Achieving High Bandwidth Efficiency Under Partial-Band Noise Jamming*

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***Abstract*— This paper examines how to achieve high bandwidth efficiency while remaining robust to partial-band noise jamming (PBNJ). The main conclusion is that using higher order modulation with moderate code rates (HMMC) results in less PBNJ loss compared to using lower order modulation and high code rates close to unity (LMHC). For example, to achieve a bandwidth efficiency of R = 1.8 bits/sym, the modulation and code rate combination (ModCod) of 8PSK 3/5 has less PBNJ loss than QPSK 9/10. Similarly, to achieve R = 2.66 bits/sym, 16APSK 2/3 has less PBNJ loss than 8PSK 8/9. In the examples shown in this paper, the HMMC options suffer less than 0.5 dB PBNJ loss, while the LMHC options suffer PBNJ loss up to 5 dB.**

This paper also presents a theoretical analysis tool for estimating the amount of PBNJ loss, which is then validated via simulation. The analytical approach takes much less computation time and predicts about 80% of the PBNJ loss, averaged over the four cases studied. The theoretical model starts with the Shannon limit for Additive White Gaussian Noise (AWGN) channel for both infinite and finite constellations and then extends to include 1) PBNJ effects on average channel capacity, 2) the effect of interleaving over finite number of hops using a method based on binomial distribution, and 3) the effects of various parameters on PBNJ loss, including number of interleaving hops, jammer power, and required code word error rate. The analysis assumes perfect SNR knowledge at the receiver. Simulations are performed using DVB-S2 forward error correction (FEC) codes.

Keywords*— *jamming; partial-band noise jamming; frequency hopping; interleaving; modulation; coding; code rate; FEC; bandwidth efficiency; capacity; mutual information;

I. Introduction

While achieving good protection, especially against partial-band-noise-jamming (PBNJ), has always been important for military communication, users are now also demanding higher data rates. To achieve high data rates in limited spectrum, high bandwidth efficiency is required.

High bandwidth efficiency may be achieved through high order modulations, such as 8PSK with 3 bits per symbol and 16APSK with 4 bits per symbol. Alternatively, it could also be achieved with high forward error correction (FEC) code rates, such as rate 8/9 or rate 9/10.

This paper compares these two methods of achieving high bandwidth efficiency while remaining robust to PBNJ. Results show that using higher order modulation with moderate code rates (HMMC) results in less PBNJ loss compared to using lower order modulation with high code rates close to unity (LMHC).

On the surface, this conclusion seems intuitive – when the code rate is higher, the jammer only has to disrupt a smaller fraction of a code word, therefore, less robust. However, to understand the jammer's optimal strategy and the amount of PBNJ loss, more analysis is needed. For example, for a rate 2/3 code, one might think the jammer should jam 1/3 of the band or less. However, results show that the optimal strategy is full band jamming in the infinite interleaving case. The reason is that, in addition to considering the fraction of symbols jammed, how much more information may be carried by the unjammed symbols due to less jammer noise must also be considered, which is limited by the modulation constellation size. Furthermore, the amount of interleaving also impacts the optimal jammer strategy and the amount of PBNJ loss.

This paper presents a theoretical analysis tool that models the finite constellation and finite interleaving effects. Simulation results show that the analytical tool predicts about 80% of the PBNJ loss, averaged over the four cases studied.

This paper is organized as follows. Section II introduces the channel model and defines the PBNJ loss metric. Section III looks at average capacity achieved under PBNJ assuming infinite interleaving and concludes that for finite constellations, there is a PBNJ loss at high code rates close to unity due to the concavity of the capacity curve. Section IV incorporates the effects of interleaving over finite number of hops using the binomial distribution and explores how PBNJ loss changes with modulation and code rate, the number of interleaving hops, jammer power, and required code word error rate. Section V verifies theoretical results presented with simulation.

II. Channel Model and PBNJ Loss Metric

A. Channel Model

We assume a frequency hopped system with a total operating bandwidth of W and a partial band noise jammer with total energy J_0W and concentrates its energy in a fraction

* This work is sponsored by the Department of the Air Force under Air Force Contract #FA8721-05-C-0002. Opinions, interpretations, conclusions, and recommendations are those of the author and are not necessarily endorsed by the United States Government

978-0-7695-5124-1/13 $31.00 © 2013 IEEE
DOI 10.1109/MILCOM.2013.195

of the frequency band of width ρW, $0 < \rho \leq 1$. Thus, the jammer signal has spectral density of J_0/ρ in the jammed band.

