Robust Beamforming for Jammers Suppression in MIMO Radar

(Invited Paper)

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***Abstract*—Robust beamforming for multiple-input multiple-output (MIMO) radar in the background of powerful jamming signals is investigated in this paper. We design two minimum variance distortionless response (MVDR) type beamformers with adaptiveness/robustness against the powerful jammers for colocated MIMO radar. Specifically, the MVDR beamformer is firstly designed for known jammers in the sector-of-interest, which maintains distortionless response towards the direction of the target while imposing nulls towards the directions of jammers. Then the adaptive/robust MVDR beamformer is designed for the general case of unknown in-sector jammers and/or out-of-sector interfering sources. Convex optimization techniques are used in both of the designs. Moreover, we derive a closed-form solution to the simplified second design. Based on this solution, we derive efficient power estimates of the desired and/or interfering sources in the context of powerful jammers and non-ideal factors such as array calibration errors and target steering vector mismatches. We demonstrate that the capability of efficient jammers suppression using these designs is unique in MIMO radar.**

I. Introduction

Multiple-input multiple-output (MIMO) radar has become the focus of intensive research [1]–[4]. Two configurations of MIMO radar have been developed in current literature. One is the MIMO radar with widely separated antennas [2], which exploits the spatial diversity of the target. This type of MIMO radar has been shown to be capable of improving the target detection and parameter estimation performance, and enhancing the ability to combat target scintillation [2], [4], [5]. The other is the MIMO radar with colocated antennas [3], which exploits the waveform diversity allowed by the transmit and receive antenna arrays. It has been shown that this type of MIMO radar facilitates improving the angular resolution and parameter identifiability, increasing the upper limit on the number of resolvable targets, extending the array aperture, and obtaining the desired transmit beampatterns [3], [6]–[10].

For MIMO radar, interfering sources can take different forms and have widely varying impacts on it. The terrain-scattered jamming is a typical interfering example that takes the form of high-power transmission and results in impairing the receiving system. It occurs when high-power jammer transmits its energy into ground. The ground reflects the energy in a dispersive manner, hence the powerful jamming signal appears at the receive array as distributed source. This situation can be more serious when the powerful jammers are present in the same directions as the desired targets, which severely affects the performance of MIMO radar such as target detection ability, parameter estimation capability, etc. It worsens even more since the powerful jammers are generally unknown. Therefore, designing simple and efficient ways of jammers suppression becomes particularly important and imperative. Moreover, these suppression methods should be robust enough against the unknown jammers.

In current literature, space-time adaptive processing (STAP) techniques [11], [12] have been exploited to mitigate the clutter and the interference in MIMO radar systems, in which two-dimensional filters operating in both the spatial and temporal domains are employed in order to adaptively impose nulls towards them. For example, prolate spherical waveform functions are used to construct the clutter subspace in [11] for fully adaptive processing, and slow-time STAP is conducted in [12] to mitigate the clutter subject to multi-path propagation between the transmit and receive arrays. Although similar STAP techniques can be used to suppress the powerful jammers with small modifications, they are not robust when there exist array calibration errors and target steering vector mismatches. Moreover, they have large computational complexity due to their two-dimensional adaptive processing.

Another category of jammer suppression strategies that can be resorted to is the beamforming technique [7], [8], [13]-[17] . In the last two decades, robust adaptive beamforming has been thoroughly investigated for traditional phased-array (PA) radar. The well-known minimum variance distortionless response (MVDR) beamformer is obtained by minimizing the variance/power of the interference and noise at the output of the adaptive beamformer while ensuring the distortionless response of the beamformer towards the direction of the desired source. To the best of our knowledge, introducing robust adaptive MVDR beamforming to MIMO radar, and further studying the capability of efficient suppression on powerful jammers, however, have not received attention yet.

In this paper, we use beamforming techniques to implement the suppression of powerful jammers for colocated MIMO radar. We show that in MIMO radar the echoes reflected from

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the targets and the intentionally radiated jamming signals have different spatial signatures even if they impinge on the receive array from the same spatial angle. Using this observation, we design two MVDR type beamformers with adaptiveness/robustness against powerful jammers for MIMO radar. We first design an MVDR beamformer for known jammers in the sector-of-interest (SOI) by maintaining distortionless response towards the direction of the target while enforcing nulls towards the directions of the jammers. Then we design another MVDR beamformer for the general case of unknown in-sector jammers and/or out-of-sector interfering sources. Both designs are cast as convex optimization problems. We propose to incorporate adaptiveness/robustness against the in-sector jammers and/or the out-of-sector interfering sources in these designs. Moreover, we derive a closed-form solution for the simplified second design. Based on this solution we further find an efficient source power estimate method in the background of powerful jammers and non-ideal factors such as array calibration errors and steering vector mismatches.

