VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ

Fakulta informačních technologií

IEL – ELEKTRONIKA PRO INFORMAČNÍ TECHNOLOGIE

2017/2018

Semestrální projekt

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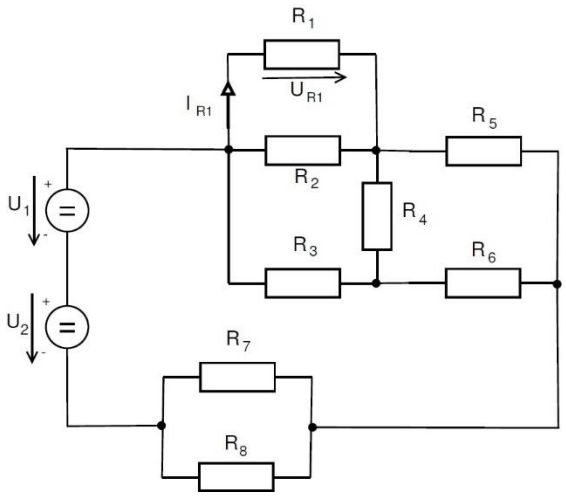
1. Úloha

Stanovte napätie U_{R1} a prúd I_{R1}. Použite metódu postupného zjednodušovania.

Sk.	U1 [V]	U2 [V]	R1 [Ω]	R2 [Ω]	R3 [Ω]	R4 [Ω]	R5 [Ω]	R6 [Ω]	R7 [Ω]	R8 [Ω]
E	115	55	485	660	100	340	575	815	255	225

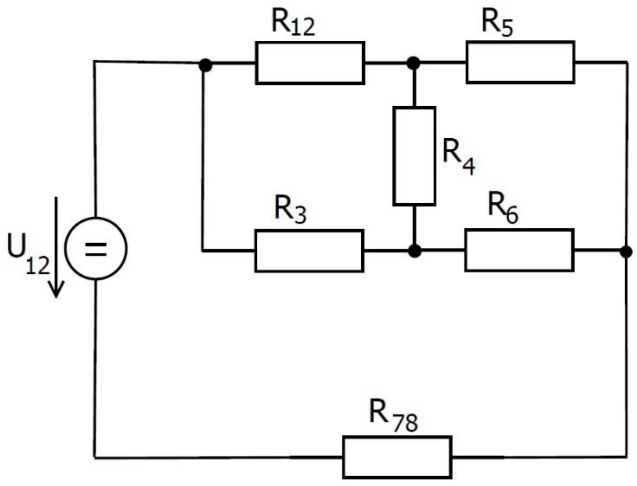


Obvod postupne zjednodušíme:

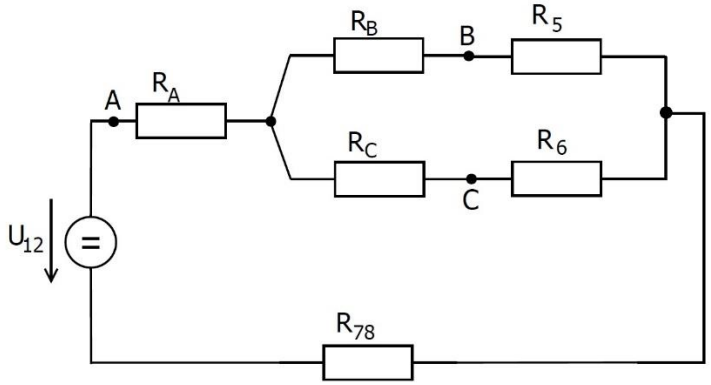
$$R_{78} = \frac{R_7 * R_8}{R_7 + R_8} = \frac{225 * 255}{225 + 255} = 119.53125\ \Omega$$

$$R_{12} = \frac{R_1 * R_2}{R_1 + R_2} = \frac{485 * 660}{485 + 660} = 279.5633\ \Omega$$

$$U_{12} = U_1 + U_2 = 115 + 55 = 170\ V$$



Vzniknutý trojuholník (R_{12}, R_3, R_4) prevedieme na hviezdu:

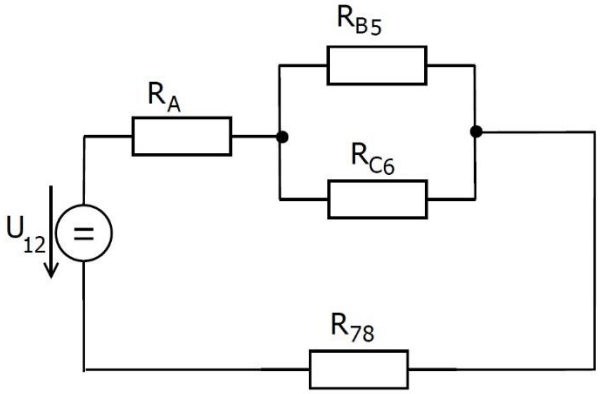


$$R_A = \frac{R_{12} * R_3}{R_{12} + R_3 + R_4} = \frac{119.53125 * 100}{119.53125 + 100 + 340} = 38.8518\ \Omega$$

$$R_B = \frac{R_{12} * R_4}{R_{12} + R_3 + R_4} = \frac{119.53125 * 340}{119.53125 + 100 + 340} = 132.0961\ \Omega$$

$$R_C = \frac{R_4 * R_3}{R_{12} + R_3 + R_4} = \frac{340 * 100}{119.53125 + 100 + 340} = 47.2508\ \Omega$$

Rezistory R_5 a R_B sú zapojené sériovo, takisto aj R_C a R_6:

