

# The Performance of Iterative Non-Coherent Slow FH-CPM with Partial-Band Jamming

Xiyun Zhu, Qiang Li, Shaoqian Li and Jun Zhou

National Key Laboratory of Communications/University of Electronic Science and Technology of  
China, Chengdu, P.R. China  
{zhuxiyun, liqiang, lsq, zhoujun\_1698} @uestc.edu.cn

**Abstract**—In this paper, a partial-band jamming suppression algorithm is proposed for a slow frequency hopping system with continuous phase modulation (CPM). The algorithm is based on a non-coherent iterative maximum a posteriori (MAP) detection and the jamming state information (JSI) estimation. The simulation results demonstrate that the algorithm performs well for the partial-band jamming suppression. Moreover, the differences of the performance due to varieties of demodulation parameters are discussed.

## I. INTRODUCTION

Continuous phase modulation (CPM) combined with the spread spectrum technique of frequency hopping (SS-FH) is attractive for power and bandwidth-limited tactical wireless military communications. The CPM signals offer a narrow spectral main lobe and extremely low spectral side lobes which satisfy the requirements of good spectral efficiency. Additionally, CPM signals make it possible to use simple but low-cost power efficient amplifiers (nonlinear). And Slow FH, multiple symbols are transmitted during one hop, can maintain phase continuous during each hop interval, which has an anti-jamming performance. However, the phase discontinuity exists between adjacent hop intervals in slow FH. Consequently, the carrier phase recovery is required in coherent demodulation [1]. And the carrier phase should be found through a suitable preamble signal when the phase is discontinuous. But this may reduce the spectral efficiency of the scheme. Therefore, to maintain the spectral efficiency, non-coherent demodulation [2] appears to be a more viable solution.

To improve the performance of non-coherent demodulation, joint iterative demodulation and decoding has been introduced [3], [4]. The benefit of this scheme is that the extrinsic information on the coded bits extracted from the output of the a posteriori probabilities (APP) decoder can be utilized again by the demodulator as updated a priori information. However, it requires a suitable soft-in soft-out (SISO) CPM demodulation algorithm. Due to the memory of CPM signal, the high demodulation complexity makes it difficult to be applied widely. In [2] and [5], a maximum

likelihood block detection method and a non-coherent iterative MAP algorithm have been proposed, respectively. But their complexity increases exponentially against the length of one hop. So it is not feasible for a long hop interval. As a development, a reduced-complexity iterative tree search algorithm has been proposed in [6], [7], in which the boundary effect at each hop is not negligible.

In this paper, a finite length of observation window [8] MAP algorithm is investigated to reduce the complexity. By this method, the complexity will not increase significantly as the length of one hop increases. What's more, we can set the length of observation window and thus enable greater freedom when dealing with complexity versus performance. By the algorithm, the anti-jam performance of joint iterative demodulation and decoding in slow FH-CPM systems has been investigated in presence of partial-band jamming (PBJ). Finally, an iterative jamming state information estimation method is exploited to improve the anti-jam performance of turbo detection systems.

The remainder of this paper is organized as follows. The model of the investigated system is presented in Section II. Section III describes the demodulation algorithm. JSI estimation is treated in Section IV. Finally, the simulation results are provided in section V.

## II. SYSTEM DESCRIPTION

A block diagram of slow FH-CPM communication system considered in this paper is shown in Fig.1. The transmitter includes a recursive systematic convolutional (RSC) encoder, a random interleaver, a CPM modulator and a frequency hopper. The receiver includes a JSI estimator, a deinterleaver, an interleaver, a non-coherent SISO demodulator and a SISO MAP decoder. At each time instant,  $m$  bits of the independent binary data sequence  $a$  are encoded by a rate  $-m/n$  RSC code resulting in the coded binary sequence  $b$ . The coded binary sequence  $b$  is then randomly interleaved in to the sequence  $\tilde{b}$ . Then the sequence  $\tilde{b}$  is mapped on the symbols  $c_k \in \{\pm 1, \pm 3, \pm 5, \dots, \pm (M-1)\}$  and the sequence is input into the CPM modulator.

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The baseband equivalent form of the full response CPM signal is given by:

$$s(t, c) = \sqrt{\frac{2E}{T}} e^{j\phi(t, c)} \quad (nT \leq t \leq (n+1)T) \quad (1)$$

where  $E$  is the energy of per symbol,  $T$  is the symbol interval, and  $c$  is the M-ary information sequences. The information is contained in the phase:

$$\phi(t, c) = \pi h \sum_{k=-\infty}^{n-1} c_k + 2\pi h c_n q(t - nT_s) \quad (2)$$

where  $h$  is the modulation index and  $q(t) = \int_0^t g(t)dt$  is the phase response function.

