GF(q) LDPC Coded Spread Dimension Scheme for Anti-jamming Communications

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***Abstract*—In this paper, a novel spread dimension (SD) communication using orthogonal patterns to transmit information is proposed to achieve the reliable communication with the smarty jamming. A soft demodulator is presented and a posterior probability (APP) according to maximum a posterior probability (MAP) estimation is derived to obtain the soft information for soft iterative decoding. The demodulator is suitable for coherent reception ,non-coherent reception with the channel state information (CSI) and non-coherent reception without CSI. Besides, the capacity of the coded modulation scheme which employs GF(q) LDPC codes is compared with the bit interleaved coded modulation (BICM) scheme using the binary LDPC codes for the SD communication system. We prove that the CM scheme is able to better utilize the soft information and owns a higher achievable rate. Simulation results show that the gain of the q-ary LDPC coded SD system is up to 0.5 dB over AWGN channel and has a very good performance with the smarty jamming.**

***Index Terms*—q-ary LDPC code, spread dimension, soft demodulation, non-coherent reception, channel capacity, jamming**

I. Introduction

Spread spectrum (SS) communication systems widely used in the license free industrial, scientific and medical bands, such as Bluetooth and wireless local area network [1]. There are three types of SS systems, namely, direct sequence spread spectrum (DS-SS), frequency hopped spread spectrum (FH-SS) and time hopping spread spectrum (TH-SS). The combination of these three techniques are frequently used in a specific circumstance [2]. The current application of SS is primarily in anti-jamming communications and it performs well with most types of intentional interference.

In traditional military anti-jamming communication system, DS-SS and frequency FH-SS techniques are often employed to mitigate the jamming or interference [3][4][5]. The working spectrum in frequency is expanded by the SS method and the jammer has to enhance jamming power to maintain efficient interference effect. The SS technique has a good performance when confronted with the typical jamming such as partial band jamming and multi-tone jamming [3][4].

However, there are two question for the SS method. Firstly, The Pseudo Noise (PN) sequence or FH pattern is known to both transmitter and receiver, thus the anti-jamming performance would dramatically deteriorate when the PN sequence or FH pattern is intercepted by the jammer. Secondly, the information rate should grow along with the expansion in frequency according to Shannon theory. But the expansion in frequency do not improve the rate of original information in the SS system. These defects makes spread spectrum technology suffer enormous challenges in the anti-jamming communications.

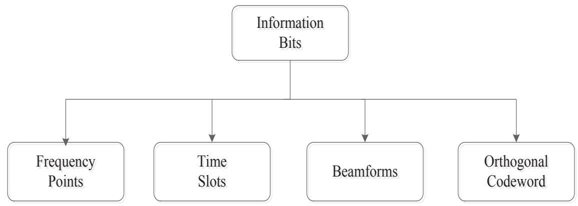

Fig. 1. Orthogonal patterns in frequency, time, space and code division

Inspired by the SS technique, we propose a a novel communication scheme which not only inherits the good performance when confronted the typical jamming, but also avoids the defects mentioned above. In this scheme, the information data is carried and embodied by different SS patterns (PN sequences or FH patterns), rather than spread by these patterns. Since these SS patterns are orthogonal in code division, frequency division and other divisions, a SS pattern could be consider a orthogonal dimension. In the proposed scheme, the signal space is expanded by using orthogonal patterns, thus, we note it as the spread dimension (SD) scheme. The SD technique not only obtains processing gain like SS method, but also utilizes information in orthogonal patterns to increase the transmission rate. As it is shown in the following part, SD method could provide reliable communication with smart jamming in which the PN sequence or FH pattern are acknowledged by the jammer.

In the SD scheme, high-performance error correction codes, such as Turbo code and the low density parity check (LDPC) code, should be employed to achieve the reliable communication [7]. Conventionally former scholars adopt the bit interleaved coded modulation (BICM) [8] method when combining binary codes and high order modulation. Additionally, to obtain a near Shannon limit performance, iterative decoding (ID) is introduced between the decoder and demodulator, noted as BICM-ID [9] - [11]. However, according to [7], the BICM capacity is still significantly inferior to the CM capacity for M-ary SD system. For example, at a $1/2$ code rate, the CM capacity outperforms the BICM capacity by approximately 2 dB for 16-ary SD in the AWGN channel. And the gap between BICM capacity and CM capacity increased along

