IP02 SELF-ERECTING INVERTED PENDULUM USER'S GUIDE

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IP-02 SELF-ERECTING, LINEAR MOTION INVERTED PENDULUM

Quanser Consulting Inc.

1.0 SYSTEM DESCRIPTION

The IP-02 consists of a motor driven cart which is equipped with two quadrature encoders. One encoder measures the position of the cart via a pinion which meshes with the track. The other encoder measures the angle of the pendulum which is free to swing in front of the cart. A weight is supplied which mounts on the cart. This weight must be used if you want to use the system in self-erecting mode. If you do not use the weight, the centrifugal force created by the swinging pendulum will cause the cart to lift off the track!

The purpose of the experiment is to design a controller that starts with the pendulum in the “down” position then swings it up and maintains it upright.

Mount the system on a table such that the pendulum swings in the front and ensure that the pendulum will not collide with any objects while it swings. Note that the pendulum will swing even while the cart is at the extremities of the track! You will need about 3 feet of clearance from either end of the track to avoid collisions. Bolt the system down to the table using the brackets mounted on the end plates.

2.0 MATHEMATICAL MODELLING

2.1 NONLINEAR MODEL

Consider the simplified model in Figure 2.1. Note that l_p is half the actual length of the pendulum ($l_p = 0.5\ L_p$).

The nonlinear differential equations are derived to be:

$$\begin{aligned} (m_p + mc)\ \ddot{x} + m_p\ \ddot{\theta}\ l_p\ \cos(\theta) - m_p\ \dot{\theta}^2\ l_p\ \sin(\theta) &= F \\ m_p\ l_p\ \cos(\theta)\ \ddot{x} + m_p\ \ddot{\theta}\ l_p^2 - m_p\ g\ l_p\ \sin(\theta) &= 0 \end{aligned} \qquad \textbf{\textit{(2.1)}}$$

where

F : input force to the cart (N)
$m_{p:}$ mass of rod (Kg)
m_c: mass of the cart (Kg)
$l_{p:}$ centre of gravity of rod (m) (half of full length)

2.2 LINEAR MODEL

The linear equations resulting from the above are:

$$\begin{bmatrix} \dot{x} \\ \dot{\theta}_u \\ \ddot{x} \\ \ddot{\theta}_u \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{m_p g}{m_c} & 0 & 0 \\ 0 & \frac{(m_p+m_c)g}{m_c l_p} & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta_u \\ \dot{x} \\ \dot{\theta}_u \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_c} \\ -\frac{1}{m_c l_p} \end{bmatrix} F \quad \textbf{(2.2)}$$

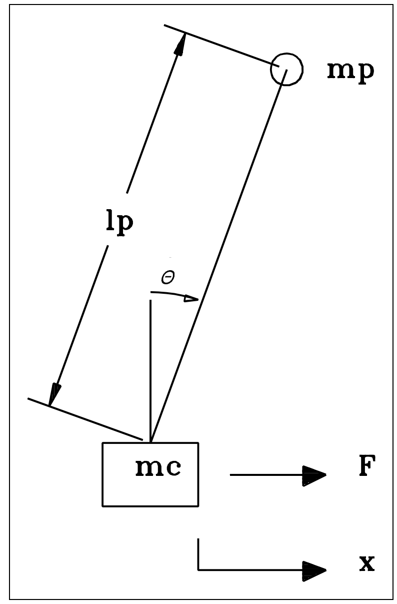


Figure 2.1 Simplified Model

Note that the zero position for all the above equations is defined as the pendulum being vertical "up"

For the DOWN position, the following equation holds:

$$\begin{bmatrix} \dot{x} \\ \dot{\theta}_d \\ \ddot{x} \\ \ddot{\theta}_d \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{m_p g}{m_c} & 0 & 0 \\ 0 & -\frac{(m_p+m_c)g}{m_c l_p} & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta_d \\ \dot{x} \\ \dot{\theta}_d \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_c} \\ \frac{1}{m_c l_p} \end{bmatrix} F \quad \textbf{(2.3)}$$

