1 Exercise 1 (Result)

3. What joint torques (τ_1, τ_2, τ_3) should be applied to the joints for holding the arm at a configuration $\boldsymbol{q} = (\theta_1, \theta_2, \theta_3) = (1, \pi/3, \pi/3)$.

The solution is found using the inverse dynamics model.

$$\begin{aligned} \boldsymbol{Q}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}) &= \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\boldsymbol{q}}} - \frac{\partial \mathcal{L}}{\partial \boldsymbol{q}} = B(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau, \\ \boldsymbol{Q}\left(\begin{bmatrix} 1 \\ \pi/3 \\ \pi/3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right) &= \begin{bmatrix} 0 \\ -10.9619 \\ 3.3845 \end{bmatrix} \text{Nm}. \end{aligned} \tag{6}$$

4. Simulate the robot arm by using `ode45` with input $\boldsymbol{Q} = (\tau_1, \tau_2, \tau_3) = -D\dot{\boldsymbol{q}}$, where $D = 5I_3$ is a diagonal matrix. Use initial condition $\boldsymbol{q} = (\theta_1, \theta_2, \theta_3) = (1, \pi/3, \pi/3)$.

To simulate the system the joint accelerations $\ddot{\boldsymbol{q}}$ has to be isolated in the model of the robot dynamics Equation 6. The robot was simulated for 5 s. The friction coefficient was $D = 5I_3$. The simulated trajectory can be seen on Figure 1.

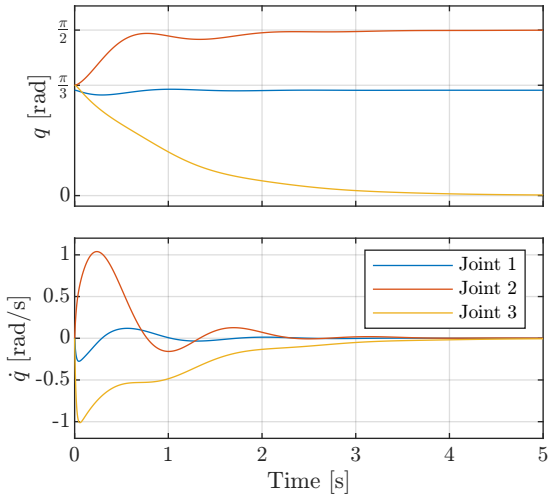

Figure 1: The simulation result.

Figure 1 was generated using Listing 1.

Listing 1: MATLAB code for plotting the simulation result.

```
set(groot,'defaultAxesTickLabelInterpreter','latex');
set(groot,'defaulttextinterpreter','latex');
set(groot,'defaultLegendInterpreter','latex');

fig = figure;
fig.Units               = 'centimeters';
fig.Position(3)         = 8; % width
fig.Position(4)         = 7; % height

subplot(4,1,1:2)
plot(t, sim_q)
ylabel('$q$ [rad]');
grid on
ylim([-0.1, 1.8])
yticks([0, pi/3, pi/2])
yticklabels({'0', '$\frac{\pi}{3}$', '$\frac{\pi}{2}$'})
xticklabels({})

subplot(4,1,3:4)
plot(t, sim_dq)
grid on
xlabel(Time [s])
ylim([-1.2, 1.2])
ylabel('$\dot{q}$ [rad/s]')
legend([Joint 1, Joint 2, Joint 3], ...
    'NumColumns', 1, ...
    'Location', 'northeast')

exportgraphics(fig,'ex1_simluation.pdf', 'BackgroundColor', 'none')
```