Frequency Hopping Acquisition Based on Variable Forgetting Factor Recursive Least Squares Algorithm

Zengmao Chen, Xianwu Zhao *, Li Lu, Rongchen Sun and Zhiguo Sun
College of Information and Communication Engineering, Harbin Engineering University
Harbin, China
Email: zhaoxianwu2019@hrbeu.edu.cn

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***Abstract*—The main drawback of frequency hopping pattern synchronization capture algorithms is time-consuming. To address this problem, variable forgetting factor recursive least square (VFFRLS) frequency hopping pattern acquisition algorithm is proposed in this paper. First, the coefficients of the adaptive spectrum estimation model are calculated iteratively used the VFFRLS algorithm. Then, instantaneous frequency of the frequency hopping signal is estimated, and the result is converted into a binary sequence. The most significant bit of the binary sequence is loaded into the linear feedback shift register. Finally, the correct frequency point could be obtained. The theoretical expressions of bit error rate, capture probability and average capture time are derived under the Additive White Gaussian Noise environment. Moreover, the capture performance is verified through numerical simulation.**

Index Terms*—frequency hopping, acquisition, spectrum estimation, variable forgetting factor

I. Introduction

Frequency hopping communication system is favored by the military due to its excellent anti-interference [1], concealment and anti-interception capabilities, and is extensively used in various military shortwave radio stations to ensure effective and reliable communication in severe channel environment [2]. Besides, frequency hopping technology has also been applied in a large range of commercied communication systems due to its excellent performance to combat frequency selective fading. The hopped frequency pattern synchronization is crucial to the performance of the hopped frequency communication system [3]. The hopped frequency pattern synchronization process is usually separated into two parts: acquisition and tracking. Acquisition is the first and the key step of frequency hopping pattern synchronization. The purpose of acquisition is to locally generate frequency hopping pattern same as that of the received frequency hopping pattern have the same frequency in the same time period. The size of time period is usually greater than half of single frequency hopping period. To further reduce the error of the frequency hopping patterns of the two communication parties, tracking is performed on the basis of capture, until almost completely aligned.

As is well known, a serial searching capture technology was proposed, which has strong anti-interference capability but is very time-consuming. A parallel searching capture technology was proposed in [4], which is a derivative of the serial searching capture algorithm [5]. It changes the single-channel processing to multi-channel simultaneous processing [6]. Parallel searching capture technology significantly reduces the capture time, but consumes too much hardware in practical applications, resulting in waste of hardware resources. The method of sync pilot is described in [7]. The time of day (TOD) is contained in sync signal. The TOD is consisted of the key number, correlation code, frequency hopping pattern, net number and time information. The TOD pilot signal is used by the frequency hopping synchronous receiver to correct the local clock, so as to accomplish frequency hopping synchronization acquisition through the method of energy detection. The means of TOD pilot signal is always employed by the frequency hopping network make-up.

In [8], a traditional frequency estimation algorithm based on eigen-decomposition was presented. This acquisition method based on maximum likelihood estimation uses two-hop signal model to estimate frequency. This algorithm can exactly estimate the frequency of the frequency hopping signal in the Additive White Gaussian Noise (AWGN) environment [9], but the frequency performance estimation is poor under Rayleigh fading channel [10]. The Least Mean Square (LMS) algorithm was proposed in [11] with strong tracking ability, but its convergence speed is slow, leading to longer capture time. The error function obtained the past input signal is used in the LMS algorithm [12], and minimum mean square error criterion is adopted without emphasis. The LMS algorithm has poor adaptability to non-stationary signals as a result of the fact that the current moment error is barely used [13]. So, the performance of LMS algorithm is poor in low signal-to-noise (SNR) ratio.

For addressing the above problems, frequency hopping pattern synchronization capture technology based on variable forgetting factor recursive least square (VFFRLS) algorithm is proposed. VFFRLS algorithm utilizes the error at not only the current moment but also the former moment by adding a forgetting factor [14], that is, the weighted sum of the squared errors of the input signal is the smallest at each moment. Therefore, the VFFRLS algorithm is more adaptable to non-stationary signals and has a fast convergence speed. VFFRLS algorithm not only overcomes the shortcomings of tracking speed and parameter imbalance, but also overcomes the problem of gain vector tending to zero. This paper also purposes

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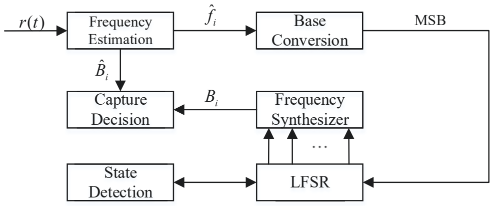

Fig. 1: Schematic diagram of frequency hopping acquisition based on adaptive spectrum estimation.

an algorithm for instantaneous frequency estimation using adaptive spectrum estimation. Moreover, theoretical derivation and simulation verification are both carried out under AWGN channel condition for the proposed algorithm.

