In [ ]:

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STAT 420 Homework 2
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In [ ]:

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Question1: a)
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In [ ]:

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Firstly, we use the matrix to generate mu and sigma, and then use the
the linear combination to calculate the mean and variance for Z_1
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In [29]:

```
mu = c(1,2,3,4)
Sigma = matrix(c(1,0.5,0.25,0.125,0.5,1,0.5,0.25,0.25,0.5,1,0.5,0.125,0.25,0.5,1),4,4)
a= c(0.5,0.25,0.125,0.125)
mean = t(mu) %*% a
var = t(a) %*% Sigma %*% a
mean
var
# Thus Z_1 ~ N(1.875,0.578125), which is also its parameters
```

```
1.875
```

```
0.578125
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In [ ]:

```
#b)
Then the pnorm() function is applied in this case for calculating the probality.
so we conclude that the probality of Z_1 is greater than 2 is 0.43470853037062
```

In [30]:

```
mu = c(1,2,3,4)
Sigma = matrix(c(1,0.5,0.25,0.125,0.5,1,0.5,0.25,0.25,0.5,1,0.5,0.125,0.25,0.5,1),4,4)
a= c(0.5,0.25,0.125,0.125)
Mean_z1 = t(mu) %*% a
Var_z1 = t(a) %*% Sigma %*% a
pnorm(2, mean = Mean_z1, sd = sqrt(Var_z1), lower.tail = FALSE)
```

```
0.43470853037062
```

In [ ]:

```
#c)
Set "a"as the A matrix that show in the MVN distribution formula, and (Z_1,Z_2)^T
is a two dimensional vector of linear combinations of X, so as the following
calculations show:
```

In [31]:

```
a = matrix(c(0.5,0.125,0.25,0.125,0.125,0.25,0.125,0.5),2,4)
mu = c(1,2,3,4)
Mean = a %*% mu
Var = a %*% matrix(c(1,0.5,0.25,0.125,0.5,1,0.5,0.25,0.25,0.5,1,0.5,0.125,0.25,0.5,1),4,4) %*% t(a)
Mean
Var
# Thus, (1.875,3.125)^T is the mean for (Z_1,Z_2)^T and
# matrix Var is for the variance of (Z_1,Z_2)^T
```

1.875
3.125

0.5781250	0.4238281
0.4238281	0.5781250

In [ ]:

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# Question 2 a)
We will use ifelse(rbinom(n, 1, 0.5), rnorm(n), rexp(n, 1)) to make sure that
all of generated number follow the distribution
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In [32]:

```
n = 1e6
x = ifelse(rbinom(n, 1, 0.5), rnorm(n), rexp(n, 1))
```

In [33]:

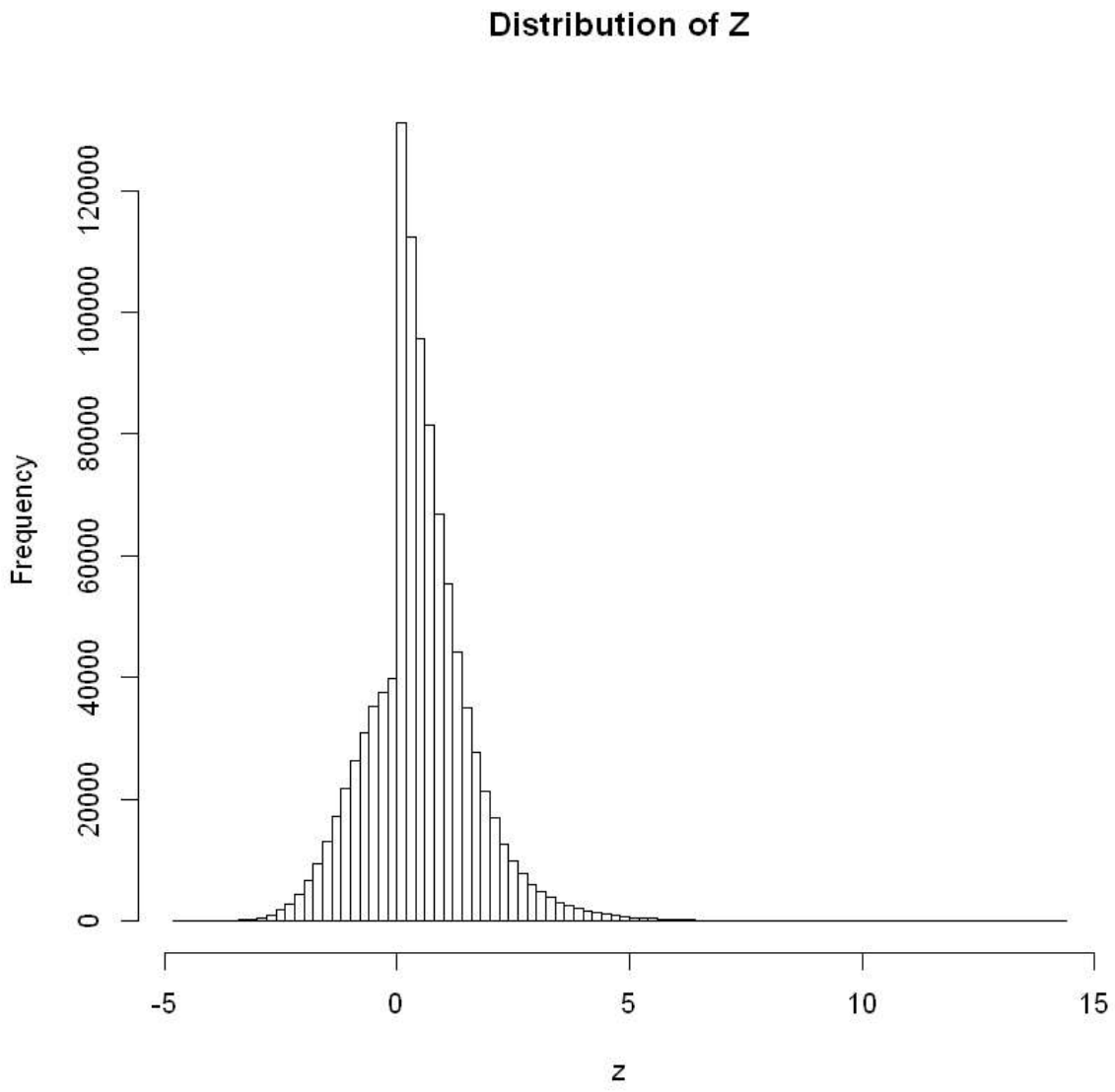
```
#b)
n = 1e6
x = ifelse(rbinom(n, 1, 0.5), rnorm(n), rexp(n, 1))
summary(x)
quantile(x,probs = 0.75)
```

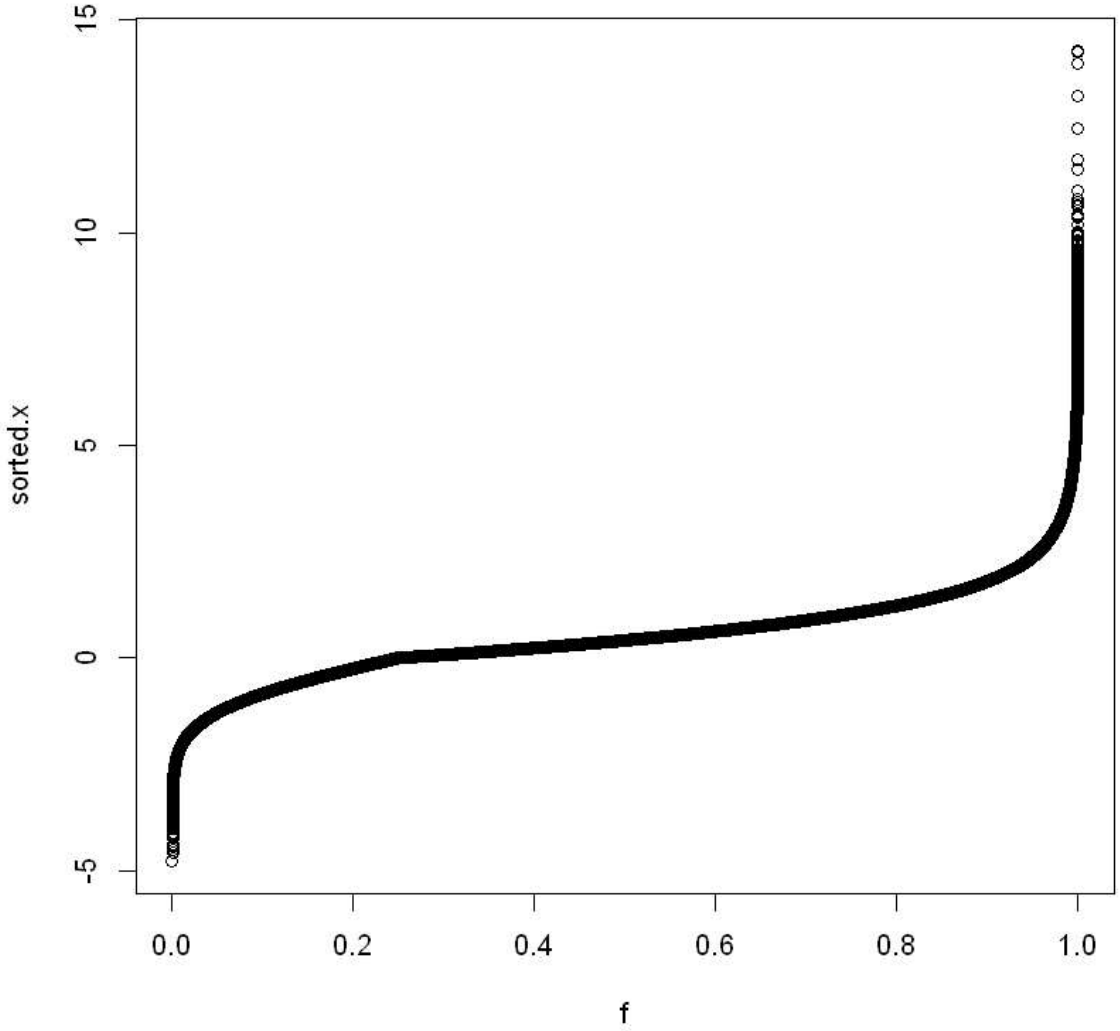
```
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max. 
-4.890000  0.000435  0.414800  0.498300  1.043000 13.570000 
```

75%: 1.0425794360655

In [34]:

```
#c)
n = 1e6
x = ifelse(rbinom(n, 1, 0.5), rnorm(n), rexp(n, 1))
sorted.x=sort(x)
n=length(x)
f=((1:n))/n
hist(sorted.x,xlab="z",main = "Distribution of Z", breaks = 100)
plot(f,sorted.x)
```




sorted.x
15
10
5
0
-5
0.0
0.2
0.4
0.6
0.8
1.0
f