Lebanese University
Faculty of Science II

SCIENTIFIC REPORT

Optics Lab P3303

Experiment V: Spectrogoniometry

By

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I. Abstract

This experiment aims to determine the wavelengths of different spectral lines of mercury, hydrogen, and an unknown lamp using a prism spectrogoniometer. It also intends to construct a standardizing (calibration) curve for accurate wavelength determination. The procedure involved measuring the positions of the spectral lines through a sighting and micrometric lens, and their corresponding wavelengths were determined along with the calculation of their errors. The unknown lamp was determined to be a high pressure Sodium vapor lamp and the Hydrogen lamp's wavelengths were used to compute the Rydberg constant as $\bar{R} \pm \Delta R_{total} = (1.11 \pm 0.11) \times 10^7/m$.

key words: *wavelength, spectrogoniometer, prism, standardization, lamp*

II. Introduction

In this experiment, a prism spectrogoniometer was used which is an instrument that separates light into its different colors so that we can measure the position and deduce the wavelength of each one. When a beam of light passes through a prism, each color bends differently because the refractive index of the material changes with the wavelength. This phenomenon is called dispersion. The aim of this experiment is to standardize a spectroscope by tracing the curve = f(X) for a given prism and a known light source (a mercury vapor lamp), and then to measure the wavelengths of the lines of an unknown and hydrogen spectrum. In order to observe lines of the hydrogen lamp we use balmer series given by the following relation:

$$\frac{1}{\lambda} = R(\frac{1}{4} - \frac{1}{n^2}) \tag{1}$$

Where:

- $n = 3, 4, 5, 6$
- Rydberg Constant $R = 1.097 \times 10^7 \text{ m}^{-1}$

After finding the experimental wavelengths of hydrogen, one can deduce the Rydberg constant R. This experiment helps us understand how light can be decomposed into its component wavelengths and how these wavelengths can be precisely measured using optical instruments.

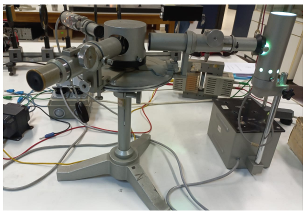
III.Procedure

1. Place the prism on the circular platform of the spectrogoniometer and make sure it is centered and stable. Adjust the three leveling screws to ensure that the platform is perfectly horizontal.
2. Slightly open the slot of the collimator and position the mercury vapor lamp a few millimeters in front of it. Look through the sighting lens and adjust the eyepiece until the edges of the slit appear clear and sharp.
3. Regulate the width of the slot and illuminate the graduated scale and adjust it so that the scale divisions are visible across the entire spectrum from violet to red.

4. Observe the different colored lines produced by the mercury lamp and note the position (X) of each line on the scale. The principal mercury lines are:
   (a) yellow: $\lambda = 5790$ Å and $\lambda = 5770$ Å
   (b) green: $\lambda = 5461$ Å
   (c) bright blue: $\lambda = 4368$ Å
   (d) bright violet: $\lambda = 4048$ Å
5. Using the recorded data, plot the calibration curve λ = f(X). This curve will be used to find the wavelengths of other lamps
6. Replace the mercury lamp with another light source . Repeat the same procedure by observing and recording the a positions of the bright lines in the new spectrum.
7. Using the calibration curve, determine the wavelengths corresponding to each measured position. Also, for the hydrogen spectrum, apply the Equation 1 to deduce R_{exp}
8. Compare your experimental results with theoretical values and discuss.

Table 1: Instrument/Material Used and Their Description.

Material / Instrument	**Description**
Spectrogoniometer	Instrument used to measure angles of spectral lines. Composed of a circular platform, collimator sighting lens for aligning light, micrometric lens for precise angle measurement, and a prism to disperse light into its component wavelengths.
Mercury Lamp	Light source emitting distinct spectral lines of mercury, used for calibration of the spectrogoniometer.
Unknown Lamp	Lamp with unknown spectral lines; analyzed to identify its type by comparing measured wavelengths with researched data.
Hydrogen Lamp	Lamp emitting hydrogen spectral lines (Balmer series), used to experimentally determine the Rydberg constant.