II. System Model

Consider a MIMO radar system equipped with linear transmit and receive arrays which contain M transmit antenna elements and N receive antenna elements. Both the transmit and receive arrays are assumed to be close enough to each other such that they share the same spatial angle of a far-field target. Let $\mathbf{\Phi}(t) = [\phi_1(t), \ldots, \phi_M(t)]^T$ be the $M \times 1$ vector that contains the complex envelopes of the transmitted waveforms $\phi_i(t),\ i = 1, \ldots, M$ which are assumed to be orthogonal, i.e., $\int_{T_p} \phi_i(t)\phi_j^*(t)dt = \delta(i-j),\ i,j = 1, \ldots, M$, where T_p is the pulse duration and $\delta(\cdot)$ is the Kronecker delta function. Here $(\cdot)^T$ and $(\cdot)^*$ stand for the transpose and complex conjugate operations, respectively.

Let us assume that L sources including the desired and interfering ones are observed in the background of noise. Then the $N \times 1$ complex vector of the received observations can be expressed as

$$\mathbf{x}(t,\tau) = \sum_{l=1}^{L} \alpha_l(\tau) \left[\mathbf{a}^T(\theta_l)\mathbf{\Phi}(t)\right] \mathbf{b}(\theta_l) + \mathbf{z}(t,\tau) \quad (1)$$

where τ is the slow time index, i.e., the pulse number, $\alpha_l(\tau)$ is the reflection coefficient of the lth source with variance σ_α^2, θ_l is the spatial angle associated with the lth source, $\mathbf{a}(\theta)$ and $\mathbf{b}(\theta)$ are the steering vectors of the transmit and receive arrays, respectively, and $\mathbf{z}(t,\tau)$ is the $N \times 1$ zero-mean white Gaussian noise term. Note that $\alpha_l(\tau)$ is assumed to remain constant during the whole pulse, but varies independently from pulse to pulse, i.e., it obeys the Swerling II target model [18].

By matched filtering the received data to the M orthogonal waveforms at the receiving end, the $MN \times 1$ virtual data vector can be obtained as

$$\begin{aligned}\mathbf{y}(\tau) &= \mathrm{vec}\left(\int_{T_p} \mathbf{x}(t,\tau)\mathbf{\Phi}^H(t)\,dt\right) \\ &= \sum_{l=1}^{L} \alpha_l(\tau)\left[\mathbf{a}(\theta_l) \otimes \mathbf{b}(\theta_l)\right] + \tilde{\mathbf{z}}(\tau)\end{aligned} \quad (2)$$

where $\tilde{\mathbf{z}}(\tau) = \mathrm{vec}\left(\int_{T_p} \mathbf{z}(t,\tau)\mathbf{\Phi}^H(t)dt\right)$ is the $MN \times 1$ noise term whose covariance is given by $\sigma_z^2\mathbf{I}_{MN}$ with $\mathbf{I}_{MN}$ denoting the identity matrix of size $MN \times MN$, $\mathrm{vec}(\cdot)$ is the operator that stacks the columns of a matrix into one column vector, $\otimes$ denotes the Kronecker product, and $(\cdot)^H$ stands for the Hermitian transpose.

Let us assume that J powerful jammers are present and transmit their energy into ground, then the signal model (2) can be rewritten as

$$\begin{aligned}\mathbf{y}(\tau) = &\sum_{l=1}^{L} \alpha_l(\tau)\left[\mathbf{a}(\theta_l) \otimes \mathbf{b}(\theta_l)\right] \\ &+ \sum_{j=1}^{J} \beta_j(\tau)\left[\mathbf{1}_M \otimes \mathbf{b}(\theta_j)\right] + \tilde{\mathbf{z}}(\tau)\end{aligned} \quad (3)$$

where $\beta_j(\tau)$ is the signal of the jth jammer, θ_j is the presumed spatial angle associated with the jth jammer, and $\mathbf{1}_M$ denotes the $M \times 1$ vector of all ones.

