Adaptive Correlation Techniques for Spread Spectrum Communication Systems

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***Abstract*— Spread spectrum communication systems can require significant computation to perform initial acquisition, searching across time and frequency to accurately detect and lock onto the desired signal. This initial acquisition problem is exacerbated in deeply spread non-binary signals lacking cyclostationary features or repetitive codes, leading to a desire for less computationally intensive approaches. This paper introduces adaptive correlation methods that employ iterative Bayesian estimation techniques to reduce the expected computational load of signal acquisition, yet still achieve desired detection probabilities. Improvements to the adaptive correlator technique are identified, permitting flexibility in the correlation search windows, making it configurable for different applications like GNSS, DSSS communications, and noise-like chaotic spread spectrum signals. This paper presents analytical predictions of adaptive correlator performance, validated by measured FPGA hardware results achieving savings of 80% over brute-force serial time-domain acquisition searches.**

I. INTRODUCTION

Correlation-based spread spectrum receivers require robust synchronization in the initial acquisition stages to efficiently establish a communications link. Deeply spread systems present difficulty by requiring precise bounds on time delay, frequency offset, and Doppler estimates to achieve sufficient coherent signal energy integration for an acquisition lock to be declared. Both time-domain and frequency-domain acquisition methods have been implemented with frequency-domain ones traditionally being better for signals with strong cyclostationary features [1] like Direct Sequence Spread Spectrum (DSSS). More advanced spreading signals like those employing digital chaotic sequence spread spectrum (CSSS) or constant amplitude zero autocorrelation (CAZAC) waveforms cannot rely on the same simplifications due to the non-repeating nature of their spreading codes.

Common techniques used to speed acquisition of spread signals include synchronized transmission of repeating short codes [2], sideband-aided acquisition schemes [3], pre-defined puncture codes that facilitate direct acquisition on a reduced chip set [4], integrating knowledge of the modulation data patterns [5], two-stage acquisition of aperiodic DSSS signals [6], spiral search parameters from previous known locks [7], parallel processed batches of search parameters [8], and matched filter correlation searches. The arithmetic required to implement these correlation techniques increases as the acquisition uncertainty (time offset, frequency offset, Doppler shift, spread bandwidth, and code length) increases. Acquisition complexity is exacerbated when the spread spectrum signal is a multi-level chipping sequence (e.g. CSSS, CAZAC) rather than the binary level signals used in DSSS modulations since "correlation" is performed by a non-binary conjugate multiplication. This paper focuses on the latter cases, where the signal of interest is taken to be a non-repeating noise-like signal [9] with bounded timing offset and frequency offset, but no other a priori knowledge (e.g., previous acquisition states, other synchronized carriers, sidebands, or puncture codes). One distinction from other correlation-based chaotic signal reception processes in [10] is that [9] helps ensure a discrete-time basis compatible with traditional DSSS systems, yet signal characteristic indistinguishable from band-limited white noise. Of particular interest for these signals is the desire to embed them well below the recipient's noise floor (>20 dB).

At the system level, correlators are typically quantified by their probability of correct detection (P_d), probability of false alarm (P_{fa}), and number of complex multiply-accumulate (CMAC) operations required to accurately locate the signal. This paper discusses a novel adaptive correlator [11] that overcomes many of the correlator efficiency problems by progressing thru successive levels of correlation confidence to achieve the same detection probabilities. The decision of whether to progress from one stage to the next or to increment correlation parameters is based on a Bayesian decision process. Efficiency gains of more than 80% have been demonstrated in live scenarios, with opportunities in the adaptive correlator control structure to change the parameter search sizes and orders based on available information, integrating results from prior work (batched processing, spiral search parameters, puncture codes) to further increase computational efficiency. This adaptive correlation technique was validated by over-the-air testing of a Field Programmable Gate Array (FPGA)-based CSSS implementation.

In developing these adaptive correlator techniques, a brief overview of the signal acquisition framework for a deeply spread signal is provided in Section II. A constructive outline of the adaptive correlation technique is presented in Section III. Concrete examples of the correlator's efficiency improvements are presented in Section IV, along with a derived lower bound for choosing initial stage correlation thresholds. Conclusions and extensions to related work are presented in Section V.