Spectrogoniometer

Hydrogen Lamp

Mercury Lamp

Figure 1: Some instruments/material used.

IV. Results and Analysis

In this section, an analysis of the mercury, hydrogen, and unknown lamp is presented. This includes wavelength calculations, spectral line positions, plots(MATLAB generated), error analysis, and a discussion of errors.

IV.1 Mercury Lamp

The experimentally measured positions(X), theoretical(λ_{exp}) and experimental(λ_{theo}) wavelengths of the different spectral lines(colors) are all shown in Table 2. The uncertainty of the position(ΔX) is as follows:

$$\Delta X = \frac{smallest\ div}{2} \tag{2}$$

In Figure 2, a standardizing(calibration) curve including its appropriate quadratic fitting was used in order to deduce λ_{exp} and its appropriate uncertainty. The equation of the fitting is under the form;

$$\lambda = 11.865X^2 - 450.110X + 8001.263 \tag{3}$$

To find the total uncertainty of the picked out experimental value of the wavelength $\Delta\lambda_{exp}$, the positional ($\Delta\lambda_{pos}$) and fitting($\Delta\lambda_{fit}$ uncertainties must be found first. To calculate $\Delta\lambda_{pos}$ at $X = 5.6cm$:

$$\Delta\lambda_{pos} = \left|\frac{\partial\lambda}{\partial X}\right| \Delta X = |(23.730X - 450.110) \times \Delta X| = 15.861 \text{ Å} \tag{4}$$

The fitting wavelength $\Delta\lambda_{fit}$ can be found using the *polyval* MATLAB function. Then the total **propagation** error with an example for $X = 5.6cm$ as:

$$\Delta\lambda_{exp} = \sqrt{\Delta\lambda_{pos}^2 + \Delta\lambda_{fit}^2} = \sqrt{15.861^2 + 186.640^2} = 187.31cm \tag{5}$$

Note that the average value of the yellow wavelength was taken.

$$\bar{\lambda}_{theo} = \frac{5770 + 5790}{2} = 5780 \text{ Å} \tag{6}$$

All other uncertainties can be found in Table 2.

Table 2: Calibration data for the mercury lamp.

Color	X (cm)	λ_{fit} (Å)	$\Delta\lambda_{pos}$ (Å)	λ_{total} (Å)	λ_{exp} (Å)	$\bar{\lambda}_{theo}$ (Å)
Yellow	5.6 ± 0.05	186.64	15.86	187	5853 ± 187	5780
Green	7.3 ± 0.05	165.19	13.84	166	5348 ± 166	5461
Blue	11.4 ± 0.05	195.43	8.98	196	4412 ± 196	4368
Violet	24.1 ± 0.05	200.27	6.09	200	4045 ± 200	4048

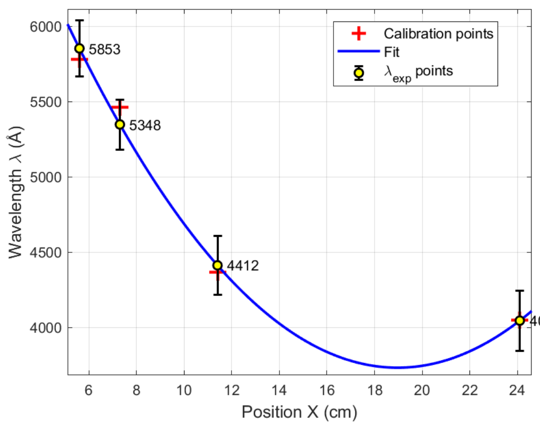


Figure 2: Standardizing curve for the mercury lamp showing experimental wavelength calibration.

IV.2 Unknown Lamp

In order to determine the type of lamp used, the previously found standardizing curve(Figure 2) can be used. Figure 3 shows the *same* curve as Figure 2 but with different positions because it is a different type of lamp.Upon further inspection, the λ_{exp} of the unknown lamp are very close to a **High-Pressure Sodium** lamp. Ranges was found for the spectrum which are:[1]

1. Red: 6150–7500 Å
2. Yellow: 5700–5900 Å
3. Green: 5150–5700 Å
4. Blue: 4500–4950 Å

All other other are in the Table 3

Table 3: Calibration data for unknown lamp.