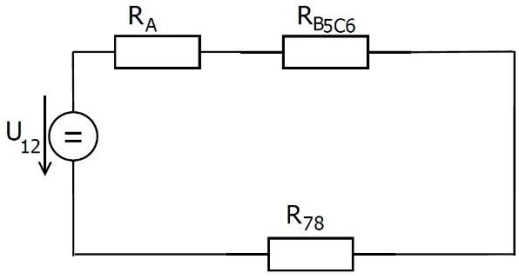


$$R_{B5} = R_B + R_5 = 132.0961 + 575 = 707.0961\ \Omega$$

$$R_{C6} = R_C + R_6 = 47.2509 + 815 = 862.2508\ \Omega$$

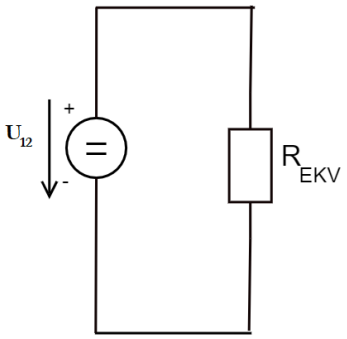
Rezistory R_{B5} a R_{C6} sú zapojené paralelne:

$$R_{B5C6} = \frac{R_{B5} * R_{C6}}{R_{B5} + R_{C6}} = \frac{707.0961 * 862.2508}{707.0961 + 862.2508} = 388.5018\ \Omega$$



Dostávame tri rezistory R_{B5C6}, R_A a R_{78} ktoré sú zapojené sériovo:

$$R_{EKV} = R_A + R_{B5C6} + R_{78} = 38.8518 + 388.5018 + 119.53125 \ = 546.88485\ \Omega$$



Celkový prúd v obvode je:

$$I = \frac{U_{12}}{R_{EKV}} = \frac{170}{546.88485} = 0.31085\ A\ = 310.85\ mA$$

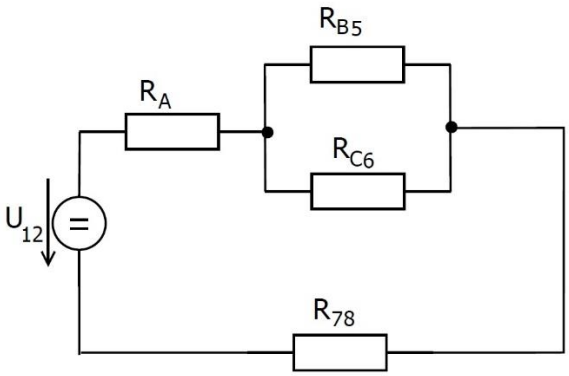
Teraz môžeme spätne vypočítať U_{R78}, U_{RB5C6} a U_{RA}:

$$U_{R78} = R_{78} * I = 0.31085 * 119.53125 = 37.1563\ V$$

$$U_{RB5C6} = R_{B5C6} * I = 0.31085 * 388.5018 = 120.7658\ V$$

$$U_{RA} = R_A * I = 0.31085 * 38.8518 = 12.0770\ V$$

Obvod postupne spätne skladáme dohromady:



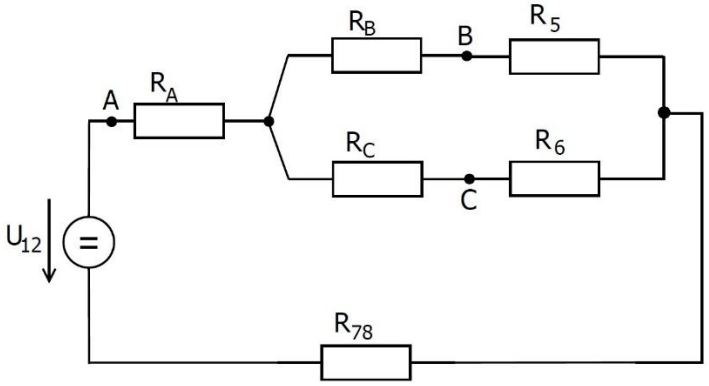
$$I_{RA} = \frac{U_{RA}}{R_A} = \frac{12.0770}{38.8518} = 0.31085\ A = 310.85\ mA$$

$$I_{RB5} = \frac{U_{RB5C6}}{R_{B5}} = \frac{120.7658}{707.0961} = 0.17079\ A = 170.79\ mA$$

$$I_{RC6} = \frac{U_{RB5C6}}{R_{C6}} = \frac{120.7658}{862.2509} = 0.14006\ A = 140.06\ mA$$

$$I_{R78} = \frac{U_{R78}}{R_{78}} = \frac{37.1563}{119.53125} = 0.31085\ A = 310.85\ mA$$

Vypočítame U_{RB}, U_{R5}, U_{RC} a U_{R6}:



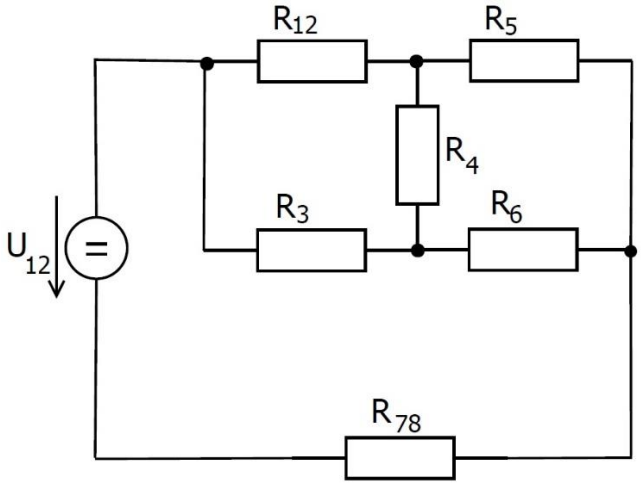
$$U_{RB} = R_B * I_{RB5} = 132.0961 * 0.17079 = 22.5607\ V$$

$$U_{R5} = R_5 * I_{RB5} = 575 * 0.17079 = 98.20425\ V$$

$$U_{RC} = R_C * I_{RC6} = 47.2509 * 0.14006 = 6.6180\ V$$

$$U_{R6} = R_6 * I_{RC6} = 815 * 0.14006 = 114.1489\ V$$

Pomocou II. Kirchoffovho zákona si zostavíme rovnice pre napätie:



$$U_{R5} - U_{R6} + U_{R4} = 0$$

$$U_{R3} + U_{R6} + U_{R78} - U_{12} = 0$$

$$U_{R12} - U_{R4} - U_{R3} = 0$$

$$U_{R4} = U_{R6} - U_{R5} = 114.1489 - 98.20425 = 15.9447\ V$$

$$U_{R3} = U_{12} - U_{R6} - U_{R78} = 170 - 114.1489 - 37.1563 = 18.695\ V$$

$$U_{R12} = U_{R4} + U_{R3} = 15.94465 + 18.6948 = 34.6395\ V$$

Nakoniec dopočítame hľadané napätie U_{R1} a prúd I_{R1}:

$$I_{R1} = \frac{U_{R12}}{R_1} = \frac{34.6395}{485} = 0.07142\ A\ = 71.42\ mA$$
$$U_{R12} = U_{R1} = 34.6395\ V$$

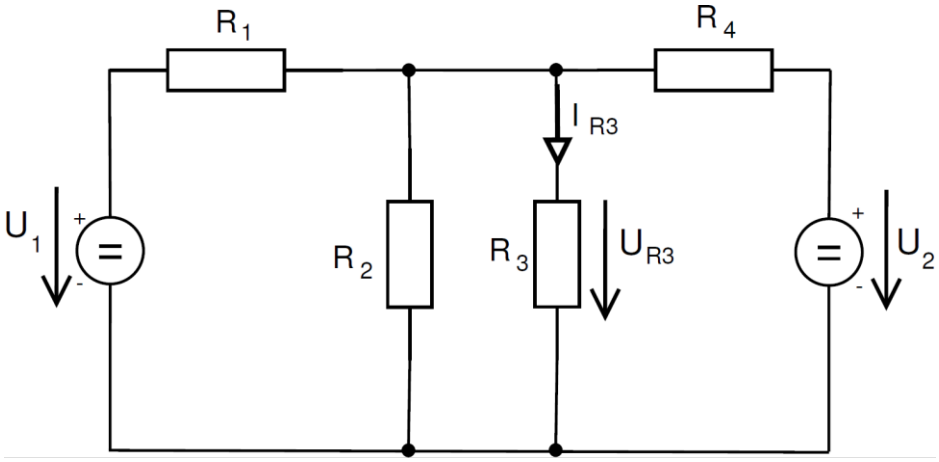
Hľadané hodnoty sú:

$$\boldsymbol{I_{R1} = 71.42\ mA}$$
$$\boldsymbol{U_{R1} = 34.6395\ V}$$

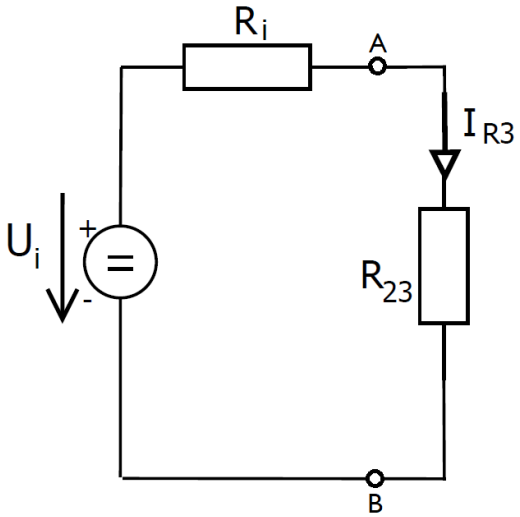
2. Úloha

Stanovte napätie U_{R3} a prúd I_{R3}. Použite metódu Théveninovej vety.

Sk.	U1 [V]	U2 [V]	R1 [Ω]	R2 [Ω]	R3 [Ω]	R4 [Ω]
C	200	70	220	630	240	450



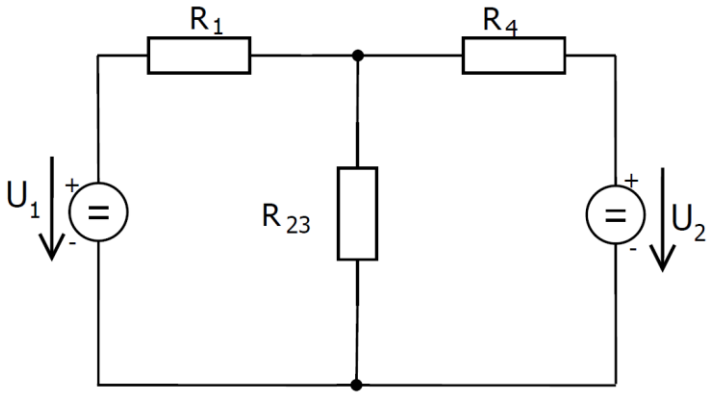
Zo zadaného obvodu si vytvoríme ekvivalentný obvod:



$$I_{R3} = \frac{U_i}{R_i + R_{23}}$$

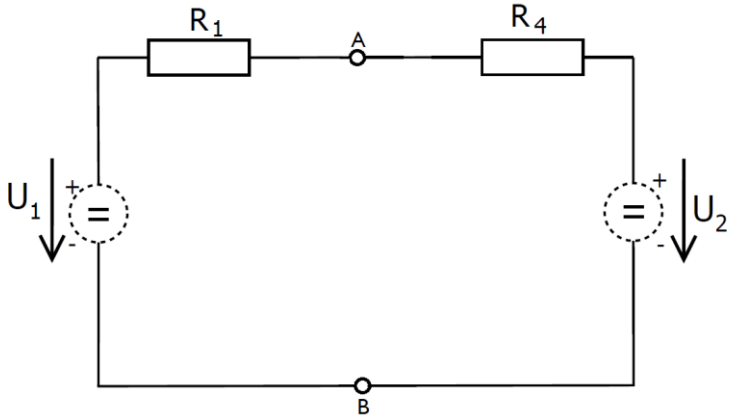
$$R_{EKV} = R_i + R_{23}$$

Rezistory R_2 a R_3 sú spojené paralelne:

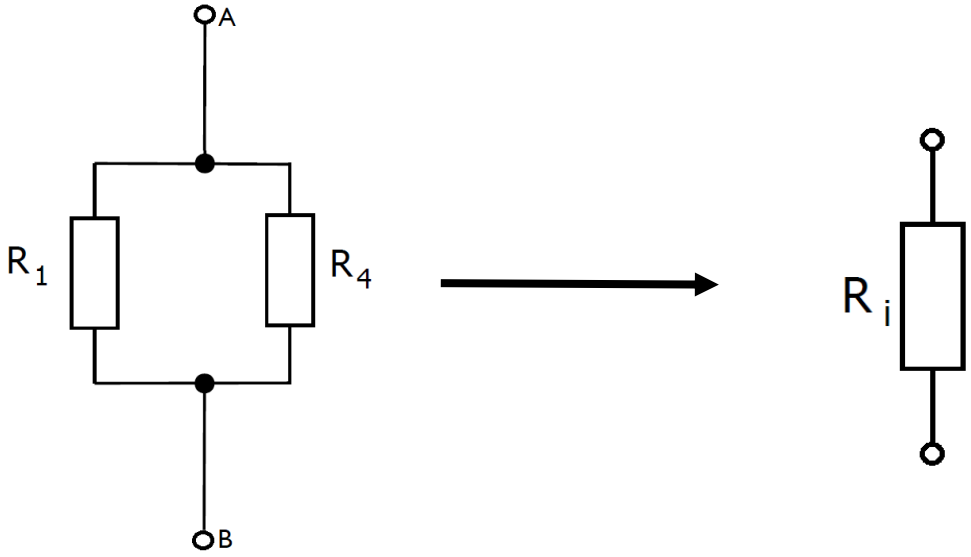


$$R_{23} = \frac{R_2 * R_3}{R_2 + R_3} = \frac{630 * 240}{630 + 240} = 173.7931\ \Omega$$

Prekreslíme obvod bez R_{23} a následne nahradíme napäťové zdroje „skratom“:

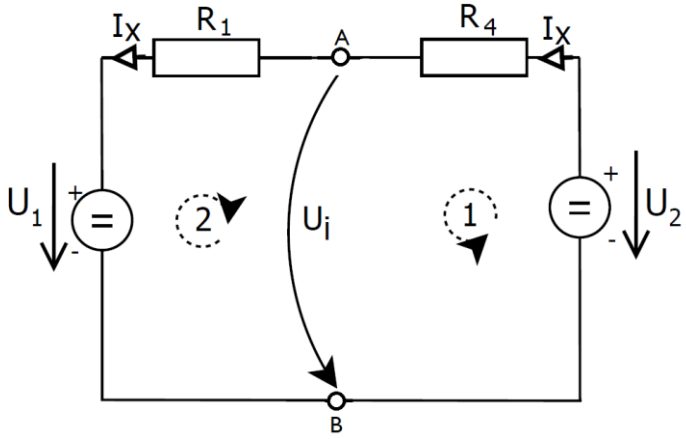


Z obrázku vidíme že rezistory R_1 a R_4 sú spojené paralelne, a dostaneme hľadané R_i :



$$R_i = \frac{R_1 * R_4}{R_1 + R_4} = \frac{220 * 450}{220 + 450} = 147.7612\ \Omega$$

Prekreslíme obvod bez R_{23} a určíme napätie „naprázdno“ medzi bodmi A, B:



Určíme si I_X podľa II. Kirchoffovho zákona:

- smyčka č. 1 = smyčka pre celý obvod

$$R_1 * I_X + U_1 - U_2 + R_4 * I_x = 0$$

$$I_X * (R_1 + R_4) + U_1 - U_2 = 0$$

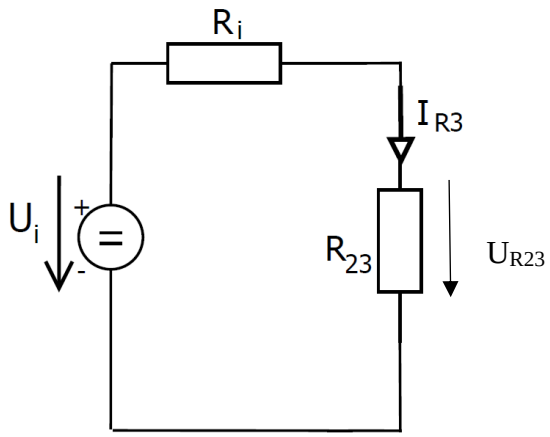
$$I_X = \frac{U_2 - U_1}{R_1 + R_4} = \frac{70 - 200}{220 + 450} = -0.1940\ A\ =\ -194\ mA$$

