Detection of Fast Frequency-Hopping Signals Using Dirty Template in the Frequency Domain

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***Abstract*—The frequency-hopping technique is widely used in commercial and military communications for its superiority in anti-jamming and low probability of interception and detection capabilities. To further lower its detection probability, a frequency-hopping signal randomly changes its carrier frequency within a transmission symbol. Conventional detection methods have poor performance in the presence of fast frequency-hopping signals. To solve this problem, we propose a dirty-template-based detection scheme. Dirty Template was originally proposed to blindly estimate the time delay in an ultra-wide-band system. We propose that it can be employed in the frequency domain to detect fast frequency-hopping signal. The decision statistic derived from the cross-correlation between the template and received signals in the frequency domain is analyzed. Using this cross-correlation, a detection scheme, based on the Neyman–Pearson test is proposed. The simulation results show that the dirty-template-based scheme outperforms the autocorrelation-based scheme when the hopping period is short.**

***Index Terms*—Detection, frequency hopping, fast frequency hopping, dirty template.**

I. Introduction

IN ORDER to achieve a low probability of interception and detection, the frequency hopping (FH)-based spread-spectrum technique is widely used in military communications. The FH system randomly changes its carrier frequency according to a pseudo-noise sequence. If an FH system changes its carrier frequency several times within one symbol, it is called a fast-frequency-hopping (FFH) system. It is a non-trivial task to detect FFH signals when the carrier frequency can change multiple times within a single detection interval.

Nevertheless, there has been much research on the detection of FH signals with special interest in FFH. FH signal detection based on energy [1]–[3], autocorrelation [4]–[6] and wavelet [7]–[9] have been studied. These conventional detection schemes require the knowledge of hopping period a priori. In practice, knowing the hopping period in advance is another challenging task. Typically, the detection interval and the hopping period cannot be matched. If the hopping period is longer than the detection interval, the exact amplitude of the frequency response can be obtained. However, if the hopping period is shorter than the detection interval, the energy is distributed among many frequency bins. Thus, there cannot be a single peak and there must be a spectral leakage. Because the energy leaks from a single bin, its detection performance must be limited.

To improve the performance of FFH-signal detection, we propose the Dirty Template (DT) scheme in the frequency domain. DT was originally proposed to blindly estimate the time delay in ultra-wide-band systems [10], [11]. It uses the received signals as a template for cross-correlation, and is called "dirty" because the template is corrupted by noise. Similar to time-domain DT, we can obtain the template from a received signal in frequency domain, collect the energy scattered by spectral leakage with correlations, and use the energy for detection purposes. Because it uses the correlation between the template and FFH signal, the energy distributed by spectral leakage can be collected. Therefore, DT can effectively cope with the spectral leakage issue of the FFH signal.

The remainder of this letter is organized as follows: In Section II, the system assumption and cross-correlation computation are presented. In Section III, the statistical characteristics of DT are analyzed and the decision statistic of the Neyman-Pearson test is derived. In Section IV, the simulation results of the proposed scheme are compared against those of the conventional schemes. Finally, we draw conclusions in Section V.

II. System Assumption and Cross-Correlation Computation

To detect FH signal, a fast Fourier transform (FFT)-based detector is considered. The carrier frequency of the FH signal randomly changes during the transmission, and only the FFH is considered in this letter. FH signal x at integer time index n is modeled as

$$x(n) = \frac{1}{T}\sqrt{\frac{E_s}{L}} \sum_{i=-\infty}^{\infty} e^{j2\pi f_i nT + \phi_i} p(n - iT), \tag{1}$$

where E_s is the energy of a symbol, L is the number of hops per symbol and T is the hopping period. f_i is the ith hopping frequency, $p(\cdot)$ denotes the rectangular pulse and ϕ indicates the uniform random phase of $[0, 2\pi]$. Define R as the N-FFT of x plus noise, then

$$R(k) = X(k) + Z(k), \quad \text{for } k = 0, 1, \ldots, N-1, \tag{2}$$

and

$$X(k) = \sqrt{\frac{E_s}{L}} \operatorname{sinc}(k - m) e^{j\phi_k}, \ \text{ for } k = 0, 1, \ldots, N-1, \tag{3}$$

