A High-Fidelity Statistical Model of Frequency Hopping Interference for Fast Simulation

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***Abstract*—Testing existing or developing new military waveforms requires computationally efficient simulations to allow for large scenarios that would not be feasible with hardware. However, fast running simulations have typically not represented wireless interference, especially frequency hopping interference, accurately. We develop the first major step toward a high-fidelity statistical model of frequency hopping interference that allows for much faster simulation times than high-fidelity deterministic models and even allows for real-time emulations of relatively large networks. We analyze the accuracy of our model by comparing it to the deterministic model it is based on, and we show some performance results by measuring computation times for a wide range of number of interferers.**

I. Introduction

One of the key tools in testing or developing new military waveforms is running computationally efficient simulations. Simulations are necessary for analyzing the performance of new wireless protocols while avoiding the high cost of real hardware tests. Simulations also allow us to test large scenarios that would not be feasible with hardware, especially at the early stages. Since tests, analysis, and performance enhancing modifications can be made with a quick turn-around time, prototypes can be developed more rapidly.

However, simulations present a challenge. Simulating the interference of wireless signals and how they affect the ability for receivers to decode is complex; particularly for the case of frequency hopping signals. In the past, simulating wireless interference has been done by either significantly simplifying the model at the expense of accuracy [1], [2], or by using a high-fidelity model at the cost of high complexity [3]. The low-complexity approach is problematic because typically a certain level of accuracy is needed for simulation. The high-complexity approach has issues because only small networks with low amounts of traffic can be simulated in a reasonable amount of time.

Complexity becomes a larger issue when using a real-time emulator. The emulator suite of CORE (Common Open Research Emulator) and EMANE (Extendable Mobile Ad-hoc Network Emulator) has been growing in popularity for military network emulation [4], [5]. CORE emulates layer 3 and above with real-time code running on light-weight Linux containers, while EMANE simulates layers 1 and 2, also running in real-time. Since EMANE and CORE expect real-time behavior, only very small networks have been tested in this emulation environment so far.

This paper details our new statistical interference model, which is based on our current deterministic model. The goal of the new model is to significantly lower computation time while maintaining a high level of accuracy. In Section II, we provide a brief overview of the deterministic interference model. In Section III, we present the theory and implementation of the new statistical model. In Section IV, we analyze the accuracy and performance of the statistical model. In Section V, we discuss the model's strengths and weaknesses and explore ideas for refining the model and generalizing it.

II. System Model

We consider a communication system employing fast frequency hopping. The transmission of each packet is divided into several pieces called hops; these hops are transmitted sequentially, each at a different frequency. In an additive white Gaussian noise (AWGN) channel, all of the hops experience the same noise distribution. In channels with time or frequency dependent noise and interference, the packet error rate can be difficult to determine. The deterministic interference model we developed previously uses the effective SINR (signal to interference plus noise ratio) mapping [6], [7], [8]. This mapping helps determine packet error rates when noise and interference are not constant over a packet. The output of our deterministic interference model is the effective SINR in an AWGN channel. From the effective SINR, a known PER curve can be referenced to determine if a packet was successful.

To determine the amount of noise at every point of a signal, the deterministic model simulates each frequency hop of each signal and measures their overlap in time and frequency (see Figure 1). The energies are then summed to obtain the per-hop total interfering energy:

$$Y_h = \sum_{i \in I(h)} X_{ih}, \tag{1}$$

X_{ih} is the interfering energy contributed by signal i on hop h of the packet of interest, and $I(h)$ is the set of all interferers on hop h. We use a capacity-based effective SINR mapping [6] to calculate the effective SINR. The per-hop channel capacity is:

$$Z_h = \ln\left(1 + \frac{E_0/\beta}{N_h + Y_h}\right), \tag{2}$$

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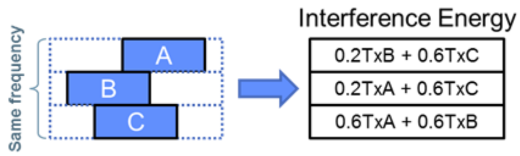

Fig. 1: The deterministic interference model totals all energy contributions by interferers on each frequency hop by measuring their overlap in time and frequency.