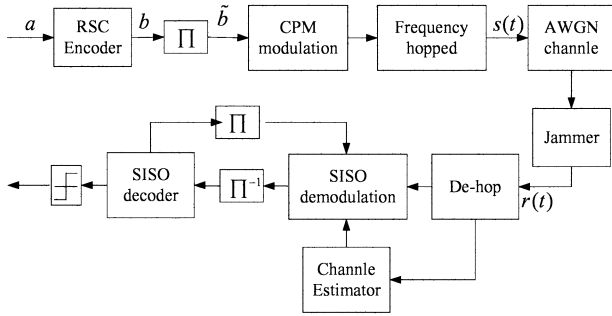


Fig. 1. The block diagram of the system.  $\Pi$  and  $\Pi^{-1}$  represent the interleaver and the deinterleaver, respectively.

After passing through the AWGN channel, the baseband equivalent received signal has the form

$$r(t) = s(t)e^{j\theta(t)} + n(t) \quad (3)$$

where  $n(t)$  is an AWGN sample and  $\theta(t)$  is an unknown carrier phase error. The phase error is assumed to be constant ( $\theta(t) = \theta$ ) over each hop interval and uniformly distributed over  $(-\pi, \pi)$ .

### III. NON-COHERENT SOFT-INPUT SOFT-OUTPUT (SISO) DEMODULATION

Without loss of generality, we consider the detection of the transmitted symbols on a hop-by-hop basis. During each hop interval, the conditional probability of  $r(t)$  given  $c_0^n$  and  $\theta$  for an n-symbol observation generalizes to

$$p\{r(t)_0^{nT} | c_0^n, \theta\} = K \exp\left(-\frac{1}{2\sigma^2} \int_0^{nT} |r(t) - s(t, c_0^n) e^{j\theta}|^2 dt\right). \quad (4)$$

The dependence on  $\theta$  can easily be removed

$$\begin{aligned} p\{r(t)_0^{nT} | c_0^n\} &= \frac{1}{2\pi} \int_0^{2\pi} p\{r(t)_0^{nT} | c_0^n, \theta\} d\theta \\ &= K' I_0\left(\frac{1}{\sigma^2} \left| \int_0^{(n+1)T} r(t) * s^*(t, c_0^n) dt \right| \right) \end{aligned} \quad (5)$$

where  $K'$  is a constant which is independent of  $s(t)$ ,  $\sigma^2$  is the noise variances and  $I_0(x)$  is the modified zero order Bessel function of the first kind with argument  $x$ .

Although an expression for the desired conditional probability is given in (5), its usefulness is limited. Since  $c_0^n$  is

not known at the receiver, optimal decoding requires that all the possibilities be explored but this is not feasible when  $n$  is large. Therefore, a finite observation window  $Z$  is used to reduce complexity. We assume that [7]

$$p\{r(t)_0^{(n+1)T} | c_0^n\} = p\{r(t)_{(n-Z)T}^{(n+1)T} | c_{n-Z}^n\}. \quad (6)$$

We build a “pseudo trellis” instead of considering the actual “phase trellis”, based on the  $Z$ . Although the system performance may degrade by limiting the size of the observation window, it does also limit the complexity of the system.

So, the (5) can be expressed in the simplified form

$$p\{r(t)_{(n-Z)T}^{(n+1)T} | c_{n-Z}^n\} = K' I_0\left(\frac{1}{\sigma^2} X_{n+1}(c_{n-Z}^n)\right) \quad (7)$$

where

$$X_{n+1}(c_{n-Z}^n) = \left| \int_{(n-Z)T}^{(n+1)T} r(t) * s^*(t, c_{n-Z}^n) dt \right|. \quad (8)$$