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where k is the frequency index, X denotes the FFT output of x and $Z(0), Z(1), \ldots, Z(N-1)$ are zero-mean complex Gaussian-distributed independent random variables with variance N_0/N. The processing gain N obtained through size N-FFT is reflected in the variance of noise Z in (2). m is the frequency-bin index of the transmitted signal and $\mathrm{sinc}(\cdot)$ denotes the normalized sinc function.

The DT-based detection scheme uses the correlation between the "dirty" template and the received signal. Our interest is in the data-aided mode, where the template is acquired over the observation time before detection. During the observation time, the energy of the signal in template E_T and noise power N_0 can also be estimated. More details on template acquisition are described in a later section. The computation of the correlation between the template and received signal results in a superposition of noise. By applying a truncated template, the issue of noise-superposition can be relived. The truncated template R_T is defined as

$$R_T(k) = \begin{cases} X(k) + Z(k) & \text{for } m - W/2 \leq k \leq m + W/2 \\ 0 & \text{else,} \end{cases} \tag{4}$$

where, W denotes the number of signal-bearing bins in the template.

Define Y as the cross-correlation between the received signal and template. Then, Y is expressed as

$$Y(\theta) = \sum_{k=0}^{N-1} R^*(k) R_T(k+\theta), \text{ for } \theta = 0, 1, \ldots, N-1, \tag{5}$$

where θ indicates the shift index in the frequency bin. When the FFH signal is presented in $R(k)$, a peak exists by the correlation between the FFH signal and template. The correct peak in Y can be observed only if the shape of the template and FFH signal are linearly dependent. The proof of the linear dependency condition can be derived by the Cauchy-Schwarz inequality [10]. The expectation and variance of Y for each hypothesis are analyzed to obtain a likelihood function of the Neyman-Pearson (NP) test. By using Y directly, the decision statistic of the NP test is developed for detection of the FFH signal.

III. Dirty Template in the Frequency Domain

A. Mean and Variance of Cross-Correlation

We apply the NP test to detect FFH signals. Before deriving the likelihood function of DT, the statistical parameters, mean and variance of Y for each hypothesis should be analyzed. Using these parameters, the likelihood function is derived and the acquisition of the template is described. Define hypothesis H_0 as the absence of an FFH signal and H_1 as the presence of an FFH signal.

1) FFH Signal Presence (H_1): In the presence of an FFH signal, Y is expected to have a peak. In the presence of an FFH signal, the expectation and variance of Y are obtained as

$$\mathrm{E}[\,Y(\theta)] = \sqrt{\frac{E_s E_T}{L^2}} \mathrm{sinc}(\theta - \hat{m}), \text{ and} \tag{6}$$

$$\mathrm{Var}[\,Y(\theta)] = \left(\frac{WE_s}{N} + E_T\right)\frac{N_0}{LN} + \frac{WN_0^2}{N^2}, \tag{7}$$

where the $\hat{m}$-th bin corresponds to the relative distance between the peak of the template and the peak of an FFH signal. Because the location of the exact frequency bin is not known, $\hat{m}$ is an unknown parameter.

2) FFH Signal Absence (H_0): When an FFH signal is absent, the peak is not observed in Y because there is no correlation between the noise and the template. The expectation and variance of Y are obtained as

$$\mathrm{E}[\,Y(\theta)] = \mathrm{E}\left[\sum_{k=0}^{N-1} Z^*(k) R_T(k+\theta)\right] = 0, \text{ and} \tag{8}$$

$$\mathrm{Var}[\,Y(\theta)] = \mathrm{E}[|\,Y(\theta)|^2] = \frac{E_T N_0}{LN} + \frac{WN_0^2}{N^2}. \tag{9}$$

