Part II - Logic and Set Theory
Theorems

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1 Propositional calculus

1.1 Semantic implication

Proposition.

(i) If v and v' are valuations with $v(p) = v'(p)$ for all $p \in P$, then $v = v'$.

(ii) For any function $W : P \to \{0, 1\}$, there is a valuation v such that $v(p) = w(p)$ for all $p \in L$, i.e. we can extend w to a full valuation.

This means "A valuation is determined by its values on P, and any values will do".

1.2 Syntactic implication

Proposition (Deduction theorem)**.** Let $S \subset L$ and $p, q \in L$. Then $S \vdash (p \Rightarrow q)$ if and only if $S \cup p \vdash q$.

"$\vdash$ behaves like the connective $\Rightarrow$ in the language"

Proposition (Soundness)**.** If $S \subset L$, $t \in L$, then if $S \vdash t$, then $S \models t$.