II. System Model

The working principle of frequency hopping acquisition based on adaptive spectrum estimation is depicted in Fig. 1. In Fig. 1, the length of linear feedback shift register (LFSR) is N. The combination of binary states of the LFSR can synthesize 2^N frequency hopping point. After the received frequency hopping signal $x(t)$ is down-converted and sampled, the discrete value $x(n)$ can be obtained, which is fed into an adaptive filter, with the auto-regressive spectral estimation acquisition technique (ASEAT) to estimation frequency point. Finally, the frequency estimation value $\hat{f}_i$ is obtained through the frequency estimation algorithm, which corresponds to the decimal state estimation value $\hat{B}_i$ of the LFSR at a certain time. Suppose B_i is the true value of LFSR decimal status, we have $0 \leqslant B_i \leqslant 2^N - 1$, $0 \leqslant \hat{B}_i \leqslant 2^N - 1$. Let $\hat{b}_i$ denote the most significant bit (MSB) of the binary value of $\hat{B}_i$. The MSB of $\hat{B}_i$ can be written as

$$\hat{b}_i = \text{int}\left[\frac{\hat{\mathrm{B}}_i}{2^{\mathrm{N}} - 1}\right] \tag{1}$$

for all i, where $\hat{b}_i$ is the estimation of b_i and int[x] means the integer part of x. Since the state of the LFSR at the next moment depends on that of the current moment, in the absence of frequency estimation error, the estimated value is equal to the true value, i.e, $\hat{B}_i = B_i$. After the N MSBs of $\{\hat{B}_1, \hat{B}_2, \ldots, \hat{B}_N\}$ are input, the state value of LFSR is utilized for initial acquisition. Assuming that the time per hop is T_c, the acquisition delay is NT_c when no error in the estimation for the N estimations $\{\hat{B}_1, \hat{B}_2, \ldots, \hat{B}_N\}$. Then, when the individual MSB's are sequentially input into the LFSR, the content of the local LFSR is consistent with the content of the sending end of LFSR, and this state is used as the initial state of the local LFSR, and subsequent generated frequencies are consistent with the sending end, namely, frequency hopping capture is completed.

When exists a frequency estimation error, the capture will fail after NT_c if the estimated $\hat{b}_i$ is incorrect. To prevent acquisition failure, the first generated state of LFSR employs the first $(N-1)$ estimation. Specifically, the first $(N-1)$ estimates of MSB are generated after $(N-1)T_c$, within which $(N-1)$ hopping frequencies are received. The original Nth and previous $(N-1)$ estimates together constitute the state estimate $\hat{B}_1$. After that, B_1 is compared with $\hat{B}_1$ to determine whether to turn to tracking. If $B_1 \neq \hat{B}_1$, the acquisition procedure continues, and $\hat{B}_1$ is served to generate the state B_2, which is compared with $\hat{B}_2$ after the $(N+1)$th hop. The acquisition procedure continues until $B_i = \hat{B}_i$, for $i = 1, 2, \ldots, N$ which is followed by the tracking process. During the acquisition process, the generated state from LFSR is compared with the estimated state. If the estimated state is not equal to the generated state, the LFSR shifts 1 bit out. Then, repeat the above acquisition process.

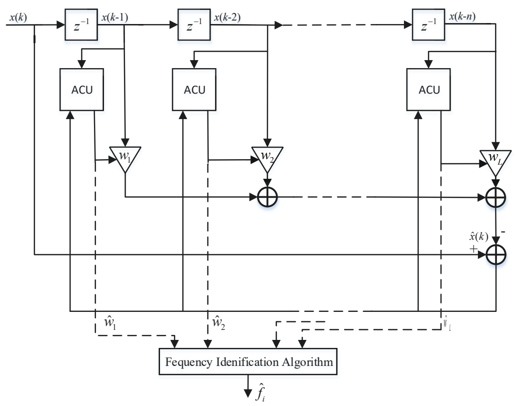

Fig. 2: Block diagram of adaptive spectrum estimation.