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II. Signal Acquisition Framework

Consider a spread spectrum communication signal, $X[n]$, where n is the spreading chip index. For simplicity, we assume the spreading ratio is an integer and that acquisition processing targets a known data preamble where data symbols are constant amplitude. The spreading sequence is also normalized to a unit variance, reducing the spread signal to a sequence of complex valued chips. When transmitted through the communications channel to the receiver, the signal is attenuated (α), corrupted by additive noise (N), and received with a frequency offset (f_o) and time delay (k), yielding a received signal of the form

$$Y[n] = \alpha e^{j2\pi f_o n} X[n-k] + N[n].$$

The noise signal N is generally assumed to be of greater magnitude than the attenuated spread signal; moreover, the receiver automatic gain control is assumed to be driven by the background noise level since the spread signal is preferably below the noise floor, allowing us to normalize the received signal according to the mean noise level. The complex valued attenuation factor α is therefore a point inside the unit circle, $|\alpha| \ll 1$. To perform matched filter reception, the receiver generates the conjugate spreading sequence X^* and then correlates against the receive signal Y with a specific frequency offset candidate $\hat{f}$. Therefore, an M-point correlation symbol S_M performed at the receiver takes the form

$$S_M = \sum_{n=1}^{M} e^{-j2\pi\hat{f}n} X^*[n] Y[n+m]$$

$$= \alpha \sum_{n=1}^{M} e^{j2\pi(f_o-\hat{f})n} X^*[n] X[n+m-k] + \sum_{n=1}^{M} X^*[n] N[n]$$

Since the signals X and N can be normalized to a unit variance, the expected coherent integration term reduces to αM at $f_o = \hat{f}$ and $m = k$, while the noise term converges to a zero-mean Gaussian random variable for large M (empirically, $M \gg 100$),

$$E[S_M]_{f_o=\hat{f}, m=k} = \alpha M + \sum_{n=1}^{M} X^*[n] N[n] \approx \alpha M + \sqrt{M}\, Z(0,1)$$

where Z(0,1) is a standard normal random variable. The actual statistics are distributed according to a modified Bessel function that is slightly skewed to the right (fatter tails) as compared to the Gaussian distribution [12]. Applying detection thresholds to this correlation statistic yields mechanisms for quantifying (P_d, P_{fa}), generally assumed to satisfy $P_d > 0.95$ and $P_{fa} < 0.01$, or equivalently choosing M sufficiently large enough to simultaneously satisfy the two constraints.

III. Adaptive Correlation Techniques

As an alternative to brute-force calculation of all possible M-point correlations, adaptive correlation seeks to reduce the search space by discarding false $(\hat{f}, m)$ early. Complementary techniques exist for winnowing the number of frequency bins or time delays based on external information [7], yet no known techniques reduce the number of points in the correlation when the signal has no cyclostationary features.

The proposed adaptive correlation mechanism reduces the total number of CMAC operations by implementing an iterative Bayesian process that allows mid-course decisions during the correlation calculation. At each stage, a decision is made to either pursue further correlation operations or to discontinue evaluation of that time/frequency combination and progress to the next pair. A three-stage decision process is shown in the adaptive correlator of Figure 1. Note that successive correlation stages perform correlations of increased length, $n_1 < n_2 < n_3$ and also that the correlations are based on independent, non-overlapping, sample sets. A binary decision is made after each reduced correlation stage to determine if the achieved correlation estimates, $\{S_{n_1}, S_{n_2}, S_{n_3}\}$ exceeds pre-defined energy thresholds $\{T_C, T_M, T_F\}$ corresponding to coarse-, medium-, and fine-resolution correlations. When the threshold is exceeded, processing transitions to the next higher resolution correlation or declares lock in the case of the fine resolution correlation; if the threshold is not met, then the correlator discontinues processing on that frequency/time combination and moves to the next set. If the entire set of possible time/frequency offsets is exhausted, then the overall correlation process is determined complete without locating a signal.

Detection thresholds are relaxed, however, such that there is a bias towards allowing false accepts in the earlier stage(s), perhaps as high as 50% for the coarse correlations, to prevent falsely discarding the correct parameter set. Since the points selected for correlation are independent (sampled at different times from an assumed stationary signal/channel), the resulting statistics are also independent. To maintain the same level of assurance obtained by the solitary M-point correlation, the total number of points must be at least as high as the traditional correlator, $M \leq n_1 + n_2 + n_3$, and likely slightly higher to compensate for the multiple potential false rejection cases.

