# Outage Probability and Performance of Moderate-Length Codes under Partial-Band Noise Jamming (PBNJ)

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*Abstract* —**This paper presents a theoretical approach for estimating the performance of moderate-length codes under partial-band noise jamming (PBNJ). The approach is based on the outage probability, which is the probability that the mutual information rate random variable is less than the code rate. This paper is an extension of previous work where we obtained a closed form expression for the outage probability of moderate-length codes under additive Gaussian noise channel (AWGN) [8]. Using the results under AWGN, we obtain a closed form expression for the outage probability under partial-band noise jamming (PBNJ). Using this result, we show that a smart jammer is driven to employ a full-band (FB) noise jamming. Relative to FB jamming, there is no advantage of PBNJ. These analytical results are compared against simulation results obtained for Protected Tactical Waveform (PTW) using Second Generation Digital Video Broadcasting Satellite (DVB-S2) forward error correction (FEC) codes under PBNJ.**

**Keywords —** ***Outage Probability; Mutual Information; Jamming; full-band noise jamming; partial-band noise jamming; Protected Tactical Waveform; DVB-S2; FEC.***

## I. Introduction

Intentional jamming has always been a major denial-of-service on satellite communication (SATCOM) users. The main objective of the jammers is to deny reliable communications. By intentionally emitting jamming signals, adversaries can disrupt communication services, resulting in throughput degradation. The objective of a jammer is to cause the most damage to the victim's signal at its minimal cost, subject to the constraint of its power. Jammers often assume a more effective jamming technique is to concentrate its power over a fraction of the total hopping bandwidth. For this reason, partial-band noise jamming (PBNJ) is commonly used. The goals of SATCOM designers are to develop robust jam-resistant or anti-jam communication systems, under the assumption the jammer has certain a priori knowledge of the victim's signal. To address this problem, the Air Force has adopted Protected Tactical Waveform (PTW) as a standard waveform to operate over commercial and Military Sattelite Communication (MILSATCOM) systems [1-2]. PTW is an advanced waveform based on the latest state-of-the-art modulation and forward error correcting (FEC) coding techniques. PTW supports very high data rates while providing secured and protected communication services as well as low probability of intercept and detection (LPI/LPD) [3]. Frequency diversity, time diversity, and FEC are integral parts of the PTW to provide robust protection against PBNJ. Frequency diversity is achieved by either frequency hopping (FH), frequency permutation, direct sequence spread spectrum (DSSS). Time diversity is achieved by time permutation and channel interleaving. A powerful FEC code is baseline technology of the Second Generation of Digital Video Broadcasting (DVB-S2) FEC [4]. DVB-S2 FEC is based on the concatenation of BCH (Bose-Chaudhuri-Hocquengham) and LDPC (Low Density Parity Check) codes. In this paper, PTW employs M-ary phased shift keying (MPSK) using DVB-S2 FEC, with numerous code rates, with a fixed code length of 16,200 coded bits.

The outage probability is the probability that the mutual information rate random variable is less than the code rate [5-7]. Mutual information rate random variable is the average mutual information over the length of the codeword. For uniform distribution of input MPSK symbols, there is no closed form expression for the average mutual information; we use a numerical approach to estimate the average mutual information. The average mutual information is then used to estimate the mutual information rate random variable, which in turn, is used to estimate the outage probability over additive white Gaussian noise (AWGN) channel. Detailed development of the outage probability to predict the performance of moderate length codes over AWGN are discussed in [8]. Results shown in [8] indicate that the outage probability provides a tight bound for DVB-S2 FEC of PTW over the AWGN channel.

Due to jamming, errors occur in bursts and an effective way to combat burst errors is to interleave the symbols such that the location of the errors appear random and is distributed over many codewords. In this way, the number of errors in each codeword is low and can be corrected by the FEC code. At the receiver, a deinterleaver is employed to undo the effect of the channel interleaver. The deinterleaver spreads the burst errors over many codewords. Sizing the channel interleaver is an important part of the code designer. Although the interleaver offers many benefits to the decoder, the larger the interleaver size, the more delay it incurs. Also there is a diminishing returns after a certain size of the interleaver.