User data are FEC encoded and modulated using a PSK modulation, such as QPSK, 8PSK, or 16APSK. Symbols are interleaved evenly over H hops. With frequency hopping, each hop is equally likely to be transmitted anywhere in the total bandwidth of W and has a likelihood of ρ of being jammed. It is assumed that the signal bandwidth is small compared to ρW, so a hop is either entirely jammed or not jammed at all.

In particular, for symbol i of hop h, the received symbol is

$$y_{h,i} = x_{h,i} + n_{h,i} \tag{1}$$

where x is the transmitted symbol and n is the combination of jammer power and background noise. Let E_s denote the average transmitted symbol energy and let N_0 denote the background noise spectral density, the signal-to-noise ratio (SNR) of each hop, for all symbols in that hop, is

$$\mathrm{SNR}_h = \begin{cases} \mathrm{SNR}_h^{\mathrm{U}} = E_s / N_0 & \text{w/ prob. } 1-\rho \\ \mathrm{SNR}_h^{\mathrm{J}} = E_s \Big/ \left(N_0 + \dfrac{J_0}{\rho} \right) & \text{w/ prob. } \rho \end{cases} \tag{2}$$

where the superscripts U and J denote unjammed and jammed. Throughout this paper, it is assumed that the receiver has perfect knowledge of the per-hop SNR.

B. *Definition of PBNJ Loss Metric*

A user wants to communicate reliably at a certain data rate using a certain combination of modulation and FEC code rate. The required E_s/N_0, denoted by $E_{s,\mathrm{req}}/N_0$ is a function of the modulation and code rate (ModCod) used, the required code word error rate (CWER), jammer-to-noise ratio (JNR=J_0/N_0), and the fraction of band jammed (ρ). The PBNJ loss metric is defined as the additional power in dB required to overcome the worse case PBNJ over full-band noise jamming (FBNJ), i.e.

$$\text{PBNJ Loss in dB} = 10 \cdot \log_{10}\left(\max_{\rho} E_{s,\mathrm{req}} / N_0 (\rho) \right) - 10 \cdot \log_{10}\left(E_{s,\mathrm{req}} / N_0 (\rho = 1) \right) \tag{3}$$

III. Average Capacity Under PBNJ

Capacity under AWGN is well understood [1,2]. For Gaussian input distribution, the channel capacity is

$$C_{\mathrm{AWGN}}(\mathrm{SNR}) = \log_2(1+\mathrm{SNR}) \tag{4}$$

For finite constellations, capacity can be computed using the definition of mutual information and numerical integration [1,2]. Fig. 1 (similar to Fig. 1 in [2]) shows capacity as functions of SNR in dB for various PSK constellations, as well as for Gaussian input distribution, i.e., the Shannon limit.

When there is PBNJ, the parts of a code word not jammed experience the higher SNR of $\mathrm{SNR}^{\mathrm{U}}$, while the parts that are jammed experience the lower SNR of $\mathrm{SNR}^{\mathrm{J}}$, as defined in (2). Let α denote the realized fraction of symbols in a code word that experience jamming. α is a random variable with mean ρ.

The realized capacity experienced by a sufficiently long code word is the average of the capacity of each channel use

$$C_{\mathrm{PBNJ}}(\alpha) = \alpha \bullet C_{\mathrm{AWGN}}(\mathrm{SNR}^{\mathrm{J}}) + (1-\alpha) \bullet C_{\mathrm{AWGN}}(\mathrm{SNR}^{\mathrm{U}}) \tag{5}$$

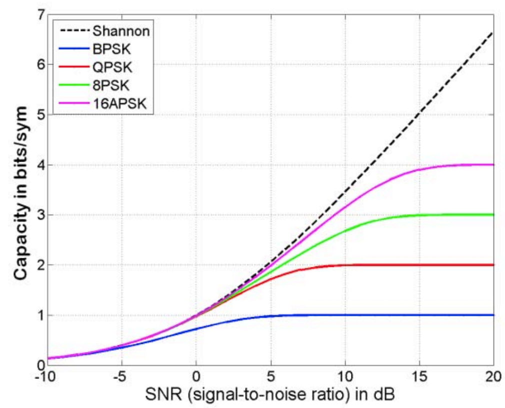

Fig. 1 Constellation constraint capacity as functions of SNR.