The average of these over all hops to get a total channel capacity over the packet is:

$$L = \frac{1}{H}\sum_{h=1}^{H} Z_h, \tag{3}$$

We then convert back from channel capacity to a total, effective SINR:

$$S = \beta\left(e^L - 1\right). \tag{4}$$

In these equations, E_0 is the energy of the signal of interest, N_h is the energy from background noise and jammers, H is the number of frequency hops in the packet, and β is a parameter used to fit the PERs resulting from (4) to those observed with our decoders under the same noise conditions. The model has been compared against a higher fidelity sample-based receiver model and found to produce the expected PERs over a wide range of interference and noise vectors.

III. New Statistical Interference Model

The deterministic model takes $O(nH)$ time to measure the interference across all the hops, which is then used to calculate the effective SINR for each desired packet; here, n is the number of interferers affecting the packet. Our new statistical model addresses this issue by treating X_{ih}, Y_h, Z_h, L, and S as random variables and approximating their distributions. Furthermore, it collects information for each packet segment (i.e. a segment of the packet during which the set of interferers is constant) instead of for each hop.

A. Theory

To approximate the distribution of X_{ih}, we assume a duty cycle of $\alpha \leq 1/2$ and that adjacent hopping frequency bands overlap by 2/5 the band. The probability of a hop getting hit directly on the same frequency by another signal is:

$$p = 2\alpha/q, \tag{5}$$

where q is the number of frequencies. When hit, the energy time overlap of the two hops is a uniform distribution, so that the energy contribution by interferer i is also uniform between 0 and E_i (the energy of that signal). The probability of a hop getting hit on an adjacent frequency is $2p$, with a uniform energy contribution between 0 and $0.4E_i$. Thus, the probability distribution function (PDF) of X_{ih} is:

$$f_{X_{ih}}(x) = (1-3p)\delta(x) + \begin{cases} 6p/E_i, & 0 < x \leq 0.4E_i \\ p/E_i, & 0.4E_i < x \leq E_i. \end{cases} \tag{6}$$

To handle send-while-receive (tx/rx) interference, we assume the system has the frequencies grouped into b sub-bands, where transmitting in one sub-band does not block receiving in any other sub-band. Following similar reasoning, the energy contributed by a tx/rx interfering signal is given by the PDF:

$$f_{X_{ih}}(x) = (1-p_T)\delta(x) + p_T/E_T, \quad 0 \leq x \leq E_T, \tag{7}$$

where E_T is the energy of the tx/rx signal and the probability of a hit is:

$$p_T = 2\alpha/b. \tag{8}$$

We approximate the distribution of Y_h as exponential with a Dirac delta function, as this produced better results than other standard non-negative distributions. The PDF of Y_h is:

$$f_{Y_h}(x) = \rho_h\delta(x) + (1-\rho_h)\lambda_h e^{-\lambda_h x}, \quad x \geq 0, \tag{9}$$

where

$$\rho_h = \begin{cases} (1-3p)^{n_h}(1-p_T), & \text{with tx/rx} \\ (1-3p)^{n_h}, & \text{without tx/rx} \end{cases} \tag{10}$$

and λ_h is found by calculating the expectation of Y_h in terms of the expectation of each X_{ih}:

$$E[Y_h] = E\left[\sum_{i\in I(h)} X_{ih}\right] \tag{11}$$

$$\frac{1-\rho_h}{\lambda_h} = \sum_{i\in I(h)} E[X_{ih}] = .9p\sum E_i + .5p_T E_T \tag{12}$$

$$\lambda_h = \frac{1-\rho_h}{.9p\sum E_i + .5p_T E_T}. \tag{13}$$

We approximate the distribution of Z_h as a truncated log-normal with a Dirac. The PDF of Z_h is:

$$f_{Z_h}(x) = \rho_h\delta(x-\bar{z}_h) + (1-\rho_h)\frac{\frac{1}{x\sqrt{2\pi}\sigma_h}\exp\left(-\frac{(\ln x-\mu_h)^2}{2\sigma_h^2}\right)}{\frac{1}{2}+\frac{1}{2}\mathrm{erf}\left[\frac{\ln \bar{z}_h-\mu_h}{\sqrt{2}\sigma_h}\right]}, \quad 0 \leq x \leq \bar{z}_h. \tag{14}$$

