

Part IA - Dynamics and Relativity

Definitions

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Basic concepts

Space and time, frames of reference, Galilean transformations. Newton's laws. Dimensional analysis. Examples of forces, including gravity, friction and Lorentz. [4]

Newtonian dynamics of a single particle

Equation of motion in Cartesian and plane polar coordinates. Work, conservative forces and potential energy, motion and the shape of the potential energy function; stable equilibria and small oscillations; effect of damping.

Angular velocity, angular momentum, torque.

Orbits: the $u(\theta)$ equation; escape velocity; Kepler's laws; stability of orbits; motion in a repulsive potential (Rutherford scattering). Rotating frames: centrifugal and coriolis forces. *Brief discussion of Foucault pendulum.* [8]

Newtonian dynamics of systems of particles

Momentum, angular momentum, energy. Motion relative to the centre of mass; the two body problem. Variable mass problems; the rocket equation. [2]

Rigid bodies

Moments of inertia, angular momentum and energy of a rigid body. Parallel axes theorem. Simple examples of motion involving both rotation and translation (e.g. rolling). [3]

Special relativity

The principle of relativity. Relativity and simultaneity. The invariant interval. Lorentz transformations in $(1+1)$ -dimensional spacetime. Time dilation and length contraction. The Minkowski metric for $(1+1)$ -dimensional spacetime. Lorentz transformations in $(3+1)$ dimensions. 4-vectors and Lorentz invariants. Proper time. 4-velocity and 4-momentum. Conservation of 4-momentum in particle decay. Collisions. The Newtonian limit. [7]

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1 Newtonian dynamics of particles

Definition (Particle). An *particle* is an object of insignificant size. It can be regarded as a point. It has a *mass* $m > 0$, and *electric charge* q .

Its position at time t is described by its *position vector*, $\mathbf{r}(t)$ or $\mathbf{x}(t)$ with respect to an origin O .

Definition (Frame of reference). A *frame of reference* is choice of coordinate axes for \mathbf{r} . The axes may be fixed, moving, or accelerating relative to another frame.

With a frame of reference, we can write \mathbf{r} in cartesian coordinates as (x, y, z)

Definition (Velocity). The *velocity* of the particle is

$$\mathbf{v} = \dot{\mathbf{r}} = \frac{d\mathbf{r}}{dt}.$$

and is tangent to the path or trajectory.

Definition (Acceleration). The *acceleration* of the particle is

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \frac{d^2\mathbf{r}}{dt^2}.$$

Definition (Momentum). The *momentum* of a particle is

$$\mathbf{p} = m\mathbf{v} = m\dot{\mathbf{r}}.$$

m is the *inertial mass* of the particle, and measures its reluctance to accelerate (c.f. Newton's Second Law)

1.1 Newton's laws of motion

Definition (Inertial frames). *Inertial frames* are frames of references in which the frames themselves are not accelerating. Newton's Laws only hold in inertial frames.

1.2 Galilean transformations

Definition (Galilean boost). A Galilean boost is a change in frame of reference by

$$\begin{aligned}\mathbf{r}' &= \mathbf{r} - \mathbf{v}t \\ t' &= t\end{aligned}$$

for a fixed, constant \mathbf{v} .

1.3 Newton's Second Law

2 Dimensional Analysis

2.1 Units

2.2 Scaling