A Fast Algorithm for Long Code Direct Acquisition

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***Abstract* — In GPS (Global Position System), direct acquisition of the GPS P(Y)-code is a two-dimension lengthy searching process in time and frequency dimension. CCPAZP-FFT (Circular Correlation by Partition And Zero Padding) uses FFT to achieve parallel correlation and frequency searching. In this way acquisition can be achieved very fast as it transfers two-dimensional search into parallel one-dimensional search. However, correlation points are still too much and Computation is heavy. This paper proposes an improvement algorithm of combination of CCPAZP-FFT and XFAST. This way Compares to CCPAZP-FFT is reduced by two thirds of the computation in order to achieve rapid long code acquisition.**

***Index Terms* — GPS, P-code, long code direct acquisition, XFAST, FFT.**

I. INTRODUCTION

GPS uses 2 ranging code: C/A code and P(Y)-code. C/A code and P-code belongs to the pseudo-random code, C/A code rate is 1.023M bit / s, P code rate of 10.23M bit /s. C/A code used as coarse ranging, while the P-code used for precision ranging.

P-code acquisition is usually use the acquisition and tracking to C/A code and demodulates TLM and HOW words in a sub-frame of navigation message, according to the synchronization information auxiliary acquire P-code. But the C/A code length is relatively short, the code rate is low, vulnerable enemy jamming and deception. in strong jamming and deception of war environment, auxiliary acquisition is difficult to achieve. P(Y)-code has better anti-interference and anti-spoofing performance than C/A code.

1999 C.Young proposed P-code direct acquisition algorithm: XFAST (Extend Replica Folding Acquisition Search Technique)[1][2]. It reduces the amount of computation of direct acquisition, but introduces noise during the superposition process. 2008 Y.Wei proposed CCPAZP-FFT (Circular Correlation by Partition and Zero Padding)[3]. This algorithm is based on the storage and zero padding segmentation correlation. This algorithm converts two dimension searches of time and frequency to one dimension search to improve the efficiency of the acquisition. But it's time-consuming in the uncertain code segment by piecewise searching.

This paper proposes an improvement algorithm of combination of CCPAZP-FFT and XFAST. This algorithm segments the total uncertain interval of every three superimposed as a mother code. Then after FFT and correlate with the segmentation receiving code which is after FFT too. After determining the code phase of mother code segment, search the mother code segment again. At this point, the code phase and frequency searches are complete. This algorithm takes advantage of XFAST to reduce the computation by 2/3, greatly reducing the acquisition time.

II. THE PRINCIPLE OF DIRECT ACQUISITION

GPS receiver received RF signal, after the RF front-end analog signal processing output IF $r(t)$. $r(t)$ by A/D samples into a digital signal, and then down-convert to the digital intermediate frequency signal into a baseband digital signal, and reduce to 4 times code rate(40.92MHz) through integral clear circuit. Superimpose the two adjacent chips to 2 times the code rate. The received digital baseband signal $r(n)$ is:

$$r(n)=\sqrt{2E_p/T_p}\,d(n-\tau)p(n-\tau)e^{j(\pi f_d T_p n+\theta)}+N(n) \quad (1)$$

Wherein the sampling time is $T_p/2$; E_p is the energy of P-code; T_p is the period of P-code; $p(n)$ is a period of 7 days of the P-code sequence; $d(n)$ is the modulated data information of 20ms period T_d ; τ is transmission delay; θ is initial phase; $N(n)$ is Noise superimposed on the received signal; f_d is Doppler frequency deviation.

The direct acquisition structure of GPS receiver as shown in Figure 1.Sent Locally generated P-code sequence $p(n)$ and receiving digital base-band signal $r(n)$ at the same rate (20.46MHz) into the direct acquisition module, output transmission delay τ and frequency deviation f_d .

