Exam 1
Stat 428
February 18, 2016

1. Consider the probability density function $f(x) = x$, for $0 < x < 1$ and $f(x) = 2 - x$ for $1 < x < 2$.
a. Propose a probability density function $g(y)$ to conduct acceptance/rejection sampling to obtain a sample $X_1, X_2, ..., X_n$ from $f(x)$.
b. Using $g(y)$ from part (a), write R code to obtain samples of size n from $f(x)$ by acceptance/rejection sampling.
c.(2 point bonus). Write R code using any method other than acceptance/rejection sampling to obtain samples of size n from $f()$.

2. Let Y be the distribution of the number of independent Bernoulli trials, each with success probability p, required to get the first success. In other words, Y has a geometric distribution with parameter p.
a. Write an R function to generate a sample of n draws from a geometric distribution with parameter p, using draws obtained from either `rbinom()` or `sample()`.
b. Using the function written in part (a), write an R function to obtain draws from a negative binomial distribution with parameters K and p. Recall that the negative binomial distribution can be thought of as the number of draws from a Bernoulli distribution with parameter p required to reach K successes.

3. Let $f(x) = 2xe^{-x^2}$ for $x > 0$.
a. Show that $f(x)$ is a probability distribution supported on $(0, \infty)$.
b. Find the cdf $F(x)$.
c. Write R code to use inverse cdf sampling to obtain samples of size n from $f(x)$.

4. Consider the integral $\theta = \int_0^1 e^{x^2} dx$

a. Write R code to estimate θ by treating it as $E[e^{X^2}]$ where $X \sim U(0,1)$.

b. Write R code to compute the standard error of $\hat{\theta}$ from part (a).

c. Next, write R code using stratified sampling with K intervals of equal length to obtain an estimate of θ.

d. Let $\phi(x) = \frac{e^x}{c}$ for $0 < x < 1$, and 0 otherwise. Find c so that $\phi()$ is a probability density function.

e. Using ϕ from part (d), write R code to compute an importance sampling estimate of θ, using ϕ as the importance function.