The probability of passing the coarse correlation stage when the desired signal is present, $P_{d,C}$, is

$$P_{d,C} = P\left(\left|\alpha n_1 + \sqrt{n_1}Z(0,1)\right| \geq T_C\right) \cong P\left(Z(0,1) \geq \frac{T_C}{\sqrt{n_1}} - \alpha\sqrt{n_1}\right).$$

The probability of passing the coarse correlation stage when the desired signal is not present, $P_{fa,C}$, is

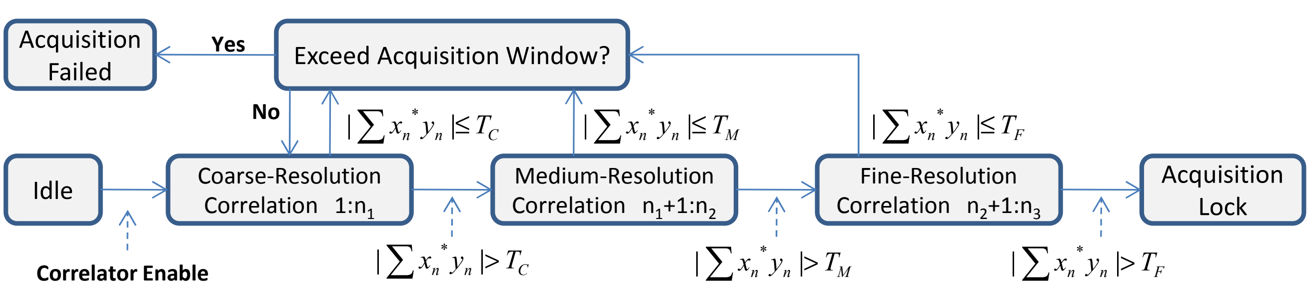

Figure 1. State Diagram for an Adaptive Correlator

$$P_{fa,C} = P\left(\left| \sqrt{n_1} Z(0,1) \right| \geq T_C \right) = P\left(Z(0,1) \geq \frac{T_C}{\sqrt{n_1}} \right).$$

Unlike traditional detection, this decision statistic represents a verdict to proceed further into the correlation processing for that combination of frequency and timing offsets. Similar decision statistics can be constructed for the medium- and fine-resolution correlations. Extending these decision statistics to the multi-stage correlation process, the overall probability of detection and false accept are given by

$$P_d = P_{d,C} \cap P_{d,M} \cap P_{d,F} = P_{d,C} P_{d,M} P_{d,F}$$

$$P_{fa} = P_{fa,C} \cap P_{fa,M} \cap P_{fa,F} = P_{fa,C} P_{fa,M} P_{fa,F}$$

The total correlation processing can then be given as a sum of the conditional probabilities of entering / exiting individual stages. Since the majority of correlations will be processed on samples where the signal is not present, that decision process, displayed conceptually in Figure 2, drives the computational efficiency of the overall correlator. All possible correlations are processed with n_1 points (n_1 CMACs); of those, a fraction $P_{fa,C}$ are processed by an additional n_2 CMACs. Likewise, a fraction $P_{fa,M}$ are processed by an additional n_3 CMACs.

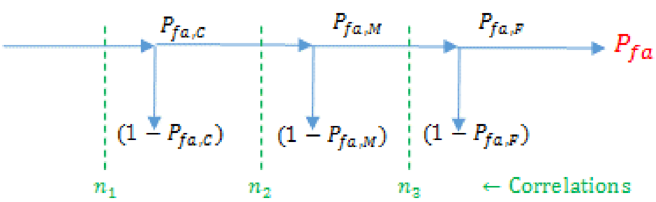

Figure 2. Iterative P_{fa} during adaptive correlation processing

The expected amount of processing for each time/frequency combination in the three-stage adaptive correlator is then

$$\text{Processing}_3 \colon n_1 + P_{fa,C} n_2 + P_{fa,C} P_{fa,M} n_3$$

extensible to

$$\text{Processing}_H \colon \sum_{t=1}^{H} \left(n_t \prod_{r<t} P_{fa,r} \right)$$

for arbitrary H-stage adaptive correlators. The number of stages can be shown to yield diminishing returns in processing gains unless signals possess highly varying SNRs, yet the processing figures hold true. To compute the anticipated efficiency savings of the three-stage adaptive correlation process, we observe that the total number of CMAC operations relative to a brute force correlator of size n_3 is

$$\eta[\text{CMACs}] = 1 - \frac{n_1 + P_{fa,C} n_2 + P_{fa,C} P_{fa,M} n_3}{n_3}$$

$$= 1 - \left(\frac{n_1}{n_3} + P_{fa,C} \frac{n_2}{n_3} + P_{fa,C} P_{fa,M} \right)$$