Due to limited signal power, users typically employ link adaptation by means of dynamic resource allocation (DRA) [9] or adaptive coding modulation (ACM) [10]. ACM provides spectral efficiency and optimizes the throughput. ACM adapts a more robust modulation order and FEC code rate (referred to as MODCOD) to compensate for changes in the link condition due to jamming. ACM requires channel information. The channel information is estimated by the system controller (SC). The SC periodically measures the link signal-to-noise (SNR) ratio. In ACM, SNR is quantized to several contiguous regions so that each level can be mapped to a MODCOD for transmission. Based on this information SC assigns the MODCOD to the transmitter.

In general, the communication signal bandwidth $W$ is much smaller than the hopping bandwidth $W_H$; partial-band jamming is easier to generate than the full-band (FB) jamming. The effect of on PBNJ on the performance of FEC is typically studied by simulation [11-13]. For FH systems, the analysis of PBNJ is similar to FB noise jamming. Some hops, the signal will be jammed; on other hops, the signal is not jammed and the only source of interference is thermal noise. For moderate length codes, the error probability is calculated using quasi-static analysis. That is, the error probability is calculated separately for thermal noise interference and thermal plus jamming noise interference, and the results are a weighted average. The weighted values are based on the probability the signal is jammed, $\rho$, and not jammed $(1-\rho)$. Using this analysis, the outage probability of the FEC under AWGN can be extended under PBNJ environment. Using this approach, this paper extends the outage probability results from AWGN [8] to PBNJ. It provides the theoretical prediction performance of moderate length codes under PBNJ. For PBNJ, there is an optimum value of $\rho$ that maximizes the error probability. The jammer has control of $\rho$, and a smart jammer will be able to adjust $\rho$ dynamically to maximize the system error probability, and therefore, cause the most damage to the system. In this paper, we show that a smart jammer can be driven to employ full-band noise jamming. Relative to full-band noise jamming, there is no advantage to PBNJ. The results are also compared with computer simulation. In this paper, for brevity, FEC is used in place of DVB-S2 FEC of PTW.

To simplify the analysis, complex baseband signaling is considered, the transponder is operating in the linear region and omitted in the paper. In addition, codeword error rate (CWER) and block error rate (BLER) are used intechangebly.

## II. System model

### *A. Channel Model*

The jammer power is fixed. The jammer power spectral frequency is:

$$J_o = \frac{J}{W_H} \tag{1}$$

where $J$ denotes jammer power, and $W_H$ denotes the total hopping bandwidth. A partial band jammer concentrates its power over a fraction of the total hopping bandwidth $\rho W_H, 0 < \rho \leq 1$. Therefore, $\rho$ represents the probability that the signal is jammed. If a hop is jammed, we assume the full duration and the bandwidth of the transmitted signal is jammed.

The discrete-time baseband of the $l^{th}$ symbol at $k^{th}$ hop of the received signal is

$$y_{k,l} = x_{k,l} + n_{k,l} + v_k w_{k,l} \tag{2}$$

where $x_{k,l}$ denotes Gray-coded MPSK transmitted symbol, $n_{k,l}$ denotes additive white Gaussian noise (AWGN) channel with zero mean and average noise power $\sigma_0^2$, $v_k \in \{0,\ 1\}$ is a binomial random variable, $v_k = 1$ with probability $\rho$ if a hop is jammed, otherwise $v_k = 0$, $w_{k,l}$ denotes additive white Gaussian jamming noise with zero mean with average power $\sigma_j^2$. Channel noise and jamming noise are statistically independent. In general, one has

$$\sigma_0^2 = N_0 W \quad and \quad \sigma_j^2 = \frac{J_0}{\rho} W \tag{3}$$

where $N_0$ denotes the power spectral density of AWGN and $W$ denotes the matched filter bandwidth of the signal at the receiver. Figure 1 shows a notional PBNJ model.

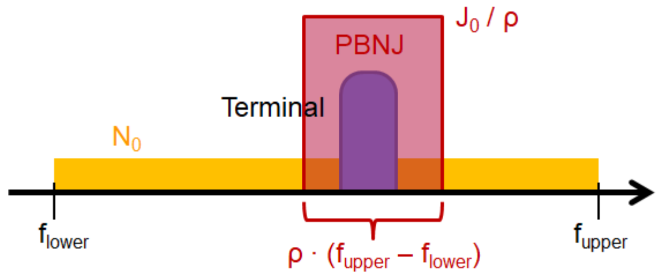


Fig. 1. A Notional PBNJ model.