B. Neyman-Pearson Test

The NP test is applied to detect the signal. N samples are collected to compute Y. As N is assumed to be a large number, Y is approximated to have a Gaussian distribution based on the central limit theorem. To simplify the equation, the expectation of Y given H_1 and the variance of Y given H_0 and H_1 are substituted with μ, σ_0^2 and σ_1^2, respectively. The likelihood function of the NP test is now obtained as

$$\frac{f(\,Y|H_1)}{f(\,Y|H_0)} = \frac{\left(1/\sqrt{2\pi\sigma_1^2}\right)^N e^{\frac{-\sum_{\theta=0}^{N-1}|\,Y(\theta)-\mu(\theta)|^2}{2\sigma_1^2}}}{\left(1/\sqrt{2\pi\sigma_0^2}\right)^N e^{\frac{-\sum_{\theta=0}^{N-1}|\,Y(\theta)|^2}{2\sigma_0^2}}}. \tag{10}$$

This leads to its log-likelihood function,

$$\sum_{\theta=0}^{N-1}\left(\frac{WN_0}{2\sigma_0^2 N^2}|\,Y(\theta)|^2 + \sqrt{\frac{E_T}{E_s}}\,Y(\theta)\mathrm{sinc}(\theta - \hat{m})\right) \underset{H_0}{\overset{H_1}{\gtrless}} \gamma, \tag{11}$$

where γ denotes a threshold. There are unknown parameters in (11), such as E_s and a peak index $\hat{m}$. As the template is also made using the received signal, it can be approximated as $\sqrt{\frac{E_T}{E_s}} \approx 1$. For the other unknown parameter, $\hat{m}$ can be difficult to estimate in the case where the pseudo-noise sequence of the FFH signal is unknown. However, it is assumed that $\hat{m}$ is the peak index of Y. Thus, $\hat{m}$ is estimated by the index that gives the maximum absolute value of $Y(\theta)$. Finally, the log-likelihood function of NP test using DT is expressed as

$$\underbrace{\left(\sum_{\theta=0}^{N-1}\frac{W}{2\left(\frac{E_T}{L} + \frac{WN_0}{N}\right)N}|\,Y(\theta)|^2\right) + \max_{\theta}(|\,Y(\theta)|)}_{=\Lambda} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma, \tag{12}$$

and we define the left term of (12), Λ, as the decision statistic of the NP test.

The difference between the centers of distributions of Λ given H_0 and H_1 becomes

$$\mathrm{E}[\Lambda; H_1] - \mathrm{E}[\Lambda; H_0] = \frac{W^2 N_0 \cdot \mathrm{SNR}/L}{2N^3(\mathrm{SNR}/L + W/N)} + \frac{E_T}{L} - 2.576\sigma_0, \tag{13}$$

where 2.576 denotes 99% uncertainty of a standard normal distribution and SNR is the SNR of the template. In (13) the first term shows the mixture of signal and noise. The second term indicates the energy of the received signal gathered by DT. The last term shows the noise superposed by DT. If the difference given by (13) is large enough to distinguish the distribution of Λ given H_1 from H_0, the FFH signal can be detected. If the difference is too small, H_0 and H_1 are difficult to distinguish. When the SNR of the FFH signal is high, $\frac{W^2 N_0 \cdot \text{ SNR}/L}{2N^3(\text{ SNR}/L+W/N)} \approx \frac{W^2 N_0}{2N^3}$ and $\sigma_0 \approx \sqrt{\frac{E_T N_0}{LN}}$. Thus, the difference increases as E_T, N, and W increase. However, if the SNR of the FFH signal is low, $\frac{W^2 N_0 \cdot \text{ SNR}/L}{2N^3(\text{ SNR}/L+W/N)} \approx 0$ and $\sigma_0 \approx \sqrt{\frac{WN_0^2}{N^2}}$. W in σ_0 shows the noise superposed by the template. The difference increases as E_T and/or N increase and W decreases. Subsequently, DT-based detection scheme gathers the energy of received signal spread by spectral leakage, but it suffers from noise superposition in low SNR.

