Rapid Timing Synchronization of hybrid DS/FH system Based on Reed-Solomon Codes

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***Abstract*—Hybrid direct sequence frequency hopping (DS-FH) is wildly used in military wireless communication. In current hybrid DS-FH communication systems, the most common approaches for timing acquisition are based on time-frequency domain search, such as matching filter and serial search. These methods set up initial timing by finding the peaks in correlation value between received signals and preamble sequence template. However, when noise or partial band jamming is present, the synchronization sequence has to be very long to combat low signal-noise-ratio (SNR). In this adverse scenario, matching filter becomes too complex to implement and serial search is too time consuming. In this paper, a fast timing acquisition approach is proposed using Reed-Solomon code. Specifically, the frequency hopping pattern is controlled by cyclic codewords. Due to the cyclic property, hard decision decoding instead of correlation can be used to set up initial timing synchronization. Moreover, erasure-decoding can be used to combat partial-band jamming. The performance is evaluated by MAT and detection probability (DP). Analysis and simulation results show improvement in MAT compared to matching filter method.**

***Index Terms*—Timing synchronization, rapid acquisition, reed-solomon codes**

I. Introduction

HYBRID direct sequence frequency hopping (DS-FH) is widely used in wireless communication, especially when partial band jamming is presented[1][2]. To set up timing synchronization at the receiver, the most widely used approach is to send synchronization words, which usually include direct-sequence spread frequency hopping pattern as preamble sequences. At the receiver, correlation and searching method is used to acquire correct timing synchronization. This often includes two stages: acquisition and tracking. At the acquisition stage, a coarse timing alignment of local template and received signal is accomplished, and the timing error should be guaranteed within the locking range of the tracking stage. In this contribution, we focus on the first stage, i.e. the timing acquisition problem.

Conventional timing acquisition methods mainly employ search and detection in time-frequency domain, including matching filter (MF) and serial search (SS)[3][4]. The problem is that more frequency hops and long hopping pattern has to be used when SNR is low for their good correlation property. In this case, both MF and SS suffer from the drawback of long mean acquisition time (MAT) [5][6]. This becomes intolerable when the sequences become long [5].

On the other hand, when partial band jamming is presented, the interference will be spread onto whole frequency band due to the correlation operation[7][8]. In this case, matching filter based on erasure (MF-ERA) is proposed to mitigate the interference, where the interfered frequency band is excluded from the correlation. The detection probability will be degraded in proportion to the excluded frequencies[11]. However, the mean acquisition time is the same as conventional MF method and becomes very long when frequency hopping pattern is long.

In this letter, a fast timing synchronization method using Reed-Solomon (RS) codes is proposed. The proposed method is well suited for military systems, but is equally applicable to other communication systems where a long frequency hopping pattern is needed and fast acquisition is required. Instead of random frequency hopping sequence, RS codewords are used as preamble signals. Taking advantage of the cyclic characteristic, only part of the positions need to be tested. Therefore, the initial phase of received sequence can be computed by partial correlation and hard decision decoding. As a result, the acquisition time can be greatly reduced.

II. Synchronization scheme description

A. system model

The generation of this preamble includes two steps. First, a (n, k) RS codeword is selected which is known to both transmitter and receiver. The selected codeword will control the generation of frequency hopping pattern.

Denoting the generator matrix as $\mathbf{G}$, the selected codewords can be written as

$$\mathbf{c_t} = \mathbf{x_t}\mathbf{G} \tag{1}$$

where $\mathbf{x_t} = [x_{k-1} \dots x_0]$, $x_i \in GF(2^m)$ and $\mathbf{c_t} = [c_{n-1} \dots c_0], x_i \in GF(2^m)$.