### ***B.*** *The Outage Probability of Codewords of Finite Length under PBNJ*

The outage probability for FEC of moderate length codes in the benign (AWGN) is given as [8]

$$P_0^N = Q\left(\frac{C(\gamma)-R_e}{\sqrt{\frac{2M\gamma}{n(\gamma+1)}}}\right), \tag{4}$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty exp\left(-\frac{t^2}{2}\right) dt, \tag{5}$$

$$Re = \log(M)\frac{k}{m}, \tag{6}$$

$$\gamma = \frac{E_s}{N_o}, \tag{7}$$

where $log(\cdot)$ denotes the natural logarithmic function, $n$ denotes the number of symbols per codeword, $M$ denotes the size of the MPSK alphabet, $R_e$ denotes the code rate in bits/symbol, $k$ denotes the number of information bits per codeword, $m$ denotes the number of coded bits per codeword, $\gamma$ denotes the signal to noise ratio (SNR) power in the benign (AWGN) environment, and $E_s$ denotes the symbol energy. Finally, $C(\cdot)$ denotes the constrained capacity of MPSK signal. As discussed in [8], there is no closed-form expression for it; a numerical integration approximation is used, and it is given as

$$C(\gamma) \approx log(M) - 1 - \frac{1}{M\pi}\sum_{j=1}^{N} w_j \sum_{i=1}^{N} v_i h(s_i, t_j), \quad (8)$$

where $s_i$, and $t_j$ are roots of the Hermite polynomial and $v_i$ and $w_j$ are associated weights. The above approximation improves as $N$ increases.

As shown in (2), the statistics of the binomial random variable $v_k$ governs the statistics of PBNJ. If $\alpha$ is a fraction of the codeword that is jammed, then $\alpha$ is a random variable. With interleaving and ACM, the average value of $\alpha$ is $\rho$. Therefore, the average outage probability of codeword is the weighted sum of the outage probability under AWGN and PBNJ, and it is given as [14]

$$P_o = \rho P_0^J + (1-\rho)P_0^N \quad (9)$$

where $P_0^N$ is given by (4), and $P_0^J$ denotes the outage probability under PBNJ. $P_0^J$ is given as

$$P_0^J = Q\left(\frac{N(\gamma_J)}{D(\gamma_J)}\right), \quad (10)$$

where

$$N(\gamma_J) = C(\gamma_J) - Re, \quad (11)$$

$$D(\gamma_J) = \sqrt{\frac{2M\gamma_J}{n(\gamma_J+1)}}, \quad (12)$$

and $\gamma_J$ denotes SNR under PBNJ and is given as

$$\gamma_J = \frac{E_s}{N_0 + J_0/\rho} = \frac{\gamma}{1+\frac{J_0/N_0}{\rho}}. \quad (13)$$

Again, for moderate-length FEC codes, it was shown in [8] that the outage probability is an upper bound for CWER. Interested readers are referred to [8] for more details.

## III. ANALYSIS

Given (13), the incremental power in dB required to maintain the desired outage probability is given as

$$\Delta_{inc} = 10log_{10}\left(1+\frac{J_0/N_0}{\rho}\right). \quad (14)$$

As shown in (9) and (14), by selecting an optimal $\rho$, a smart jammer can maximize (increase) the outage probability and force the system to require more power. Under a jamming environment, the receiver operates at high SNR; the second term of (9) on the right hand side is negligible. The outage probability is reduced to

$$P_o = \rho P_0^J. \quad (15)$$

Again, $P_0^J$ is the outage probability under PBNJ; it is given by (10). Under high jamming power $\left(\frac{J_0}{N_0} \gg 1\right)$, one obtains

$$\gamma_J = \frac{\gamma}{1+\frac{J_0/N_0}{\rho}} \cong \frac{\rho\gamma}{J_0/N_0}, \quad (16)$$

and,

$$\frac{\gamma_J}{\gamma_J+1} \cong \frac{\gamma}{\gamma+\left(\frac{J_0/N_0}{\rho}\right)}. \quad (17)$$

As shown in (16) and (17), given $J_0/N_0$, equations (11) and (12) are functions of $\gamma$ and $\rho$, i.e.

$$N(\gamma_J) = N\left(\gamma,\rho;\frac{J_0}{N_0}\right) \ \ and \ \ D(\gamma_J) = D\left(\gamma,\rho;\frac{J_0}{N_0}\right). \quad (18)$$

As $\rho$ increases, $\frac{J_0/N_0}{\rho}$ decreases, (17) increases, therefore $D\left(\gamma,\rho;\frac{J_0}{N_0}\right)$ increases, i.e.

$$As \ \ \rho \ increases \ \Rightarrow D\left(\gamma,\rho;\frac{J_0}{N_0}\right) \ increases, \ 0<\rho\le 1. \quad (19)$$

Also, the constrained capacity can be approximated as [15]

$$C(\gamma) \approx log_2(M)(1-\sum_{i=1}^{L} a_i e^{-b_i\gamma}) \quad (20)$$

where $\gamma$ is given by (7), $a_i, b_i,$ and $L$ are the fitting curve parameters. Approximation parameters for QPSK and 8PSK are summarized in Table I.