From this efficiency measure, it should be noted that the design choices for the initial/early stage(s), $(n_1, P_{fa,C})$, are the significant drivers of the overall computational efficiency. Subsequent stages help increase the rejection of falsely accepted signal candidates in this first stage, yet they also degrade P_d performance set bounded in the first stage.

IV. Adaptive Correlator Examples

A. Hardware Proof-of-Concept

To help validate the expected performance of the adaptive correlator, a three-stage embodiment was constructed in a Virtex-4 LX60 FPGA and used to receive a coherent digital CSSS signal [12]. This proof-of-concept system employed a spread ratio of M=200, a spread bandwidth of 10 MHz, and a QPSK-based 100 kbps modulation. Given a target operating point of +10 dB (≈3 dB implementation loss) despread SNR, the effective channel signal level was -10 dB. Based on an assumed correlation statistic threshold of 20 dB to achieve sufficient P_d, the correlation window was configured to support off-line processing of ≈5 symbol durations (1024 chips). The chosen adaptive correlator parameters were:

$$|\alpha| \geq \sqrt{0.1}, \;\; n_1 = 32, \;\; n_2 = 224, \;\; n_3 = 768,$$

$$T_C = 0, \;\; T_M = 32, \;\; T_F = 175.$$

The resulting total number of correlation points was $n_1 + n_2 + n_3 = 1024$, and predicted (P_d, P_{fa}) statistics were

$$P_{d,C} = 0.963, P_{d,M} = 0.995, P_{d,F} = 0.993$$

$$P_{fa,C} = 0.50, P_{fa,M} = 0.0163, P_{fa,F} = 1.3 \cdot 10^{-10}$$

$$P_d = 0.951 \quad , \quad P_{fa} \approx 10^{-12}.$$

The predicted computational savings for this prototype was

$$\eta = 1 - \left(\frac{32}{768} + 0.5 \cdot \frac{224}{768} + 0.5 \cdot 0.0163 \right) = 80.4\%$$

These estimated savings were validated in live over-the-air hardware testing of a vectorized adaptive correlator, achieving measured performance of 78%, which is well within the uncertainty level of the measured SNR in the hardware setup. Hardware utilization of the prototype (8 parallel lanes) accounted for 42% of the embedded multipliers, 23% of block RAMs (offline sample storage), and ≈21% of the computational logic. While significant for the LX60 device, newer devices are substantially more capable. Of particular importance in the CSSS prototype was the ability to quickly lock onto the desired signal with a short acquisition preamble.

B. Correlator Threshold Selections

The next goal of the design process is to determine the optimal choice of correlation lengths and thresholds for signals at different SNRs, spreading ratios, and spread characteristics. As a first bound, the minimum coarse correlation length to achieve the desired P_d can be estimated using the Q^{-1} function and a quadratic formula reduction on $\sqrt{n_1}$. Note that n_1 must be positive, enabling simplification of the quadratic solutions.

$$n_1 \geq \left(\frac{-Q^{-1}(P_d) + \sqrt{\left(Q^{-1}(P_d)\right)^2 + 4\alpha T_C}}{2\alpha} \right)^2 \tag{1}$$

A simplifying option is to choose $T_C = 0$, which equates to a 50% false accept rate for the subsequent stage (i.e. half of the search window is eliminated by the coarse correlator), and assume that the desired $P_d > 0.5$, giving a simpler closed form bound for n_1, given the desired P_d.