The truncation occurs because (2) can only output positive values and has a maximum when $Y_h = 0$:

$$\bar{z}_h = \ln\left(1 + \frac{E_0/\beta}{N_h}\right). \tag{15}$$

The parameters μ_h and σ_h are the mean and standard deviation of the normal distribution corresponding to this log-normal distribution. When the cumulative distribution function (CDF) of $\ln Z_h$ is computed exactly from the assumption of Y_h taking the form of (9), we get that the corresponding normal distribution (whose random variable we will call Φ_h) should be fit to the following CDF:

$$F_{\Phi_h}(x) = \exp\left(\frac{E_0\lambda_h/\beta}{e^{e^x}-1}\right). \tag{16}$$

We have tested fitting a normal to Φ_h over all practical values of the parameter $r_h := E_0\lambda_h/\beta$ with very low error for most common values and still relatively low error for extreme values. Our method of fit is to compute the mean and standard deviation from the inverse CDF of Φ_h, $Q_{\Phi_h}(x)$, by assuming its percentiles are close to that of a normal. Specifically,

$$Q_{\Phi_h}(x) = \ln \ln \left(1 - \frac{E_0 \lambda_h / \beta}{\ln x}\right), \tag{17}$$

$$\Phi_h^+ = Q_{\Phi_h}(0.841), \qquad \Phi_h^- = Q_{\Phi_h}(0.159), \tag{18}$$

$$\mu_h = \frac{1}{2}\left(\Phi_h^+ + \Phi_h^-\right), \tag{19}$$

$$\sigma_h = \frac{1}{2}\left(\Phi_h^+ - \Phi_h^-\right). \tag{20}$$

We approximate the distribution of L (see (3)) as a log-normal, with a PDF of:

$$f_L(x) = \frac{1}{x\sqrt{2\pi}\sigma_L} \exp\left(-\frac{(\ln x - \mu_L)^2}{2\sigma_L^2}\right). \tag{21}$$

To fit μ_L and σ_L, we first calculate the 1st and 2nd moments of Z_h:

$$E[Z_h] = \left(\rho_h + (1-\rho_h)\frac{\text{erfcx}(A_h + B_h)}{\text{erfcx}(A_h)}\right)\bar{z}_h, \tag{22}$$

$$E[Z_h^2] = \left(\rho_h + (1-\rho_h)\frac{\text{erfcx}(A_h + 2B_h)}{\text{erfcx}(A_h)}\right)\bar{z}_h^2, \tag{23}$$

where

$$A_h = \frac{\mu_h - \ln \bar{z}_h}{\sqrt{2}\sigma_h} \tag{24}$$

and

$$B_h = \sigma_h / \sqrt{2}. \tag{25}$$

We then match the mean and variance of L. The moments of L in terms of the moments of Z are:

$$E[L] = \frac{1}{H}\sum_h E[Z_h] \tag{26}$$

and

$$E[L^2] - E[L]^2 = \frac{1}{H^2}\sum_h \left(E[Z_h^2] - E[Z_h]^2\right) \tag{27}$$

$$E[L^2] = E[L]^2 + \frac{1}{H^2}\sum_h \left(E[Z_h^2] - E[Z_h]^2\right). \tag{28}$$

In terms of the log-normal distribution, they are:

$$E[L] = e^{\mu_L + \sigma_L^2/2}, \tag{29}$$

$$E[L^2] = e^{2(\mu_L + \sigma_L^2)}. \tag{30}$$

Taking the logarithm of both sides results in a system of linear equations with the following solution:

$$\mu_L = 2\ln E[L] - \frac{1}{2}\ln E[L^2], \tag{31}$$

$$\sigma_L^2 = \ln E[L^2] - 2\ln E[L]. \tag{32}$$

The CDF and inverse CDF of S are computed from L:

$$
\begin{aligned}
F_S(x) &= \Pr(S \le x) && (33)\\
&= \Pr\left(\beta\left(e^L - 1\right) \le x\right) && (34)\\
&= \Pr\left(L \le \ln\left(\frac{1}{\beta}x + 1\right)\right) && (35)\\
&= F_L\left(\ln\left(\frac{1}{\beta}x + 1\right)\right) && (36)\\
&= \frac{1}{2}\text{erfc}\left(\frac{\mu_L - \ln\ln\left(\frac{1}{\beta}x + 1\right)}{\sqrt{2}\sigma_L}\right), && (37)
\end{aligned}
$$

$$x = \frac{1}{2}\text{erfc}\left(\frac{\mu_L - \ln\ln\left(\frac{1}{\beta}Q_S(x) + 1\right)}{\sqrt{2}\sigma_L}\right) \tag{38}$$

$$Q_S(x) = \beta\left(\exp\exp\left(\mu_L - \sqrt{2}\sigma_L \text{erfc}^{-1}(2x)\right) - 1\right). \tag{39}$$