III. ACQUISITION SYSTEM

A. Model of ASEAT

The block diagram of adaptive spectral estimation is depicted in Fig. 2, where $x(n)$ stands for the discrete signal of the received frequency hopping signal after analog-to-digital conversion. And the linear prediction of the receipt signal at the kth moment is defined as

$$\hat{x}(k) = \sum_{l=1}^{L} w_l x(k-l) \quad k = 1, 2, \ldots, n \tag{2}$$

In the Eq.(2), L denotes the order of the linear predictor, n is sampling number in the hopping period, $x(k)$ is the samples of the hopped frequency signal $x(t)$ and $\hat{x}(k)$ is the predicted values of the hopped frequency signal $x(t)$ at kT_s. w_l denotes the coefficient of the lth linear prediction which is adjusted by adaptive correction unit (ACU). Through the error between estimated value $\hat{x}(k)$ and input value $x(k)$, the coefficient of the linear prediction iteratively update to estimate adaptively the signal spectrum. The adaptive spectral estimation can be described by the following equation

$$\hat{S}(f) = \frac{G^2}{\left|1 - \sum\limits_{l=1}^{L} w_l \exp(-j2\pi l f)\right|^2} \tag{3}$$

where $G^2 = E[(x(k) - \hat{x}(k))^2]$ is estimated mean-square error. The optimal factors w_l, for $l = 1, 2, \ldots, L$, can be obtained from the VFFRLS algorithm. Consequently, the frequency point is estimated from the pole model of Eq.(3) which is maximize $\hat{S}(f)$ in the acquisition process with the ASEAT [15].

B. Acquisition algorithm

Based on the VFFRLS algorithm with faster tracking capability while taking into account the smaller parameter estimation error, the error of estimation is as follows

$$e(k) = x(k) - \hat{x}(k) \tag{4}$$

In the Eq.(4), $e(k)$ is the estimation error between the input value $x(k)$ and the predicted value $\hat{x}(k)$ at the kth moment.

$$\mathbf{w}(k) = \mathbf{w}(k-1) + \mathbf{I}(k)e(k) \tag{5}$$

In the Eq.(5), $\mathbf{w}(k)$ is the update of tapped power vector at the kth moment.The prediction coefficient $\mathbf{w}(k)$ is updated accordingly, when the adaptive filter enters a new data. where the $\mathbf{I}(k)$ is the gain vector which is defined as

$$\mathbf{I}(k) = \frac{\mathbf{P}(k-1)\mathbf{u}(k)}{\lambda(k) + \mathbf{u}(k)^{\mathrm{T}}\mathbf{P}(k-1)\mathbf{u}(k)} \tag{6}$$

here $\mathbf{u}(k)$ is frequency hopping vector loaded into the adaptive filter at moment k, and where $\mathbf{P}(k)$ is

$$\mathbf{P}(k) = \frac{1}{\lambda(k)}[\mathbf{P}(k-1) - \mathbf{I}(k)\mathbf{u}(k)^{\mathrm{T}}\mathbf{P}(k-1)] \tag{7}$$

The variable forgetting factor is controlled by Eq.(8).

$$\lambda(k) = \lambda_{min} + (1-\lambda_{min})2^{[\mu e^2(k)]} \tag{8}$$

here μ is the stepping factor, $\lambda(k)$ denotes variable forgetting factor, which may changes in each iteration. Meanwhile, $[0.8, 1]$ is the range of values of λ. The smaller the forgetting factor λ is, the greater the tracking ability of the system is. Likewise, the larger the forgetting factor λ is, the weaker the tracking ability of the system is, but system is not sensitive to noise, estimation error will be small after convergence.

IV. Performance Analysis

The state estimation value of LFSR can be defined as the true value of the decimal state plus the additive white noise. It is deducted that the probability of estimation error of MSB and the mean capture time by analyzing the relationship between the generated state value and the estimated state value of the LFSR.

$$\hat{B}_i = B_i + W_i \quad i = 1, 2, \ldots, N \tag{9}$$

In the Eq.(9), the variance of W_i represents the mean-square estimation error, which is assumed as a zero-mean random process. The probability density function of W_i determines the probability of MSB errors

P_{e1} and P_{e2} represent two kinds of error probabilities respectively [16]. When $2^{n-1} \leq \hat{B}_i \leq 2^n - 1$, $1 \leq B_i \leq 2^{n-1} - 1$.