Table I. Approximation parameters of (37) for QPSK and 8PSK modulations.

| Modulation | $a$ | $b$ | $L$ |
|---|---|---|---|
| QPSK | 1 | 0.6507 | 1 |
| 8PSK | 0.6130<br>0.3855 | 0.1681<br>0.8992 | 2 |

In addition, the Shanon capacity or channel capacity is given as

$$C^0(\gamma) = log(1+\gamma). \quad (21)$$

Figure 2 shows the approximated constrained capacity [15] for QPSK and 8PSK and the Shannon capacity.

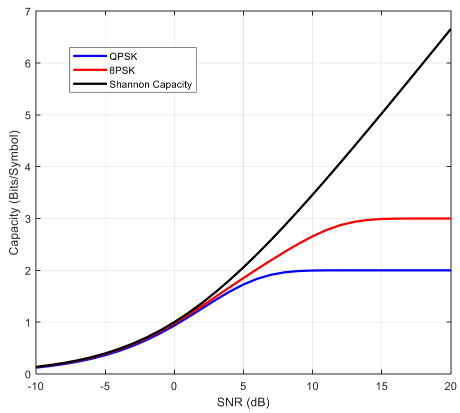


Fig. 2. Approximated constrained capacity for QPSK, 8PSK, and Shannon capacity as a function of SNR.

Substituting $\gamma_J$ into (20), the constrained capacity under PBNJ is given by

$$C(\gamma_J) \approx log_2(M)(1 - \sum_{i=1}^{L} a_i e^{-b_i \gamma_J}). \quad (22)$$

Substituting (22) into (11); one obtains

$$N\left(\gamma, \rho; \frac{J_0}{N_0}\right) = G - H \sum_{i=1}^{L} a_i e^{-\rho c_i}, \quad (23)$$

where

$$G = log_2(M) - Re > 0\,, \quad (24)$$

$$H = log_2(M) > 0, \quad (25)$$

$$c_i = \frac{b_i \gamma}{J_0/N_0} = b_i \frac{E_s}{J_o} > 0. \quad (26)$$

Using (23), taking the partial derivative of $N\left(\gamma, \rho; \frac{J_0}{N_0}\right)$ with respect to $\rho$, one obtains

$$\frac{\partial N}{\partial \rho} = H \sum_{i=1}^{L} a_i c_i e^{-\rho c_i}\,, \quad 0 < \rho \leq 1. \quad (27)$$

$\frac{\partial N}{\partial \rho}$ is a positive function, when $\rho$ increases, $\frac{\partial N}{\partial \rho}$ decreases. Therefore,

$$As\ \rho\ increases \implies N\left(\gamma, \rho; \frac{J_0}{N_0}\right) \ decreases, \quad 0 < \rho \leq 1. \quad (28)$$

An approximation of the $Q(\cdot)$ function is

$$Q(x) \approx \frac{1}{\sqrt{2\pi}x} e^{-\frac{x^2}{2}}. \quad (29)$$

The above approximation improves as $x$ inceases. Combining (19), (28), and (29), we have.

As $\rho$ increases, $\frac{N\left(\gamma,\rho;\frac{J_0}{N_0}\right)}{D\left(\gamma,\rho;\frac{J_0}{N_0}\right)}$ decreases, $Q\left(\frac{N(\gamma,\rho)}{D(\gamma,\rho)}\right)$ increases.

Therefore,

$$As\ \ \rho\ increases \implies P_0^J\ increases, \quad 0 < \rho \leq 1\,. \quad (30)$$

Equations (15) and (30) indicate that a smart jammer is driven to employ a full-band noise jammer ($\rho = 1$) to cause maximum degradation to the receiver. Relative to full-band noise jamming, there is no advantage to employ PBNJ.

Consider $J_0/N_0 = 20\ dB$. Using (16), Figure 3 shows the outage probability for QPSK, rate 1/4, under PBNJ.