B. Implementation Considerations

The statistical model implementation keeps track of packet segments instead of hops. Each hop in a packet segment has i.i.d. random variables Z_h, so that $E[L]$ and $E[L^2]$ become weighted sums over the packet segments, where the weights are the number of hops in each segment. For each packet segment, the model keeps track of the following:

- E: The vector of received energies for each signal interfering with this packet segment.
- w: The (non-integer) number of hops in this segment.
- J: The interfering energy contributed by barrage jammers.
- E_T: The energy of an outgoing signal, if it exists (used for tx/rx interference).

Additionally, the following information is either collected for the entire packet or calculated for the entire simulation:

- E_0: The received energy of the desired signal.
- N_0: The interfering energy contributed by background noise.
- w_0: The (non-integer) number of hops in the packet for which there is no interference (friendly or barrage jamming).
- p: The probability that an interferer will overlap (at least partially) with a given hop of the desired signal.
- p_T: The probability of (at least partial) overlap from an outgoing signal, whenever present in a particular packet segment.
- β: The scaling parameter for the effective SINR.

The collection of the above information takes $O(n)$ time, since the maximum number of packet segments is $2n+1$. After collecting the information, the effective SINR is calculated according to the algorithm given in Figure 2. The algorithm takes directly from the equations developed in Section III-A, with a few exceptions. All occurrences of $\ln(1+x)$ use a log1p algorithm, all occurrences of $e^x - 1$ use an expm1 algorithm, and (22–23) are calculated logarithmically to avoid problems with numerical precision.

for $k = 1$ to size(E) **do**
  $\rho[k] \leftarrow (1-3p)^{n[k]}$ # Equation (10)
  if $E_T[k] > 0$ **then**
    $\rho[k] \leftarrow \rho[k](1-p_T)$ # tx/rx
  end if
  $\lambda[k] \leftarrow \dfrac{1-\rho[k]}{.9p\sum E[k] + .5p_T E_T[k]}$ # Eq. (13)
  $\bar{z}[k] \leftarrow \text{log1p}\left(\dfrac{E_0/\beta}{N_0 + J[k]}\right)$ # Eq. (15)
  $\Phi^+[k] \leftarrow \log\left(\text{log1p}\left(-\frac{E_0\lambda[k]/\beta}{\log(.841)}\right)\right)$ # Eq. (17–18)
  $\Phi^-[k] \leftarrow \log\left(\text{log1p}\left(-\frac{E_0\lambda[k]/\beta}{\log(.159)}\right)\right)$ # Eq. (17–18)
  $\mu[k] \leftarrow 0.5(\Phi^+[k] + \Phi^-[k])$ # Eq. (19)
  $\sigma[k] \leftarrow 0.5(\Phi^+[k] - \Phi^-[k])$ # Eq. (20)
  $A[k] \leftarrow \frac{\mu[k] - \log\bar{z}[k]}{\sqrt{2}\sigma[k]}$ # Eq. (24)
  $B[k] \leftarrow \sigma[k]/\sqrt{2}$ # Eq. (25)
  $\text{foo} \leftarrow \log\text{erfcx}(A[k]+B[k]) - \log\text{erfcx}(A[k])$
  $EZ[k] \leftarrow \left(\rho_h + (1-\rho_h)e^{\text{foo}}\right)\bar{z}[k]$ # Eq. (22)
  $\text{bar} \leftarrow \log\text{erfcx}(A[k]+2B[k]) - \log\text{erfcx}(A[k])$
  $EZ2[k] \leftarrow \left(\rho_h + (1-\rho_h)e^{\text{bar}}\right)\bar{z}[k]^2$ # Eq. (23)
end for
$EZ_0 \leftarrow \text{log1p}\left(\frac{E_0/\beta}{N_0}\right)$ # EZ when no interference
$EL \leftarrow \frac{1}{H}(w \cdot EZ + w_0 EZ_0)$ # Eq. (26) with weights
$EL2 \leftarrow EL^2 + \frac{1}{H^2}\left(w \cdot (EZ2 - EZ^2)\right)$ # Eq. (28)
$\mu_L \leftarrow 2\log EL - 0.5\log EL2$ # Eq. (31)
$\sigma_L \leftarrow \sqrt{\log EL2 - 2\log EL}$ # Eq. (32)
$x \leftarrow$ uniform random number between 0 and 1
$SINR \leftarrow \beta\text{expm1}\left(e^{\mu_L - \sqrt{2}\sigma_L \text{erfc}^{-1}(2x)}\right)$ # Eq. (39)
return $SINR$