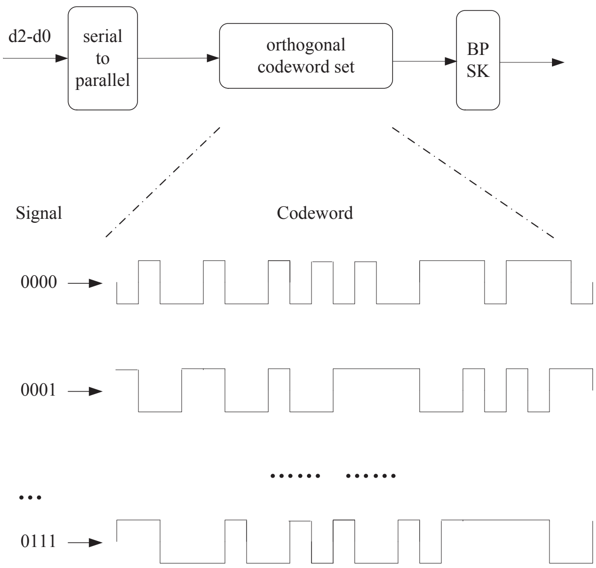

Fig. 2. 8-ary spread dimension system

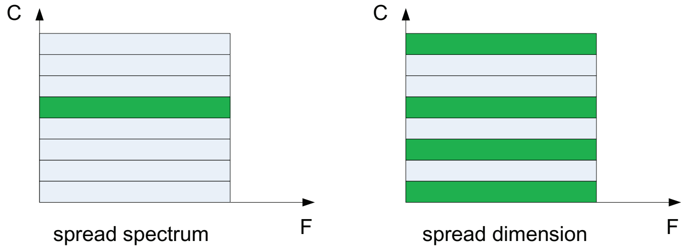

Fig. 3. Comparison between spread spectrum and spread dimension in frequency and codeword.(F stands for frequency, C stands for codeword and codeword depicted in green means the orthogonal sequence that is being used)

with increasing M. Therefore, a symbol-wise mapping scheme is proposed for the coded SD communication system where GF(q) LDPC code is employed to obtain a CM scheme effect in the SD system.

The organization and contribution of this paper is as follows. The principle of spread dimension and the structure of GF(q) LDPC coded SD system are described in Section II. Section III illustrates how to make the soft demodulation with coherent and non-coherent reception for iterative decoding. And then, theoretical analysis of capacity are presented to prove the capacity of the symbol-wise mapping is higher than the bit-wise mapping strategy. Numerical results and simulations are presented in Section IV. Finally, Section V concludes this paper.

II. System Model of Spread Dimension Communication

A. Principles of Spread Dimension System

Inspired by DS-SS and FH-SS techniques, we think the transmitted information should be expanded to as many dimensions as possible in order to enhance its anti-jamming performance. As it is presented in Fig.1, these different dimensions which is chosen by information bits are orthogonal in code space, frequency space and other spaces. The transmitted information is extended to a space with large dimensions, thus making it difficult for the jammer to intercept the transmitted information which is embodied in orthogonal patterns. In the rest of this paper, we take orthogonal patterns in code division as example for the notation simplicity, however, our analysis could be easily extended to other kinds of orthogonal patterns.

Unlike traditional SS method, the information bits is embodied by orthogonal patterns other than spread by these patterns. Fig.2 shows, in a 8-dimension SD system, the bit stream of user is converted to 8-ary symbol by every 3 bits and orthogonal sequence is chosen by this symbol. We propose a new concept named spread dimension. In spread dimension system, data of different user is directly mapped into selected orthogonal sequences, rather than spread by these sequences in CDMA. Taking 8BOK for example, the bit stream of user is converted to 8-ary symbol by every 3 bits and orthogonal sequence is chosen by this symbol.

The relationship between SD and SS could be depicted in Fig.3, In SS method, the PN sequence is acknowledged and fixed by both transmitter and receiver and the frequency of information symbol is spread by this PN sequence. However, in SD method, different PN sequences are selected by transmitted symbols and the PN sequence is not fixed, which makes it difficult for jammer to intercept.