Color	X (cm)	$\Delta\lambda_{\text{fit}}$ (Å)	$\Delta\lambda_{\text{pos}}$ (Å)	$\Delta\lambda_{\text{total}}$ (Å)	λ_{exp} (Å)
Red	3.9 ± 0.05	240.30	17.88	241	6426 $\pm$241
Yellow	5 ± 0.05	202.04	16.57	203	6047 $\pm$203
Green	6.19 ± 0.05	176.95	15.27	178	5697 $\pm$178
Blue	10.9 ± 0.05	190	9.57	190	4505 $\pm$190

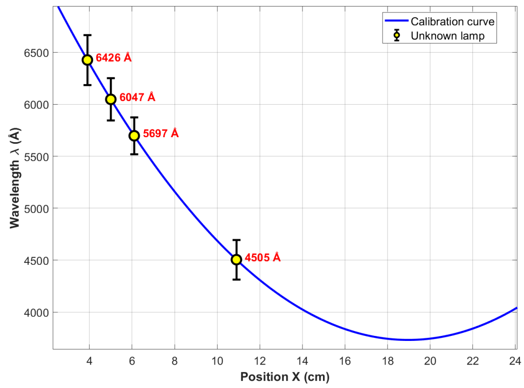


Figure 3: Previously found Standardizing Curve with Unknown lamp wavelength points.

IV.3 Hydrogen Lamp

The mercury standardization curve was once more used to find all the λ_{exp} of hydrogen from the measured X(Figure 4). The same uncertainty calculations(since same curve) were performed but at different positions once more. All data will be placed in Table 4.

Table 4: Calibration data for hydrogen lamp.

Color	X (cm)	$\Delta\lambda_{\text{fit}}$ (Å)	$\Delta\lambda_{\text{pos}}$ (Å)	$\Delta\lambda_{\text{total}}$ (Å)	λ_{exp} (Å)	λ_{theo} (Å)
Red	2.4 ± 0.05	310.6	19.7	311	6989 $\pm$311	6561
Green	9.4 ± 0.05	174	11.3	174	4819 $\pm$174	4860
Blue	11.9 ± 0.05	200.6	8.4	201	4325 $\pm$201	4339
Violet	18.3 ± 0.05	213.2	0.8	213	3738 $\pm$213	4101

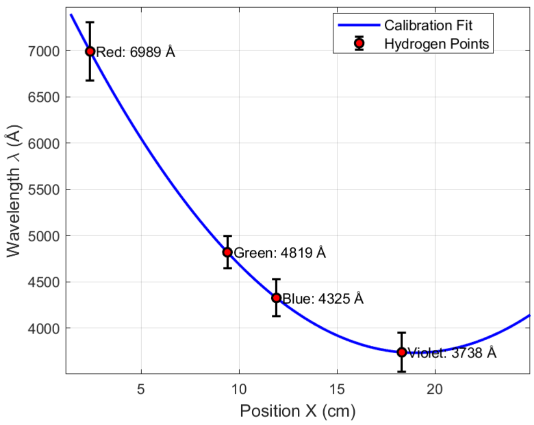


Figure 4: Standardizing Curve for Hydrogen Lamp (Balmer Series).

From Eqaution 1, the Balmer-Rydberg relation for the Balmer series at $m = 2$ becomes;

$$\frac{1}{\lambda} = R(\frac{1}{4} - \frac{1}{n^2}) \tag{7}$$

Rearranging this relation:

$$R = \frac{1}{\lambda(\frac{1}{4} - \frac{1}{n^2})} \tag{8}$$

For n=3, $R_{exp} = 1.030 \times 10^7/m$. Now for the uncertainty on the Rydberg constant, taking the Equation 1, applying the natural logarithm on both sides and deriving gives:

$$\Delta R = \frac{R_{exp}\Delta\lambda_{total}}{\lambda_{exp}} = 4.584 \times 10^5/m \tag{9}$$

This calculation was done for position of the red spectral line at n=3. All other values are found in Table 5.

