

# Part IB - Electromagnetism

## Theorems with Proof

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### **Electromagnetism and Relativity**

Review of Special Relativity; tensors and index notation. Lorentz force law. Electromagnetic tensor. Lorentz transformations of electric and magnetic fields. Currents and the conservation of charge. Maxwell equations in relativistic and non-relativistic forms. [5]

### **Electrostatics**

Gauss's law. Application to spherically symmetric and cylindrically symmetric charge distributions. Point, line and surface charges. Electrostatic potentials; general charge distributions, dipoles. Electrostatic energy. Conductors. [3]

### **Magnetostatics**

Magnetic fields due to steady currents. Ampere's law. Simple examples. Vector potentials and the Biot-Savart law for general current distributions. Magnetic dipoles. Lorentz force on current distributions and force between current-carrying wires. Ohm's law. [3]

### **Electrodynamics**

Faraday's law of induction for fixed and moving circuits. Electromagnetic energy and Poynting vector. 4-vector potential, gauge transformations. Plane electromagnetic waves in vacuum, polarization. [5]

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# 1 Introduction

## 1.1 Charge and Current

**Law** (Continuity equation).

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0$$

## 1.2 Forces and Fields

**Law** (Lorentz force law).

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

**Law** (Maxwell's Equations).

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{E}}{\partial t} = 0$$

$$\nabla \times \mathbf{B} - \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{j}$$

where

–  $\varepsilon_0 = 8.85 \times 10^{-12} \text{ m}^{-3} \text{ kg}^{-1} \text{ s}^2 \text{ C}^2$  is the electric constant

–  $\mu_0 = 4\pi \times 10^{-6} \text{ m kg C}^{-2}$  is the magnetic constant

## 2 Electrostatics

### 2.1 Gauss' Law

**Law** (Gauss' law).

$$\int_S \mathbf{E} \cdot d\mathbf{S} = \frac{Q}{\varepsilon_0},$$

where  $Q$  is the total charge inside  $V$ .