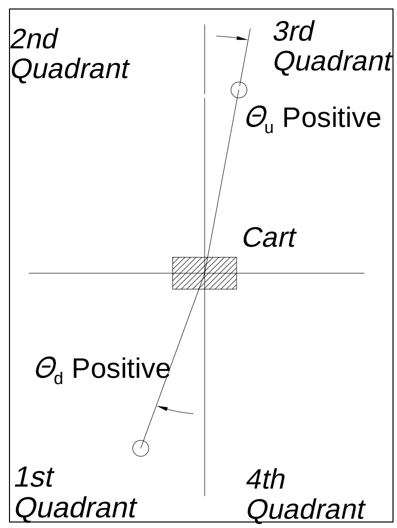


Figure 2.2 Definitions of θu and θd

where θ_d =0 is defined for the pendulum suspended and its motion is relative to the "down" axis and is positive for counterclockwise rotations.

3.0 CONTROL SYSTEM DESIGN

The controller will consist of two main parts. One will be the "swing up" controller while the second is the "balance" controller. The swing up controller will oscillate the cart until it has built up enough energy in the pendulum that it is almost upright at which point the "balance" controller is turned on and is used to maintain the pendulum vertical.

3.1 Swing up control

Swing up control will essentially control the position of the cart in order to destabilize the "down" position. Assume the cart position can be commanded via x_d. Then the feedback:

$$x_d = P\,\theta_d + D\,\dot{\theta}_d$$

can be made to destabilize the system with the proper choice of the gains P & D. This means we want to command the cart based on the position and rate of the pendulum. This approach is intuitive: it simply makes sense that by moving the cart back and forth one can eventually bring up the pendulum.

For the cart to track the desired cart position, we design a PD controller for the cart:

$$V_d = K_p\,(x - x_d) + K_d\,\dot{x}$$

This is a position control loop that controls the voltage applied to the motor so that x tracks xd.

Designing a swing-up controller in this fashion ensures that the cart is not commanded to move beyond the track limits. By limiting xd, we ensure that the cart does not reach the side supports of the track.

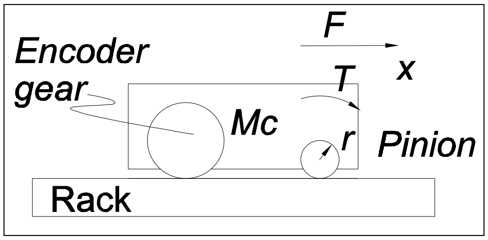


Figure 2.3 Motor torque and force applied to cart

The motor equations are:

$$V = I_m R_m + K_m K_g \omega_g = I_m R_m + K_m K_g \frac{\dot{x}}{r}$$

where

V (volts) :	Voltage applied to motor
I_m (Amp):	Current in motor
K_m(V/ (rad sec $^{-1)}$):	Back EMF Constant
K_g:	Gear ratio in motor gearbox
ω_g(Rad/sec):	Motor output angular velocity
x (m/sec):	Cart velocity
r (m) :	Radius of motor pinion that meshes with the track

The torque generated by the motor is:

$$T = K_m K_g I_m$$

which is transmitted as a force to the cart via the pinion by:

$$F = \frac{T}{r}$$

This force results in an acceleration of the cart governed by the equation:

$F = m_c\ \ddot{x}$ (Note m_c is the mass of the cart *plus* the weight placed on the cart)

Combining the above equation results in the open loop transfer function:

$$\frac{x}{V} = \frac{1}{s\ \left(\frac{m_c\ R_m\ r}{K_m\ K_g}\ s + \frac{K_m K_g}{r}\right)}$$

Substituting system parameters we obtain:

$$\frac{x}{V} = \frac{1}{s\ (0.49s+4.47)}$$

and closing the loop we have the cart position control transfer function:

$$\frac{x}{x_d} = \frac{K_p}{-0.49s^2 + (K_d-4.47)\ s + K_p}$$

which has a characteristic polynomial of the type:

$$s^2 + 2\ \zeta\ \omega_o\ s + \omega_o^2 \qquad and \qquad t_p = \frac{\pi}{\omega_o\sqrt{(1-\zeta^2)}}$$

where ζ is the damping ratio and t_p is the peak time.