$$n_{1|T_C=0} \geq \frac{|Q^{-1}(P_d)|^2}{|\alpha|^2}$$

Returning to the prototype example, plugging in $P_d = 0.95$, we calculate $Q^{-1}(0.95) = -1.6449$, leading to a lower recommended bound of $n_{1|T_C=0} > 27.1$. Rounding up to 32 is based on wanting a slightly higher $P_{d,C}$ (knowing subsequent stages will not be 100%) and also to use binary powers that are easy to implement in digital logic. From (1), it should be noted that the lower bound of values for n_1 depends on the SNR and the chosen coarse threshold. The dependence on SNR confirms the expectation that fewer points are required in the correlation to obtain a desired P_d when the SNR is higher; likewise, choosing a larger threshold value permits more selective (lower P_{fa}), yet more faulty, decisions. However, the use of a $T_C > 0$ quickly creates a condition whereby the number of points in the first stage rapidly approaches that of a brute force correlator for the same P_d. To demonstrate these facts, a lower bound on n_1 was calculated over a range of despread SNRs from [3,20] dB, a spreading ratio of 10^3, and 5 steps of the coarse threshold, T_C; these simulated results are shown in Figure 3. Note that increasing T_C does significantly reduce P_{fa}, as displayed in Figure 4.

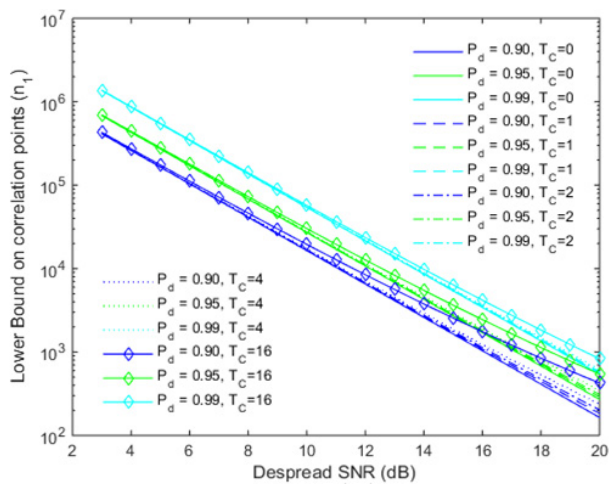

Figure 3. Minimum coarse correlation length to achieve {90%,95%,99%} $P_{d,C}$ as a function of SNR; spread ratio = 1000

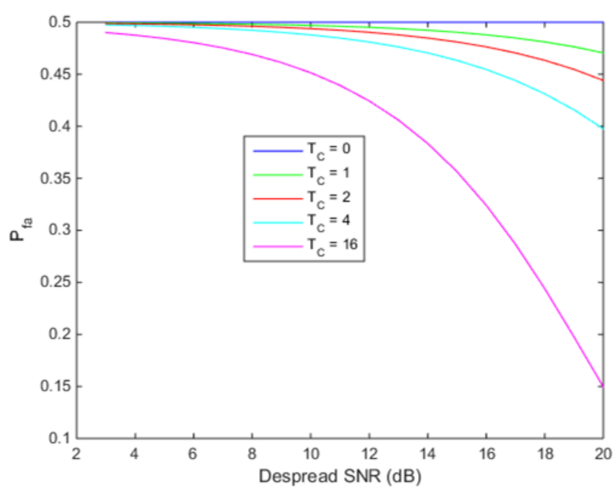

Figure 4. P_{fa} as a function of SNR for different coarse thresholds

V. Adaptive Correlator Conclusions

The adaptive correlator method described in this paper has been shown analytically and in prototype hardware to support more efficient time-domain correlation processing of low probability of detection (LPD) spread spectrum signals operating at relevant SNRs. With savings of >90% demonstrated in some deeply spread systems, these techniques offer significant advantages over traditional methods that rely on cyclostationary signal features, inherently defeating the benefit of LPD systems. Moreover, the adaptive correlation techniques are easily extensible, permitting an adaptable number of stages, integration of spiral techniques that re-order which time and frequency offsets are tested, and parallel processing of multiple time/frequency combinations. Future work will focus on developing heuristics for the relative sizes of $\{n_1, n_2, n_3\}$, creating algorithms to dynamically adjust the thresholds based on prior search results, and integrating complementary techniques [7] for spiral search processes.

Limitations of the adaptive correlator techniques include the need for real-time storage of a captured preamble while acquisition processing occurs, an inferred lower bound on the system SNR by choice of detection thresholds, and a more complex state machine that relies on partial detection statistic results. For channels with severe fading, the adaptive correlator thresholds will likely need to be chosen as less selective, yet that greater uncertainty is expected actually to yield greater processing advantages over traditional time-domain correlators operating in the same channel conditions.

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