Let $\mathbf{v}(\theta_l) \triangleq \mathbf{a}(\theta_l) \otimes \mathbf{b}(\theta_l)$ and $\tilde{\mathbf{v}}(\theta_j) \triangleq \mathbf{1}_M \otimes \mathbf{b}(\theta_j)$ be the virtual steering vectors of the lth target and the jth jammer, respectively, then (3) can be expressed as

$$\mathbf{y}(\tau) = \sum_{l=1}^{L} \alpha_l(\tau)\mathbf{v}(\theta_l) + \sum_{j=1}^{J} \beta_j(\tau)\tilde{\mathbf{v}}(\theta_j) + \tilde{\mathbf{z}}(\tau). \quad (4)$$

Note that the reason that the virtual steering vector of the jammer contains $\mathbf{1}_M$ is because the terrain-scattered jammer is independent of the transmitted waveforms.

III. Robust Beamforming Design for Jammers Suppression

We assume that the desired targets are located within a known SOI $\mathbf{\Theta}$ [10] where powerful jamming sources are also present and can even have the same spatial angles as the targets. Meanwhile, we assume that the interfering sources are present outside the SOI.

The first beamforming design is for the case when all the in-sector jammers are known, for example, they can be estimated beforehand. To achieve the goal of jammers suppression, deep null notches should be formed towards the spatial directions of the jammers while maintaining distortionless response towards the direction of the target(s). Thus, the corresponding optimization problem can be written as

$$\min_{\mathbf{w}} \quad \mathbf{w}^H\mathbf{R}\mathbf{w} \quad (5a)$$

$$\text{s.t.} \quad \mathbf{w}^H\mathbf{v}(\theta_t) = 1 \quad (5b)$$

$$\mathbf{w}^H\tilde{\mathbf{v}}(\theta_j) = 0,\ j = 1, \ldots, J \quad (5c)$$

where $\mathbf{R}$ is the covariance matrix of the interference plus jammer and noise, $\mathbf{w}$ is the designed beamforming weight vector, and θ_t is the spatial angle of the desired target.

The constraints (5c) impose nulls towards the powerful jamming signals, and they are expected to annihilate all the jammer components of the received signal even if the jammers have the same spatial angle as the desired target. The principle of robust adaptive beamforming can be incorporated in the considered case. Indeed, if there is an uncertainty about the in-sector target and jammer locations in the presence of the out-of-sector interfering signals, the distortionless response constraint (5b) and the null constraints (5c) can be extended to all steering vectors defined by a ball (or ellipse) centered around θ_t and θ_j [15]–[17].

The second beamforming design is based on the general case that the in-sector jammers and the out-of-sector interfering sources are unknown. Deep null notches should be formed towards all the possible directions of the jammers and, if needed, the out-of-sector interfering source attenuation should be kept to an acceptable level. Hence, the corresponding optimization problem can be written as

$$\min_{\mathbf{w}} \quad \mathbf{w}^H \mathbf{R} \mathbf{w} \tag{6a}$$

$$\text{s.t.} \quad \mathbf{w}^H \mathbf{v}(\theta_t) = 1 \tag{6b}$$

$$\left|\mathbf{w}^H \tilde{\mathbf{v}}(\theta_i)\right| \leq \delta, \ \theta_i \in \mathbf{\Theta}, \ i = 1, \ldots, Q \tag{6c}$$

$$\left|\mathbf{w}^H \mathbf{v}(\bar{\theta}_k)\right| \leq \gamma, \ \bar{\theta}_k \in \bar{\mathbf{\Theta}}, \ k = 1, \ldots, K \tag{6d}$$

where $\bar{\mathbf{\Theta}}$ combines a continuum of all out-of-sector directions, $\{\bar{\theta}_k \in \bar{\mathbf{\Theta}}, \ k = 1, \ldots, K\}$ is the angular grid chosen to properly approximate the out-of-sector $\bar{\mathbf{\Theta}}$ by a finite number K of directions, $\{\theta_i \in \mathbf{\Theta}, \ i = 1, \ldots, Q\}$ is a grid of angles used to approximate $\mathbf{\Theta}$ by Q directions, $\gamma > 0$ is the parameter of the user choice that characterizes the worst acceptable out-of-sector attenuation of interfering targets, $\delta > 0$ is the parameter that characterizes the worst acceptable level of the jamming power radiation in the desired sector $\mathbf{\Theta}$, and $|\cdot|$ denotes the magnitude of a complex quantity.