Now we need to select the damping ratio and peak time to obtain Kp & Kd.

Consider the pendulum in the system. Its parameters are:

lp(m) : 0.305 m (centre of gravity is located at half of full length)
m_p(Kg): 0.210 Kg

The natural period for small oscillations is given by:

$$T = 2\ \pi\ \sqrt{\frac{l_p}{g}}$$

We want the cart to react to these movements. Therefore the closed loop response of the cart should be considerably faster that the natural frequency of the pendulum. The natural period is 1.1 seconds. It would then be reasonable to design a closed loop controller for the cart which has the following specifications:

$$t_p = \frac{T}{8}\ (0.138\text{sec}) \quad \textit{and} \quad \zeta = 0.707$$

This means we want the peak time to be 8 times faster then the natural period of the pendulum and the system to be slightly underdamped. The factor of 8 is rather arbitrary and could have been 5 or 10.

Using this approach (see ***cartpos.m***) , we obtain the gains:

$$K_p = -5.05 \;\; V/cm$$
$$K_d = -0.17 \;\; V\,/(cm\ \text{sec}^{-}1)$$

A SIMULINK block diagram that simulates (**s_down.m**) the swing up controller is shown in Figure 3.1. The "open loop" system consists of the state space system given in equation ***2.3***. the outputs are converted to cm and deg within the block. The input to the system is a voltage which should be saturated to +/- 5 Volts in order to obtain a realistic simulation. To run the simulation you must run the m file **down.m** first which loads the model parameters into the MATLAB workspace.

The results of the simulation are shown in Figure 3.2. The simulation indicates that this method of swing up control can be used to bring up the pendulum. This is not exact analysis however since the simulation is based on a linear model and once the angle exceeds a few degrees, the model is not valid. Conceptually however, the simulation shows that it can be done.