- smyčka c. 2 = smyčka len po U_i

$$U_i - U_1 - R_1 * I_x = 0$$

$$U_i = R_1 * I_x + \ U_1 = 220 * (-0.1940) + 200 = 157.3134\ V$$

Zo získaných hodnôt vieme vypočítať U_{R3} a I_{R3}:



$$I_{R23} = \frac{U_i}{R_i + R_{23}} = \frac{157.3134}{147.7612 + 173.7931} = 0.4892\ A\ =\ 489.2\ mA$$

$$U_{R23} = R_{23} * I_{R23} = 173.7931 * 0.4892 = 85.0196\ V$$

$$I_{R3} = \frac{U_{R23}}{R_3} = \frac{85.0196}{240} = 0.3542\ A = 354.2\ mA$$

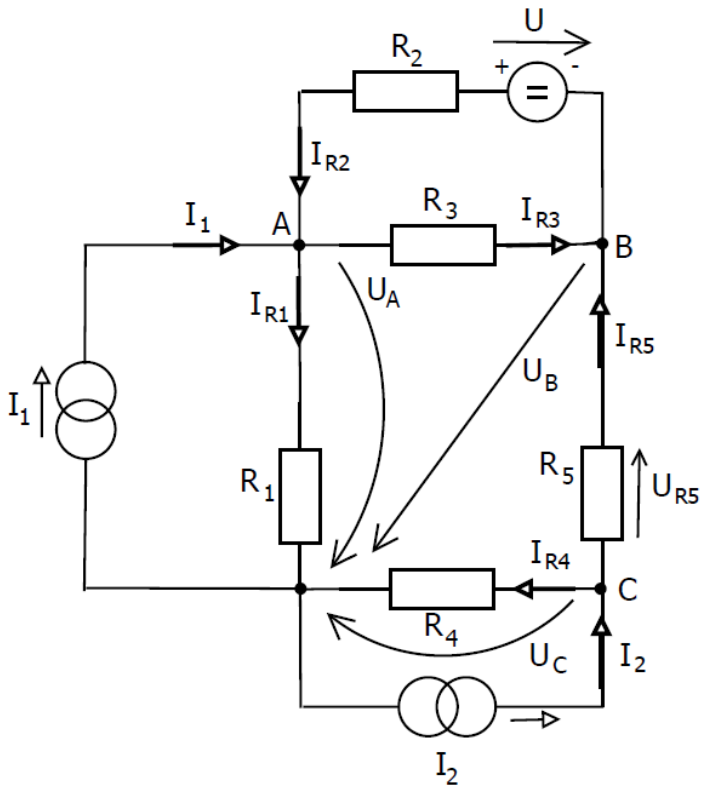
Hľadané hodnoty sú:

$$\boldsymbol{I_{R3} = 354.2\ mA}$$
$$\boldsymbol{U_{R3} = 85.0196\ V}$$

3. Úloha

Stanovte napätie U_{R5} a prúd I_{R5}. Použite metódu uzlových napätí (U_A, U_B, U_C).

Sk.	U [V]	I1 [A]	I2 [A]	R1 [Ω]	R2 [Ω]	R3 [Ω]	R4 [Ω]	R5 [Ω]
A	120	0.9	0.7	53	49	65	39	32



Podľa I. Kirchoffovho zákona zostavíme rovnice pre uzly A, B, C:

$$A: I_1 + I_{R2} - I_{R1} - I_{R3} = 0$$

$$B: I_{R3} + I_{R5} - I_{R2} = 0$$

$$C: I_2 - I_{R5} - I_{R4} = 0$$

Vyjadríme si jednotlivé prúdy pomocou uzlových napätí a zavedieme substitúciu $G_X = \frac{1}{R_X}$:

$$I_{R1} * R_1 - U_A = 0 \Rightarrow I_{R1} = \frac{U_A}{R_1} = G_1 * U_A$$

$$I_{R2} * R_2 + U_A - U_B = 0 \Rightarrow I_{R1} = \frac{U + U_B - U_A}{R_2} = G_2 * (U + U_B - U_A)$$

$$I_{R3} * R_3 + U_B - U_A = 0 \Rightarrow I_{R1} = \frac{U_A - U_B}{R_3} = G_3 * (U_A - U_B)$$

$$I_{R4} * R_4 - U_C = 0 \Rightarrow I_{R1} = \frac{U_C}{R_4} = G_4 * U_C$$

$$I_{R5} * R_5 + U_B - U_C = 0 \Rightarrow I_{R5} = \frac{U_C - U_B}{R_5} = G_5 * (U_C - U_B)$$

