

Part IA - Dynamics and Relativity

Theorems

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Basic concepts

Space and time, frames of reference, Galilean transformations. Newton's laws. Dimensional analysis. Examples of forces, including gravity, friction and Lorentz. [4]

Newtonian dynamics of a single particle

Equation of motion in Cartesian and plane polar coordinates. Work, conservative forces and potential energy, motion and the shape of the potential energy function; stable equilibria and small oscillations; effect of damping.

Angular velocity, angular momentum, torque.

Orbits: the $u(\theta)$ equation; escape velocity; Kepler's laws; stability of orbits; motion in a repulsive potential (Rutherford scattering). Rotating frames: centrifugal and coriolis forces. *Brief discussion of Foucault pendulum.* [8]

Newtonian dynamics of systems of particles

Momentum, angular momentum, energy. Motion relative to the centre of mass; the two body problem. Variable mass problems; the rocket equation. [2]

Rigid bodies

Moments of inertia, angular momentum and energy of a rigid body. Parallel axes theorem. Simple examples of motion involving both rotation and translation (e.g. rolling). [3]

Special relativity

The principle of relativity. Relativity and simultaneity. The invariant interval. Lorentz transformations in $(1+1)$ -dimensional spacetime. Time dilation and length contraction. The Minkowski metric for $(1+1)$ -dimensional spacetime. Lorentz transformations in $(3+1)$ dimensions. 4-vectors and Lorentz invariants. Proper time. 4-velocity and 4-momentum. Conservation of 4-momentum in particle decay. Collisions. The Newtonian limit. [7]

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1 Newtonian dynamics of particles

1.1 Newton's laws of motion

Law (Newton's First Law of Motion). A body remains at rest, or moves uniformly in a straight line, unless acted on by a force. (This is in fact Galileo's Law of Inertia)

Law (Newton's Second Law of Motion). The rate of change of momentum of a body is equal to the force acting on it (in both magnitude and direction).

Law (Newton's Third Law of Motion). To every action there is an equal and opposite reaction: the forces of two bodies on each other are equal and in opposite directions.

1.2 Galilean transformations

Law (Galilean relativity). The *principle of relativity* asserts that the laws of physics are the same in inertial frames.

1.3 Newton's Second Law

Law. The *equation of motion* for a particle subject to a force F is

$$\frac{d\mathbf{p}}{dt} = \mathbf{F},$$

where $\mathbf{p} = m\mathbf{v} = m\dot{\mathbf{r}}$ is the (linear) momentum of the particle. We say m is the (inertial) mass of the particle, which is a measure of its reluctance to accelerate.

2 Dimensional Analysis

2.1 Units

2.2 Scaling

3 Forces

3.1 Force and potential energy in one dimension

Proposition. Suppose the equation of a particle satisfies

$$m\ddot{x} = -\frac{dV}{dx}. \quad (*)$$

Then the total energy

$$E = T + V = \frac{1}{2}m\dot{x}^2 + V(x)$$

is conserved, i.e. $\dot{E} = 0$.

3.2 Motion in a potential