Both of the MVDR beamforming designs (5) and (6) incorporate adaptiveness towards the out-of-sector interferences and robustness towards the in-sector jammers. The first design is simple and efficient for the case of known in-sector jammers whose spatial angles have been already known beforehand, while the second design adapts to the general case that the in-sector jammers are unknown, making it more robust than the first design. Although the robust in-sector jammers suppression and the out-of-sector interferences attenuation result in increased computational complexity, the latter design is a flexible and useful strategy. It overcomes the challenge that there is no prior information about the in-sector jammers and allows the jammers to be varying. Thus, in this sense, it serves as a universal strategy for powerful jammers suppression using MIMO radar. In addition, the out-of-sector attenuation facilitates the design that requires controlling the sidelobes in practice.

IV. Source Power Estimation

The standard MVDR beamforming problem, i.e., the problem (5) without the constraint (5c), leads to the following closed-form solution [19]

$$\mathbf{w}_{\text{opt}} = \frac{\mathbf{R}^{-1}\mathbf{v}(\theta_t)}{\mathbf{v}^H(\theta_t)\mathbf{R}^{-1}\mathbf{v}^H(\theta_t)} \tag{7}$$

when $\mathbf{v}(\theta_t)$ is known. Using (7), the standard MVDR beamforming yields the estimate of the source power σ_0^2 as

$$\sigma_0^2 = \frac{1}{\mathbf{v}^H(\theta_t)\mathbf{R}^{-1}\mathbf{v}(\theta_t)}. \tag{8}$$

Powerful jamming source that locates at the same direction as the desired target results in serious performance degradation of the standard MVDR beamforming technique. It becomes even worse when the knowledge of $\mathbf{v}(\theta_t)$ is imprecise because the standard MVDR beamformer attempts to suppress the desired target as if it was an interfering source. This happens in practice especially when there are array calibration errors and mismatches between the presumed and actual target steering vectors. In what follows, we derive a solution of the simplified second proposed beamforming design with robustness against both the in-sector jammers and the aforementioned non-ideal factors simultaneously and, furthermore, provide the source power estimate for the robust MVDR beamforming design.

To simplify the derivation, we ignore the out-of-sector interfering source attenuation constraint (6d). Let us introduce an $MN \times Q$ matrix $\tilde{\mathbf{V}}$ whose ith column is defined as $\tilde{\mathbf{v}}(\theta_i)$, $i \in \{1, \ldots, Q\}$, i.e., $\tilde{\mathbf{V}} \triangleq [\tilde{\mathbf{v}}(\theta_1), \ldots, \tilde{\mathbf{v}}(\theta_Q)]$. Then the optimization problem (6) without (6d) can be expressed as

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H \mathbf{R} \mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}^H \mathbf{v}(\theta_t) = 1 \\ & \|\mathbf{w}^H \tilde{\mathbf{V}}\|_\infty \leq \delta \end{aligned} \tag{9}$$

where $\|\cdot\|_\infty$ denotes the chebyshev norm.

Using the fact that $\|\mathbf{w}^H\tilde{\mathbf{V}}\|_\infty \geq \|\mathbf{w}^H\tilde{\mathbf{V}}\|/\sqrt{MN}$ where $\|\cdot\|$ denotes the Euclidean norm, and assuming that the mismatch between the actual target steering vector $\mathbf{v}(\theta)$ and the presumed target steering vector $\bar{\mathbf{v}}(\theta)$ is bounded as $\|\bar{\mathbf{v}}(\theta) - \mathbf{v}(\theta)\|^2 \leq \epsilon$, where ϵ is the given parameter that characterizes the worst allowable steering vectors error, then the optimization problem (9) can be approximated by the following strengthened optimization problem

$$\min_{\mathbf{w}} \quad \mathbf{w}^H \mathbf{R} \mathbf{w} \tag{10a}$$

$$\text{s.t.} \quad \mathbf{w}^H \bar{\mathbf{v}}(\theta_t) = 1 \tag{10b}$$

$$\|\mathbf{w}^H \tilde{\mathbf{V}}\|^2 \leq \tilde{\delta} \tag{10c}$$

where $\tilde{\delta} \triangleq MN\delta^2$ is the new parameter that characterizes the worst acceptable level of the in-sector jamming power radiation for the new Euclidean norm based constraint.