B. System Model

In an M-ary SD system in Fig.4, the q-ary LDPC is employed to achieve reliable communication and q=M is required to avoid the loss of soft information when decoding. The coded symbol in the output of encoder is $s_n \in GF(q)$ and it is mapped to an orthogonal sequence $\mathbf{x}_n = (x_{n,0}...x_{n,M-1})$. The corresponding received symbol is $\mathbf{r}_n$ and $\mathbf{n}_n$ is the complex additive white Gaussian noise (AWGN).

Considering the random phase introduced by non-coherent reception and the channel fading fading, the received sequences in each branch satisfy

$$r_{n,k} = \lambda x_{n,k} \exp(j\theta) + n_{n,k} \tag{1}$$

where θ is the random phase which is introduced by non-coherent reception and λ is the fading depth. We could also note it in a vector format.

$$\mathbf{r}_n = \lambda \mathbf{X}_n \exp(j\theta) + \mathbf{N}_n \tag{2}$$

$\mathbf{x}_n$ is a column of Hadamard matrix and $\mathbf{x}_n = \mathbf{H}_k$ is assumed.

Received sequences are passed through a bank of matched filters and the correlations are performed to calculate the soft information for iterative decoding.

$$y_{n,m} = \mathbf{H}_m^T \mathbf{Y}_n = \lambda \mathbf{H}_m^T \mathbf{H} \exp(j\theta) + \mathbf{H}_m^T \mathbf{N}_k, \tag{3}$$

where the orthogonal attribute of the Hadamard matrix is employed, $\mathbf{H_m^T H_m} = \mathbf{M}$ and $\mathbf{H_m^T H_k} = \mathbf{0}$ when $m \neq k$.

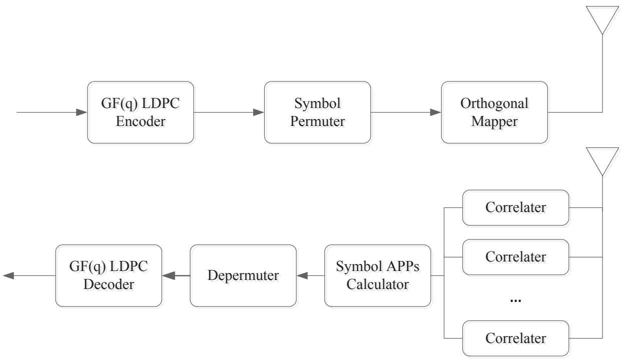

Fig. 4. Block diagram of spread dimension system

$\mathbf{H_m^T N_k}$ is a linear transformation of Gaussian random variable, therefore $n_{n,m} = \mathbf{H_m^T N_k}$ is also a Gaussian variable.

$$y_{n,m} = \begin{cases} \lambda M \exp(j\theta) + n_{n,m} & , m = k \\ n_{n,m} & , m \neq k \end{cases} \tag{4}$$

III. Soft Demodulation of Spread Dimension

A posterior probability (APP) should be obtained from the demodulator to initialize the soft iterative decoding. By using Bayes Theorem, APP is

$$\begin{aligned} p(s_n = m|\mathbf{y}_n) &= p(\mathbf{x}_n|\mathbf{y}_n) \\ &= \frac{p(\mathbf{y_n}|\mathbf{x_n} = m)}{\sum_{i=0}^{M-1} p(\mathbf{y_n}|\mathbf{x_n} = i)} \end{aligned} \tag{5}$$

where $p(s_n = m) = p(\mathbf{x}_n = m) = 1/M$ is assumed.

Since the demodulator operates on a symbol by symbol basis, the subscript n of s_n, $\mathbf{x}_m$, $\mathbf{r}_m$, $\mathbf{y}_m$could be dropped without introducing any ambiguity.

A. Coherent Detection

For a coherent reception, the CSI including the fading coefficient λ and the random phase θ are known at the receiver, thus, according to (4), $p(\mathbf{y}_n|\mathbf{x}_n)$ is conditionally Gaussian distributed and is given as

$$p(\mathbf{y}|\mathbf{x} = m) = \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^M \exp\left(-\frac{\sum\limits_{k=1,k\neq m}^{M} |y_k|^2}{2\sigma^2}\right) * \exp\left(-\frac{|y_m - M|^2}{2\sigma^2}\right) \tag{6}$$

Thus, the soft information at the end of soft output demodulator is derived for coherent reception by substituting (6) to (5)

$$p(\mathbf{x} = m|\mathbf{y}) = \frac{\exp\left(\frac{My_m}{\sigma^2}\right)}{\sum_{i=1}^{M} \exp\left(\frac{My_i}{\sigma^2}.\right)} \tag{7}$$

B. Non-coherent Detection with CSI

For non-coherent detection with CSI, the fading coefficient λ is deterministic and the phase of different codewords is unknown to us. Without loss of generality, we make the following assumptions:

(1) θ of different codewords are i.i.d.