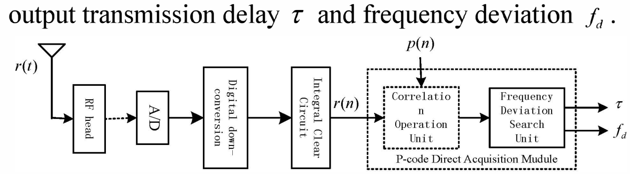

Figure 1 Direct Acquisition Structure of GPS Receiver

III. P-CODE DIRECT ACQUISITION MODULE

A. Correlation Operation Unit

Double sample the local P-code (time uncertain interval of ± 1s), the uncertain chips number is 4.092×10^7. Each bit of data code corresponds to the total number of phase is

$M' = 2T_d / T_p = 409200$. Correlate the uncertain interval of local with M points. The correlation operation output SNR_o and digital baseband signal SNR_{in} s follow the formula:

$$SNR_o = SNR_{in} + 10\log_{10} M \qquad (2)$$

GPS signal is input SNR minimum 32dB. To make SNR_o greater than 10dB, Seen by the formula (2) that $10\log_{10} M$ >42dB. If M is too large, it will increase the correlation time and can't achieve quick acquisition. Considering the above two conditions, segment the local phase with 1/10 period of data code ($0.1T_d$ =2ms). Corresponding to the total number of the search phases is : $M = 0.2T_d / T_p = 40920$.

Assuming a known starting point for each data cycle is aligned with a bit synchronization signal. The local signal phase every 10 above 40920 points will align a bit synchronization signal. Aligned with local signal to produce a set of interval segmentation flag signal mark for 40919 phase signal with the Sec, and every 10 Sec and a segmentation signal overlap (data code changes). Receiver storage local uncertain code segments aligned with P-code segments flag signal Sec. Due to uncertain interval corresponding to the starting point is not necessarily aligned with the Sec signal, worst case, Sec in uncertain interval comes before point $M-1$, it must start from that point backwards search the Sec. The first Sec signals corresponding to the local phase as the start of uncertain code. A segment contains 40920 points, the last one contains the last point of uncertain interval. So it's need to storage 1003 segments. Uncertain code segment corresponding to the phase shown in Figure 2. Add 2 segments to uncertain code, the total number of uncertain code is 1005. Every 3 segments superimpose to generate 335 segments mother code.

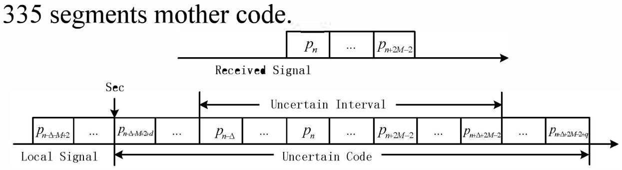

Figure 2 Uncertain Code

Receive real-time digital signal $r(n)$, get point A sample values stored in memory 1, while get the uncertain code correspond to local signal $p(n)$ stored in memory 2. Under the control of the control logic, get $p(n)$ and $r(n)$ from memory and sent to correlation operation unit. The structure of correlation operation unit shown in Figure 3:

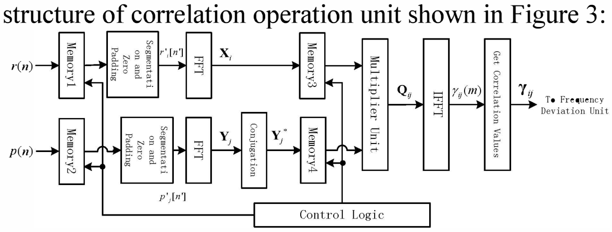

Figure 3 Structure of Correlation Operation Unit

Control logic control phase search in the entire process. Set up 3 counters i, j, k to achieve control of the phase search process. The logic diagram shown in Figure 4:

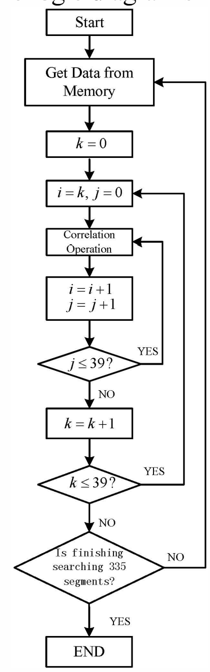

Figure 4 Control Logic Diagram

Under the control of the control logic, get $2M-1$ points receiver signal $r(n)$ and the first segment of uncertain code $p(n)$ to send to correlation operation unit. The first segment $p(n)$ of uncertain code contains 40920 points, divided $p(n)$ into 40 segments on average at length $W = 1023$. $L = 40920/1023 = 40$. The first point of received $2M-1$ points signal $r(n)$ as the starting point of every W phase points to segment. The length of every segment is $2W-1 = 2045$, a total of $2L-1 = 79$ segments. After segmentation, the i-th of received sequence $r_{i(2W-1)}[n']$ and the j-th of local sequence $p_{jW}[n']$ ($i = 0,1,\cdots 78$, $j = 0,1,\cdots 39$) can be expressed as:

$$r_{i(2W-1)}[m'] = r[iW + m'] \qquad m' = 0,\ 1,\ \cdots,\ 2W-2 \qquad (3)$$

$$p_{jW}[n'] = p[jW + n'] \qquad n' = 0,\ 1,\ \cdots,\ W-1 \qquad (4)$$

Zero padding $r_{i(2W-1)}[m']$ and $p_{jW}[n']$, both are complementary to the N points ($N > 2W$ and $N = 2^s$, s is an integer), $W = 1023$, $N = 2048$. $r'_i[m']$ and $p'_j[n']$ resulting from the zero padding:

$$r'_i[n'] = \begin{cases} r[iW + m'] & m' = 0,\ 1,\ \cdots,\ 2W-2 \\ 0 & m' = 2W-1,\ 2W,\ \cdots,\ N-1 \end{cases} \qquad (5)$$

$$p'_j[n'] = \begin{cases} p[jW + n'] & n' = 0,\ 1,\ \cdots,\ W-1 \\ 0 & n' = W,\ W+1,\ \cdots,\ N-1 \end{cases} \qquad (6)$$

Set $\mathbf{X}_i$ and $\mathbf{Y}_j$ express 2048 points FFT results of 2 column vectors $(r'_i[0], r'_i[1], \cdots, r'_i[2047])^T$ and $(r'_i[0], r'_i[1], \cdots, r'_i[2047])^T$. As shown in Figure 3, send $\mathbf{X}_i$ to memory 3 and send $\mathbf{Y}_j$ to memory 4 after conjugation. The counter i, j, k is set to zero. Get $\mathbf{X}_i$ and $\mathbf{Y}_j$ from memory and send them to Multiplier. $i = k, k+1, \ldots, k+39$, $j = 0, 1, \ldots 39$, $k = 0, 1, \ldots, 39$. The change of values as shown in Figure 4. Get vector $\mathbf{Q}_{ij}$.

Do N point IFFT operations on $\mathbf{Q}_{ij}$. Set first W values of IFFT results corresponding to each segment of $\mathbf{Q}_{ij}$ is $\gamma_{ij}(m)$:

$$\begin{aligned}\gamma_{ij}(m) &= R_W[m]\sum_{n'=0}^{N-1} r'_i[m']p'_j[n'-m] \\ &= R_W[m]\sum_{n'=0}^{2W-2} r[iW+m']p[jW+n'-m] \\ &= R_W[m]\sum_{n'=0}^{2W-2} r[iW+n']p[jW+n'-m]\end{aligned} \tag{7}$$

$R_W[m]$ is a gate function of length W:

$$R_W[m] = \begin{cases} 1 & m = 0,\ 1,\ \cdots,\ W-1 \\ 0 & \text{else} \end{cases} \tag{8}$$

