Part IA - Dynamics and Relativity
Theorems

Lectured by G. I. Ogilvie

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Basic concepts
Space and time, frames of reference, Galilean transformations. Newton's laws. Dimensional analysis. Examples of forces, including gravity, friction and Lorentz. [4]

Newtonian dynamics of a single particle
Equation of motion in Cartesian and plane polar coordinates. Work, conservative forces and potential energy, motion and the shape of the potential energy function; stable equilibria and small oscillations; effect of damping.

Angular velocity, angular momentum, torque.

Orbits: the $u(\theta)$ equation; escape velocity; Kepler's laws; stability of orbits; motion in a repulsive potential (Rutherford scattering). Rotating frames: centrifugal and coriolis forces. *Brief discussion of Foucault pendulum.* [8]

Newtonian dynamics of systems of particles
Momentum, angular momentum, energy. Motion relative to the centre of mass; the two body problem. Variable mass problems; the rocket equation. [2]

Rigid bodies
Moments of inertia, angular momentum and energy of a rigid body. Parallel axes theorem. Simple examples of motion involving both rotation and translation (e.g. rolling). [3]

Special relativity
The principle of relativity. Relativity and simultaneity. The invariant interval. Lorentz transformations in $(1+1)$-dimensional spacetime. Time dilation and length contraction. The Minkowski metric for $(1+1)$-dimensional spacetime. Lorentz transformations in $(3+1)$ dimensions. 4-vectors and Lorentz invariants. Proper time. 4-velocity and 4-momentum. Conservation of 4-momentum in particle decay. Collisions. The Newtonian limit. [7]

Contents

1 Newtonian dynamics of particles

1.1 Newton's laws of motion

Law (Newton's First Law of Motion)**.** A body remains at rest, or moves uniformly in a straight line, unless acted on by a force. (This is in fact Galileo's Law of Inertia)

Law (Newton's Second Law of Motion)**.** The rate of change of momentum of a body is equal to the force acting on it (in both magnitude and direction).

Law (Newton's Third Law of Motion)**.** To every action there is an equal and opposite reaction: the forces of two bodies on each other are equal and in opposite directions.

1.2 Galilean transformations

Law (Galilean relativity)**.** The *principle of relativity* asserts that the laws of physics are the same in inertial frames.

1.3 Newton's Second Law

Law. The *equation of motion* for a particle subject to a force F is

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = \mathbf{F},$$

where $\mathbf{p} = m\mathbf{v} = m\ddot{\mathbf{r}}$ is the (linear) momentum of the particle. We say m is the (inertial) mass of the particle, which is a measure of its reluctance to accelerate.

2 Dimensional Analysis

2.1 Units

2.2 Scaling

3 Forces

3.1 Force and potential energy in one dimension

Proposition. Suppose the equation of a particle satisfies

$$m\ddot{x} = -\frac{\mathrm{d}V}{\mathrm{d}x}. \tag{*}$$

Then the total energy

$$E = T + V = \frac{1}{2}m\dot{x}^2 + V(x)$$

is conserved, i.e. $\dot{E} = 0$.

3.2 Motion in a potential

3.3 Equilibrium points

3.4 Force and potential energy in three dimensions

Proposition. If $\mathbf{F}$ is conservative, then the energy

$$\begin{aligned} E &= T + V \\ &= \frac{1}{2}m|\mathbf{v}|^2 + V(\mathbf{r}) \end{aligned}$$

is conserved. Then the work done is equal to the change in potential energy, and is independent of the path taken between the end points.

In particular, if we travelled around a closed loop, no work is done.

3.5 Central forces

Proposition. $\nabla r = \hat{\mathbf{r}}$.

Proposition. Let $\mathbf{F} = -\nabla V(r)$ be a central force. Then

$$\mathbf{F} = -\nabla V = -\frac{\mathrm{d}V}{\mathrm{d}r}\hat{\mathbf{r}},$$

where $\hat{\mathbf{r}} = \mathbf{r}/r$ is the unit vector in the radial direction pointing away from the origin.

Proposition. Angular momentum is conserved by a central force.

3.6 Gravity

Law. If a particle of mass M is fixed at a origin, then a second particle of mass m experiences a potential energy

$$V(r) = -\frac{GMm}{r},$$

where $G \approx 6.67 \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$ is the *gravitational constant.*

The gravitational force experienced is then

$$F = -\nabla V = -\frac{GMm}{r^2}\hat{\mathbf{r}}.$$

Proposition. The gravitational potential due to many fixed masses M_i at points $\mathbf{r}_i$ is is

$$\Phi_g(\mathbf{r}) = -\sum_i \frac{GM_i}{|\mathbf{r} - \mathbf{r}_i|}.$$

Again, $V = m\Phi_g$ for a particle of mass m.

Proposition. The external gravitational potential of a spherically symmetric object of mass M is the same as that of a point particle with the same mass at the center of the object, i.e.

$$\Phi_g(r) = -\frac{GM}{r}.$$