The second step is to modulate the spreading sequence on each hopping frequency. The hopping sequence is generated according to selected RS codeword. For example, if the number of available frequencies is $q = 2^m$, we need to generate a RS codeword on $GF(q)$, and assign a frequency to each symbol on $GF(q)$. Without loss of generality, we propose a method for generating frequency hopping sequence as followed. Denote the symbols on $GF(q)$ as $s_i \in GF(q)$ and the available frequency band of DS-FH system is $[W_L, W_H]$,

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we have that the central frequency assigned to s_i takes the form as

$$f(s_i) = W_L + (s_i + 1/2)\frac{W_H - W_L}{q} \tag{2}$$

Denote the spreading sequence is $\mathbf{w} = [w_0, \dots w_{N_c-1}]$, $w_i \in [-1, 1]$, the transmitted signals can be written as

$$s(t) = \sqrt{E_c}\sum_{i=0}^{n-1}\sum_{k=0}^{N_c-1} w_k cos[2\pi f(c_j)(t - jT_h)]g(t - kT_c - jT_h), \tag{3}$$

where $f(c_j)$ is the frequency generated by c_j according to (2), T_h is the dwell time of each hop, T_c is the chip duration, $g(t)$ is the shaping pulse.

For partial band interference channel with additive white Gaussian noise (AWGN), the received signals become $r(t) = s(t-\tau)e^{j\theta} + n(t) + I(t)$, where τ is the time delay, $n(t)$ is Gaussian random process with spectral density $N_0/2$ and $I(t)$ denotes the interference. The term θ models the carrier phase at the receiver for conventional spread spectrum systems. As this letter aims to address the problem of fast acquisition algorithm, coherent detection approach is assumed for analysis simplicity. Therefore, in the following analysis, it is assumed that $\theta = 0$.

We assume that the receiver samples the received signal at a time interval of T_c/N, where N indicates the oversampling rate. As the locking range of tracking loop are usually $T_c/2$, a successful acquisition can be declared if $|\hat{\tau} - \tau| \leq T_c/2$. The perfect sampling position can be estimated at tracking stage provided there are a sufficient number of samples per chip [12]. As only the acquisition stage is considered, it is reasonable to assume that $N = 1$ and the sampling position is perfect to simplify the analysis. Assuming the time delay is $\tau = dT_c$, and $p = \lfloor \frac{d}{M} \rfloor, q = mod(d, M)$, the mth sample on jth frequency hop can be written as

$$r_{jm} = \begin{cases} \sqrt{E_c}w_{m+q} + n_{jm} + z_{jm}, & c_{\lfloor m/N_c \rfloor - p} = j \\ n_{jm} + z_{jm}, & c_{\lfloor m/N_c \rfloor - p} \neq j \end{cases}. \tag{4}$$

where n_{jm} is a Gaussian distributed random variable with variance N_0, z_{jm} is sample of interfere signal which will be addressed later.

Sampled signal will be correlated with spreading sequence, and the correlated value is used to decide if correct timing has been acquired. On each hopping frequency, there will be n correlated values and the output of correlators can be written as

$$\mathbf{Y} = \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ y_{q1} & y_{q2} & \cdots & y_{qn} \end{pmatrix} \tag{5}$$

where y_{ji} is the accumulated energy on the jth hopping frequency at the ith hopping dwell time, which can be further written as

$$y_{ji} = \frac{1}{N_c}\sum_{m=iNc}^{(i+1)N_c-1} r_{jm}w_m \tag{6}$$

According to (4), (6) can be further written as

$$y_{ji} = A_{ji} + \varepsilon_{ji} + \vartheta_{ji} \tag{7}$$

Assuming $\mathbf{w}$ has perfect self-correlation property, we have

$$A_{ji} = \; = \begin{cases} \sqrt{E_c}, & j = c_{i-p}, q = 0 \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

B. synchronization algorithm under AWGN channel

Assuming the channel is AWGN, (4) can be simplified as

$$y_{ji} = \begin{cases} \sqrt{E_c} + \varepsilon_{ji}, & j = c_{i-p}, q = 0 \\ \varepsilon_{ji}, & \text{otherwise} \end{cases} \tag{9}$$

where ε_{ji} are independent and identically distributed random variable with Gaussian distribution and the variance is $\sigma^2 = N_0/N_c$. To simplify the analysis, we denote the decision variable for each symbol as $\mathbf{y}_i = [y_{0i}, y_{1i}, \dots, y_{(q-1)i}]^{\mathrm{T}}$, which is the accumulated energy on all hopping frequencies at ith dwell time in the following description.