Due to formula (7) we know that $\gamma_{ij}(m)$ is the results of received signal $r(n)$ and local signal $p(n)$ W points correlation[4]. Define the vector $\boldsymbol{\gamma}_{ij} = (\gamma_{ij}(0), \gamma_{ij}(1), \cdots, \gamma_{ij}(W-1))^T$. As shown in figure 3, when k is constant, change i, j, $i = k, k+1, \ldots, k+39$, $j = 0, 1, \ldots 39$, get L groups of correlation values. Then Generate a merged by line $W \times L$ matrix:

$$\mathbf{R}_k = \left(\boldsymbol{\gamma}_{k,0}, \boldsymbol{\gamma}_{(k+1),1}, \cdots, \boldsymbol{\gamma}_{(k+L-1),(L-1)}\right) = \begin{pmatrix} \gamma_{k,0}(0), & \gamma_{k+1,1}(0), & \cdots, & \gamma_{k+L-1,L-1}(0) \\ \gamma_{k,0}(1), & \gamma_{k+1,1}(1), & \cdots, & \gamma_{k+L-1,L-1}(1) \\ \vdots & \vdots & \cdots, & \vdots \\ \gamma_{k,0}(W-1), & \gamma_{k+1,1}(W-1), & \cdots, & \gamma_{k+L-1,L-1}(W-1) \end{pmatrix}_{W \times L} \tag{9}$$

Each line of the matrix corresponding to the same search phase, each column represents a correlation vector $\boldsymbol{\gamma}_{ij}$. L correlation values of a line correspond the same search phase.

k values vary from 0-39, as shown in figure 4. Generate 40 groups $W \times L$ correlation matrix $\mathbf{R}_0, \mathbf{R}_1, \cdots, \mathbf{R}_{39}$. Generate a merged by column $M \times L$ matrix $\mathbf{R}$ is expressed as formula:

$$\mathbf{R} = (\mathbf{R}_0{}^T, \mathbf{R}_1{}^T, \cdots, \mathbf{R}_{39}{}^T)^T = \begin{pmatrix} \gamma_{0,0}(0) & \gamma_{1,1}(0) & \cdots & \gamma_{39,39}(0) \\ \gamma_{0,0}(1) & \gamma_{1,1}(1) & \cdots & \gamma_{39,39}(1) \\ \vdots & \vdots & \cdots & \vdots \\ \gamma_{0,0}(W-1) & \gamma_{1,1}(W-1) & \cdots & \gamma_{39,39}(W-1) \\ \gamma_{1,0}(0) & \gamma_{2,1}(0) & \cdots & \gamma_{40,39}(0) \\ \vdots & \vdots & \cdots & \vdots \\ \gamma_{1,0}(W-1) & \gamma_{2,1}(W-1) & \cdots & \gamma_{40,39}(W-1) \\ \vdots & \vdots & \cdots & \vdots \\ \gamma_{39,0}(0) & \gamma_{40,1}(0) & \cdots & \gamma_{78,39}(0) \\ \vdots & \vdots & \cdots & \vdots \\ \gamma_{39,0}(W-1) & \gamma_{40,1}(W-1) & \cdots & \gamma_{78,39}(W-1) \end{pmatrix}_{M \times L} \tag{10}$$

M=40920, $\mathbf{R}$ represents the 40920 search phase correlation results of local signal $p(n)$ and received signal $r(n)$. Get $\mathbf{R}$ stored in memory 5. The control logic set counter i, j, k zero at this time.

Under the control of the control logic, get the 40920 points of next segment uncertain code of local signal and send to system to search. Repeat the process until complete local search uncertain code section 335.

B. Frequency Deviation Search Unit

There is frequency deviation between received signal $r(n)$ and local signal $p(n)$. Get values by line from matrix $\mathbf{R}$ and send to frequency deviation search unit. The structure of frequency deviation search unit is shown as figure 5:

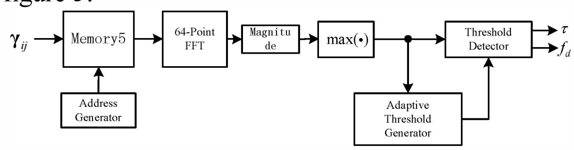

Figure 5 The Structure of Frequency Deviation Search Unit

Get the L points correlation results corresponding to the same search phase from memory 5. Assuming the received signal and the local signal phase has been aligned. At this time the L correlation values are $\gamma(l),\ l = 0,\ 1,\ \cdots,\ L-1$:

$$\begin{aligned}\gamma(l) &= \sum_{n=lW}^{lW+W-1} r(n)p(n) \\ &= \sum_{n=lW}^{lW+W-1} [\sqrt{E_p/T_p}\, d(n)p(n)p(n)e^{j(\pi f_d T_p n+\theta)} + N(n)p(n)] \\ &= \sum_{n=lW}^{lW+W-1} [\sqrt{E_p/T_p}\, d(n)e^{j(\pi f_d T_p n+\theta)} + N(n)p(n)] \qquad l = 0,\ 1,\ \cdots,\ L-1\end{aligned} \tag{11}$$