We first examine the case of infinite interleaving in this section; finite interleaving is treated in the next section. With sufficiently large H, α approaches ρ in probability. So the realized channel capacity is always $C_{\mathrm{PBNJ}}(\rho)$.

Let the capacity associated with full band noise jamming be $C_{\mathrm{FBNJ}} = C_{\mathrm{PBNJ}}(\rho = 1) = C_{\mathrm{AWGN}}(E_s/(N_0+J_0))$. To understand the relationship between C_{FBNJ} and $C_{\mathrm{PBNJ}}(\rho)$, we define noise-to-signal ratio NSR = 1/SNR and $C'(\mathrm{NSR}) = C(\mathrm{SNR})$. $C'_{\mathrm{AWGN}}(\mathrm{NSR})$ is shown in Fig. 2.

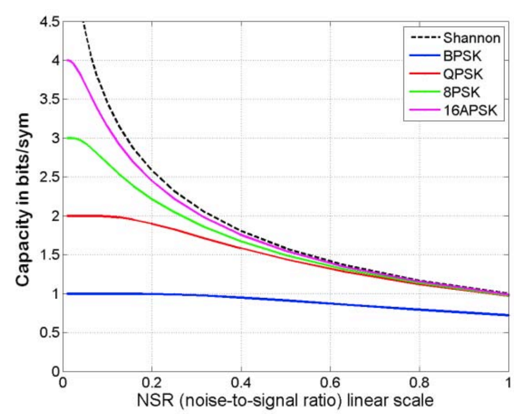

Fig. 2 Constellation constraint capacity as functions of NSR

Note that $\rho \cdot NSR^J + (1-\rho) \cdot NSR^U = (N_0+J_0)/E_s$, which is independent of ρ. C_{FBNJ} and $C_{PBNJ}(\rho)$ can be expressed as

$$C_{FBNJ} = C'_{AWGN}(\rho \bullet NSR^J + (1-\rho) \bullet NSR^U)$$
$$C_{PBNJ}(\rho) = \rho \bullet C'_{AWGN}(NSR^J) + (1-\rho) \bullet C'_{AWGN}(NSR^U). \quad (6)$$

Equation (6) shows that C_{FBNJ} averages the inputs to the function C'_{AWGN}, while $C_{PBNJ}(\rho)$ averages the outputs of the function C'_{AWGN} with the same weights. Thus, when the function C'_{AWGN} (NSR) is concave, $C_{PBNJ}(\rho)$ is less, i.e., it is more advantageous for the jammer to perform PBNJ. Otherwise, C_{FBNJ} is less, and the jammer should use FBNJ.

This is also illustrated graphically in Fig. 3, which is a portion of Fig. 2. At R = 1.8 bits/sym, the QPSK (red) curve is concave. Therefore, $C_{PBNJ}(\rho)$, which averages the outputs of the C'_{AWGN} function, indicated by the red square on the red dashed line, is less than C_{FBNJ}, indicated by the red circle on the red solid curve. In contrast, the 8PSK (green) curve is convex, so C_{FBNJ} is less. This shows that while operating with QPSK 9/10 suffers from PBNJ, 8PSK 3/5 does not, while both achieves the bandwidth efficiency of 1.8 bits/sym. Moving from QPSK 9/10 to 8PSK 3/5 takes us from the concave part of the QPSK curve to the convex part of the 8PSK curve, avoiding the PBNJ loss. In contrast, the Shannon capacity curve (black dashed) in Fig. 2 is always convex, therefore, would not suffer from PBNJ loss.

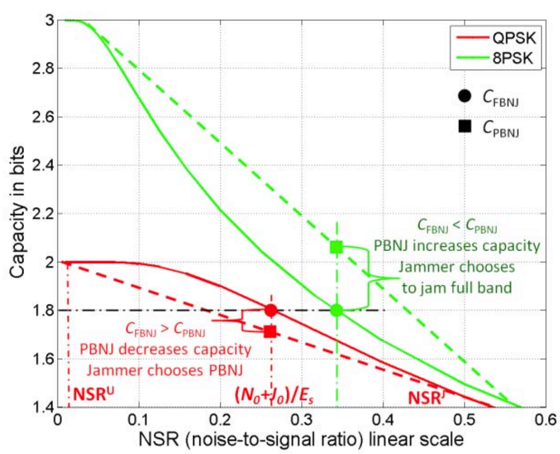

Fig. 3 Comparison of full band noise jamming capacity C_{FBNJ} (circles) and partial band noise jamming capacity $C_{PBNJ}(\rho)$ (squares) for QPSK (red) and 8PSK (green) at R = 1.8 bits/sym.