$$P_{e1} = P(\hat{b_i} = 1, b_i = 0) \tag{10}$$

and when $(1 \leq \hat{B}_i \leq 2^{n-1} - 1, 2^{n-1} \leq B_i \leq 2^n - 1$

$$P_{e2} = P(\hat{b_i} = 0, b_i = 1) \tag{11}$$

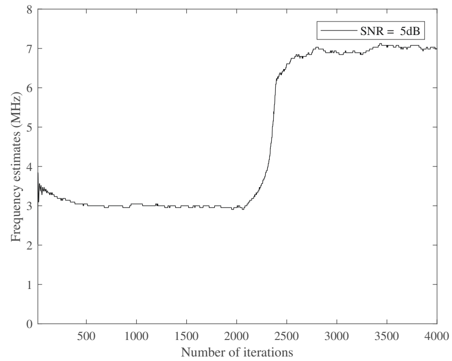

Fig. 3: convergence curve at different frequency.

under fixed B_i, when $1 \leq B_i \leq 2^{n-1} - 1$, the above error probabilities can be expressed as

$$P_{e1} = \int_{2^{n-1}-B_i}^{2^n-1-B_i} f_W(x)\mathrm{d}x \tag{12}$$

and when $2^{n-1} \leq B_i \leq 2^n - 1$

$$P_{e2} = \int_{1-B_i}^{2^{n-1}-B_i} f_W(x)\mathrm{d}x \tag{13}$$

Due to the pseudo-randomness of the sequence, B_i follows uniform distribution on $[0, 2^n - 1]$. And for any B_i, the error probabilities of MSB can be expressed as

$$P_{e1} = \sum_{B_i=1}^{2^{n-1}-1} \frac{1}{2^n-1} \int_{2^{n-1}-B_i}^{2^n-1-B_i} f_W(x)\mathrm{d}x \tag{14}$$

$$P_{e2} = \sum_{B_i=2^{n-1}}^{2^n-1} \frac{1}{2^n-1} \int_{1-B_i}^{2^{n-1}-B_i} f_W(x)\mathrm{d}x \tag{15}$$

Therefore, the total error probability of MSB is

$$P_e = P_{e1} + P_{e2} \tag{16}$$

Then, the correct capture probability is obtained as

$$P_c = [1-(1-P_e)^n]^{i-1}(1-P_e)^n \quad i \geq 1 \tag{17}$$

As a consequence, the average delay after input $(N-1)$ MSB estimation is

$$E[i] = \sum_{i=1}^{\infty} iP_c(i) = \frac{1}{(1-P_e)^n} \tag{18}$$

According to the Initial input time and the average delay, the total mean acquisition time can be obtained as follows

$$\begin{aligned}\overline{T}_{acq} &= (n-1)T_c + E[i]T_c \\ &= (n-1)T_c + \frac{1}{(1-P_e)^n}T_c\end{aligned} \tag{19}$$

According to Eq.(19), the average capture time mainly depends on the length of LFSR in the case where estimated error probability is less than 10^{-1}.

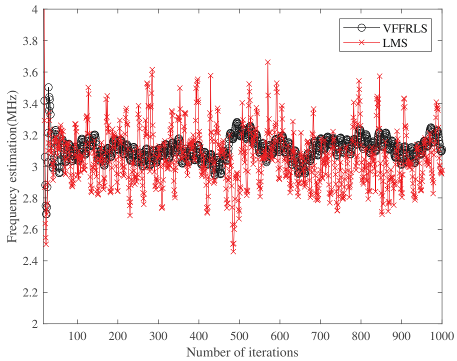

Fig. 4: Comparison curves frequency estimates based on LMS and VFFRLS algorithm.

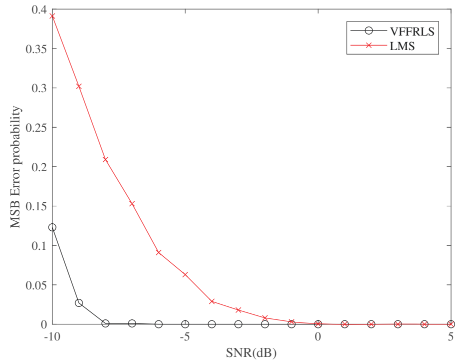

Fig. 6: Error probability of MSB frequency estimation based on LMS and VFFRLS algorithm at different SNRs.

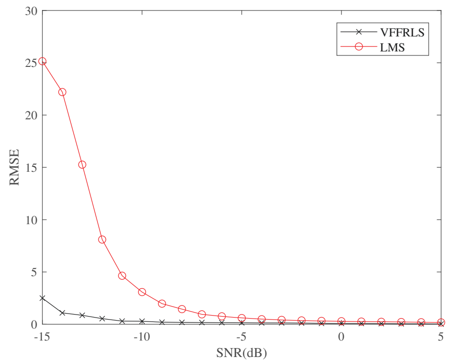

Fig. 5: RMSE of two algorithms at different SNRs.