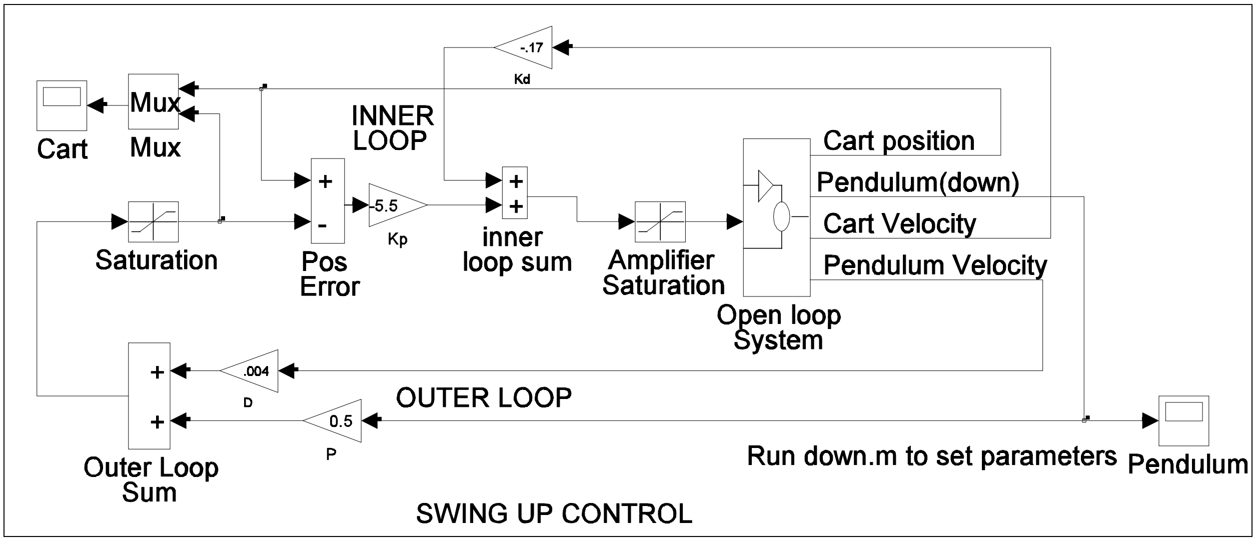


Figure 3.1 SIMULINK simulation of swing up control

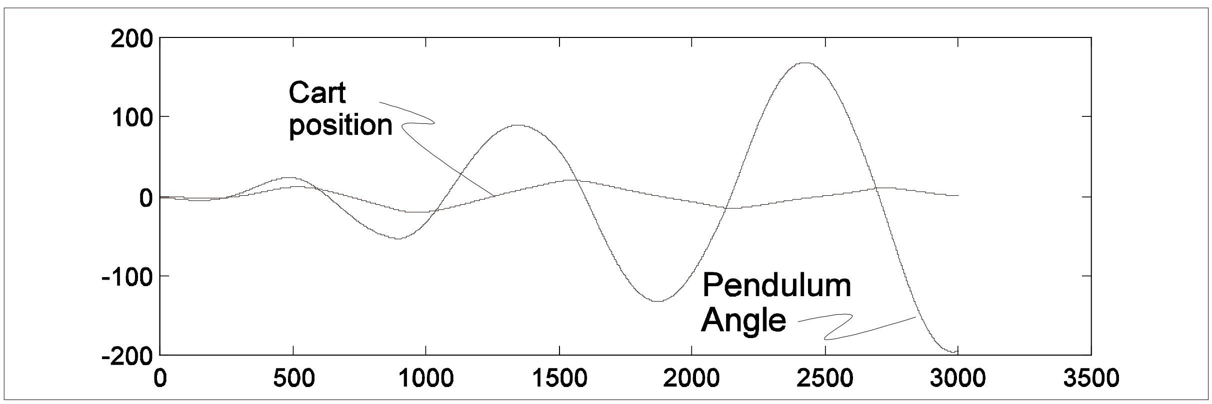


Figure 3.2 Simulation of swing up control

3.2 Balance control

Assuming the pendulum is almost upright, a state feedback controller can be implemented that would maintain it upright (and handle disturbances up to a certain point). The state feedback controller is designed using the linear quadratic regulator and the linear model of the system.

The linear model that was developed is based on a force ***F*** applied to the cart. The actual system however is voltage driven. From the motor equations derived above we obtain:

$$F = \frac{K_m K_g}{Rr} V - \frac{K_m^2 K_g^2}{Rr^2} \dot{x}$$

substituting this into the matrix equation and inserting the parameter values results in

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -2.4 & -9.1 & 0 \\ 0 & 40.1 & 29.8 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2.03 \\ -6.66 \end{bmatrix} V$$

which is the desired representation.

Defining the LQR weighting matrices:

$$Q = diag\ (\ [\ 0.25\ \ 4\ \ 0\ \ 0]\ )$$
$$r = .003$$

results in the feedback gains:

$$K = [\ -0.2887\ \ -2.5620\ \ -0.3354\ \ -0.2706]$$

(Note that gains have been converted to V/ cm and V / deg. See ***d_ip02.m***)

3.3 Mode Control

The MODE controller determines when to switch between the two controllers. This requires some intuition (and a few tricks!).

Experimentally, we note that if you maintain the destabilizing motion to a fixed saturation amplitude (as in the simulation), there will come a time when the cart is moving too much and it actually starts reducing the amplitude of oscillations (*beating*). So, to start from rest, you apply a few large movements until the amplitude of oscillation is large enough and then you switch to a smaller cart command amplitude that will slowly bring the pendulum higher.

