STAT 428
Fall 2017
Exam 1
10/09/2017
Time Limit: 50 minutes

Name (Print): _____

NetID _____

SOLUTION

This exam contains **6 pages** (including this cover page) of exam material consisting of **3 problems**. Check to see if any problems are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes or calculator on this exam. You should simplify any numerical answers as much as possible.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Where applicable, write your final answer in the space provided.**
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Problem	Points	Score
1	12	
2	12	
3	6	
Total:	30	

When finished turn in your exam and exit quietly.
Do not write in the table to the right.

Academic Integrity The University statement on your obligation to maintain academic integrity is:

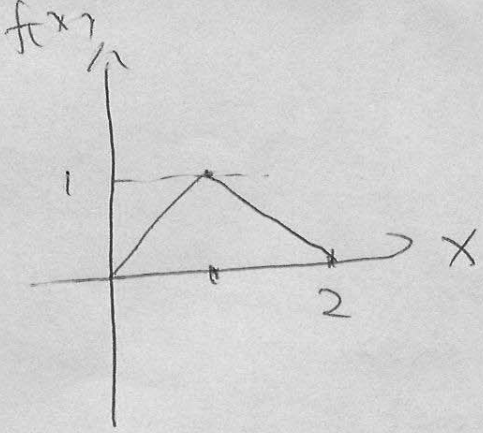
- If you engage in an act of academic dishonesty, you become liable to severe disciplinary action. Such acts include cheating; falsification or invention of information or citation in an academic endeavor; helping or attempting to help others commit academic infractions; plagiarism; offering bribes, favors, or threats; academic interference; computer related infractions; and failure to comply with research regulations.

Rule 33 of the Code of Policies and Regulations Applying to All Students gives complete details of rules governing academic integrity for all students. You are responsible for knowing and abiding by these rules.

1. Consider a random variable X with the density $f(x)$ given by

$$f(x) = \begin{cases} x & \text{for } 0 \le x \le 1 \\ 2 - x & \text{for } 1 \le x \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

(a) (2 points) Draw the density



(b) (3 points) List steps you would use to simulate a random variable X by using an accept-reject algorithm.(Hint: keep it simple)

Instrumental distribution: $g(x) = \frac{1}{2}$ $x \in [0, 2]$, set $c = 2$

Accept-reject algorithm:

① generate x from $g(x)$

② generate $u \sim \text{unif}(0, 1)$

③ if $u \le \frac{f(x)}{c\, g(x)}$, accept the sample.

otherwise, repeat from step ①

(c) (2 points) What is the acceptance probability of your method? Explain

acceptance probability: $\frac{1}{c} = \frac{1}{2}$.

(c) _____

(d) (2 points) Would it be appropriate to use inverse CDF method to simulate from X? If so, write pseudo code for it. (*You can assume you have acccess to a function invCDF that gives you the inverse CDF*). If not, argue why not.

n: sample size x: generated sample

code: $u = \text{runif}(n)$

$x = \text{invCDF}(u)$

$$f(x) = \begin{cases} x & 0 \le x \le 1 \\ 2-x & 1 \le x \le 2 \end{cases}$$

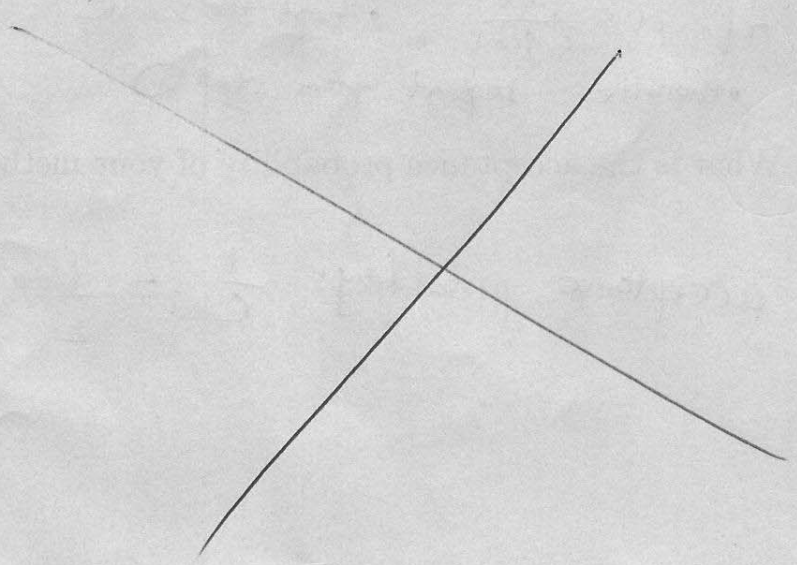
$$F(x) = \begin{cases} \frac{1}{2}x^2 & 0 \le x \le 1 \\ -\frac{1}{2}x^2 + 2x - 1 & 1 \le x \le 2 \end{cases}$$

Inverse CDF:

$$X = F^{-1}(u) = \begin{cases} \sqrt{2u} & 0 \le u \le \frac{1}{2} \\ 2 - \sqrt{2-2u} & \frac{1}{2} \le u \le 1 \end{cases}$$

(d) _____

(e) (3 points) Would it be appropriate to use importance sampling method to simulate from X? If yes, state a good importance sampling function. If no, explain.



(e) _____

2. Estimate $\theta = E(X^2)$ using importance sampling when X has density given by

$$g(x) = e^{-x-e^{-x}}, \quad -\infty < x < \infty$$

(a) (4 points) What importance function would you use? Explain

Double exponential: $\frac{1}{2}e^{-|x|}$

Reason: 1. support: $-\infty < x < \infty$

2. thicker tail: exponential tail.

(Normal, Gamma should be fine.
actually, Normal distribution is not good)
it i

(b) (8 points) Write pseudo code to compute the estimate $\hat{\theta}$ and its variance.

1. Generate (size n) sample x from double exponential distribution

2. calculate weight $w = e^{-x-e^{-x}} / (\frac{1}{2}e^{-|x|})$.

$\hat{\theta} = \text{mean}(x^2 w)$

$\text{Var}(\hat{\theta}) = \text{Var}(x^2 w)/n$

Or, we can estimate $\hat{\theta}$ m times and
calculate $\text{Var}(\hat{\theta})$ based on m estimates of $\hat{\theta}$
(m should be large)

3. State whether the following statements are True or False.

(a) (1 point) To generate a random observation X based on inverse transform method, first generate a $Uniform(0,1)$ variate u, find the cdf F_X and its inverse F_X^{-1} and deliver the value $F_X^{-1}(u)$. **Circle one: Yes No**

(b) (1 point) To apply transformation methods to generate discrete random variable X based on random variable Y, random variable Y must be a discrete random variable. **Circle one: Yes No**

(c) (1 point) To apply importance sampling to estimate $\int_0^\infty \frac{e^{-x}}{1+x^2}$ we can use $Uniform(0,1)$ as the importance function. **Circle one: Yes No**

(d) (1 point) By using stratified importance sampling we would obtain better estimates with similar standard error compared with regular importance sampling. **Circle one: Yes No**

(e) (1 point) Inverse transform method can be applied for generating continuous or discrete random variables. **Circle one: Yes No**

(f) (1 point) Suppose we want to use importance sampling to estimate $E(X^2)$, where X has the density that is proportional to $e^{\frac{-|x|^3}{3}}$, $-\infty < x < \infty$ then $N(0,1)$ is a valid proposal distribution. **Circle one: Yes No**

How was this exam? (a)Very Hard (b)Hard (c)OK (d)Easy (e)Very easy