Another way to compare operating using LMHC vs. using HMMC is by evaluating the PBNJ loss metric as defined in (3). This is done by plotting $E_{s,req}/N_0$ as a function of ρ as shown in Fig. 4. The x-axis, ρ, goes from 0 to 1, so the right edge corresponds to FBNJ. The two red curves are for R = 1.8 bits/sym. The higher one with × marker is for QPSK 9/10, while the lower one with ○ marker is for 8PSK 3/5, corresponding to the two cases in Fig. 3. Using the definition of PBNJ loss in (3), 8PSK 3/5 suffers no PBNJ loss as it attains its maximum with FBNJ; while QPSK 9/10 suffers a 1.7 dB loss at ρ = 0.2. The pair of blue curves are for R = 2.66 bits/sym with 8PSK 8/9 (×) and16APSK 2/3 (○). Again, while the HMMC option, 16 APSK 2/3, suffers no PBNJ loss, the LMHC option, 8PSK 8/9, suffers a PBNJ loss of 0.7 dB at ρ = 0.4. The PBNJ loss results are summarized in Table 1.

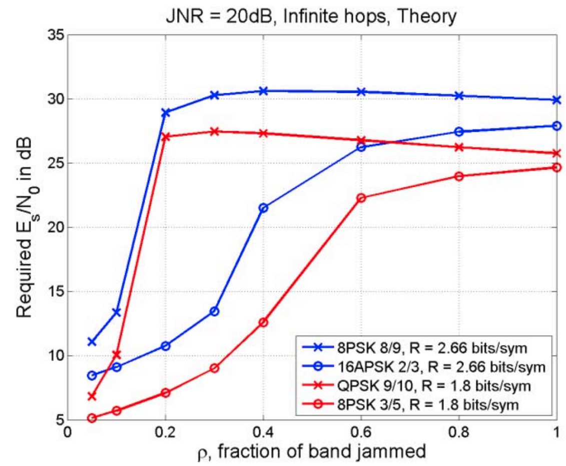

Fig. 4 Comparison of PBNJ loss between using lower order modulation with high code rate (×) vs. using higher order modulation with moderate code rate (○) for R = 2.66 bits/sym (blue) and R = 1.8 bits/sym (red). JNR=20dB.

Fig. 3 and Fig. 4 demonstrate the main conclusion of this paper in the ideal infinite hop case, i.e., using HMMC leads to less PBNJ loss compared to using LMHC. This loss is due to the constellation-constraint capacity curve becoming concave for high enough code rate as the curve is forced to bend flat and deviate from Shannon capacity, as shown in Fig.2 and Fig.3. In contrast, for higher order modulation, the unjammed symbols are able to carry more information, stay closer to the Shannon limit.

IV. Modeling Finite Number of Hops

This section builds upon the previous section and adds modeling for finite number of hops. The key difference between having finite number of hops vs. infinite number of hops is in the distribution of α, the realized fraction of symbols in a code word that experiences jamming. With infinite hops, α approaches ρ in probability; with finite number of hops, α is sometimes greater than ρ and sometimes less.

A. Binomial Distribution

Under the assumption that each hop is jammed independently and identically according to (2), α follows a binomial distribution:

$$\Pr\left[\alpha = \frac{a}{H}\right] = \binom{H}{a} \cdot \rho^a \cdot (1-\rho)^{H-a} \quad (7)$$

where a = 0, 1, 2, …, H is the actual number of hops jammed.

Fig. 5 shows the binomial distribution corresponding to the parameters H = 100 hops and ρ = 0.2. The mean of the distribution is $H \cdot \rho = 20$, i.e., on average 20 out of the 100 hops are jammed. However, unlike the infinite interleaving case, this distribution has a noticeable spread. While the probability that α is 30 or less is 99.4%, there is still a small probability that α is even larger. This introduces an additional element of randomness.

