Spații vect. euclidiene.

$u \in V \Rightarrow [u]_B = (x_1, x_2, \dots, x_n)_B$.

$v \in V \Rightarrow [v]_B = (y_1, y_2, \dots, y_n)_B$.

$$\langle u, v \rangle = [x_1\ x_2\ x_3 \dots x_n] \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = x_1 y_1 + x_2 y_2 + \dots + x_n y_n$$

$$\|v\| = \sqrt{\langle \bar{v}, \bar{v} \rangle} \geq 0$$

$$|\langle u, v \rangle| \leq \|u\| \cdot \|v\| \quad \Big|\ \|u\| \cdot \|v\|$$

$$\frac{|\langle u, v \rangle|}{\|u\| \cdot \|v\|} \leq 1 \qquad \langle u, v \rangle = \|u\| \cdot \|v\| \cdot \cos(\widehat{u, v})$$

$$-1 \leq \frac{\langle u, v \rangle}{\|u\| \cdot \|v\|} \leq 1 \qquad = \cos(\widehat{u, v})$$

$u \perp v \Leftrightarrow \langle u, v \rangle = 0$

$u \parallel v \Leftrightarrow \dfrac{x_1}{y_1} = \dfrac{x_2}{y_2} = \dots = \dfrac{x_n}{y_n}$

$$\|(1, 2, 3)\| = \sqrt{(1\ 2\ 3) \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}} = \sqrt{1^2 + 2^2 + 3^2}$$

Dist. 2 vect:

$\|u - v\|$

$\|v\| = 1 \Rightarrow v$ - versor.

Bază ortogonală:

$v_i \perp v_j \quad (\forall)\ i \neq j.$

Bază ortonormată:

$v_i \perp v_j \quad ; (\forall)\ i \neq j$

$\|v_i\| = 1 \quad ; (\forall)\ i.$

$\bar{u} = (x_1, x_2, x_3)_{Bc}.$

$\bar{v} = (y_1, y_2, y_3)_{Bc}.$

$$\bar{u} \times \bar{v} = \begin{vmatrix} \bar{e}_1 & \bar{e}_2 & e_3 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} =$$

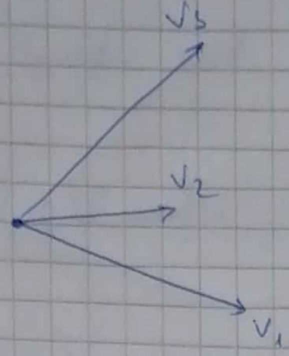
$$= \bar{e}_1 \cdot \begin{vmatrix} x_2 & x_3 \\ y_2 & y_3 \end{vmatrix} - \bar{e}_2 \begin{vmatrix} x_1 & x_3 \\ y_1 & y_3 \end{vmatrix} + \bar{e}_3 \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix}$$

$\bar{u} = (1, 0, -1)$

$\bar{v} = (0, 2, 1).$

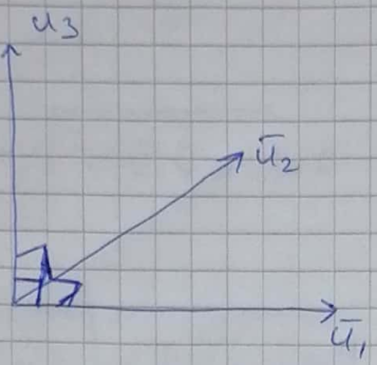
$$\bar{u} \times \bar{v} = \begin{vmatrix} \bar{e}_1 & \bar{e}_2 & \bar{e}_3 \\ 1 & 0 & -1 \\ 0 & 2 & 1 \end{vmatrix} = \bar{e}_1 \cdot \begin{vmatrix} 0 & -1 \\ 2 & 1 \end{vmatrix} - \bar{e}_2 \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} + \bar{e}_3 \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix}$$

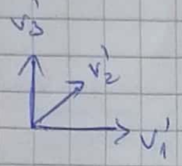
$$= \bar{e}_1 \cdot 2 - \bar{e}_2 \cdot 1 + e_3 \cdot 2 = (2, -1, 2)_{Bc}$$



baze.

$\bar{v}_1 = \bar{u}_1$





$B = \{ v_1 = (2, 0, 0), v_2 = (-4, 3, 0), v_3 = (6, -3, 5) \}$

$\bar{u}_1 = (2, 0, 0)$

$\lambda_{21} = \frac{\langle v_2, u_1 \rangle}{\langle u_1, u_1 \rangle} =$

v apare 1 $\quad u$ apare 3

$\bar{u}_2 = v_2 - \lambda_{21} \cdot u_1$

$\bar{u}_2 = (-4, 3, 0) + 2(2, 0, 0)$

$\bar{u}_2 = (0, 3, 0)$

$\bar{u}_3 = v_3 - \lambda_{31} \cdot u_1 - \lambda_{32} u_2 = \frac{(-4, 3, 0)\begin{pmatrix}2\\0\\0\end{pmatrix}}{(2, 0, 0)\begin{pmatrix}2\\0\\0\end{pmatrix}} = \frac{-8}{4} = -2$

$\bar{u}_3 = (6, -3, 5) - 3(2, 0, 0) +$

$+ 1(0, 3, 0)$

$\lambda_{31} = \frac{\langle v_3 \cdot u_1 \rangle}{\langle u_1, u_1 \rangle} = \frac{12}{4} = 3$

$\bar{u}_3 = (0, 0, 5)$

$\lambda_{32} = \frac{\langle v_3 \cdot u_2 \rangle}{\langle u_2, u_2 \rangle} = \frac{-9}{9} = -1$

$u_1 \perp u_2$

$u_2 \perp u_3$

$u_1 \perp u_3$.

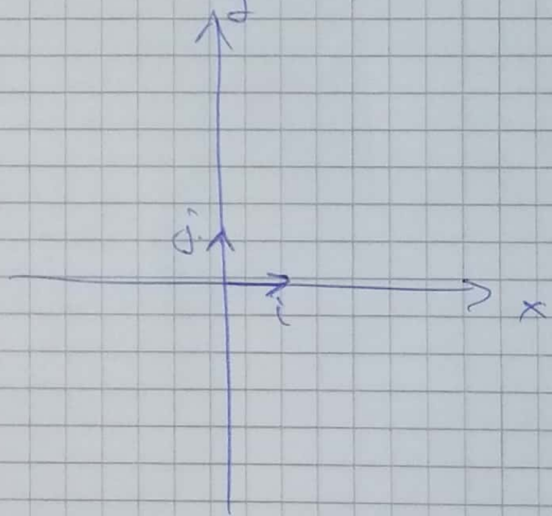
$$v_1' = \frac{1}{\|u_1\|} \cdot u_1 = \frac{1}{\sqrt{2^2+0^2+0^2}} \cdot (2,0,0) = (1,0,0).$$

$$v_2' = \frac{1}{\|u_2\|} \cdot u_2 = \frac{1}{\sqrt{3^2}} \cdot (0,3,0) = (0,1,0).$$

$$v_3' = \frac{1}{\|u_3\|} \cdot u_3 = \frac{1}{\sqrt{5^2}} \cdot (0,0,5) = (0,0,1)$$

$B' = \{v_1', v_2', v_3'\}$ - bază ortonormată.

Rest: $\begin{cases} \text{Diagonalizare} \\ \text{Formă pătratică} \\ \text{un Gram-Schmidt.} \end{cases}$



$(2,0,0), (0,1,1), (0,1,-1)$

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