Dosadíme jednotlivé prúdy do rovníc:

$$I_1 + G_2 * (U + U_B - U_A) - G_1 * U_A - G_3 * (U_A - U_B) = 0$$

$$G_3 * (U_A - U_B) + G_5 * (U_C - U_B) - G_2 * (U + U_B - U_A) = 0$$

$$I_2 - G_5 * (U_C - U_B) - G_4 * U_C = 0$$

Upravíme rovnice:

$$U_A * (-G_2 - G_1 - G_3) + U_B * (G_2 + G_3) + 0 * U_c = -I_1 - G_2 * U$$

$$U_A * (G_3 + G_2) + U_B * (-G_3 - G_5 - G_2) + G_5 * U_C = G_2 * U$$

$$0 * U_A + U_B * G_5 + U_C * (-G_4 - G_5) = -I_2$$

Dosadíme čísla:

$$U_A * \left(-\frac{1}{49} - \frac{1}{53} - \frac{1}{65}\right) + U_B * \left(\frac{1}{49} + \frac{1}{65}\right) + 0 * U_c = -0.9 - \frac{1}{49} * 120$$

$$U_A * \left(\frac{1}{49} + \frac{1}{65}\right) + U_B * \left(-\frac{1}{49} - \frac{1}{65} - \frac{1}{32}\right) + U_C * \frac{1}{32} = \frac{1}{49} * 120$$

$$0 * U_A + U_B * \frac{1}{32} + U_C * \left(-\frac{1}{39} - \frac{1}{32}\right) = -0.7$$

Číselné hodnoty dosadíme do matice a pomocou Cramerovho pravidla vypočítame determinanty:

$$A = \begin{pmatrix} -0.05466 & 0.03579 & 0 & \vdots & -3.34898 \\ 0.03579 & -0.06704 & 0.03125 & \vdots & 2.44898 \\ 0 & 0.03125 & -0.05689 & \vdots & -0.7 \end{pmatrix}$$

$$det\ A = \begin{vmatrix} -0.05466 & 0.03579 & 0 \\ 0.03579 & -0.06704 & 0.03125 \\ 0 & 0.03125 & -0.05689 \end{vmatrix} = -8.22174 * 10^{-5}$$

$$det\ U_B = \begin{vmatrix} -0.05466 & -3.34898 & 0 \\ 0.03579 & 2.44898 & 0.03125 \\ 0 & -0.7 & -0.05689 \end{vmatrix} = -3.99156 * 10^{-4}$$

$$det\ U_C = \begin{vmatrix} -0.05466 & 0.03579 & -3.34898 \\ 0.03579 & -0.06704 & 2.44898 \\ 0 & 0.03125 & -0.7 \end{vmatrix} = -1.2309 * 10^{-3}$$

$$U_B = \frac{det\ U_B}{det\ U_A} = \frac{-3.99156 * 10^{-4}}{-8.22174 * 10^{-5}} = 4.85488\ V$$

$$U_C = \frac{det\ U_C}{det\ U_A} = \frac{-1.2309 * 10^{-3}}{-8.22174 * 10^{-5}} = 14.97128\ V$$

Zo získaných hodnôt vypočítame hľadané hodnoty U_{R5} a I_{R5} :

$$I_{R5} = \frac{U_C - U_B}{R_5} = \frac{14.97128 - 4.85488}{32} = 0.3161\ A = 316.1\ mA$$

$$U_{R5} = I_{R5} * R_5 = 0.3161 * 32 = 10.1152\ V$$

Hľadané hodnoty sú:

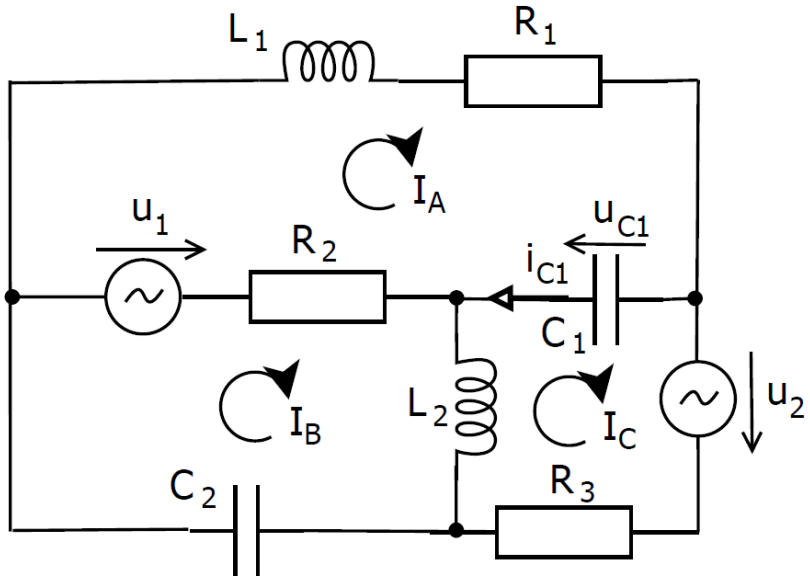
$$\boldsymbol{U_{R5} = 10.1152\ V}$$

$$\boldsymbol{I_{R5} = 0.3161\ A = 316.1\ mA}$$

4. Úloha

Pre napájacie napätie platí: $u_1 = U_1 * sin(2\pi ft)$, $u_2 = U_2 * sin(2\pi ft)$. Vo vzťahu pre napätie $u_{C1} = U_{C1} * \sin(2\pi ft + \varphi_{C1})$ určite $|U_{C1}|$ a φ_{C1}. Použite metódu smyčkových prúdov.