C. Acquisition of the Template

When the correlation between the template and received signal is computed, noise is superimposed. Thus, directly computing the received signal as a template can deteriorate the detection performance. In order to mitigate the noise-superposition issue, a template is obtained from the truncated FFT outputs of the received signal through the following steps. Firstly, the FFT outputs of the received signal are divided into $\lceil \frac{N}{W} \rceil$ blocks. The block size W denotes the number of consecutive frequency bins and each block has the same W frequency bins. Hence, the product of block size and the number of blocks always becomes the FFT size. Secondly, detection based on the NP test is applied to find a signal-bearing block in the received signal. For each block, (12) is computed with the following received signal. To estimate the energy of the received signal, $\max_{\theta}(|Y(\theta)|)$ is chosen to calculate Λ. Finally, if the FFH signal presents within one of the $\lceil \frac{N}{W} \rceil$ blocks, Λ of that block should be larger than the other blocks. By doing so, we find the largest Λ block among the $\lceil \frac{N}{W} \rceil$ blocks and compare it with appropriate threshold.

IV. Performance Analysis

A. Simulation Results

To determine the probability of detection and false alarm, the analytical or numerical solution of Λ should be found. Unfortunately, it is difficult to find analytical solutions with a known probability distribution. For an alternative method, a Monte Carlo simulation is performed. The distribution of Λ is obtained, and the probability of detection and false alarm are calculated with a suitable threshold. We compare the receiver operating characteristic (ROC) of the proposed detector utilizing the DT-based and autocorrelation-based detection schemes [6]. DT produces an NP test with the cross-correlation, whereas the autocorrelation detection scheme makes test statistics with autocorrelation. Due to the similarity between these two schemes, the autocorrelation detection scheme is chosen for comparison. The modulation of the FFH signal is assumed to be binary frequency-shift keying. The

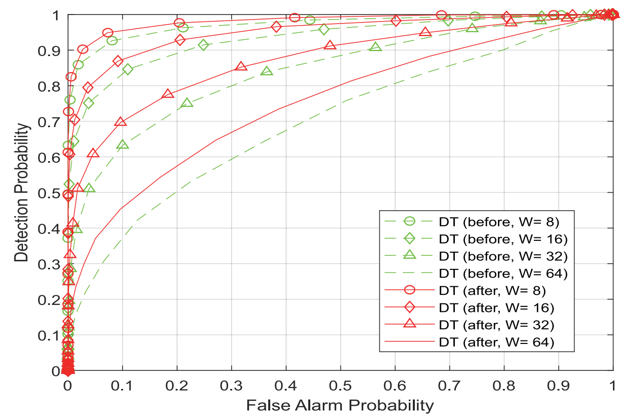

Fig. 1. ROC of Dirty Template of before and after the observation time over different block size at SNR $= -18$ dB and $L = 1$.

number of hopping bands is 100, the symbol rate is 6000 symbols/second, and the hopping rate is 6000 $\times$ L hops/s. The size of the FFT N is 2048.

We compare the two DT-based detection schemes whose templates are obtained differently. In Fig. 1, comparisons of ROC with respect to W are shown. DT (before) denotes the template is obtained before the observation time through the method described in Section III-C, whereas DT (after) indicates that the template is given after the observation time using perfectly estimated received signal energy. The performance of DT (before) and DT (after) are not much different. Before the observation time, DT estimates the energy of the received signal through a simple max operation. During the observation time, the template is constructed and the energy of the received signal is estimated. After the observation time, a perfectly estimated template can be obtained. Eventually, the performance of DT (before) converges to that of DT (after). It is observed that the performance increases as W decreases (the number of blocks increases). If the block size is too big, the signal-bearing-block contains too much noise and it decreases the detection probability. On the other hand, too small block sizes reduce the energy of received signal. When the size of the block is perfectly matched to the received signal, the performance of DT would be optimized. However, in practice, knowing the symbol rate or hopping rate of the received signal is a challenging task. In this simulation, we assume that $W = 8$ shows the best performance.