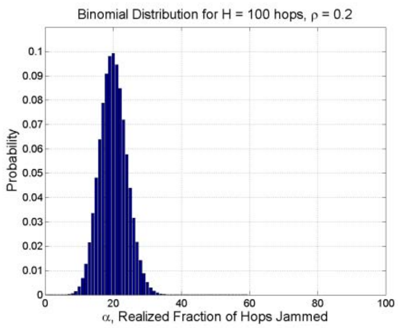

Fig. 5 Distribution of α, the realized fraction of symbols in a codeword that experience jamming, for the case of interleaving over 100 hops and 20% of the spectrum jammed

B. Evaluate E_s/N_0 Required

This subsection evaluates the E_s/N_0 required to achieve a required CWER using a particular ModCod with H hops, and under a jammer operating with JNR=J_0/N_0 and ρ.

To model the error performance under finite hop interleaving, this section assumes that the FEC used is capacity achieving, so that the resulting CWER is

$$\text{CWER} = \begin{cases} 0\% & \text{if } C_{\text{PBNJ}}(\alpha) \geq R \\ 100\% & \text{if } C_{\text{PBNJ}}(\alpha) < R \end{cases} \tag{8}$$

where R is the bandwidth efficiency corresponding to the ModCod choice, and $C_{\text{PBNJ}}(\alpha)$ is the capacity achievable when the realized fraction of symbols jammed is α.

Since $C_{\text{PBNJ}}(\alpha)$ monotonically decreases as α increases, (8) may be rewritten as

$$\text{CWER} = \begin{cases} 0\% & \text{if } \alpha \leq \alpha^* \\ 100\% & \text{if } \alpha > \alpha^* \end{cases} = \Pr[\alpha > \alpha^*] \tag{9}$$

where α^* is defined via $C_{\text{PBNJ}}(\alpha^*) = \text{R}$.

By the definition of $C_{\text{PBNJ}}(\alpha)$ in (5), it increases with E_s/N_0. Therefore, E_s/N_0 must be high enough, such that α^* is large enough (tolerate enough jammed hops), such that $\Pr[\alpha>\alpha^*]$ is less than the required CWER.

The following procedure illustrates how to compute the E_s/N_0 required to achieve a particular combination of CWER, ModCod, H, JNR, and ρ.

1. From H and ρ, compute the binomial distribution $\Pr[\alpha = a/H]$ using (7)
2. Given a required CWER, identify the smallest α^* such that $\Pr[\alpha>\alpha^*]$ is less than the required CWER
3. Find the E_s/N_0 such that $C_{\text{PBNJ}}(\alpha^*)$ = R. Note that $C_{\text{PBNJ}}(\alpha^*)$ is also a function of JNR, ρ, and the modulation.

Example: Consider the case of required CWER = 10^{-3}, ModCod QPSK 9/10, R = 1.8 bits/sym, H = 100 hops, and a jammer with 20dB JNR=J_0/N_0 and ρ = 0.2. Following the three steps

1. Fig. 5 shows the binomial distribution for H = 100 and ρ = 0.2
2. $\Pr[\alpha > 33/100] = \Pr[\alpha \geq 34/100] = 0.0007 < 10^{-3}$, while $\Pr[\alpha \geq 33/100] = 0.0016 > 10^{-3}$, so α^* is 33/100
3. Found that at E_s/N_0 = 29.6 dB, which results in SNR^{U} = 29.6 dB and SNR^{J} = 2.6 dB, $C_{\text{PBNJ}}(0.33) = 0.33 \cdot C_{\text{AWGN,Q}}(2.6\text{dB}) + 0.67 \cdot C_{\text{AWGN,Q}}(29.6\text{dB}) = 0.33 \cdot 1.38 + 0.67 \cdot 2 = 1.8$ bits/sym

Note that the 33% jammed symbols still contribute to about 25% (=0.33*1.38/1.8) of the information.

C. PBNJ Loss with Finite Number of Hops

Using the method for evaluating the E_s/N_0 required laid out in Section VI.B, Fig. 6 shows the E_s/N_0 required to achieve 10^{-3} CWER, using 100 hop interleaving, under 20dB JNR, for the same four ModCod options as in Fig. 4. The original curves in Fig. 4 corresponding to infinite interleaving are reproduced as dashed lines for comparison.