By using Bayes' rule with (7), it follows that

$$\begin{aligned} p\{r(t)_{nT}^{(n+1)T} | c_{n-Z}^n, r(t)_{(n-Z)T}^{nT}\} &= \frac{p\{r(t)_{(n-Z)T}^{(n+1)T} | c_{n-Z}^n\}}{p\{r(t)_{(n-Z)T}^{nT} | c_{n-Z}^n\}} \\ &= \frac{I_0\left(\frac{1}{\sigma^2} X_{n+1}(c_{n-Z}^n)\right)}{I_0\left(\frac{1}{\sigma^2} X_n(c_{n-Z}^{n-1})\right)} \end{aligned} \quad (9)$$

where  $K''$  is a constant. Using the pseudo trellis, with the branch metrics given in (9), it is straightforward to calculate the APP's. Note that

$$p(c_n = c | r(t)) = p(c_n = c) \sum_{(m, m') \in S} \alpha_n(m) \gamma_n(m, m') \beta_{n+1}(m') \quad (10)$$

where  $S$  defines the state space and the state at time  $n$  as

$$S_n = c_{n-Z}^{n-1}.$$

$$\alpha_n(m) = p(S_n = m | r(t)_0^{nT}) \quad (11)$$

$$\beta_{n+1}(m') = \frac{p(r(t)_{(n+1)T}^{nT} | S_{n+1} = m', r(t)_0^{(n+1)T})}{p(r(t)_{(n+1)T}^{nT} | r(t)_0^{(n+1)T})} \quad (12)$$

$$\begin{aligned} \gamma(m, m') &= p(r(t)_{nT}^{(n+1)T} | S_n = m, S_{n+1} = m', r(t)_0^{nT}) \\ &= p(r(t)_{nT}^{(n+1)T} | S_n = m, S_{n+1} = m', r(t)_{(n-Z)T}^{nT}) \\ &= p(r(t)_{nT}^{(n+1)T} | c_{n-Z}^n, r(t)_{(n-Z)T}^{nT}) \end{aligned} \quad (13)$$

$\alpha_n(m)$  and  $\beta_{n+1}(m')$  can be computed recursively using the APP algorithm. That is

$$\begin{aligned} \alpha_{n+1}(m') &= p(S_{n+1} = m' | r_0^{nT}) = \sum_{m \in S} \sum_{c \in M} p(S_n = m, c_n = c | r_0^{nT}) \\ &= \sum_{m \in S} \sum_{c \in M} p(r_{(n-1)T}^{nT} | S_n = m, c_n = c) / p(r_{(n-1)T}^{nT} | r_0^{(n-1)T}) \\ &= \frac{p(c_n = c)}{p(r_{(n-1)T}^{nT} | r_0^{(n-1)T})} \sum_{m \in S} \sum_{c \in M} \alpha_n(m) \times \gamma_n(m, m'). \end{aligned} \quad (14)$$

The recursion is initialized with  $\alpha_0(m) = 1$  for  $m = 0$  and  $\alpha_0(s) = 0$  otherwise, since the encoder is initialized to the zero state. Similarly

$$\beta_n(m) = \frac{p(c_n = c)}{p(r_{(n-1)T} | r_0^{(n-1)T})} \sum_{c \in M} \gamma(m, m') \beta_{n+1}(m') \quad (15)$$

with the initial condition of  $\beta_{N+1} = 1/M^Z$  for all terminate state.

From the above analysis we know that for each hop interval, The complexity of [5], which is based on the maximum likelihood block detection, is  $O(2^{Nh})$  ( $Nh$  is the number of symbols per hop), correspondingly a finite observation window detector's complexity is  $O(2^Z)$ . In slow FH system,  $Nh \gg Z$ , so it is obvious that the complexity of the latter much lower than the former.

To reduce complexity, we make use of the Log-MAP algorithm in computing (10). Furthermore we can use following equation making further simplification.

$$\log(I_0(x)) \approx x \quad ; x \gg 1. \quad (16)$$

#### IV. JAMMING STATE INFORMATION ESTIMATION

Let  $W_j$  is the bandwidth of the jamming signal and  $W_{sig}$  is the bandwidth of the signal. The duty factor is  $\rho = W_j / W_{sig}$ . It means that a certain fraction of the transmission band is assumed to be jammed. And the duty factor  $\rho$  can also be viewed as the probability that a hop has been jammed. We assumed that  $z = 1$  When a hop is jammed, otherwise  $z = 0$ .

Let the received signal be represented in a hop-by-hop way by  $\mathbf{r} = [\mathbf{r}^0, \mathbf{r}^1, \dots, \mathbf{r}^{N_f}]$ , where  $\mathbf{r}^i = [r_0^i, r_1^i, \dots, r_{N_h}^i]$ .  $N_h$  is the number of symbols per hop, and  $N_f$  is the total number of hops in a frame.

If JSI is known, the channel branch transfer metric can be calculated as:

$$P(r_k | x_k, z_k = z) = \frac{1}{\sqrt{2\pi\sigma_z^2}} \exp\left\{-\frac{|r_k - x_k|^2}{2\sigma_z^2}\right\} \quad (17)$$

where  $\sigma_z^2 = N_0/2 + zN_j/2\rho$  and denotes the received noise variances including background thermal noise variances and jamming noise variances.