(2) θ is uniformly distributed in [0,2π]. the probability density of θ is $p(\theta) = 1/2\pi, \theta \in [0, 2\pi]$.

The conditional likelihood probability can be expressed as

$$\begin{aligned} p(\mathbf{y}|\mathbf{x} = m, \theta) &= \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^M \exp\left(-\frac{\sum\limits_{k=1,k\neq m}^{M} (y_m^r)^2 + (y_m^i)^2}{2\sigma^2}\right) \\ &* \exp\left(-\frac{(y_m^r - M\cos\theta)^2 + (y_m^i - M\sin\theta)^2}{2\sigma^2}\right) \end{aligned} \tag{8}$$

where y_k^r and y_k^i are the real part and imaginary part of y_k. Given the assumptions about θ, the conditional likelihood probability could be marginalized with respect to θ,

$$\begin{aligned} p(\mathbf{y}|\mathbf{x} = m) &= \int_0^{2\pi} p(\mathbf{y}|\mathbf{x} = m, \theta) p(\theta) d\theta \\ &= \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^M \exp\left(-\frac{1}{2\sigma^2}(\sum\nolimits_{k=1}^{M} |y_k|^2 + M^2)\right) \\ &* I_0(\frac{M}{\sigma^2}|y_m|) \end{aligned} \tag{9}$$

where $I_0(\cdot)$ is the zero order modified Bessel function. Therefore, (5) could be expressed by

$$p(\mathbf{x} = m|\mathbf{y}) = \frac{I_0(\frac{M}{\sigma^2}|y_m|)}{\sum_{i=0}^{M-1} I_0(\frac{M}{\sigma^2}|y_i|)} \tag{10}$$

C. Non-coherent detection without CSI

For the non-coherent decttion without CSI, both the phase θ and the fading amplitude λ are unknown. θ is uniformly distributed between 0 and 2π and fading amplitude λ is a Rayleigh variable. The conditional likelihood probability is derived from (7)

$$\begin{aligned} p(\mathbf{y}|\mathbf{x} = m, \theta, \lambda) &= \prod\nolimits_{k=0}^{M-1} p(y_k|\mathbf{x} = m, \lambda, \theta) \\ &= \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^M \exp\left(-\frac{1}{2\sigma^2}\left(\sum\nolimits_{k=0}^{M-1} |y_k|^2 + M^2\lambda^2\right)\right. \\ &\left. + \frac{\lambda M}{\sigma^2}|y_m|\cos(\theta - \alpha)\right) \end{aligned} \tag{11}$$

where $\alpha = \arctan(y_m^i/y_m^r)$. If distributions of θ and λ are known, then two parameters can be integrated out, yielding a simple APP to achieve symbol level iterative decoding in (12) as follows

$$\begin{aligned}p(\mathbf{y}|\mathbf{x}=m) &= E_{\theta,\lambda}(p(\mathbf{y}|\mathbf{x}=m,\theta,\lambda)) \\ &= \int_0^{2\pi}\int_0^{\infty} p(\mathbf{y}|\mathbf{x}=m,\theta,\lambda)p(\lambda)p(\theta)d\lambda d\theta \\ &= C\frac{2\sigma^2}{M^2+2\sigma^2}\exp\left(\frac{M^2\left|y_m\right|^2}{2\sigma^2(2\sigma^2+M^2)}\right)\end{aligned} \tag{12}$$

where C is a constant. If formula (12) is applied to (5), we might obtain the APP

$$p(\mathbf{x}=m|\mathbf{y}) = \frac{\exp\left(\frac{M^2|y_m|^2}{2\sigma^2(2\sigma^2+M^2)}\right)}{\sum_{i=1}^{M}\exp\left(\frac{M^2|y_i|^2}{2\sigma^2(2\sigma^2+M^2)}\right)} \tag{13}$$