In Fig. 2, the ROC curves of the DT-based and autocorrelation-based detection schemes are compared at $L = 4$. In this simulation, the spectral leakage of the FFH signal is observed because its hopping period is smaller than the length of the detection interval. Owing to spectral leakage, the ROC curve of the autocorrelation-based detection scheme is worse than that of the theoretical model in [6]. The superiority of the DT-based detection scheme can be seen in Fig. 2.

ROC curves of the DT-based and autocorrelation-based detection schemes over different L with SNR $= -16$ dB are shown in Fig. 3. The ROC curves of the two schemes are almost the same at $L = 1$ and $L = 2$. Nevertheless, as L increases, the ROC curve of the autocorrelation-based detection scheme sharply moves to the center. The reason for this

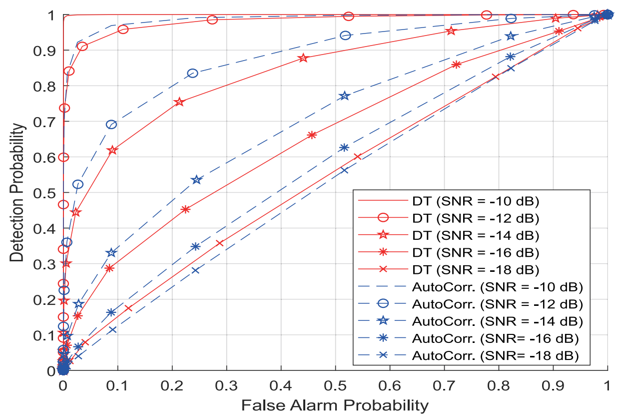

Fig. 2. ROC of Dirty Template and the autocorrelation detection scheme over different SNR at $L = 4$.

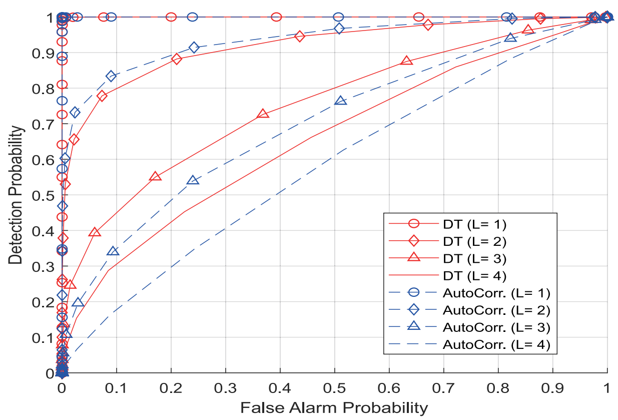

Fig. 3. ROC of Dirty Template and the autocorrelation detection scheme over different L with SNR $= -16$ dB.

sharp movement is the energy reduction and spectral leakage. As L is the number of hops per symbol, it separates the energy of the symbol by a factor of L. This leads to a reduction in the amplitude of the peak frequency bin. Furthermore, a shorter signal creates more spectral leakage. For these reasons, the performance of the autocorrelation-based detection scheme deteriorates when an FFH signal is transmitted. In contrast, because of the energy collected by the DT-based detection scheme, its ROC curve moves rather slowly to the center. When L is smaller than 1, it becomes a slow frequency hopping and we do not consider this case in this letter. We only confirm that the performance of $L < 1$ is the same as $L = 1$ through the simulation.

B. Computational Complexity Analysis

Because correlation needs to be computed, the autocorrelation-based and DT-based detection schemes have high computational complexity. A correlation process consists of N multiplications and N additions. The autocorrelation-based detection scheme calculates N of the correlation processes. Hence, the computational complexity of the autocorrelation-based detection scheme becomes $\mathcal{O}(N^2)$, where $\mathcal{O}(\cdot)$ denotes Big-O notation. The computational complexity of DT can be calculated in a similar way. Computing $Y(\theta)$ takes $\mathcal{O}(N)$ operations and Λ consists of N of $Y(\theta)$. Therefore, the computational complexity for a DT-based detection scheme also becomes $\mathcal{O}(N^2)$.