Next, once you have attained an acceptable amplitude of pendulum oscillations, you want to start the stabilizing controller at the right time. Given that the stabilizing controller is linear, it has a small region in which it can stabilize the moving pendulum. Experimentally, we determine that it can stabilize the pendulum when it is about 15 degrees (θk) from the vertical and not moving faster than 100 deg/sec. So, the "swing-up" controller should ensure that when the pendulum is almost upright, it is not moving too quickly. This is controlled by adjusting the gain P & D in the swing up controller.

Once the pendulum is up, it is easy to keep it up. But what if someone disturbs it? What should the system do? With the above scheme, if the disturbance causes the pendulum to swing beyond the 15 degrees then the mode switches to 0 and the swing up controller kicks in.... but this is not very good because the disturbance may be too large and the cart would move to the end of the track. So it would be better to switch the mode to 0 when the angle is greater than maybe two or three degrees. Once then pendulum stabilizes, it would be better to make the criterion a little more strict. This is done by changing the value of θk dynamically. To start, we make θk = 15 degrees but once the system has stabilized we change θk to 2 degrees. We should also ensure the value of MODE does not bounce. It is a switch after all and switches need "de-bouncing". This is done by rate limiting the output of the AND gate and feeding it through a "deadband". The parameters of the rate limiters and the deadband are obtained experimentally.

4.0 CONTROL SYSTEM IMPLEMENTATION

The system is implemented in SIMULINK. The controller is then run in WinCon which runs the SIMULINK diagram in realtime under Windows.

Figure 4.1 shows the SIMULINK controller ***ip02_se.m*** supplied with the system. You can run this controller directly from WinCon by simply loading ***ip02_se.wcl*** into WinCon. If you want to perform changes to the parameters you need MATLAB, SIMULINK and Realtime Workshop. If you want to design other controller you also need Watcom C++.

The subsystems of the controller are now described in detail:

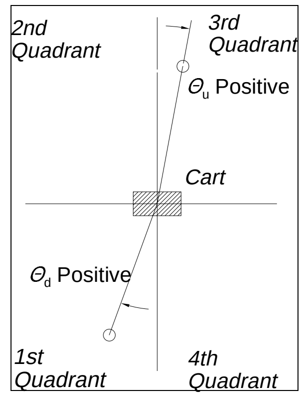
4.1 Calibration and differentiation

This block simply converts the count value measured from the board to the appropriate units. The cart position encoder measures -0.00454 cm/count and the pendulum encoder measures -0.08789 degrees per count. The values are also fed through a high pass filter which essentially differentiates the signals.

4.2 Pendulum Up/Down measurements

This block converts the measurement of the pendulum angle from the “Full” position to two other frames of reference.

4.2.1 θf (Full) : The full position measures values which start at zero (hanging down) and increases in the positive direction when the pendulum rotates clockwise and decreases when the pendulum rotates counterclockwise. The value of the angle measured can exceed 360 degrees many times over in both directions. This value is obtained directly from the encoder counter.