When the testing cell satisfies the condition $\tau = pMT_c$, i.e. $q = 0$, it is referred to as an in-phase cell; otherwise, it is called an out-of-phase cell.

Denote $\mathbf{c_r} = [\hat{c}_0, \dots, \hat{c}_{n-1}]$, where $\hat{c}_i$ is the hard-decision symbol estimated according to

$$\hat{c}_i = \arg\max_j \mathbf{y}_i \tag{10}$$

It can be seen that $\mathbf{c_r}$ takes the form of

$$\mathbf{c_r} = [c_p, \dots, c_{n-1}, c_0, \dots, c_{p-1}] + [E_0, \dots, E_{n-1}]. \tag{11}$$

The first part of $\mathbf{c_r}$ is a cyclic shifted version of $\mathbf{c_t}$ given by

$$\rho(\mathbf{c_t}, p) = \mathbf{c_r} = [c_p, \dots, c_{n-1}, c_0, \dots, c_{p-1}]. \tag{12}$$

According to the cyclic property of RS codes, $\rho(\mathbf{c_t}, p)$ is also a legal codeword of the chosen (n, k) RS code. The second part of $\mathbf{c_r}$ is an error pattern denoted as $\mathbf{E}$. The number of nonzero elements in $\mathbf{E}$ is the number of errors, which can be denoted as N_e. Denoting the decoded codeword by $\mathbf{c_d}$, if we have $N_e < (n-k)/2$, the decoding will succeed and $\mathbf{c_d} = \rho(\mathbf{c_t}, p)$. Otherwise, the decoding process fails.

If the decoding succeeds, there are two ways to estimate p if the transmitted codeword is known to the receiver. First method is to find the location of $\mathbf{x_t}$ in $\rho(\mathbf{c_t}, p)$. For systematic RS code, $\mathbf{x_t} = [c_0, \dots, c_{k-1}]$. Therefore, according to (12), p can be estimated once $\mathbf{x_t}$ is located. This can be realized by simple shift and comparison operations. The other method is to use look-up table (LUT). As RS code is cyclic code, the first k symbols in $\rho(\mathbf{c_t}, p)$ uniquely determine the value of p. The first method is simple but need extra time. The time needed will not exceed nT_c. The second method won't need extra time, but a ROM will be needed to implement the LUT. As only n positions need to be looked up, the size of this ROM is negligible. Therefore, the LUT approach may be favorable when rapid synchronization is required.

If the decoding process fails, the observed sequence will be shifted by T_s, and the decoding process and the correlation is repeated. If the incoming sequence is a preamble sequence, a successful timing acquisition will be declared with at most N_cT_c/T_s shifts. Therefore, if acquisition is not successfully declared after all N_cT_c/T_s positions are tested, it is considered that this incoming sequence is not a preamble and the local observation window will shift by nN_cT_c.

C. synchronization algorithm under interference channel

In this paper, we assume the partial band jamming is a wide band noise, which can be modelled as

$$I(t) = \int_{-\infty}^{\infty} \sqrt{P_j} N(t) * w(t-\tau) d\tau \tag{13}$$

where $*$ indicate the operation of convolution, P_j is the transmit power of the jammer and $w(t)$ is a window function with spectrum as

$$W(j\omega) = \begin{cases} 1, & W_0 \le \omega \le W_1 \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