Parameter estimation of the autocorrelation-based detection scheme uses test statistics [6] and it can be computed as $\mathcal{O}(N^2)$. For the template acquisition of DT, additional computational complexity is required. Although the template acquisition has an additional max process, the computational complexity of the additional process is $\mathcal{O}(\lceil \frac{N}{W} \rceil)$. Thus, the computational complexity of the template acquisition converges to $\mathcal{O}(N^2)$ calculated from Λ. Consequently, it can be considered that the computational complexity of the autocorrelation-based and DT-based detection schemes are the same.

V. Conclusion

In this letter, we propose an FFH-signal-detection scheme applying DT in the frequency domain. By using DT, a signal can be detected without its hopping period. Furthermore, the template makes the detection robust against spectral leakage. A decision statistic of the NP test is proposed, and ROCs of DT-based and autocorrelation-based detection schemes are compared through simulation. The simulation results show that the performance of the DT-based scheme is better than that of the autocorrelation-based scheme. When the hopping period is short, the performance of the DT-based scheme decreases less than that of the autocorrelation detection scheme, which implies that the DT-based detection scheme has better performance in FFH detection.

References

[1] R. A. Dillard and G. M. Dillard, "Likelihood-ratio detection of frequency-hopped signals," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 32, no. 2, pp. 543–553, Apr. 1996.

[2] J. J. Lehtomaki, "Maximum based detection of slow frequency hopping signals," *IEEE Commun. Lett.*, vol. 7, no. 5, pp. 201–203, May 2003.

[3] F. Liu, M. W. Marcellin, N. A. Goodman, and A. Bilgin, "Compressive sampling for detection of frequency-hopping spread spectrum signals," *IEEE Trans. Signal Process.*, vol. 64, no. 21, pp. 5513–5524, Nov. 2016.

[4] A. Polydoros and K. Woo, "LPI detection of frequency-hopping signals using autocorrelation techniques," *IEEE J. Sel. Areas Commun.*, vol. SAC-3, no. 5, pp. 714–726, Sep. 1985.

[5] J. Hampton and J. Oetting, "A frequency hopping sequential detection technique for in-net coarse acquisition," in *Proc. IEEE Military Commun. Conf. (MILCOM)*, vol. 1, Oct. 1992, pp. 43–47,

[6] H. Fan, Y. Guo, and Y. Xu, "A novel algorithm of blind detection of frequency hopping signal based on second-order cyclostationarity," in *Proc. IEEE Congr. Image Signal Process.*, vol. 5, May 2008, pp. 399–402.

[7] H. Luan and H. Jiang, "Blind detection of frequency hopping signal using time-frequency analysis," in *Proc. IEEE Int. Conf. Wireless Netw. Mobile Comput.*, Sep. 2010, pp. 1–4.

[8] M. Sirotiya and A. Banerjee, "Detection and estimation of frequency hopping signals using wavelet transform," in *Proc. IEEE 2nd U.K.–India IDRC Int. Workshop Cogn. Wireless Syst. (UKIWCWS)*, Dec. 2010, pp. 1–5.

[9] J. Lv and W. Qu, "Application of the wavelet rearrangement algorithm in the detection of noncooperative frequency hopping signals," in *Proc. IEEE 11th Int. Conf. Signal Process.*, vol. 1, Oct. 2012, pp. 263–266.

[10] L. Yang and G. B. Giannakis, "Timing ultra-wideband signals with dirty templates," *IEEE Trans. Commun.*, vol. 53, no. 11, pp. 1952–1963, Nov. 2005.

[11] R. Alhakim, K. Raoof, and E. Simeu, "Detection of UWB signal using dirty template approach," *Signal Image Video Process.*, vol. 8, no. 3, pp. 549–563, Mar. 2014.