The data code $d(n)$ corresponding to the 40920 points of every segment uncertain code remains unchanged. Therefore, polarity reversal does not occur with $d(n)$ of $\gamma(l),\ l = 0,\ 1,\ \cdots,\ L-1$::

$$\begin{aligned}\gamma(l) &= d\sqrt{E_p/T_p}\sum_{n=lW}^{lW+W-1} e^{j(\pi f_d T_p n+\theta)} + \sum_{n=lW}^{lW+W-1} N(n)p(n) \\ &= d\sqrt{E_p/T_p}\left[e^{j\frac{\pi}{2}f_d T_p(W-1)}\frac{\sin(\frac{\pi}{2}f_d T_p W)}{\sin(\frac{\pi}{2}f_d T_p)}\right]e^{j(\pi f_d T_p lW+\theta)} + I_l \qquad l = 0,\ 1,\ \cdots,\ L-1\end{aligned} \tag{12}$$

I_l represents interference signal output by noise generated:

$$I_l = \sum_{n=lW}^{lW+W-1} N(n)p(n) \tag{13}$$

Same phase corresponds to L correlation values. That is it corresponds L correlation values of a line of the correlation matrix. Zero padding the L correlation values to K ($K > L$ and $K = 2^t$, t is an integer) points. So get T_k:

$$\begin{aligned}T_k &= \sum_{l=0}^{K-1}\gamma(l)e^{-j\frac{2\pi}{K}lk} \\ &= \sum_{l=0}^{L-1} d\sqrt{E_p/T_p}\left[e^{j\frac{\pi}{2}f_d T_p(W-1)}\frac{\sin(\frac{\pi}{2}f_d T_p W)}{\sin(\frac{\pi}{2}f_d T_p)}\right]e^{j(\pi f_d T_p lW+\theta)} \bullet e^{-j\frac{2\pi}{K}lk} + \sum_{l=0}^{K-1} I_l e^{-j\frac{2\pi}{K}lk} \\ &= d\sqrt{E_p/T_p}\, e^{j(\theta+\frac{\pi}{2}f_d T_p(W-1)+\frac{\pi}{2}f_d T_p W(L-1)+\frac{\pi k}{K}(1-L))}\frac{\sin(\frac{\pi}{2}f_d T_p W)}{\sin(\frac{\pi}{2}f_d T_p)}\frac{\sin((\frac{\pi}{2}f_d T_p W-\frac{\pi k}{K})L)}{\sin(\frac{\pi}{2}f_d T_p W-\frac{\pi k}{K})} + I_k \\ &= G_k + I_k \qquad k = 0,\ 1,\ \cdots,\ K-1\end{aligned} \tag{14}$$

Do not consider effects of noise, get the magnitude of G_k then send to threshold detector. If the maximum of G_k -- $|G_k|_{\max}$ is greater than the threshold, expand the corresponding mother code to 3 sub-segments. Use the same method to search them, get:

$$|G_k| = \left| d\sqrt{E_p/T_p} e^{j(\theta + \frac{\pi}{2} f_d T_p (W-1) + \frac{\pi}{2} f_d T_p W (L-1) + \frac{\pi k}{K}(1-L))} \frac{\sin(\frac{\pi}{2} f_d T_p W)}{\sin(\frac{\pi}{2} f_d T_p)} \frac{\sin(\frac{\pi}{2} f_d T_p W - \frac{\pi k}{K})L}{\sin(\frac{\pi}{2} f_d T_p W - \frac{\pi k}{K})} \right| \quad (15)$$

$$= \sqrt{E_p/T_p} \left| \frac{\sin(\frac{\pi}{2} f_d T_p W)}{\sin(\frac{\pi}{2} f_d T_p)} \right| \left| \frac{\sin(\frac{\pi}{2} f_d T_p W - \frac{\pi k}{K})L}{\sin(\frac{\pi}{2} f_d T_p W - \frac{\pi k}{K})} \right|$$