Table 5: Experimental and theoretical uncertainties of the Rydberg constant for the Balmer series.

Color	n	$R_{\mathbf{exp}}$ **(**$\times 10^7$ $\mathbf{m^{-1}}$**)**	ΔR ($\times 10^5$ $\mathbf{m^{-1}}$)
Red	3	1.030	4.584
Green	4	1.107	4.005
Blue	5	1.101	5.112
Violet	6	1.203	6.866

For Rydberg constant, the average value is:

$$\bar{R} = \frac{1.030 + 1.107 + 1.101 + 1.203}{4} \times 10^7 = 1.110 \times 10^7/m \tag{10}$$

To find the total error:

$$\Delta R_{total} = \sqrt{\sum \Delta R^2} = 1.05 \times 10^6/m \tag{11}$$

So the final answer of the Rydberg constant is:

$$\bar{R} \pm \Delta R_{total} = (1.11 \pm 0.11) \times 10^7/m$$

IV.4 Errors and Results Discussion

In order to see the difference between the theoretical and experimental values of the data, one must compute the percent error:

$$\%Error(\lambda) = \frac{|\lambda_{theo} - \lambda_{exp}|}{\lambda_{theo}} \times 100 = \frac{|5780 - 5853|}{5780} \times 100 = 1.3\% \tag{12}$$

$$\%Error(R) = \frac{|R_{theo} - R_{exp}|}{R_{theo}} \times 100 = \frac{|1.09 - 1.11|}{1.09} \times 100 = 1.2\% \tag{13}$$

For the average yellow spectral line wavelength of Mercury lamp and the Rydberg constant. All other wavelengths for the different lamps are in the table below.

Table 6: Percent error of wavelength for different colors and lamps.

Color	Mercury Lamp (%)	Hydrogen Lamp (%)
Red	N/A	6.5
Yellow	1.3	N/A
Green	2.0	0.8
Blue	1.0	0.3
Violet	0.1	8.9

Mercury and Hydrogen lamps have very negligible and barely visible red and yellow spectral lines respectively.
For the **Sodium lamp**, all the wavelengths except for yellow($\lambda_{exp} = 6047 \pm 203$ Å) were in the provided range(5700–5900 Å for yellow).
Other sources errors include *reading* errors like recording the positions of the spectral lines, *instrumental* errors like misalignment of the prism and an unfocused sighting lens, and random errors. In order to *reduce* these sources, one must perform the experiment in a controlled environment, correctly calibrate all parts of the spectrogoniometer, adjust the prism, always double check the readings and properly focus the sighting lens.

V. Conclusion

To sum up, the standardizing curve was successfully determined using the experimental values of the Mercury spectral lines' wavelength which were all in the range of the determined total uncertainty according to the expected values. Then, using this curve, the experimental wavelengths of the determined Sodium vapor and Hydrogen lamps.For Sodium, the red, blue and green wavelengths were found in the given range while yellow was not. For Hydrogen, the green and blue wavelengths were found in the range of uncertainty but the red and violet ones were not. The discrepancies in the experimental values of wavelengths were due to a combination of reading and instrumental errors. In addition, the error between the theoretical and experimental value of the Rydberg constant was 1.2% because of same errors as those of the wavelengths.

VI. Appendix

Take the logarithm of

$$\frac{1}{\lambda} = R(\frac{1}{4} - \frac{1}{n^2})$$

To get:

$$-\ln(\lambda) = \ln(R) + \ln(\frac{1}{4} - \frac{1}{n^2})$$

As λ and R are the only variables and taking the absolute value of each term both sides:

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta R}{R} \tag{14}$$

VII. References

[1] M. Fortes, A. Pereira, A. Fragoso and G. Tavares, Some Considerations about LED Technology in Public Lighting, IEEE Chilecon, p. 1–6, (2015).

[2] P3303 Physics Laboratory Manual, Department of Physics and Electronics, Lebanese University, 2025.