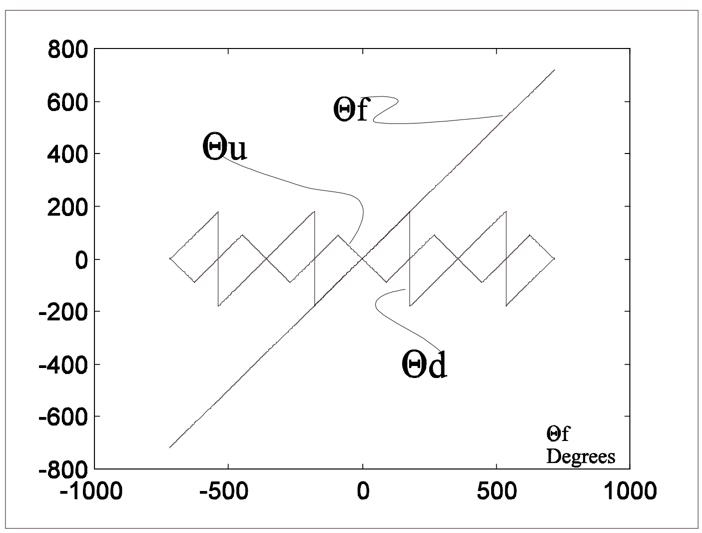


Up and down positions and their directions

4.2.2 θu (UP) : The UP position is relative to the vertical axis in the up position. It gives a zero value when the pendulum is straight up and a positive value when the pendulum is tilting to the right of the vertical and a negative value when it is to the left of the vertical. It is obtained by performing the following operation:

$$\theta_u = -\sin^{-1}(\sin(\theta_f)) \quad \textit{(note } \sin^{-1} \textit{ returns values between } \pm\pi/2 \textit{)}$$

4.2.3 θd (DOWN) : The DOWN position is essentially the same as the FULL position expect that it is limited to values between ±180 degrees (*NOT SATURATED*!). This is the value used in the “swing-up”controller since the feedback is relative to the down position. If we feed back the FULL value and the pendulum does a full swing then swing up control will be ineffective! The value θd is obtained using the following equation:



Up / Down and Full measurements

$$\theta_d = \tan2^{-1}(\sin(\theta_f), \cos(\theta_f)) \quad \textit{(note } \tan2^{-1} \textit{ returns values between } \pm\pi \textit{)}$$

Note that all trigonometric operations are performed on the angles converted to radians. The results are then converted back to degrees.

4.3 Swing up control

This is the "positive feedback loop" used to swing the pendulum up. It consists of two loops. The inner loop performs position control of the cart :

$$V_d = K_p\,(x - x_d) + K_d\,\dot{x}$$

The outer loop of the system generates the command x_d via the following equation:

$$x_d = P\,\theta_d + D\,\dot{\theta}_d$$

with P = 0.5 cm/deg and D = .004 cm / (deg/sec). These two values are crucial in bringing up the pendulum smoothly. You can tune the value of D to adjust the "damping" in the system.

Note also that x_d is limited to ±3 cm. For the first 2.75 (T_{high}) seconds of control however, x_d is multiplied by (1+Kh) resulting in a "high gain" system for a short time. This effectively speeds up the initial response to get maximum swing in the shortest possible time. Ideally, you want to move the cart back and forth a minimum number of times and bring the pendulum up quickly. If you do not have a high gain period, the system will still come up but it may take up to 20 oscillations of the cart to bring the pendulum up.

4.4 Mode control

This block determines which of the two voltages should be fed to the motor. Initially, at startup, we want to feed the voltage computed by the swing up controller. When all of the conditions listed below are met, we want to switch to the "stabilizing" controller output voltage.

Stabilizing conditions:

$|\theta_u| < \theta_k$ (*ie angle small enough*)

$|x| < x_k$ (*ie cart position small enough*

$\cos(\theta_f) < 0$ (*ie pendulum is up*)

$|\dot{\theta}_f| < \dot{\theta}_k$ (*ie pendulum moving slowly enough*)

When all of the above conditions are true, the output of the AND gate is 1. We limit the rate of change of the output and pass it through a backlash block in order to "de-bounce" it. This output is the signal **MODE**. When MODE is 0, the voltage fed to the motor is obtained from the "swing-up" controller, When MODE is 1, the voltage to the motor is obtained from the stabilizing controller.

Furthermore, MODE is fed to a delay block (1.8s / (s+1.8)) and its output is fed to a comparator. When the output of the comparator is 1, the value of θ_k is reduced by 13 degrees. This now changes the stabilizing conditions to the following "***let go***" conditions. When any of the following conditions is NOT TRUE, the system switches MODE to 0 and goes back to swing up mode.