Assuming $\lambda = \frac{W_1 - W_0}{W_H - W_L}$, $\lambda 2^m$ frequencies be interfered at the receiver. Without loss of generality, we assume that $f(0), \dots, f(\lambda M - 1)$ is interfered, where $M = 2^m$. Therefore, (6) becomes

$$y_{ji} = \begin{cases} \sqrt{E_c} + \varepsilon_{ji} + \vartheta_{ji}, & c_i = j, j < \lambda M - 1 \\ \sqrt{E_c} + \varepsilon_{ji}, & c_i = j, j > \lambda M - 1 \\ \varepsilon_{ji} + \vartheta_{ji}, & c_i \ne j, j < \lambda M - 1 \\ \varepsilon_{ji}, & c_i \ne j, j > \lambda M - 1 \end{cases} \tag{15}$$

Due to the presence of partial band noise, the symbol error rate will increase dramatically. To improve the probability of successful decoding, we propose a synchronization algorithm based on generalized minimum distance (GMD) algorithm. This algorithm erase interfered signal by estimating the reliability of the hard decision. The diagram of proposed synchronization algorithm is shown in Fig. 2. At the receiver, the posterior probability of ith symbol is given by

$$\begin{aligned} P(c_i = x|\mathbf{y}_i) &= \frac{P(c_i = x, \mathbf{y}_i)}{\sum_{j=0}^{q-1} P(c_i = x, \mathbf{y}_i)} \\ &= \frac{P(\mathbf{y}_i|c_i = x)P(c_i = x)}{\sum_{j=0}^{q-1} P(\mathbf{y}_i|c_i = x)P(c_i = x)} \end{aligned} \tag{16}$$

Assuming the transmitted symbol is uniformly distributed, we have $P(c_i = x) = 1/q$. Therefore $P(\mathbf{y}_i|c_i = x)$ can be further written as

$$P(y_{ji}|x_i = c_j) = \begin{cases} \mathcal{N}(y_{ji}, \sqrt{E_c}, \sqrt{\sigma^2 + P_j/(\lambda M)}), & c_i = j, j < \lambda M - 1 \\ \mathcal{N}(y_{ji}, \sqrt{E_c}, \sigma), & c_i = j, j > \lambda M - 1 \\ \mathcal{N}(y_{ji}, 0, \sqrt{\sigma^2 + P_j/(\lambda M)}), & c_i \ne j, j < \lambda M - 1 \\ \mathcal{N}(y_{ji}, 0, \sigma), & c_i \ne j, j > \lambda M - 1 \end{cases} \tag{17}$$

where $\mathcal{N}(y, \mu, \gamma) = \frac{1}{\gamma\sqrt{2\pi}} \exp \frac{-(y-\mu)^2}{\gamma^2}$ is the probability density function of normal distributed random variable.

When the $f(c_i)$ is not interfered, i.e. $c_i > \lambda M$, $P(y_{ji}|x_i = c_j)$ should be greater than the posterior probability of other