Fig. 2: Statistical interference model's algorithm for calculating the effective SINR for a single packet of interest.

IV. Statistical Model Accuracy and Performance

We implemented the statistical model in EMANE using the same waveform model as our deterministic model. Since the statistical model is an approximation of the deterministic model, which has already had its accuracy vetted, we use the deterministic model as the benchmark for the statistical model's accuracy. Figure 3 shows the CDF of the effective SINR for 100 samples produced in simulation by the two models where the background noise is -103 dBm, the power of the desired signal is -67 dBm, and there are 25 interfering signals overlapping (with random start times between 0 and 1 hop's worth including down time) each at -42 dBm. The difference between the two models in this case is negligible. We found the statistical model to be a very good match in cases where there are few interferers or the power levels of the interferers are not spread over a wide range.

However, in some cases when the power levels of the interferers are spread over a wide range, the discrepancy between the two models is very large. Figure 4 shows the CDF of the effective SINR for 100 samples produced in simulation by the two models where the background noise is -43 dBm, the power of the desired signal is -22 dBm, and there are 25 interfering signals overlapping at a wide range of power levels, from -75 dBm to -18 dBm. The two curves are offset by approximately 4.5 dB, with the statistical model being pessimistic. With other power spreads, we have seen discrepancies of up to 8 dB. However, there were also many cases where a similarly wide spread of power over many interferers did not result in a significant discrepancy between the two models, especially when the noise floor was low, in the region of -100 dBm.

Another set of scenarios that resulted in a large discrepancy between the two models was with tx/rx interference. The statistical model consistently predicted SINRs several dB lower than the deterministic model.

To study the performance gain of the statistical model versus the deterministic model, we compared the time it takes each model to both collect the necessary information and calculate the SINR on the exact same set of interferers on a single desired signal. For each run, we calculated the statistical model's performance gain as the time it took the deterministic model divided by the time it took the statistical model. In other words, the data collected for a single run is how many times faster the deterministic model is. We then collected samples from 5200 runs for every number of interferers we wished to plot and took the mean over the 5200 samples. To keep the comparison fair, a raw time of one model in one run was not compared against any raw times of the other model in other runs, only against the raw time in the same run to produce the ratio or performance gain. Instead of averaging over raw times, we averaged over the gains, which had little variance. Looking at the gain also mostly negates effects of using a particular computer system.

Figure 5 shows the performance gain of the statistical model over the deterministic model. Even with just one interferer, the statistical model runs approximately 140 times faster. With as many as 50 interferers, it runs approximately 470 times faster.

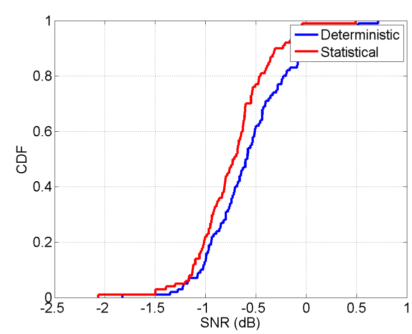

Fig. 3: CDF of the effective SINR for 100 samples produced in an EMANE simulation by the deterministic interference model (blue) and the statistical model (red), where the background noise is -103 dBm, the power of the desired signal is -67 dBm, and there are 25 interfering signals overlapping (with random start times between 0 and 1 hop's worth including down time) each at -42 dBm.