We define the log likelihood ratio of the  $m$ -th iteration of the  $i$ -th hop as [5]:

$$L^m(z^i) = \log \left[ \frac{P(r^i | z^i = 1)P(z^i = 1)}{P(r^i | z^i = 0)P(z^i = 0)} \right]. \quad (18)$$

The conditional probability of  $z^i$  is given by:

$$P(r^i | z^i = z) = \frac{1}{(2\pi\sigma_z^2)^{N_h/2}} \exp\left\{-\frac{1}{2\sigma_z^2} \sum_{l=0}^{N_h-1} |r_l^i - \text{sign}(r_l^i)|^2\right\} \quad (19)$$

where  $i = 0, 1, \dots, N_f - 1$ . Let  $\rho = 0.5$  at the beginning, for not knowing any jammer information. From (19), we define  $V^i = \sum_{l=0}^{N_h-1} |r_l^i - \text{sign}(r_l^i)|^2 / 2N_h$ , thus (18) can be simplified as:

$$L^m(z^i) = \ln \left\{ \frac{\rho}{1-\rho} \cdot \frac{\sigma_0^{N_h}}{\sigma_1^{N_h}} \cdot \exp\left[(\sigma_0^{-2} - \sigma_1^{-2})N_h V^i\right] \right\}. \quad (20)$$

It's obvious that  $L^m(z^i)$  is a monotonic function of  $V^i$  referring to (20). Let  $L^m(z^i)$  equal to zero, and the zero point  $(\varepsilon, 0)$  of function  $L^m(z^i)$  is calculated as.

$$\varepsilon = K_\sigma \cdot \left\{ \frac{1}{2} \log\left(\frac{\sigma_1^2}{\sigma_0^2}\right) + \frac{1}{N_h} \log\left(\frac{1-\rho}{\rho}\right) \right\} \quad (21)$$

where  $K_\sigma = \sigma_1^2 \sigma_0^2 / (\sigma_1^2 - \sigma_0^2)$ . If  $V^i > \varepsilon$ , let  $z^i = 1$ , or  $z^i = 0$ . The duty factor  $\rho$  is updated by  $\rho = N_1 / N_f$ . Here  $N_1$  denotes the number of jammed hops. The Gaussian noise variances and the jamming variances can be estimated by:

$$\begin{aligned} \sigma_1^2 &= \frac{1}{N_1} \sum_{i \in z_j} \sum_{l=0}^{(i+1)N_h-1} |r_l^i - \text{sign}(r_l^i)|^2 / 2N_h \\ \sigma_0^2 &= \frac{1}{N_1} \sum_{i \notin z_j} \sum_{l=0}^{(i+1)N_h-1} |r_l^i - \text{sign}(r_l^i)|^2 / 2N_h. \end{aligned} \quad (22)$$

Since  $|r_l^i - \text{sign}(r_l^i)|^2 \leq |r_l^i - s_l^i|^2$ , the original values of variances are generally greater than the estimated one, which, however, becomes more accurate by multiplying a modifying factor. The factor is determined by the distribution of the noise and jamming.

#### V. SIMULATION RESULTS

In this section, the BER performance of the proposed noncoherent systems is investigated by means of computer simulation. The system is tested with a rate 1/2, the RSC generating matrix  $G = (17, 15)_8$  and full-response binary CPM with modulation index  $h = 1/2$ . An interleaver of length 2000 is used and the maximum number of iterations at the receiver is set to 5. A signal-to-thermal noise density ratio of  $E_b / N_0 = 20\text{dB}$  is used for all simulation.

From the results shown in Fig.2, we can see how the duty factor affects the bit error probability. When  $E_b / N_j$  is less than  $2.9\text{dB}$ , the BER performance improves as the duty factor decreases. But this situation is almost inversed when  $E_b / N_j$  becomes larger, and the BER performance is close to each other. Notably, the performance is quite different from the others when  $\rho = 1.0$ . Although almost every hop is jammed; the total average power of jammer is constant. So the result of the Fig. 2 is reasonable.

Fig.3 shows the BER performances with different number of iterations by setting the duty factor ( $\rho = 0.6$ ). The BER performances after 2 iterations are unacceptable. But, as the number of iterations increasing, the BER performance is improved obviously. After 5 iterations the BER becomes  $10^{-3}$  at  $2\text{dB}$ . However the system performance may not always increase with the rise of the iterative number because the extrinsic information has been completely used.