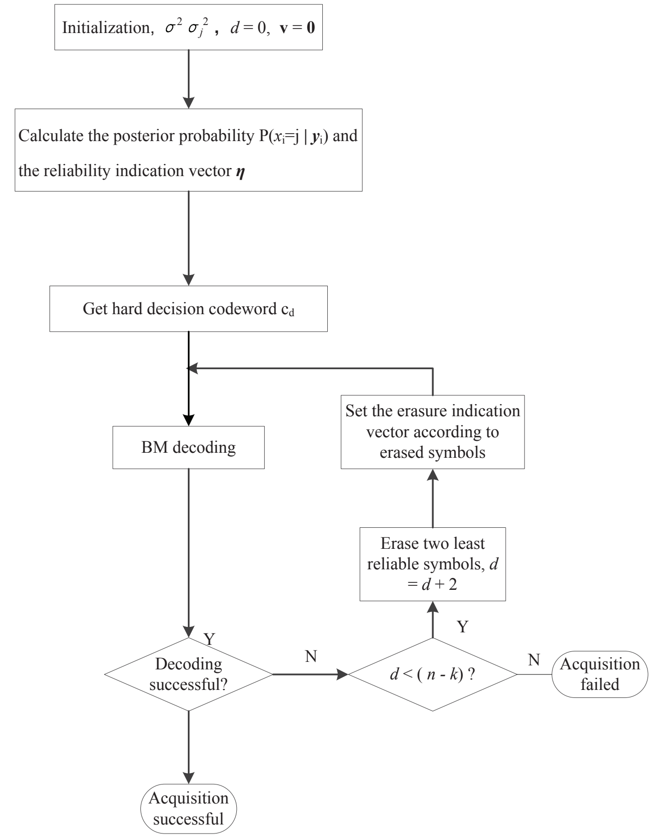

Fig. 1. Synchronization algorithm under partial band interference channel

hopping frequency. Based on this principle, the reliability of $\mathbf{y}_i$ can be written as

$$\eta = \frac{max_j P(c_i = j|\mathbf{y}_i)}{avg P(c_i = j|\mathbf{y}_i)} \tag{18}$$

where

$$avg P(c_i = j|\mathbf{y}_i) = \frac{1}{q} \sum_{j=0}^{q-1} P(c_i = x|\mathbf{y}_i) \tag{19}$$

The larger the value of η is, the more reliable the hard decision of $\mathbf{y}_i$ will be.

In GMD algorithm, erasure and decoding is implemented iteratively. At each iteration, two least reliable symbol is erased, and the hard decision codeword and erasure indication vector is passed to BM decoding algorithm. Assuming there is t erasures and e errors, then decoding process will succeed as long as $2t + e + 1 < n - k + 1$.

III. Performance analysis

In this section, based on Neyman-Pearson criterion [15], the DP and false alarm rate (FAR) are analyzed. A false alarm happens when the testing cell is an out-of-phase cell but mistaken as an in-phase cell. In this case, the decision variable for each symbol takes the form of

$$y_{ji} = \varepsilon_{ji} \tag{20}$$

under AWGN channel and

$$y_{ji} = \begin{cases} \varepsilon_{ji} + \vartheta_{ji}, & c_i \neq j, j < \lambda M - 1 \\ \varepsilon_{ji}, & c_i \neq j, j > \lambda M - 1 \end{cases} \quad (21)$$

under interference channel. It can be seen that each decision element in $\mathbf{y}_i$ is independent and identically distributed (i.i.d) random variable. The decision of ith symbol $\hat{c}_i$ can be written as

$$P(\hat{c}_i = m) = P(\max(\mathbf{y}_i) = y_{mi}) = \frac{1}{M}, \quad (22)$$

where $P(\cdot)$ denotes the probability density function for continuous variables and probability distribution function for discrete variables. As y_{mi} are i.i.d variable, $\hat{c}_i$ is uniformly distributed over $GF(2^m)$. Therefore, the false alarm happens when a random sequence is decoded as the $\mathbf{c_t}$ or $\rho(\mathbf{c_t}, j)$, which can be written as

$$P_{fa} = mnq^{(k-n)} \sum_{i=0}^{(n-k)/2} \binom{n}{i}, \quad (23)$$

where $(n-k)/2$ is the number of errors that can be corrected by decoding. This is the un-detectable error rate of RS code. It happens when the distance of a random sequence between the desired codeword is less than $(n-k)/2$.

For synchronization algorithm based on erasure decoding, the erased symbols are also uniformly distributed random variable. Assuming the transmitted frequency hopping pattern was chosen from m RS codewords, then the false alarm rate can be written as

$$P_{fa} = mnq^{(-k)} \quad (24)$$

On the other hand, a successful acquisition happens when there are less than $(n-k)/2$ errors in $\mathbf{c_d}$. Therefore, the probability of detection can be written as

$$P_d = \sum_{j=0}^{(n-k)/2} B(j, n, P_e) \quad (25)$$

where $B(i,n,p) = \binom{n}{i} p^i (1-p)^{n-i}$, P_e is the symbol error rate (SER) under AWGN channel which can be further written as

$$P_e = 1 - \int_{-\infty}^{\infty} Pr(y^m = r) \prod_{j \neq m} Pr(v^j < r) dr \quad (26)$$