Sk.	U1 [V]	U2 [V]	R1 [Ω]	R2 [Ω]	R3 [Ω]	L1 [mH]	L2 [mH]	C1 [µF]	C2 [µF]	f [Hz]
E	50	30	14	13	14	130	60	100	65	90



Vypočítame uhlovú rýchlosť ω:

$$\omega = 2\pi f = 2 * \pi * 90 = 565.48678\, rad/s$$

Vypočítame impedanciu:

$$Z_{L1} = j\omega L_1 = 565.48678 * 130 * 10^{-3} j = 73.5133 j\Omega$$

$$Z_{L2} = j\omega L_2 = 565.48678 * 60 * 10^{-3} j = 33.9292 j\Omega$$

$$Z_{C1} = \frac{-1}{j\omega C_1} j = \frac{-1}{565.48678 * 100 * 10^{-6}} j = -17.6839 j\Omega$$

$$Z_{C2} = \frac{-1}{j\omega C_2} j = \frac{-1}{565.48678 * 65 * 10^{-6}} j = -27.206 j\Omega$$

Zostavíme si rovnice podľa II. Kirchoffovho zákona pre napätie v smyčkách:

$$I_A: Z_{L1} * I_A + R_1 * I_A + Z_{c1} * (I_A - I_C) + R_2 * (I_A - I_B) - U_1 = 0$$

$$I_B: U_1 * R_2 * (I_A - I_B) + Z_{L2} * (I_B - I_C) + Z_{L2} * I_B = 0$$

$$I_C: R_3 * I_C + Z_{L2} * (I_C - I_B) + Z_{C1} * (I_C - I_A) + U_2 = 0$$

Rovnice si upravíme:

$$I_A * (Z_{L1} + R_1 + Z_{C1} + R_2) - I_B * R_2 - I_C * Z_{C1} = U_1$$

$$-I_A * R_2 + I_B * (R_2 + Z_{L2} + Z_{C2}) - I_C * Z_{L2} = -U_1$$

$$-I_A * Z_{C1} - I_B * Z_{L2} + I_C * (R_3 + Z_{L2} + Z_{C1}) = -U_2$$

Dosadíme si číselné hodnoty:

$$I_A * (73.5133j + 14 - 17.6839j + 13) - I_B * 13 - I_C * -17.6839j = 50$$

$$-I_A * 13 + I_B * (13 + 33.9292j - 27.206j) - I_C * 33.9292j = -50$$

$$-I_A * -17.6839j - I_B * 33.9292j + I_C * (14 + 33.9292j - 17.6839j) = -30$$

Číselné hodnoty dosadíme do matice a pomocou Cramerovho pravidla vypočítame determinanty:

$$A = \left(\begin{array}{ccc|c} 27 + 55.8294j & -13 & 17.6839j & 50 \\ -13 & 13 + 6.7232j & -33.9292j & -50 \\ -17.6839j & -33.9292j & 14 + 16.2453j & -30 \end{array}\right)$$

$$det\ A = \begin{vmatrix} 27 + 55.8294j & -13 & 17.6839j \\ -13 & 13 + 6.7232j & -33.9292j \\ 17.6839j & -33.9292j & 14 + 16.2453j \end{vmatrix} = 2.1011 * 10^3 + 7.5934 * 10^4 j$$

$$det\ I_A = \begin{vmatrix} 50 & -13 & 17.6839 \\ -50 & 13 + 6.7232j & -33.9292j \\ -30 & -33.9292j & 14 + 16.2453j \end{vmatrix} = 1.8532 * 10^4 - 1.6294 * 10^3 j$$

$$det\ I_C = \begin{vmatrix} 27 + 55.8294j & -13 & 50 \\ -13 & 13 + 6.7232j & -50 \\ 17.6839j & -33.9292j & -30 \end{vmatrix} = 1.0646 * 10^5 - 5.097 * 10^4 j$$

$$I_A = \frac{det\ I_A}{det\ A} = \frac{1.8532 * 10^4 - 1.6294 * 10^3 j}{2.1011 * 10^3 + 7.5934 * 10^4 j} = -0.01469 - 0.24446j$$

$$I_C = \frac{det\ I_C}{det\ A} = \frac{1.0646 * 10^5 - 5.097 * 10^4 j}{2.1011 * 10^3 + 7.5934 * 10^4 j} = -0.63196 - 1.41946j$$

Zo získaných hodnôt vypočítame $|U_{C1}|$ a φ_{C1}:

$$U_{C1} = Z_{C1} * (I_A - I_C) = -17.6839j * (0.6173 + 1.175j) = 20.7786 - 10.9163j$$

$$|U_{C1}| = \sqrt{20.7786^2 + 10.9163^2} = 23.4716\ V$$

4.kvadrant (-y, +x):

$$\varphi C1 = arctan \frac{Im(U_{C1})}{Real(U_{C1})} = arctan \frac{-10.9163}{20.7786} = -0.48373\ rad$$

Hľadané hodnoty sú:

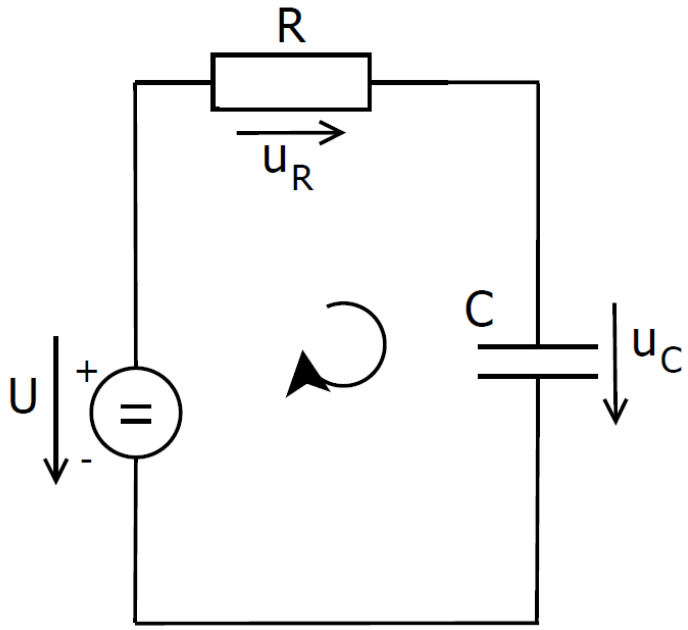
$$\boldsymbol{\varphi_{C1} = -0.48373\ rad * \frac{180}{\pi} = -27.7157°}$$