Fig.4 and Fig.5 show the effect of  $N_h$  and  $Z$  on the BER performances. If  $N_h \geq 10$  the algorithm works well, otherwise, the loss of performance is prominent. When the  $N_h$  and  $Z$  increase, we can notice a giant improvement of the performance and this effect is more noticeable when  $E_b/N_j$  is large which comes at the expense of a more complicated system. But it is still acceptable comparing with [5].

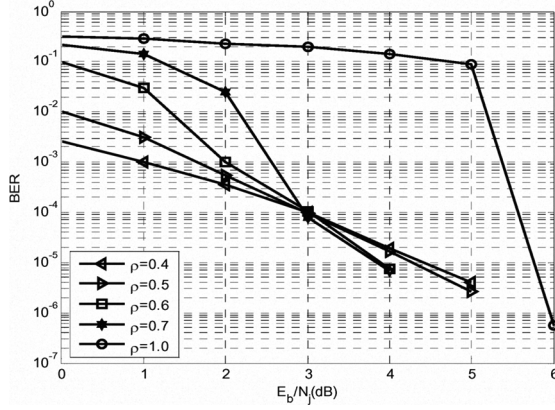


Fig. 2. The anti-jamming performance under the PBJ for different duty factor with  $N_h = 20$  and  $Z = 5$ .

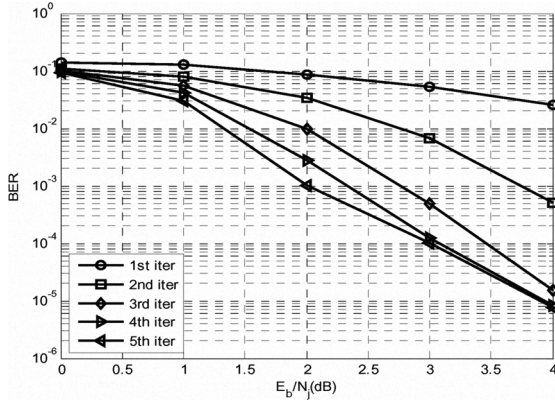


Fig. 3. The anti-jamming performance under the PBJ for different iteration with  $N_h = 20$ ,  $\rho = 0.6$  and  $Z = 5$ .

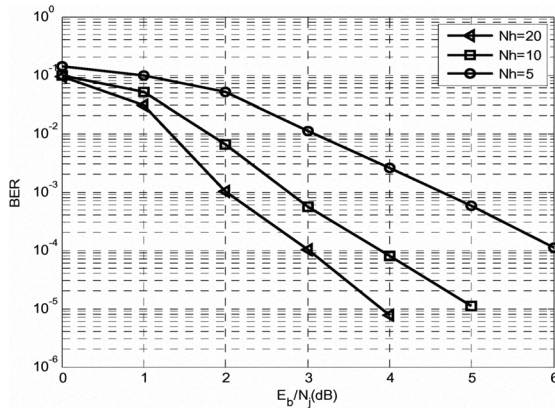


Fig. 4. The anti-jamming performance of an iterative non-coherent detector for different numbers of symbols per hop.

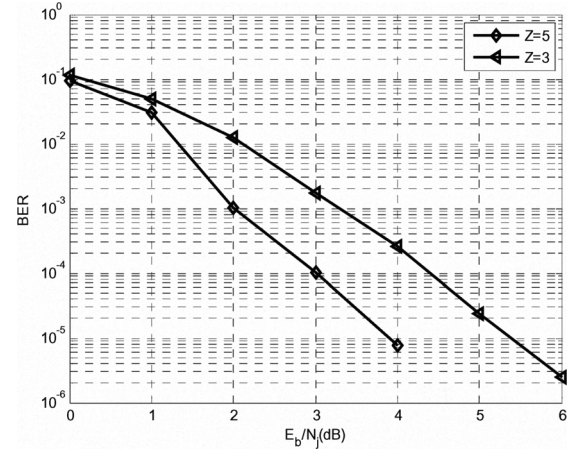


Fig. 5. Simulation results of anti-jamming performance with different length of observation window under the PBJ.

## VI. CONCLUSION

This paper has introduced and evaluated a non-coherent iterative MAP detector combined with JSI estimation scheme using CPM for application in wireless, frequency hopping, and military communication system. It is shown that the scheme can achieve good performance through iterative and appropriate parameters. It is also shown that the scheme can have greater freedom when dealing with complexity versus performance. In addition, the scheme can be employed in tactical wireless military communications to suppress jam.

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