Part IA - Dynamics and Relativity

Definitions

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Lent 2015

Basic concepts
Space and time, frames of reference, Galilean transformations. Newton's laws. Dimensional analysis. Examples of forces, including gravity, friction and Lorentz. [4]

Newtonian dynamics of a single particle
Equation of motion in Cartesian and plane polar coordinates. Work, conservative forces and potential energy, motion and the shape of the potential energy function; stable equilibria and small oscillations; effect of damping.

Angular velocity, angular momentum, torque.

Orbits: the $u(\theta)$ equation; escape velocity; Kepler's laws; stability of orbits; motion in a repulsive potential (Rutherford scattering). Rotating frames: centrifugal and coriolis forces. *Brief discussion of Foucault pendulum.* [8]

Newtonian dynamics of systems of particles
Momentum, angular momentum, energy. Motion relative to the centre of mass; the two body problem. Variable mass problems; the rocket equation. [2]

Rigid bodies
Moments of inertia, angular momentum and energy of a rigid body. Parallel axes theorem. Simple examples of motion involving both rotation and translation (e.g. rolling). [3]

Special relativity
The principle of relativity. Relativity and simultaneity. The invariant interval. Lorentz transformations in $(1+1)$-dimensional spacetime. Time dilation and length contraction. The Minkowski metric for $(1+1)$-dimensional spacetime. Lorentz transformations in $(3+1)$ dimensions. 4-vectors and Lorentz invariants. Proper time. 4-velocity and 4-momentum. Conservation of 4-momentum in particle decay. Collisions. The Newtonian limit. [7]

Contents

1 Newtonian dynamics of particles

Definition (Particle)**.** An *particle* is an object of insignificant size. It can be regarded as a point. It has a *mass* $m > 0$, and *electric charge* q.

Its position at time t is described by its *position vector*, $\mathbf{r}(t)$ or $\mathbf{x}(t)$ with respect to an origin O.

Definition (Frame of reference)**.** A *frame of reference* is choice of coordinate axes for $\mathbf{r}$. The axes may be fixed, moving, or accelerating relative to another frame.

With a frame of reference, we can write $\mathbf{r}$ in cartesian coordinates as (x, y, z)

Definition (Velocity)**.** The *velocity* of the particle is

$$\mathbf{v} = \dot{\mathbf{r}} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}.$$

and is tangent to the path or trajectory.

Definition (Acceleration)**.** The *acceleration* of the particle is

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \frac{\mathrm{d}^2\mathbf{r}}{dt^2}.$$

Definition (Momentum)**.** The *momentum* of a particle is

$$\mathbf{p} = m\mathbf{v} = m\dot{\mathbf{r}}.$$

m is the *inertial mass* of the particle, and measures its reluctance to accelerate (c.f. Newton's Second Law)

1.1 Newton's laws of motion

Definition (Inertial frames)**.** *Inertial frames* are frames of references in which the frames themselves are not accelerating. Newton's Laws only hold in inertial frames.

1.2 Galilean transformations

Definition (Galilean boost)**.** A Galilean boost is a change in frame of reference by

$$\mathbf{r}' = \mathbf{r} - \mathbf{v}t$$
$$t' = t$$

for a fixed, constant $\mathbf{v}$.

1.3 Newton's Second Law

2 Dimensional Analysis

2.1 Units

2.2 Scaling

3 Forces

3.1 Force and potential energy in one dimension

Definition (Potential energy)**.** Given a force field $F = F(x)$, we define the *potential energy* to be a function $V(x)$ such that

$$F = -\frac{\mathrm{d}V}{\mathrm{d}x}.$$

or

$$V = -\int F \,\mathrm{d}x.$$

V includes an arbitrary additive constant.

3.2 Motion in a potential