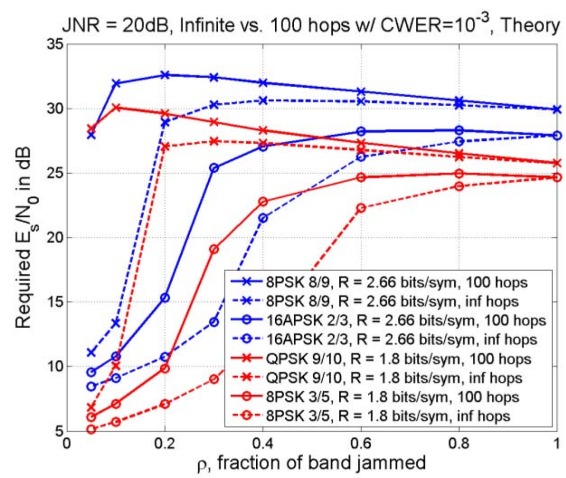

Fig. 6 Comparison of PBNJ loss between using infinite hops (dashed, previously shown in Fig. 4) and using finite number of hops (solid, 100 hops, required code word error rate of 10^{-3}). JNR = 20dB.

Focusing on the solid lines first, comparison of the pair of blue solid curves and comparison of the pair of red solid curves both confirm the earlier conclusion of LMHC suffering more PBNJ loss. In particular, QPSK 9/10 suffers 4.3 dB PBNJ loss at ρ = 0.1, while 8PSK 3/5 only suffers 0.3 dB loss; 8PSK 8/9 suffers 2.7 dB PBNJ loss at ρ = 0.2, while 16APSK 2/3 only suffers 0.4 dB loss. These results are summarized in Table 1.

Comparing the solid and dashed curves, using finite hops always require higher E_s/N_0 as expected. The resulting PBNJ losses are also higher (see Table 1). Furthermore, the ρ values at which the worst case PBNJ loss occurs, ρ′, move to the left for the finite-hop cases (e.g., from 0.4 to 0.2 for 8PSK 8/9, from 0.3 to 0.1 for QPSK 9/10). The smaller ρ′ is, the longer E_s/N_0 keeps increasing starting from the right end at ρ = 1.

D. PBNJ Loss Trends

Using the tools established, changes in PBNJ loss as functions of number of hops, jammer power, required CWER, may be examined. One benefit of this theoretical method proposed is that for most scenarios, finding the E_s/N_0 required only takes seconds on a typical desktop computer, much quicker than running full FEC simulations. As a result, a large multi-dimensional grid of scenarios may be run in just hours. The following results are run with a full grid of four ModCod, eight values of ρ as in Fig. 4, five values of H from 25 to 400 at factor of 2 steps, nine JNR levels from -10 dB to 30 dB at 5 dB steps, six CWER levels from 10^{-1} to 10^{-6} at factor of 10 steps, totaling 4×8×5×9×6 = 8640 scenarios, finished in six hours.

Fig. 7 shows PBNJ loss as functions of number of hops, H, for the cases with JNR=20 dB and CWER = 10^{-3}. Similar to Fig. 4, the pair of blue curves are for R = 2.66 bits/sym and the pair of red curves are for R = 1.8 bits/sym. The curves with × markers use LMHC; the curves with ○ markers use HMMC. All four curves have a downward trend, meaning more interleaving hops reduces PBNJ loss, as it is well known. At both bandwidth efficiency levels (red and blue), the LMHC options (× markers) suffer significantly more PBNJ loss than the HMMC options (○ markers). In particular, while 16APSK 2/3 and 8PSK 3/5 suffer less than 2 dB loss with as little as 25 hop interleaving, QPSK 9/10 suffers about 3 dB loss even with 400 hops of interleaving.

Fig. 8 shows PBNJ loss as functions of JNR for the same four ModCod as in Fig. 7, with H = 100 and CWER = 10^{-3}. All four curves have an upward trend, meaning higher jammer power leads to more PBNJ loss, which is as expected. Again, the curves with × markers are much higher than the curves with ○ markers showing that LMHC suffer more PBNJ loss. In particular, while 16APSK 2/3 and 8PSK 3/5 suffer less than 0.5 dB PBNJ loss under a 30 dB JNR jammer (1000 times as strong as the background noise), QPSK 9/10 suffers more than 3 dB loss under just a 5 dB JNR jammer (3.16 times as strong as the background noise).

