

Part IB - Electromagnetism

Theorems with Proof

Lectured by David Tong

Lent 2015

Electromagnetism and Relativity

Review of Special Relativity; tensors and index notation. Lorentz force law. Electromagnetic tensor. Lorentz transformations of electric and magnetic fields. Currents and the conservation of charge. Maxwell equations in relativistic and non-relativistic forms. [5]

Electrostatics

Gauss's law. Application to spherically symmetric and cylindrically symmetric charge distributions. Point, line and surface charges. Electrostatic potentials; general charge distributions, dipoles. Electrostatic energy. Conductors. [3]

Magnetostatics

Magnetic fields due to steady currents. Ampere's law. Simple examples. Vector potentials and the Biot-Savart law for general current distributions. Magnetic dipoles. Lorentz force on current distributions and force between current-carrying wires. Ohm's law. [3]

Electrodynamics

Faraday's law of induction for fixed and moving circuits. Electromagnetic energy and Poynting vector. 4-vector potential, gauge transformations. Plane electromagnetic waves in vacuum, polarization. [5]

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1 Introduction

1.1 Charge and Current

Law (Continuity equation).

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0$$

1.2 Forces and Fields

Law (Lorentz force law).

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Law (Maxwell's Equations).

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{E}}{\partial t} = 0$$

$$\nabla \times \mathbf{B} - \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{j}$$

where

– $\varepsilon_0 = 8.85 \times 10^{-12} \text{ m}^{-3} \text{ kg}^{-1} \text{ s}^2 \text{ C}^2$ is the electric constant

– $\mu_0 = 4\pi \times 10^{-6} \text{ m kg C}^{-2}$ is the magnetic constant

2 Electrostatics

2.1 Gauss' Law

Law (Gauss' law).

$$\int_S \mathbf{E} \cdot d\mathbf{S} = \frac{Q}{\varepsilon_0},$$

where Q is the total charge inside V .

2.2 Electrostatic potential

2.2.1 Point charge

2.2.2 Dipole

2.2.3 General charge distribution