APP obtained from (7), (10) and (13) is implemented to initialize the first step of belief propagation (BP) algorithm. The BP algorithm is applied to Tanner graph that corresponds to the H matrix. According to the non-zero elements of H, the Tanner graph involves a set of variable nodes and check nodes. The probability messages are transmitted along the edges to get the correct code. In paper [6], Fast Fourier Transformation (FFT) is employed in decoding to enhance the efficiency of decoding algorithm. The four steps of FFT based BP algorithm is

Step 1: Initialization, we use APP to initialize the variable information;

Step 2: Updating check information;

Step 3: Updating variable information;

Step 4: Tentative decoding, If the tentative decoding result satisfies the constraint of H, the iterative process will halt, otherwise, continue step 2.

IV. Comparison of Capacity Between the CM and BICM Scheme

In the previous section, the q-ary (q=M) LDPC code is adopted to work in the SD system in order to achieve the reliable communication. However, the reason why we choose the q-ary LDPC code whose complexity is relatively higher than the binary LDPC code is not illustrated. The increase in computational complexity should be compensated by the promotion of performance.

In the SD communication system, there are two kinds of schemes to combine the channel coding with the modulation: the coded modulation (CM) scheme and the bit interleaved coded modulation (BICM) scheme.

For M-ary (M=2^m) SD system, given the symmetry of orthogonal constellations, the CM capacity does not depend on the signal labeling and it can be arbitrarily chosen to obtain the CM effect. Thus,the CM capacity under uniform input constraint could be expressed as

$$C_{CM}=\log M - E_{\mathbf{x},\mathbf{y}}\left\{\log\left(\frac{\sum_{\mathbf{x}\in\mathbf{S}} p_\theta(\mathbf{y}|\mathbf{x})}{p_\theta(\mathbf{y}|\mathbf{x})}\right)\right\} \tag{14}$$

where Θ is the channel state information and $p_\Theta(\mathbf{y}|\mathbf{x})$ is the channel transition PDF given the channel state.

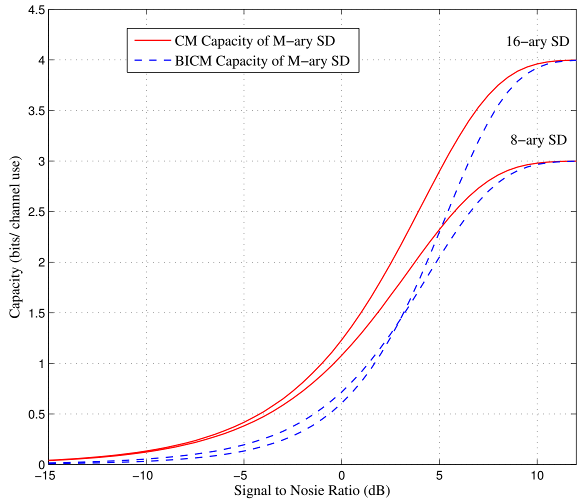

Fig. 5. Capacity versus SNR for 8-ary and 16-ary SD with coherent reception

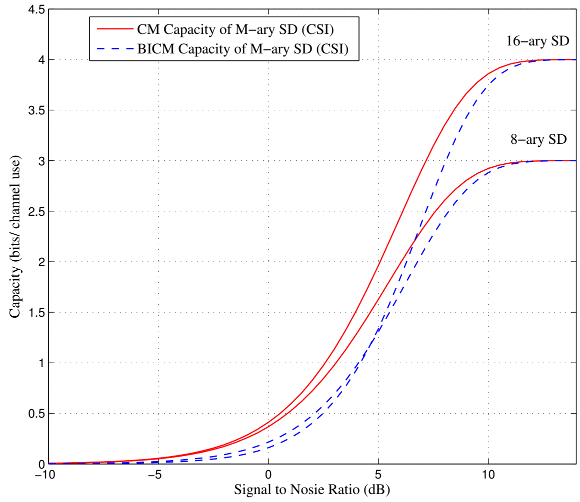

Fig. 6. Capacity versus SNR for 8-ary and 16-ary SD with non-coherent reception with CSI

For a M-ary OM with BICM, a equivalent model for BICM with ideal interleaving is a set of m parallel independent binary channels. Therefore, the BICM capacity of OM is equal to the sum of binary channel capacities [**?**]

$$C_{BICM}=\log M - \sum_{i=1}^{m} E_{b,\mathbf{y}}\left\{\log\left(\frac{\sum_{\mathbf{x}\in\mathbf{S}} p_\theta(\mathbf{y}|\mathbf{x})}{\sum_{\mathbf{x}\in\mathbf{S}_b^i} p_\theta(\mathbf{y}|\mathbf{x})}\right)\right\} \tag{15}$$

where $\mathbf{S}_b^i$ is the set of orthogonal signals whose label has a value of b at i-th bit.