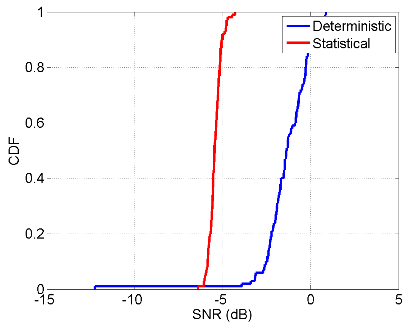

Fig. 4: CDF of the effective SINR for 100 samples produced in simulation by the two models where the background noise is -43 dBm, the power of the desired signal is -22 dBm, and there are 25 interfering signals overlapping at a wide range of power levels, from -75 dBm to -18 dBm.

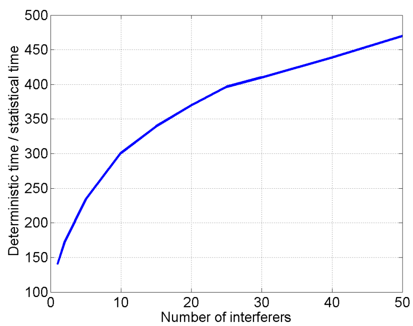

Fig. 5: Performance gain of the statistical interference model over the deterministic model, calculated as the deterministic model's computation time over the statistical's time, with respect to the number of interferers. Each data point in the plot is an average over 5200 samples.

V. Discussion

The overall effectiveness of the statistical model depends heavily on the simulation or emulation scenarios that are run. As we saw in the previous section, the statistical model matches the deterministic model quite well when there are few interfering signals at any given time or the interferers mostly have similar power levels, but it may not match well for some cases of many interferers with power levels over a wide range, and it consistently mismatches when there is tx/rx interference. Thus, the effectiveness of the statistical model depends on how frequently such cases are encountered in the simulation scenario. Furthermore, what ultimately matters in the simulation is whether or not the packet is received successfully. Sometimes the two models can return very different SINR values but still result in the same packet reception outcome. For example, if the deterministic model returns 20 dB while the statistical model returns 10 dB, but all packets above 2 dB are received successfully, the discrepancy in SINR is moot. In the waveforms that we have modeled, we have typically seen the case of tx/rx interference surface regularly, while having many interferers all sending at the exact same time with a wide range of power levels only occasionally.

We can discuss more thoroughly why it is that the statistical model may not match well when there is a wide range of power levels for the interferers. In a frequency hopping system, the combined effect of a weak interferer and a strong interferer is not the same as the effect of two equal-power interferers such that the sum power of the interferers is the same as with the weak and the strong. But the statistical model treats these two scenarios as equivalent. To see this, note that (9) only has one parameter, λ_h, in order to fit the distribution of the sum of interfering energies. With only one parameter, and with our fitting criterion as matching the mean between the exact distribution and the approximation, there is no room in the fit for expressing different variances. The statistical model can only express the sum of the interfering power levels, not any differences between those power levels. Similarly, the statistical model must express the very different effects of tx/rx interference using the same distribution (9) with a single parameter.

One way to enhance the statistical model to fix the mentioned discrepancies is to expand (9) to have a separate exponential term for each interferer, including a separate term for tx/rx interference. More precisely:

$$f_{Y_h}(x) = (1-3p)^{n_h}\delta(x) + (1-(1-3p)^{n_h}) \cdot \left[\epsilon\lambda_h e^{-\lambda_h x} + (1-\epsilon) \sum_{i \in I(h)} \lambda_{ih} e^{-\lambda_{ih} x} \right], \tag{40}$$

where each λ_{ih} is fit to its corresponding interferer i according to

$$\lambda_{ih} = \frac{1/n_h}{.9pE_i + .5p_T E_T}, \tag{41}$$

ϵ is a weight between the λ_h term and the other terms used for fitting. This effectively gives us an extra parameter λ_{ih} for each interferer i, allowing for a much more precise fit of Y_h.

