Part IB - Electromagnetism

Definitions

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Electromagnetism and Relativity
Review of Special Relativity; tensors and index notation. Lorentz force law. Electromagnetic tensor. Lorentz transformations of electric and magnetic fields. Currents and the conservation of charge. Maxwell equations in relativistic and non-relativistic forms. [5]

Electrostatics
Gauss's law. Application to spherically symmetric and cylindrically symmetric charge distributions. Point, line and surface charges. Electrostatic potentials; general charge distributions, dipoles. Electrostatic energy. Conductors. [3]

Magnetostatics
Magnetic fields due to steady currents. Ampre's law. Simple examples. Vector potentials and the Biot-Savart law for general current distributions. Magnetic dipoles. Lorentz force on current distributions and force between current-carrying wires. Ohm's law. [3]

Electrodynamics
Faraday's law of induction for fixed and moving circuits. Electromagnetic energy and Poynting vector. 4-vector potential, gauge transformations. Plane electromagnetic waves in vacuum, polarization. [5]

Contents

1 Introduction

1.1 Charge and Current

Definition (Charge density)**.** The *charge density* is the charge per unit volume. The total charge in a region V is

$$Q = \int_V \rho(\mathbf{x}, t)\ \mathrm{d}^3x$$

Definition (Current and current density)**.** For any surface S, the integral

$$I = \int_S \mathbf{J} \cdot d\mathbf{S}$$

counts the charge per unit time passing through S. I is the *current*, and $\mathbf{J}$ is the *charge density*, "current per unit area".

1.2 Forces and Fields

2 Electrostatics

2.1 Gauss' Law

Definition (Flux through surface)**.** The *flux* of $\mathbf{E}$ through the surface S is defined to be

$$\int_S \mathbf{E} \cdot \mathrm{d}\mathbf{S}.$$

2.2 Electrostatic potential

Definition (Electrostatic potential)**.** If $\mathbf{E} = -\nabla\phi$, then ϕ is the *electrostatic potential*.

2.2.1 Point charge

2.2.2 Dipole

Definition (Dipole)**.** A *dipole* consists of two point charges, $+Q$ and $-Q$ at $\mathbf{r} = 0$ and $\mathbf{r} = -\mathbf{d}$ respectively. By the principle of superposition,

$$\phi = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r} - \frac{Q}{|\mathbf{r}+\mathbf{d}|}\right).$$

Definition (Electric dipole moment)**.** We define the *electric dipole moment* is

$$\mathbf{p} = Q\mathbf{d}.$$

By convention, it points from -ve to +ve.

2.2.3 General charge distribution