For a preamble sequence of length $L = nN_c$, conventional synchronization algorithm will declare an acquisition only when the time delay is multiples of LT_c. However, the proposed algorithm can successfully declare acquisition as long as the time delay is multiples of LT_c/n due to the cyclic property of RS codes. Therefore, the MAT can be greatly reduce. As the false alarm rate is extremely low using this method, there is no confirmation state in this algorithm. Thus the mean acquisition time is

$$E\{T_{acq}^{Pro.}\} = \sum_{k=0}^{\infty} \Big(knN_c + \sum_{i=1}^{N_c} \frac{N_c - i}{N_c} \Big) P_d \big(1 - P_d\big)^k \quad (27)$$

It is seen that considering the most optimistic case where $P_d = 1$, the MAT of proposed algorithm is reduced by n times compared with matching filter.

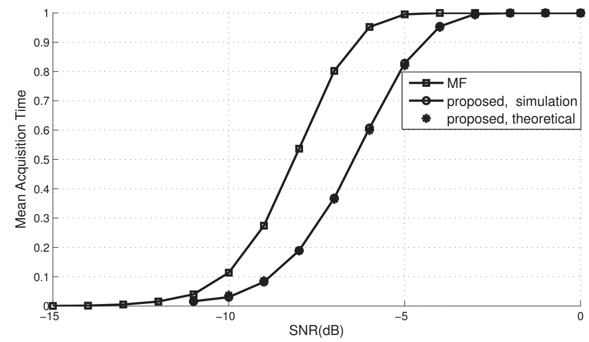

Fig. 2. DP under AWGN channel. (15,3) RS code is used in proposed algorithm. $N_c = 1$. $SNR = E_c/\sigma^2 = N_c E_c / N_0$.

IV. Simulation results

In this section, some numerical simulation results are presented. Without loss of generality, N_c is assumed to be 1.

The performance of acquisition under AWGN channel is presented in fig. 2 and fig. 3. A $(15,3)$ RS codeword is used as frequency hopping pattern. According to (21), the false alarm rate is 5×10^{-10}. The performance of proposed algorithm is compared with matching filter. For the sake of fairness, the frequency hopping pattern length is 15 and the false alarm rate is set as 5×10^{-10}. It can be seen in fig. 2 that the proposed algorithm is slightly worse then matching filter for about $2dB$. The simulation results is in accordance with the theoretical analysis, which proves the correctness of our analysis. On the other hand, the mean acquisition time shown in fig. 3 shows great improvement over matching filter. It can be seen that as SNR increases, the detection probability becomes 1, and the MAT of proposed algorithm is 1 while the MAT of matching filter is directly proportional to the length of frequency hopping pattern.

Fig. 4 and fig. 5 shows the detection probability in presence of pulse jamming with $\lambda = 0.3$ and $\lambda = 0.5$ respectively. The jamming-signal-ratio is set as $JSR = P_j/(N_c E_c) = 15dB$. $(31,5)$ RS codeword is used as hopping pattern. In this case, the false alarm rate is $P_{fa} = 9 \times 10^{-7}$. The performance is compared with matching filter (MF) and matching filter with erasure (MF-ERA). It can be seen that the detection probability of MF is mainly affected by the partial band jamming and remains under 50% even when SNR is high. The detection MF-ERA is greatly improved compared to MF, while the proposed algorithm is better than MF and worse than MF-ERA.

As for the mean acquisition time, it can be seen in fig. 6 that the acquisition of MF takes the longest time due to low detection probability, and the MAT of MF-ERA is directly proportional to the pattern length when SNR is high. The MAT of proposed algorithm is slightly larger than that of MF-ERA when SNR is low but becomes much smaller as SNR increases.