Fig. 9 shows PBNJ loss as functions of CWER with H = 100 and JNR = 20dB. All four curves have a downward trend, meaning tolerating higher CWER leads to less PBNJ loss. Again, LMHC options (× markers) suffer significantly more PBNJ loss than HMMC options (○ markers). In particular, while 16APSK 2/3 and 8PSK 3/5 suffer less than 1 dB PBNJ loss with a 10^{-6} CWER, QPSK 9/10 suffers more than 3 dB PBNJ loss even at a poor CWER of 10^{-2}.

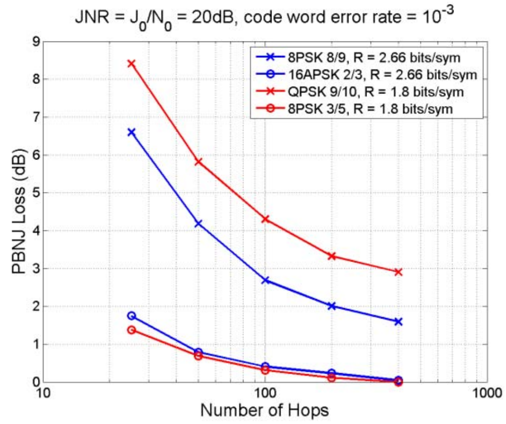

Fig. 7 PBNJ loss in dB as functions of number of hops for interleaving.

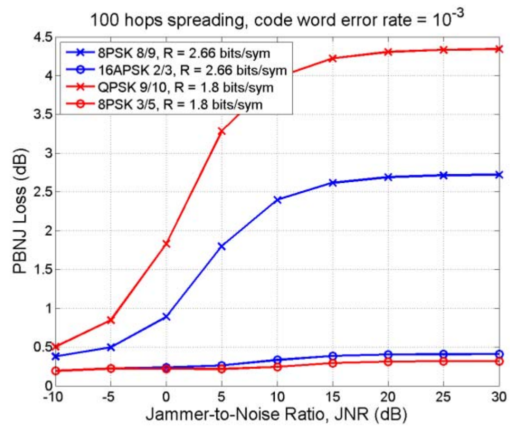

Fig. 8 PBNJ loss in dB as functions of jammer-to-noise ratio JNR =J_0/N_0

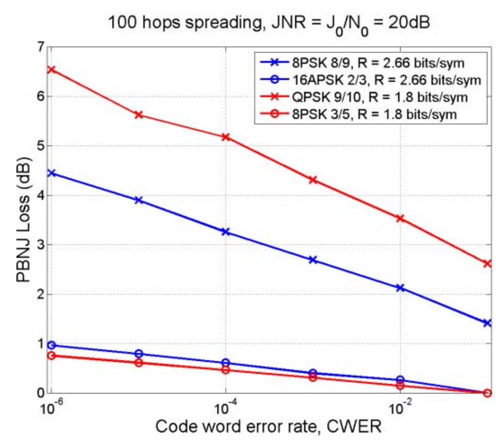

Fig. 9 PBNJ loss in dB as functions of code word error rate

V. Simulation Results

To validate the theoretical results presented in Section IV, simulations were performed using DVB-S2 (Digital Video Broadcasting, 2nd generation) standard [3]. The FEC coding is a low-density-parity-check (LDPC) code with a BCH code as an outer code. The output code word block length is 64800 bits; this long code length allows this FEC to achieve performance within 1 dB of theoretical capacity. The same four ModCod presented in earlier sections are simulated, namely, QPSK 9/10, 8PSK 3/5, 8PSK 8/9, and 16APSK 2/3. The 16APSK constellation used here and in the previous sections is the one defined in the DVB-S2 standard for rate 2/3 operation.

Fig. 10 shows the E_s/N_0 required to achieve 10^{-3} CWER, using H = 100, under 20 dB JNR, for the same four ModCod options as in Fig. 4 and Fig. 6. The solid curves in Fig. 6 corresponding to the 100-hop theoretical results are reproduced as dashed lines for comparison. The distance between each pair of solid and dashed curves are about 1 dB for most values of ρ. Larger gaps occur at the "knees" of the curves, where the required E_s/N_0 starts to decrease as ρ decreases. In particular, for 16APSK 2/3 at ρ = 0.2, the gap is ~10dB. However, this does not impact the worst case PBNJ loss which occurred at higher values of ρ. For the LMHC options (× markers), as ρ decreases, all four curves increase steadily until ρ ≤ 0.2, where the theoretical curve starts the descent slightly early than the simulation curves. It is possible that this difference is due to the finite-length non-capacity-achieving FEC used.