In the BICM scheme, the symbol level APPs should be converted to bit level APPs to initialize the soft iterative decoding. This conversion omits the relationship between bits

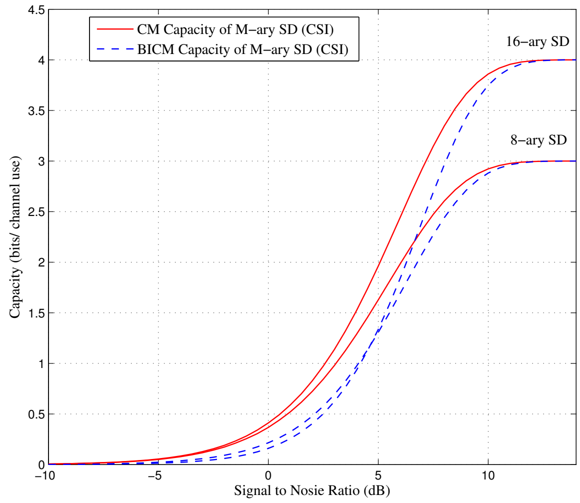

Fig. 7. Capacity versus SNR for 8-ary and 16-ary SD with non-coherent reception without CSI

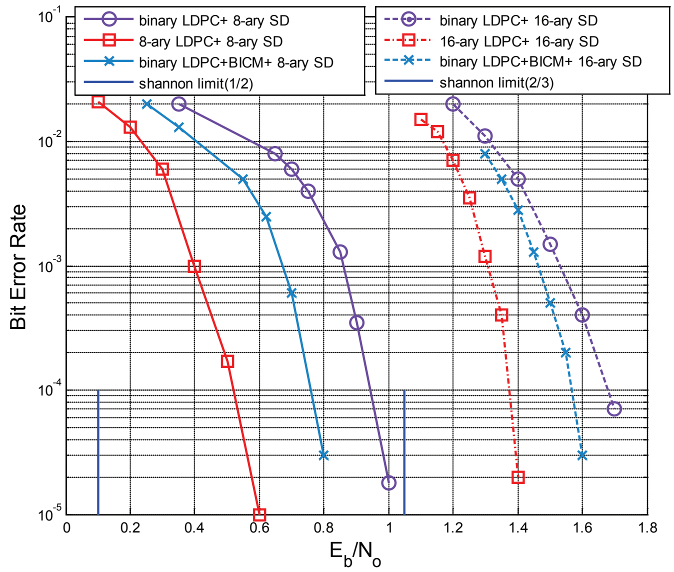

Fig. 8. BER performance of q-ary LDPC coded SD system in AWGN channel.

within a symbol and incurs the performance degradation of the soft compared to the CM scheme. The capacities are derived and presented in Fig. 5, Fig. 6 and Fig. 7 for coherent detection, non-coherent detection with CSI and non-coherent detection without CSI, respectively. The capacity of the CM scheme with symbol-level mapping outperforms that of the BICM schem with bit-level mapping in all the three cases. Therefore, q-ary LDPC code, rather than binary LDPC code, should be chosen to work in the SD communication system.

It can be explained by the fact that, when using binary code, the log-likelihood ratio (LLR) of the i-th bit in symbol is

$$LLR_i = log\frac{\Pr(x_{n,i}=0|Y=y)}{\Pr(x_{n,i}=1|Y=y)} \tag{16}$$

however, in order to approach the capacity of independent identically distributed (i.i.d) channel, the LLR should be

$$LLR_i = log\frac{\Pr(x_{n,i}=0|Y=y;X\ x_{n,i})}{\Pr(x_{n,i}=1|Y=y;X\ x_{n,i})} \tag{17}$$

In summary, the capacity when using the binary code

$$\sum\nolimits_{i=0}^{M-1} I(x_{n,i};Y) \leq I(x_{n,0},x_{n,1}...x_{n,M-1};Y) \tag{18}$$

where $I(x_{n,0},x_{n,1}...x_{n,M-1};Y)$ is the capacity of i.i.d channel, because bitwise mapping strategy omits the information of other bits within this symbol.