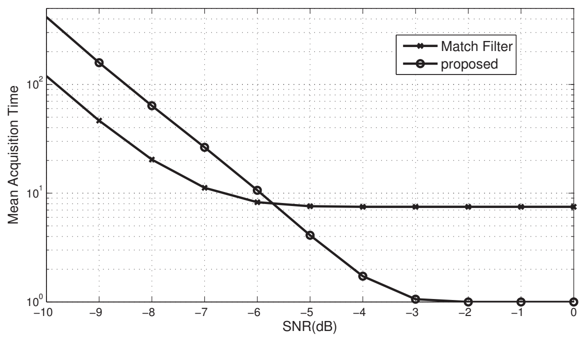

Fig. 3. Mean Acquisition time under AWGN channel. (15,3) RS code is used in proposed algorithm. $N_c = 1$. $SNR = E_c/\sigma^2 = N_cE_c/N_0$.

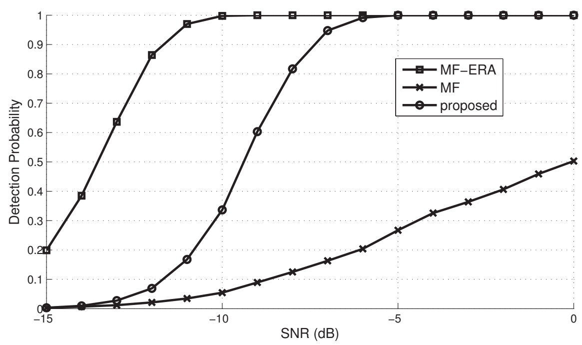

Fig. 4. DP under partial band interference channel. (31,5) RS code is used in proposed algorithm. $N_c = 1$. $SNR = E_c/\sigma^2 = N_cE_c/N_0$. JSR = P_J/E_s=15dB, $\lambda = 0.3$.

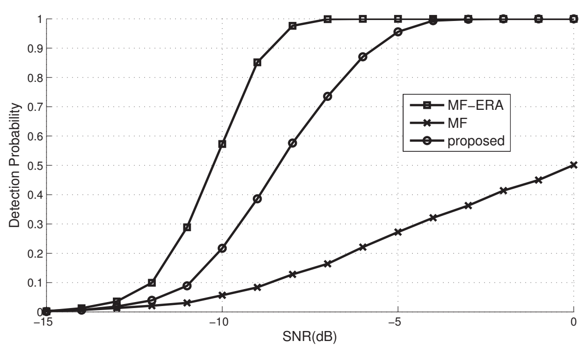

Fig. 5. DP under partial band interference channel. (31,5) RS code is used in proposed algorithm. $N_c = 1$. $SNR = E_c/\sigma^2 = N_cE_c/N_0$. JSR = P_J/E_s=15dB, $\lambda = 0.5$.

V. CONCLUSION

A fast timing acquisition method for DS-FH system was proposed in this paper. Instead of random sequences, RS codeword is used to control the generation of frequency hopping pattern for synchronization. At the receiver, the time delay can be estimated by partially correlating received sequences and hard decision decoding algorithm. The MAT and DP were analyzed both theoretically and numerically under AWGN as well as partial-band interference. Simulation results show that the performance of both the MAT are improved significantly under both circumstances.

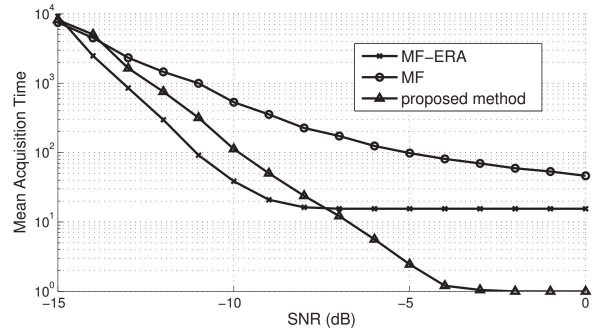

Fig. 6. DP under partial band interference channel. (31,5) RS code is used in proposed algorithm. $N_c = 1$. $SNR = E_c/\sigma^2 = N_cE_c/N_0$. JSR = P_J/E_s=15dB, $\lambda = 0.5$.

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