Let $\lambda > 0$ and μ be the real-valued Lagrange multipliers with μ being arbitrary. Then we define the Lagrangian of the optimization problem (10) as

$$\begin{aligned} & L(\mathbf{w}, \lambda, \mu) \\ & = \mathbf{w}^H \mathbf{R} \mathbf{w} + \lambda \left(\|\mathbf{w}^H \tilde{\mathbf{V}}\|^2 - \tilde{\delta} \right) + \mu \left(-\Re\left\{ \mathbf{w}^H \bar{\mathbf{v}}(\theta_t) \right\} + 1 \right) \end{aligned} \tag{11}$$

where $\Re\{\cdot\}$ denotes the real part of a complex quantity. Let $\mathbf{R}_{\tilde{\mathbf{V}}} \triangleq \tilde{\mathbf{V}}\tilde{\mathbf{V}}^H$, then (11) can be rewritten as

$$\begin{aligned} & L(\mathbf{w}, \lambda, \mu) \\ & = \mathbf{w}^H \mathbf{R} \mathbf{w} + \lambda \left(\mathbf{w}^H \mathbf{R}_{\tilde{\mathbf{V}}} \mathbf{w} - \tilde{\delta} \right) + \mu \left(-\Re\left\{ \mathbf{w}^H \bar{\mathbf{v}}(\theta_t) \right\} + 1 \right) \end{aligned} \tag{12}$$

which satisfies the following inequality

$$L(\mathbf{w},\lambda,\mu) \le \mathbf{w}^H \mathbf{R}\mathbf{w} \quad \text{for any } \mathbf{w} \in D \tag{13}$$

where D is the feasible set defined by the constraints in (10). The equality in (13) holds on the boundary of D.

The solution of (10) can be derived under two conditions. The first condition is that

$$\frac{\bar{\mathbf{v}}^H(\theta_t)\mathbf{R}^{-1}\mathbf{R}_{\tilde{\mathbf{V}}}\mathbf{R}^{-1}\bar{\mathbf{v}}(\theta_t)}{\left[\bar{\mathbf{v}}^H(\theta_t)\mathbf{R}^{-1}\bar{\mathbf{v}}(\theta_t)\right]^2} \le \tilde{\delta} \tag{14}$$

and based on it we can derive the solution of (10) directly according to (7) by replacing the actual target steering vector with the presumed one, i.e.,

$$\bar{\mathbf{w}} = \frac{\mathbf{R}^{-1}\bar{\mathbf{v}}(\theta_t)}{\bar{\mathbf{v}}^H(\theta_t)\mathbf{R}^{-1}\bar{\mathbf{v}}^H(\theta_t)}. \tag{15}$$

This means that in this case λ is equal to 0 and hence the constraint (10c) is not necessary.

If (14) is not satisfied, then the second condition described by the following inequality

$$\frac{\bar{\mathbf{v}}^H(\theta_t)\mathbf{R}^{-1}\mathbf{R}_{\tilde{\mathbf{V}}}\mathbf{R}^{-1}\bar{\mathbf{v}}(\theta_t)}{\left[\bar{\mathbf{v}}^H(\theta_t)\mathbf{R}^{-1}\bar{\mathbf{v}}(\theta_t)\right]^2} > \tilde{\delta} \tag{16}$$

holds, which sets the upper bound on $\tilde{\delta}$ and hence leads to a new solution. Realizing that (12) can be expressed as

$$\begin{aligned} L(\mathbf{w},\lambda,\mu) = &\left[\mathbf{w} - \mu\left(\mathbf{R}+\lambda\mathbf{R}_{\tilde{\mathbf{V}}}\right)^{-1}\bar{\mathbf{v}}(\theta_t)\right]^H \left(\mathbf{R}+\lambda\mathbf{R}_{\tilde{\mathbf{V}}}\right) \\ &\times \left[\mathbf{w} - \mu\left(\mathbf{R}+\lambda\mathbf{R}_{\tilde{\mathbf{V}}}\right)^{-1}\bar{\mathbf{v}}(\theta_t)\right] - \mu^2\bar{\mathbf{v}}(\theta_t)^H \\ &\times \left(\mathbf{R}+\lambda\mathbf{R}_{\tilde{\mathbf{V}}}\right)^{-1}\bar{\mathbf{v}}(\theta_t) - \lambda\tilde{\delta} + 2\mu \end{aligned} \tag{17}$$