We investigated several methods of fitting more parameters and decided to pursue the above fit further for several reasons. First, it reduces to the original statistical model in (9) when the interferers have the same power and there is no tx/rx interference, thus preserving the accuracy of the original model in those cases. Second, without the λ_h term, (40) would consistently return SINR values that were somewhat high. Introducing the λ_h term allows us to include influence from the original model, which would estimate low. We found that tuning the weight ϵ between the two sets of terms resulted in the right balance of high and low estimators. We currently use $\epsilon = 0.2$. Third, this particular fit shows promising initial results. Figure 6 shows, for each data point, the error in mean SINR (in dB) of the first and second statistical models over 100 samples for a particular set of conditions. The errors are plotted against the actual SINR (dB) of each run. The battery of tests consisted of each combination of noise at -100 dBm; the desired signal's power at -100 and -50 dBm; interfering powers at 20 dB less than the desired signal, same as the desired signal, 20 dB more, and a random spread between 32 less and 25 more than the desired signal; 1, 25, and 50 interferers; and both with and without tx/rx interference. The original statistical model's poorest results are cases with tx/rx interference. As seen in the figure, this modified approach

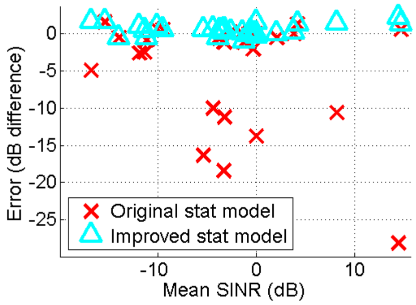

Fig. 6: The improved statistical model compared to the original statistical model. Each data point is the error of the mean SINR plotted against the actual mean SINR, over 100 samples of a particular set of interference conditions. The conditions range over several power levels, numbers of interferers, and whether there is send-while-receive interference. See the body of the paper for more details.

shows great promise in maintaining an accuracy of ± 2 dB and often well within ± 1 dB.

The benefit of using the statistical model is readily apparent in the performance gains measured in the previous section. The major problem with running larger simulations with more traffic and more nodes is that the computation grows quadratically as each node contributes more interference and also must calculate interference and desired signals from all its neighbors. Because the performance gain of the statistical model grows steadily with the number of interferers, it has the potential to counteract the growth in the computational requirements for larger networks. In future work, we intend to run several large scale network simulations with the statistical model using a full CORE/EMANE distributed setup to see realistically how large of a network it can handle.

Another limitation of the statistical model is that it only looks at interference from signals of the same waveform, background noise, and barrage jammers. It would be beneficial to simulate the effects of other friendly waveforms or jammers that have a partial band overlap with the desired signal or a different frequency hopping spectrum or duty cycle. This results in an extra task of determining how each waveform affects a single hop, i.e., determining the distribution of X_{ih}. We believe that extending the statistical model to include other waveforms could be somewhat straightforward by following this method, and this is another area of future research.

VI. Conclusion

In this paper, we have presented a large step toward having a statistical interference model of frequency hopping signals that maintains nearly the same accuracy as a per-hop deterministic model while drastically speeding up simulation time. The statistical model calculates the distribution of the effective SINR of a packet and includes effects like barrage jammer noise and send-while-receive interference. In the past, it has been difficult to simulate large networks using military waveforms because either the interference model made the simulation prohibitively complex and time consuming or the model has been so simple the results often cannot be trusted. With a statistical interference model, simulations of large networks could potentially be run in real time without sacrificing accuracy.

We implemented our first statistical interference model in EMANE, based on our previously developed deterministic model which uses the same overall equation for calculating the effective SINR. We analyzed the accuracy of the statistical model by comparing its empirical CDF of the SINR over a given set of conditions for many samples with the CDF of the deterministic model. We found that the statistical model matches the deterministic model quite well (offsets much less than 1 dB) when there are few interfering signals at any given time or the interferers mostly have similar power levels, but it sometimes does not match well if there are many interferers with power levels over a wide range (offsets as high as 8 dB). We also found that the statistical model estimates very pessimistically when there is send-while-receive interference.

We have obtained initial results in a modified statistical interference model that addresses the issues found in the first. By introducing a separate parameter for each interferer in the fitting distribution for total interference, we have found that the model is more flexible and able to handle the cases mentioned above. Testing over a wide range of conditions, we found that the improved model maintains an accuracy within 2 dB of the deterministic model.

The performance benefit of the statistical model was measured at about 140 times faster computation than the deterministic model with just one interferer, and the gain rose steadily to about 470 times faster with 50 interferers. This has the potential of running very large networks.

For future work, we are considering ways to enhance the statistical model to handle the discrepancy with a wide range of interfering powers. Among our considerations is an approach of approximating certain distributions with a larger set of fitting parameters, based more closely on mathematical derivations from the individual distributions. We are also considering ways to apply the model to heterogeneous waveforms.

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