Part IA - Dynamics and Relativity

Definitions

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Basic concepts
Space and time, frames of reference, Galilean transformations. Newton's laws. Dimensional analysis. Examples of forces, including gravity, friction and Lorentz. [4]

Newtonian dynamics of a single particle
Equation of motion in Cartesian and plane polar coordinates. Work, conservative forces and potential energy, motion and the shape of the potential energy function; stable equilibria and small oscillations; effect of damping.

Angular velocity, angular momentum, torque.

Orbits: the $u(\theta)$ equation; escape velocity; Kepler's laws; stability of orbits; motion in a repulsive potential (Rutherford scattering). Rotating frames: centrifugal and coriolis forces. *Brief discussion of Foucault pendulum.* [8]

Newtonian dynamics of systems of particles
Momentum, angular momentum, energy. Motion relative to the centre of mass; the two body problem. Variable mass problems; the rocket equation. [2]

Rigid bodies
Moments of inertia, angular momentum and energy of a rigid body. Parallel axes theorem. Simple examples of motion involving both rotation and translation (e.g. rolling). [3]

Special relativity
The principle of relativity. Relativity and simultaneity. The invariant interval. Lorentz transformations in $(1+1)$-dimensional spacetime. Time dilation and length contraction. The Minkowski metric for $(1+1)$-dimensional spacetime. Lorentz transformations in $(3+1)$ dimensions. 4-vectors and Lorentz invariants. Proper time. 4-velocity and 4-momentum. Conservation of 4-momentum in particle decay. Collisions. The Newtonian limit. [7]

Contents

1 Newtonian dynamics of particles

Definition (Particle)**.** An *particle* is an object of insignificant size. It can be regarded as a point. It has a *mass* $m > 0$, and *electric charge* q.

Its position at time t is described by its *position vector*, $\mathbf{r}(t)$ or $\mathbf{x}(t)$ with respect to an origin O.

Definition (Frame of reference)**.** A *frame of reference* is choice of coordinate axes for $\mathbf{r}$. The axes may be fixed, moving, or accelerating relative to another frame.

With a frame of reference, we can write $\mathbf{r}$ in cartesian coordinates as (x, y, z)

Definition (Velocity)**.** The *velocity* of the particle is

$$\mathbf{v} = \dot{\mathbf{r}} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}.$$

and is tangent to the path or trajectory.

Definition (Acceleration)**.** The *acceleration* of the particle is

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \frac{\mathrm{d}^2\mathbf{r}}{dt^2}.$$

Definition (Momentum)**.** The *momentum* of a particle is

$$\mathbf{p} = m\mathbf{v} = m\dot{\mathbf{r}}.$$

m is the *inertial mass* of the particle, and measures its reluctance to accelerate (c.f. Newton's Second Law)

1.1 Newton's laws of motion

Definition (Inertial frames)**.** *Inertial frames* are frames of references in which the frames themselves are not accelerating. Newton's Laws only hold in inertial frames.

1.2 Galilean transformations

Definition (Galilean boost)**.** A Galilean boost is a change in frame of reference by

$$\begin{aligned} \mathbf{r}' &= \mathbf{r} - \mathbf{v}t \\ t' &= t \end{aligned}$$

for a fixed, constant $\mathbf{v}$.

1.3 Newton's Second Law

2 Dimensional Analysis

2.1 Units

2.2 Scaling

3 Forces

3.1 Force and potential energy in one dimension

Definition (Potential energy)**.** Given a force field $F = F(x)$, we define the *potential energy* to be a function $V(x)$ such that

$$F = -\frac{\mathrm{d}V}{\mathrm{d}x}.$$

or

$$V = -\int F\ \mathrm{d}x.$$

V includes an arbitrary additive constant.

3.2 Motion in a potential

3.3 Equilibrium points

Definition (Equilibrium point)**.** A particle is in *equilibrium* if it has no tendency to move away. It will stay there for all time. Since $m\ddot{x} = -V'(x)$, the equilibrium points are the stationary points of the potential energy, i.e.

$$V'(x_0) = 0;$$

3.4 Force and potential energy in three dimensions

Definition (Power)**.** The *power* is the rate at which work is done on a particle by a force. It is given by

$$P = \mathbf{F} \cdot \mathbf{v}.$$

Definition (Work done)**.** The *work done* on a particle by a force is the change in kinetic energy caused by the force. The work done on a particle moving from $\mathbf{r}_1 = \mathbf{r}(t_1)$ to $\mathbf{r}_2 = \mathbf{r}(t_2)$ along a trajectory C is the line integral

$$W = \int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r} = \int_{t_1}^{t_2} \mathbf{F} \cdot \dot{\mathbf{r}}\ \mathrm{d}t = \int_{t_1}^{t_2} P\ \mathrm{d}t.$$

Definition (Conservative force and potential energy)**.** A *conservative force* is a force field $\mathbf{F}(\mathbf{r})$ that can be written in the form

$$\mathbf{F} = -\nabla V.$$

V is the *potential energy function*.

3.5 Central forces

Definition (Central force)**.** A central force is a force with a potential $V(r)$ that depends only on the distance from the origin, $r = |\mathbf{r}|$. Note that a central force can be both attractive or repulsive.

Definition (Angular momentum)**.** The *angular momentum* of a particle is

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = m\mathbf{r} \times \dot{\mathbf{r}}.$$

Definition (Torque)**.** The *torque* G of a particle is the rate of change of momentum.

$$\mathbf{G} = \frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = \mathbf{r} \times \mathbf{F}.$$

3.6 Gravity

Definition (Gravitaional potential and field)**.** The *gravitational potential* is the gravitational potential energy per unit mass. It is

$$\Phi_g(r) = -\frac{GM}{r}.$$

Note that *potential* is different from *potential energy*.

The *gravitational field* is the force per unit mass,

$$\mathbf{g} = -\nabla\Phi_g = -\frac{GM}{r^2}\hat{\mathbf{r}}.$$

These are properties of the mass M alone.

Then the potential energy of a second particle is $V = m\Phi_g$.