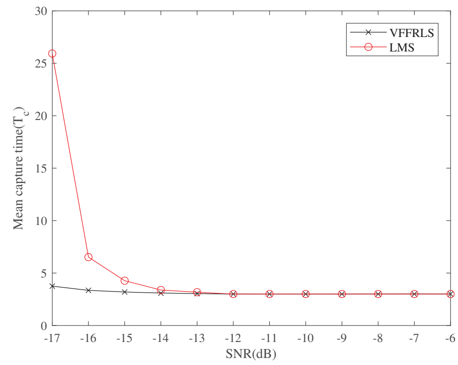

Fig. 7: Mean capture time frequency estimation based on LMS and VFFRLS algorithm at different SNRs.

V. Simulation Results

The following parameters are utilized in the study of the noise immunity of the algorithm based on VFFRLS acquisition procedure. According to the algorithm based on VFFRLS acquisition procedure, the LFSR length is 4; the number of hopping frequency points is 16; the duration of one frequency hop is $T_c = 2$ ms; the number of sampling points of each hopping signal is 2048; the current hopping frequency to be estimated is 3 MHz; the next hopping frequency to be estimated is 7 MHz. And the order of Linear predictor is 16, minimum value of variable forgetting factor $\lambda_{min} = 0.8$, and stepping is $\mu = 0.001$, matrix initialization parameters $\delta = 0.001$.

Fig. 3 demonstrates the simulation of frequency estimation based on the VFFRLS spectral estimation method for two hops of the frequency hopping signal under AWGN channel condition. The frequencies of the two hops are 3 MHz and 7 MHz, respectively. As seen from Fig. 3, the estimation interval of the two frequency estimates is within the allowed range of the number of iterations, the frequency estimates are accurate, and the VFFRLS algorithm has good tracking performance for different frequencies of change. By observing the simulation results, it is found that the VFFRLS algorithm can converge to 3 MHz quickly, and it converges to 3 MHz frequency after approximately 200 iterations, meanwhile it converges to 7 MHz frequency after approximately 240 iterations. The convergence stability is good at the SNR of 5 dB.

Fig. 4 depicts frequency estimation based on LMS and VFFRLS algorithm respectively when SNR is -5 dB. The LMS-based frequency estimation algorithm varies widely around 3 MHz. But the VFFRLS-based frequency estimation algorithm is better stability. Fig. 5 depicts root mean squared error (RMSE) of frequency estimation based on LMS and VFFRLS at different SNRs. From Fig. 5, the LMS-based frequency estimation algorithm of RMSE is 5, but the VFFRLS-based frequency estimation algorithm has tiny RMSE when SNR is -12 dB. The RMSE of the LMS-based frequency estimation algorithm at SNR of -5 dB is approximately equal to that of the VFFRLS-based frequency estimation algorithm at -12 dB. Therefore, the noise immunity based on VFFRLS algorithm is better.

Fig. 6 shows the error probability of MSB frequency estimation based on LMS and VFFRLS algorithm at different SNRs. It is clearly concluded that the error probability of MSB declines when the SNR increases. The error probability of MSB of the VFFRLS-based frequency estimation algorithm reach 0 at -8 dB, however, the error probability of MSB of the LMS-based frequency estimation algorithm reach 0 at -1 dB. Obviously, the VFFRLS-based frequency estimation algorithm has better capture performance in frequency hopping synchronization. As seen in Fig. 7, the simulation result shows that the mean capture time decreases as the SNR increases. At low MSB error probability, the mean capture time is

$4T_c$. Below -14 dB, mean capture time of the frequency estimation LMS-based algorithm is visibly more because of MSB error probability is larger. The frequency estimation VFFRLS-based algorithm use shorter mean capture time at low SNR environment.

VI. Conclusions

In this paper, a fast capture technique for frequency hopping pattern synchronization based on VFFRLS with adaptive frequency estimation is proposed. The simulation results show that the VFFRLS-based frequency estimation algorithm can converge under low SNR conditions through characterizing the statistic of the frequency estimation error. Compared with the LMS-based frequency estimation algorithm, the VFFRLS-based frequency estimation algorithm has better noise immunity, and MSB error probability is lower and mean capture time is shorter. Therefore, the VFFRLS-based frequency estimation algorithm has better adaptability in the environment of frequency hopping communication. It can be certified that the proposed algorithm can accurately estimate the instantaneous frequency of the frequency hopping signal, and frequency hopping acquisition based on VFFRLS algorithm can achieve fast capture by the numerical simulation.

VII. ACKNOWLEDGMENT

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