we find that the following solution

$$\bar{\mathbf{w}}_{\lambda,\mu} = \mu\left(\mathbf{R}+\lambda\mathbf{R}_{\tilde{\mathbf{V}}}\right)^{-1}\bar{\mathbf{v}}(\theta_t) \tag{18}$$

yields the minimum value of the Lagrangian (12) for fixed λ and μ, i.e.,

$$\begin{aligned} &L(\bar{\mathbf{w}}_{\lambda,\mu},\lambda,\mu) \\ &\quad = -\mu^2\bar{\mathbf{v}}(\theta_t)^H\left(\mathbf{R}+\lambda\mathbf{R}_{\tilde{\mathbf{V}}}\right)^{-1}\bar{\mathbf{v}}(\theta_t) - \lambda\tilde{\delta} + 2\mu. \end{aligned} \tag{19}$$

The maximization of (19) with respect to μ is achieved by letting its derivative with respect to μ be equal to 0, which gives the following optimal value for μ

$$\bar{\mu} = \frac{1}{\bar{\mathbf{v}}(\theta_t)^H\left(\mathbf{R}+\lambda\mathbf{R}_{\tilde{\mathbf{V}}}\right)^{-1}\bar{\mathbf{v}}(\theta_t)}. \tag{20}$$

By substituting (20) into (19), the Lagrangian can be rewritten as

$$L(\bar{\mathbf{w}}_{\lambda,\mu},\lambda,\bar{\mu}) = \frac{1}{\bar{\mathbf{v}}(\theta_t)^H\left(\mathbf{R}+\lambda\mathbf{R}_{\tilde{\mathbf{V}}}\right)^{-1}\bar{\mathbf{v}}(\theta_t)} - \lambda\tilde{\delta}. \tag{21}$$

Similarly, the maximization of (21) with respect to λ is achieved by letting its derivative with respect to λ be equal to 0. It leads to the following equality for the given parameter $\tilde{\delta}$

$$\tilde{\delta} = \frac{\bar{\mathbf{v}}(\theta_t)^H\left(\mathbf{R}+\bar{\lambda}\mathbf{R}_{\tilde{\mathbf{V}}}\right)^{-1}\mathbf{R}_{\tilde{\mathbf{V}}}\left(\mathbf{R}+\bar{\lambda}\mathbf{R}_{\tilde{\mathbf{V}}}\right)^{-1}\bar{\mathbf{v}}(\theta_t)}{\left[\bar{\mathbf{v}}(\theta_t)^H\left(\mathbf{R}+\bar{\lambda}\mathbf{R}_{\tilde{\mathbf{V}}}\right)^{-1}\bar{\mathbf{v}}(\theta_t)\right]^2}. \tag{22}$$

The right-hand side of (22) is a monotonically decreasing function of $\bar{\lambda}$, meaning that $\bar{\lambda} > 0$ is unique and it can be achieved using an efficient searching algorithm. Hence, substituting (20) and $\bar{\lambda}$ that is found numerically as described above into (18), we obtain the solution of the optimization problem (10) under the condition (16) as

$$\bar{\mathbf{w}} = \frac{\left(\mathbf{R}+\bar{\lambda}\mathbf{R}_{\tilde{\mathbf{V}}}\right)^{-1}\bar{\mathbf{v}}(\theta_t)}{\bar{\mathbf{v}}(\theta_t)^H\left(\mathbf{R}+\bar{\lambda}\mathbf{R}_{\tilde{\mathbf{V}}}\right)^{-1}\bar{\mathbf{v}}(\theta_t)} \tag{23}$$

and the corresponding source power estimate as

$$\bar{\sigma}_0^2 = \frac{\bar{\mathbf{v}}(\theta_t)^H\left(\mathbf{R}+\bar{\lambda}\mathbf{R}_{\tilde{\mathbf{V}}}\right)^{-1}\mathbf{R}\left(\mathbf{R}+\bar{\lambda}\mathbf{R}_{\tilde{\mathbf{V}}}\right)^{-1}\bar{\mathbf{v}}(\theta_t)}{\left[\bar{\mathbf{v}}(\theta_t)^H\left(\mathbf{R}+\bar{\lambda}\mathbf{R}_{\tilde{\mathbf{V}}}\right)^{-1}\bar{\mathbf{v}}(\theta_t)\right]^2}. \tag{24}$$

The source power estimate (24) can be use for the first case when (14) is satisfied only if $\bar{\lambda} = 0$.