Define

$$\varepsilon_1(f_d) = \left| \frac{\sin(\frac{\pi}{2} f_d T_p W)}{W \sin(\frac{\pi}{2} f_d T_p)} \right| \quad (16)$$

$$\varepsilon_2(f_d, k) = \left| \frac{\sin(\frac{\pi}{2} f_d T_p W - \frac{\pi k}{K})L}{L \sin(\frac{\pi}{2} f_d T_p W - \frac{\pi k}{K})} \right| \quad (17)$$

Formula (15) can be expressed as:

$$|G_k| = LW\sqrt{E_p/T_p}\,\varepsilon_1(f_d)\varepsilon_2(f_d, k) \quad (18)$$

$\varepsilon_1(f_d)$ is only related to the carrier frequency deviation. Fix f_d, if there is $|G_k|_{\max}$, k must follow the formula:

$$k = \left\lfloor \frac{1}{2} K f_d T_p W \right\rfloor \quad (19)$$

$\lfloor \cdot \rfloor$ represents the rounding operation.

Send $|G_k|_{\max}$ to threshold detector, if $|G_k|_{\max}$ is greater than threshold, assume local phase p_{aq} corresponds to $|G_k|_{\max}$ and the time deviation is:

$$\tau = [p_{aq} - (\Delta + M - 2)]\frac{T_p}{2} \quad (20)$$

At this time frequency deviation f_d corresponding to k can be expressed as:

$$f_d = \frac{2k}{KT_p W} \quad (21)$$

Now we get the transmission delay τ and frequency deviation f_d, long code direct acquisition is completed.

IV. THE SIMULATION RESULTS

In this paper, all simulation results are based on Monte-Carlo simulation. In the simulation results, channel is AWGN channel , SNR=-20dB , CCPAZP-FFT simulation curve is shown in figure 6 , CCPAZP-FFT combines with XFAST simulation curve is shown in figure 7. The correlation peak r between received signal and local signal changes as delay time τ (μs). We can see when the local signal and the received signal is aligned on a time there is a significant correlation peak.

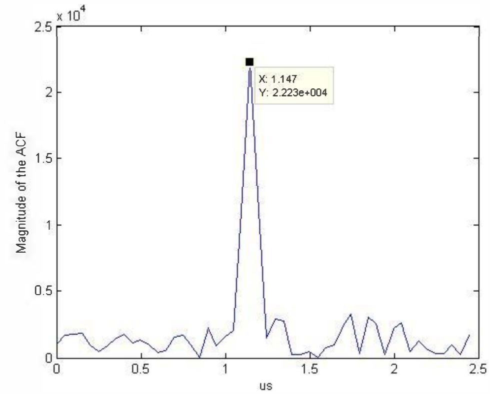

Figure 6 Correlation Peak Curve of CCPAZP-FFT

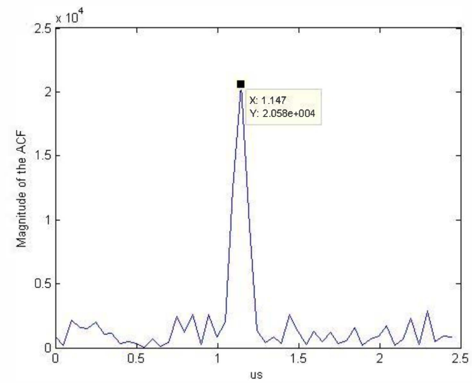

Figure 7 Correlation Peak Curve of CCPAZP-FFT combines with XFAST

VII. CONCLUSION

Through the above analysis, we can see that due to the introducing noise of XFAST chip superposition process, the correlation peak amplitude decreases. It has certain influence on acquisition probability, but the loss is not large. Computation cost to 1/3 of the original, the new algorithm reduces the computation greatly.

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