$|\theta_u| < \theta_k$ (*ie angle disturbed too much*)
$|x| < x_k$ (*ie cart position tool far*
$\cos(\theta_f) < 0$ (*ie pendulum is not up*)
$|\dot{\theta}_f| < \dot{\theta}_k$ (*ie pendulum moving too quickly*)

4.5 Balance control

This is the stabilizing state feedback controller. It simply feeds back the voltage:

$$V_s = -(k_1\, x + k_2\, \theta + k_3\, \dot{x} + k_4\, \dot{\theta})$$

as obtained in the LQR design.

This voltage maintains the pendulum upright

5.0 RESULTS

Typical results are shown in Figure 5.1,5.2 and 5.3. These are all obtained from the same run. Always start the system with the cart in the middle and the pendulum in the DOWN position. The controller begins by resetting the encoder counters to zero and thus the initial position is defined as "zero".

The controller starts by applying a de-stabilizing cart position command limited to ±3*(1+Kh) cm. as shown in Figure 5.1 This results in the pendulum swinging with increasing amplitude. At T = 2.75 seconds (as set in Figure 4.4) the cart command is limited to ±3 cm. The destabilizing controller continues however to increase the amplitude of the pendulum oscillations.

At some point, all the following conditions are met:

- Pendulum UP angle < 15 degrees
- Pendulum speed < 100 deg/sec
- Cart position < 25 cm
- Pendulum Full angle in 2nd or 3rd quadrant

This sets MODE = 1 and the state feedback controller output V_s is fed to the motor rather than the destabilizing voltage V_d. This stabilizes the pendulum and keeps it upright.

The Mode is maintained to the value 1 unless one of the following conditions becomes false:

- Pendulum UP angle less than 2 degrees
- Cart position < 25 cm
- Pendulum velocity < 100 deg/sec
- Pendulum Full angle is in 2nd or 3rd quadrant as defined in Figure 2.1

If you apply a tap to the pendulum, it will remain upright unless the tap was so hard that one of the above conditions is not maintained. If MODE switches to zero, then the pendulum falls and the cart will start oscillating again until the pendulum comes up and MODE switches to 1. Note however that there may be situations when the tap was so hard that the pendulum swinging in full rotations and never stabilizes. It is of course possible to design a controller that avoids this situation. One way would be to implement a controller that detects full rotations and turns off the system momentarily until the pendulum is simply swinging in the down mode and then resume to the swing up controller in low gain.

6.0 System Parameters

Parameter	Symbol	Value	Units
Motor Torque constant (and back emf constant in SI units)	Km	0.00767	Nm/Amp V/ rad sec^{-1}
Motor Armature Resistance	Rm	2.6	Ω
Gearbox ratio	Kg	3.7	N/A
Motor pinion # teeth	NA	24	teeth
Motor pinion diameter	r	0.00635	m
Cart mass	mc	0.455	Kg
Weight mass	mw	0.360	Kg
Pendulum true length	Lp	0.61	m
Pendulum mass	mp	0.210	Kg
Rack pitch	N/A	6.01	teeth/cm
Cart encoder pinion # teeth	N/A	56	N/A
Cart encoder resolution	N/A	512 * 4	count/rev
Cart encoder calibration constant	N/A	.00454	cm/count
Pendulum angle encoder resolution	N/A	1024 * 4	count/rev
Pendulum angle calibration constant	N/A	.08789	deg/count

7.0 WIRING

Wire the system as shown in Figure 7.1. The following cables are supplied with the system:

Wire type	From	To
5 Pin Din - 5 Pin Din	Cart position encoder	Enc #0 on MultiQ terminal board
5 Pin Din - 5 Pin Din	Pendulum angle encoder	Enc #1 on MultiQ terminal board
2 Pin Din - 2 Banana	Power module RED to OUT BLACK to GND FORK to (-)	Motor on cart
2 Pin Din - RCA	Quick Connect	MultiQ D/A #0 (Analog output)

Questions? Contact us at help@quanser.com