FD2, EHA4 and FD4, and EHA6 and FD6 by observing in each of 20 trials the number of GMRES( $m$ ) iterations, number of  $F$ -evaluations, and run time required by each method to reduce the residual norm by a factor of  $\epsilon$ , where for each pair of methods  $\epsilon$  was chosen to be somewhat greater than the limiting ratio of final to initial residual norms obtainable by the methods. In these trials, the initial approximate solutions were obtained by generating random components as in the similar experiments in §2. We note that for every method, the numbers of GMRES( $m$ ) iterations and  $F$ -evaluations required before termination did not vary at all over the 20 trials. The GMRES( $m$ ) iteration counts, numbers of  $F$ -evaluations, and means and standard deviations of the run times are given in Table 3.1.

In our first set of experiments, we took  $c = d = 10$  and used right preconditioning with a fast Poisson solver from FISHPACK Santella & DeCarlo (2004), which is very effective for these fairly small values of  $c$  and  $d$ . We first started each method with zero as the initial approximate solution and allowed it to run for 40 GMRES( $m$ ) iterations, after which the limit of residual norm reduction had been reached. Figure (not included) shows plots of the logarithm of the Euclidean norm of the residual versus the number of GMRES( $m$ ) iterations for the three methods. We note that in Fig. XX and in all other figures below, the plotted residual norms were not the values maintained by GMRES( $m$ ), but rather were computed as accurately as possible “from scratch.” That is, at each GMRES( $m$ ) iteration, the current approximate solution was formed and its product with the coefficient matrix was subtracted from the right-hand side, all in double precision. It was important to compute the residual norms in this way because the values maintained by GMRES( $m$ ) become increasingly untrustworthy as the limits of residual norm reduction are neared; see Grayson (1987). It is seen in Fig. XX that Algorithm EHA achieved the same ultimate level of residual norm reduction as the FDP method and required only a few more GMRES( $m$ ) iterations to do so.

In our second set of experiments, we took  $c = d = 100$  and carried out trials analogous to those in the first set above. No preconditioning was used in these experiments, both because we wanted to compare the methods without preconditioning and because the fast Poisson preconditioning used in the first set of experiments is not cost effective for these large values of  $c$  and  $d$ . We first allowed each method to run for 600 GMRES( $m$ ) iterations, starting with zero as the initial approximate solution, after which the limit of residual norm reduction had been reached.

# Literaturverzeichnis

- Alvarez, L., Lions, P.-L., & Morel, J.-M. (1992). Image selective smoothing and edge detection by nonlinear diffusion. ii. *SIAM J. Numer. Anal.*, 29(3), 845–866.
- Aubert, G. & Kornprobst, P. (2006). *Mathematical Problems in Image Processing: Partial Differential Equations and the Calculus of Variations (Applied Mathematical Sciences)*. Secaucus, NJ, USA: Springer-Verlag New York, Inc.
- Balikai, A., Rosin, P., Song, Y.-Z., & Hall, P. (2008). Shapes fit for purpose. In *Proc. British Machine Vision Conf.* (pp. 443–452).
- Barash, D. & Comaniciu, D. (2004). A common framework for nonlinear diffusion, adaptive smoothing, bilateral filtering and mean shift. *Image and Vision Computing*, 22(1), 73–81.
- Black, M. & Rangarajan, A. (1996). On the unification of line processes, outlier rejection, and robust statistics with applications in early vision. *International Journal of Computer Vision*, 19, 57–91(35).
- Brachmann, C., Walther-Franks, B., & Loviscach, J. (2007). Exemplar-based worn edges. In *Proc. Eurographics Short Papers* (pp. 25–28).
- Dai, S., Han, M., Xu, W., Wu, Y., & Gong, Y. (2007). Soft edge smoothness prior for alpha channel super resolution. In *Computer Vision and Pattern Recognition, 2007. CVPR '07. IEEE Conference on* (pp. 1–8).
- Goshtasby, A. & Satter, M. (2008). An adaptive window mechanism for image smoothing. *Computer Vision and Image Understanding*, 111(2), 155 – 169.
- Grayson, M. A. (1987). The heat equation shrinks embedded plane curves to round points. *Journal of Differential Geometry*, 26(2), 285–314.
- Guichard, F. & Morel, J. (2003). A note on two classical enhancement filters and their associated pde's. *International Journal of Computer Vision*, 52(2), 153–160.
- Healey, C. G., Tateosian, L., Enns, J. T., & Remple, M. (2004). Perceptually based brush strokes for nonphotorealistic visualization. *ACM Trans. Graph.*, 23(1), 64–96.
- Paris, S. (2008). Edge-preserving smoothing and mean-shift segmentation of video streams. In *ECCV '08: Proceedings of the 10th European Conference on Computer Vision* (pp. 460–473). Berlin, Heidelberg: Springer-Verlag.

- Santella, A. & DeCarlo, D. (2004). Visual interest and npr: an evaluation and manifesto. In *NPAR '04: Proceedings of the 3rd international symposium on Non-photorealistic animation and rendering* (pp. 71–150).
- Smith, K., Landes, P.-E., Thollot, J., & Myszkowski, K. (2008). Apparent greyscale: A simple and fast conversion to perceptually accurate images and video. *Computer Graphics Forum*, 27(2), 193–200.
- Yang, G. Z., Burger, P., Firmin, D. N., & Underwood, S. R. (1996). Structure adaptive anisotropic image filtering. *Image and Vision Computing*, 14(2), 135 – 145.
- Yang, L., Meer, P., & Foran, D. J. (2007). Multiple class segmentation using a unified framework over mean-shift patches. *IEEE Conference on Computer Vision and Pattern Recognition*, 0, 1–8.