$$\boldsymbol{U_{C1} = 23.4716\ V}$$

5. Úloha

Zostavte diferenciálnu rovnicu popisujúcu chovanie obvodu na obrázku, ďalej upravte dosadením hodnoty parametrov. Vypočítajte analytické riešenie $u_C = f(t)$. Preveďte kontrolu výpočtu dosadením do zostavenej diferenciálnej rovnice.

Sk.	U [V]	C [F]	R [Ω]	$u_c(0)$ [V]
C	60	5	30	7



Podľa II. Kirchoffovho zákona si zostavíme rovnicu a vyjadríme si prúd:

$$u_R + u_C - u = 0$$

$$R * I + u_C - u = 0$$

$$I = \frac{u - u_c}{R}$$

Známe hodnoty dosadíme do rovnice ktorá platí pre tento obvod:

$$u'_C = \frac{1}{C} I$$

$$u'_C = \frac{u - u_C}{R * C}$$

Dosadíme si známe hodnoty:

$$u'_C + \frac{u_C}{R * C} = \frac{u}{R * C}$$

$$u'_C + \frac{u_C}{30 * 5} = \frac{60}{30 * 5}$$

$$u'_C + u_C \frac{1}{150} = \frac{60}{150}$$

Vypočítame λ z charakteristickej rovnice:

$$\lambda + \frac{1}{R * C} = 0$$

$$\lambda = -\frac{1}{R * C} = -\frac{1}{30 * 5} = -\frac{1}{150}$$

Očakávané riešenie je v tvare:

$$u_c(t) = k(t)e^{\lambda t}$$

$$u_c(t) = k(t)e^{-\frac{t}{R*C}}$$

Dosadíme známe hodnoty a následne zderivujeme:

$$u_c(t) = k(t)e^{-\frac{1}{150}t}$$

$$u'_c(t) = k(t)e^{-\frac{1}{150}t} - \frac{1}{150}k(t)e^{-\frac{1}{150}t}$$

Známe hodnoty dosadíme do rovnice popisujúcej obvod:

$$u'_C + \frac{u_C}{R * C} = \frac{u}{R * C}$$

$$u'_C + u_C \frac{1}{150} = \frac{60}{150}$$

$$k'(t)e^{-\frac{1}{150}t} - \frac{1}{150}k(t)e^{-\frac{1}{150}t} + \frac{1}{150}k(t)e^{-\frac{1}{150}t} = \frac{60}{150}$$

Po vykrátení dostávame:

$$k'(t)e^{-\frac{1}{150}t} = \frac{2}{5}$$

Vyjadríme k(t):

$$k'(t)e^{-\frac{1}{150}t} = \frac{2}{5}$$

$$k'(t) = \frac{2 * e^{\frac{1}{150}t}}{5}$$

$$\int k'(t)dt = \frac{2 * e^{\frac{1}{150}t}}{5}$$

$$k(t) = u * e^{\frac{1}{150}t} + \mathrm{A}$$

$$k(t) = 60e^{\frac{1}{150}t} + A$$

Dosadíme si k(t) do očakávaného riešenia:

$$u_c(t) = k(t)e^{\lambda t}$$

$$u_c(t) = (60e^{\frac{1}{150}t} + A) * e^{-\frac{1}{150}t}$$

$$u_c(t) = 60 + A * e^{-\frac{1}{150}t}$$

Dosadíme počiatočnú podmienku:

$$u_c(0) = 60 + A * e^{-\frac{1}{150}*0}$$

$$7 = 60 + A$$

$$A = -53$$

Hľadané riešenie je:

$$\boldsymbol{u_c(t) = 60 - 53 * e^{-\frac{1}{150}t}}$$

Skúška: Dosadíme u_c, u'_c do rovnice popisujúcej obvod:

$$u'_C + u_C \frac{1}{150} = \frac{60}{150}$$

$$u_C(t) = 60 - 53 * e^{-\frac{1}{150}t}$$

$$u'_C(t) = \frac{53}{150} e^{-\frac{1}{150}t}$$

$$\frac{53}{150} e^{-\frac{1}{150}t} + \frac{1}{150} * \left(60 - 53 * e^{-\frac{1}{150}t}\right) = \frac{60}{150}$$

$$\frac{53}{150} e^{-\frac{1}{150}t} + \frac{60}{150} - \frac{53}{150} e^{-\frac{1}{150}t} = \frac{60}{150}$$

$$\frac{60}{150} = \frac{60}{150}$$

$$0 = 0$$

6. Výsledky

Príklad	Skupina	Výsledok
1.	E	$I_{R1} = 71.42\ mA$ $U_{R1} = 34.6395\ V$
2.	C	$I_{R3} = 354.2\ mA$ $U_{R3} = 85.0196\ V$
3.	A	$U_{R5} = 10.1152\ V$ $I_{R5} = 0.3161\ A = 316.1\ mA$
4.	E	$\varphi_{C1} = -0.48373\ rad = -27.7157°$ $\mathrm{U_{C1}} = 23.4716\ \mathrm{V}$
5	C	$u_c(t) = 60 - 53 * e^{-\frac{1}{150}t}$