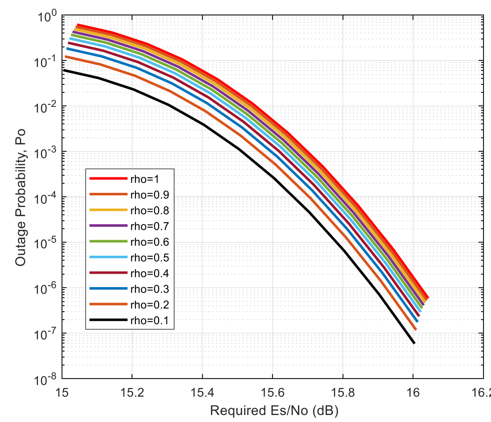


Fig. 3. Outage probability $P_0$ under PBNJ for QPSK, rate 1/4, under PBNJ, $\frac{J_0}{N_0} = 20\ dB$.

Results shown in Fig. 3 indicates that FB noise jamming is the optimal jamming and there is no advantage for PBNJ.

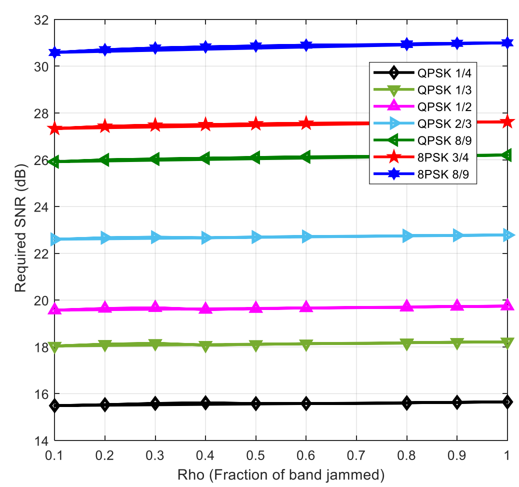


Fig. 4. Required SNR to achieve $P_0 = 10^{-3}$ for QPSK and 8PSK, rate ={1/4, 1/3, 1/2, 2/3, 3/4, 8/9}, under PBNJ, $\frac{J_0}{N_0} = 20\ dB$.

Figure 4 shows the required SNR to achieve the outage probability of $10^{-3}$, for QPSK and 8PSK with numerous code rates under PBNJ.

Fig. 4 confirms the analysis previously shown in (30), relative to FB noise jamming; there is no advantage of PBNJ.

Figure 5 shows the computer simulation results to achieve CWER of $10^{-3}$ [13].

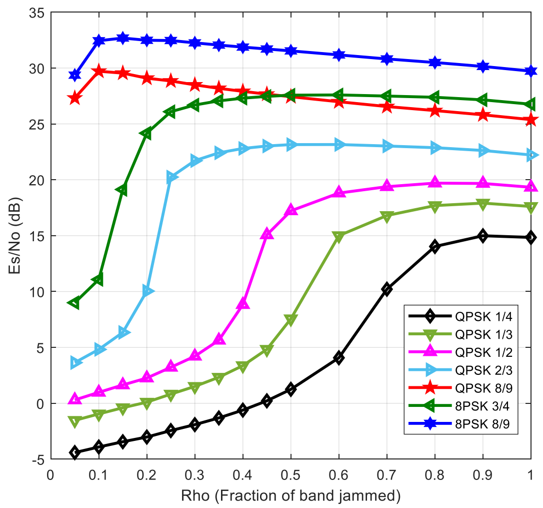


Fig. 5 Computer Simulation, Required SNR to achieve CWER = $10^{-3}$ for QPSK and 8PSK, rate ={1/4, 1/3, 1/2, 2/3, 3/4, 8/9}, under PBNJ, $\frac{J_0}{N_0} = 20\ dB$.

Except for high code rate 8/9, results shown in Fig. 5 also indicate that FB noise jamming is the optimal jamming and there is no advantage of PBNJ against PTW. For high code rate 8/9, the interleaver is not big enough to reduce the burst errors sufficiently to force the jammer to operate FB noise jamming.

## IV. Conclusion

Using closed-form expression of the outage probability under benign (AWGN) environment, we derived a closed form expression for the outage probability under PBNJ environment. Using this result, under high jammer power, we showed that there is no jamming advantage of PBNJ against FB noise jamming when interleaving and DRA or ACM are employed. Previous PTW simulation results confirmed these analytical results.


## Acknowledgment

The authors would like to thank the reviewers for all the comments.


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