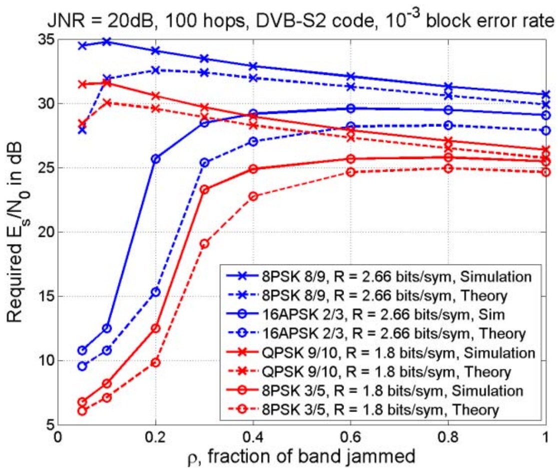

Fig. 10 Comparison of PBNJ loss between analytical results (dashed, previously shown in Fig. 6 as solid) and simulation results using DVB-S2 codes (solid). JNR = 20dB. H = 100 hops. Code word error rate = 10^{-3}.

Table I summarizes the PBNJ losses for the results presented in Fig. 4, Fig. 6, and Fig. 10, for the cases of theoretical infinite-hop results, theoretical 100-hop result, and simulation 100-hop results, for the four ModCod. Comparing the last two columns, the theoretical result is about 80% of the simulation values, averaged over the four cases. For 8PSK 3/5 and 16APSK 2/3, the HMMC options, the gaps are no more than 0.1 dB. This small gap is because, as ρ decreases, these curves start off nearly flat from the FBNJ point. The worst case PBNJ loss occurs during this flat portion where the simulation and theoretical curves have good agreement. On the other hand, for QPSK 9/10 and 8PSK 8/9, the LMHC cases, although theoretical results are able to predict most of the PBNJ losses seen in the simulation results, there is notable differences between the simulation and theoretical results. This is because, as ρ decreases, the curves keep going up from the FBNJ points, the worst case PBNJ losses occur right around the "knees" where the gaps get large. Since the simulation curves start their descent later, the PBNJ losses are higher.

TABLE I. TABLE TYPE STYLES

Modulation and Code Rates			PBNJ Losses in dB		
Modu-lation	***Code Rate***	***R (bits/sym)***	***Theory Inf. Hops***	***Theory 100 Hops***	***Simulation 100 Hops***
QPSK	9/10	1.8	1.7	4.3	5.2
8PSK	3/5	1.8	0	0.3	0.3
8PSK	8/9	2.66	0.7	2.7	4.1
16APSK	2/3	2.66	0	0.4	0.5

While the simulation results presented here assumed perfect per-hop SNR knowledge at the receiver, preliminary simulation results have shown that with modest number of reference symbols, per-hop SNR estimation could be performed at the receiver and achieve similar PBNJ losses.

VI. SUMMARY AND CONCLUSIONS

The main conclusion is that to achieve high bandwidth efficiency in a partial-band noise jamming environment, using higher order modulation with moderate code rates (HMMC) suffers less PBNJ loss compared to using lower order modulation with high code rates (LMHC) that achieves the same bandwidth efficiency. This loss is rooted in the fact that the constellation-constraint capacity curve becomes concave for higher code rate as the curve is forced to bend flat and deviate from Shannon capacity. Two examples were shown. For R=1.8 bits/sym, 8PSK 3/5 suffers less PBNJ loss than QPSK 9/10; for R=2.66 bits/sym, 16APSK 2/3 suffers less PBNJ loss than 8PSK 8/9. Thus, code rates close to unity should be avoided for operations under PBNJ; while with moderate code rates, PBNJ loss can be nearly eliminated.

This paper also demonstrates the benefit of using a theoretical analysis tool based on binomial distribution. Since the theoretical tool requires a significantly lower amount of computation compared to simulation and provides a reasonably good approximation to simulation results, it may be used to quickly survey a large number of scenarios, get a sense of whether the PBNJ loss would be large or small. Results are presented on how PBNJ losses change as functions of number of hops, jammer-to-noise ratio, and code word error rate.

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