V. Simulation and Numerical Results

In this section, we address a special kind of irregular code named the quasi-regular LDPC code and three categories of quasi-regular LDPC codes are adopted: (GF(8), N=1334, R=1/2), (GF(16), N=1000, R=2/3) and (GF(32), N=800, R=1/3), where R is the code rate and N is block length of the LDPC code.

In order to make a comparison between bitwise mapping method and our symbol-wise mapping method, we compare the performance of GF(2)+8-ary SD, GF(2)+BICM+8-ary SD, GF(8)+8-ary SD, GF(2) +16-ary SD, GF(2)+BICM+16-ary SD and GF(16)+16-ary SD with two different code rates. The maximum iteration time is set in 50 times in order to obtain a good decoding performance. As we have discussed above, the noise is AWGN. Simulation results are presented in Fig.8, the CM scheme with LDPC codes over GF(q) outperforms the BICM scheme with binary LDPC codes.

As we has stated in Section I, The performance of traditional DS-SS and FH-SS would dramatically degrade when facing with smarty jamming in which the PN sequence or frequency hopping pattern is acquired by the jammer. When it comes to SD method, the information is carried by these dimension patterns (in the codeword or frequency) and in the SD system the information is carried by different PN sequences, which makes the jammer difficult to intercept all the PN sequences used in communication. Additionally, the jamming in the working pattern would increase the power of transmitted signal. Therefore, the smarty jamming would do little harm to our communication. Fig.9 shows the BER performance of GF(q) LDPC coded M-ary SD system when existing jamming in some occupied dimensions. Compared to binary LDPC coded 32 dimension SD system, the performance of 32-ary LDPC coded 32 dimension SD system suffers little degradation when facing with the 3 dimension jamming.

VI. Conclusion

In this paper, we studied GF(q) LDPC coded M-ary spread dimension system to achieve reliable communication with jammings. Specifically, a GF(q) LDPC coded spread dimension communication system is proposed. APPs to initialize the iterative decoding are derived for coherent detection,

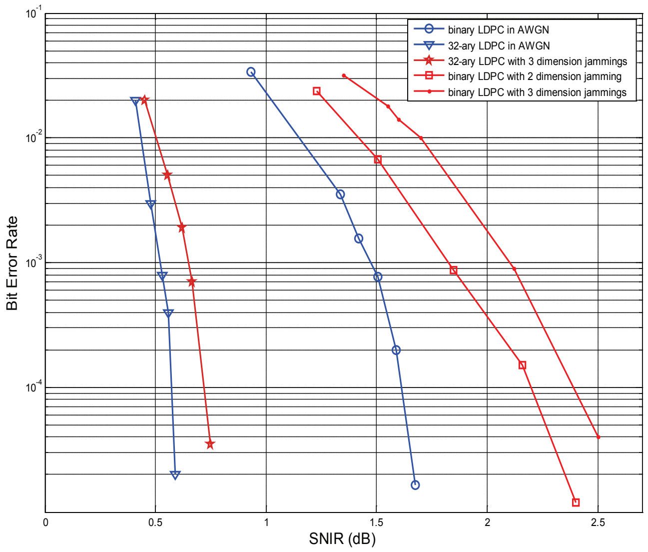

Fig. 9. BER performance of q-ary LDPC coded SD system and binary LDPC coded SD system with jamming in used dimensions.

non-coherent detection with CSI and non-coherent detection without CSI. Besides, we proved that the GF(q) LDPC coded SD system basically obtained a CM effect and outperformed the BICM scheme using binary LDPC codes in capacity. Simulation results showed that the LDPC coded SD system has a better performance than the coded SS system and the LDPC coded SD system also performs well with the smarty jamming.

Acknowledgment

This work was partially supported by National Natural Science Foundation of China (60972019, 61128001), National S&T Major Project (2011ZX03004-001-01), Tsinghua University Initiative Scientific Research Program (2011Z05117), Tsinghua National Laboratory for Information Science and Technology (TNList) and Co-innovation Laboratory of Aerospace Broadband Network Technology.

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