Note that throughout the derivation of the power estimate, we require $\mathbf{R}$ and $\mathbf{R}+\bar{\lambda}\mathbf{R}_{\tilde{\mathbf{V}}}$ to be positive definite in order to guarantee the invertibility. Generally, the matrix $\mathbf{R}_{\tilde{\mathbf{V}}}$ is positive semidefinite due to the fact that $\mathrm{rank}\{\mathbf{R}_{\tilde{\mathbf{V}}}\} \le \mathrm{rank}\{\tilde{\mathbf{V}}\}$, thus $\mathbf{R}+\bar{\lambda}\mathbf{R}_{\tilde{\mathbf{V}}}$ will be close to singular when $\bar{\lambda}$ is large enough. To avoid this, we can employ a small diagonal loading to the matrix $\mathbf{R}_{\tilde{\mathbf{V}}}$.

It is also worth noting that the equality (22) gives the lower bound on $\tilde{\delta}$, i.e., the limit when $\bar{\lambda}$ goes to infinity. Hence, (22) together with (14) imply that $\tilde{\delta}$ should be chosen from the following interval

$$\frac{1}{\bar{\mathbf{v}}(\theta_t)^H\mathbf{R}_{\tilde{\mathbf{V}}}^{+}\bar{\mathbf{v}}(\theta_t)} \le \tilde{\delta} < \frac{\bar{\mathbf{v}}^H(\theta_t)\mathbf{R}^{-1}\mathbf{R}_{\tilde{\mathbf{V}}}\mathbf{R}^{-1}\bar{\mathbf{v}}(\theta_t)}{\left[\bar{\mathbf{v}}^H(\theta_t)\mathbf{R}^{-1}\bar{\mathbf{v}}(\theta_t)\right]^2} \tag{25}$$

where $\mathbf{R}_{\tilde{\mathbf{V}}}^{+}$ denotes the Moore-Penrose pseudoinverse of the matrix $\mathbf{R}_{\tilde{\mathbf{V}}}$. As $\tilde{\delta}$ increase, the performance of source power estimate using the above-mentioned method approaches that of the standard MVDR beamformer, i.e., the performance of (8).

V. Simulation Results

In our simulations, we use uniform linear arrays of $M = 10$ transmit and $N = 10$ receive antenna elements spaced half a wavelength apart from each other. The presumed SOI area is $\boldsymbol{\Theta} = [10°, 25°]$ and the out-of-sector area is $\bar{\boldsymbol{\Theta}} = [-90°, 0°] \cup [35°, 90°]$. We assume that there is one desired target located in the SOI with the direction-of-arrival (DOA) $\theta_t = 18°$ and four interfering sources located outside the SOI with DOAs $\theta_l = -35°,\ -20°,\ 0°$, and $50°$, respectively. The signal-to-noise ratio (SNR), interference-to-noise ratio (INR), and jammer-to-noise ratio (JNR) are assumed to be equal to $5\ dB$, $40\ dB$, and $50\ dB$, respectively.

The jammer suppression performance of the two proposed MVDR beamformers is validated by showing the ideal beampatterns and the source power estimates in the presence of powerful in-sector jammers as well as array calibration errors and target steering vector mismatches. Each beampattern is normalized to its maximal value. The mismatches are introduced by perturbing both the transmit and receive steering

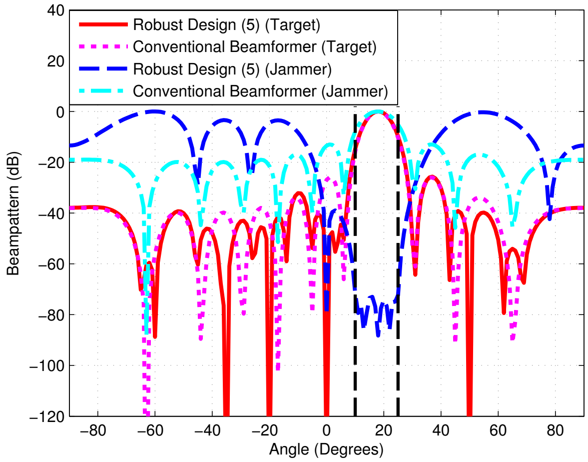

Fig. 1. Beampatterns versus angle for the first example.

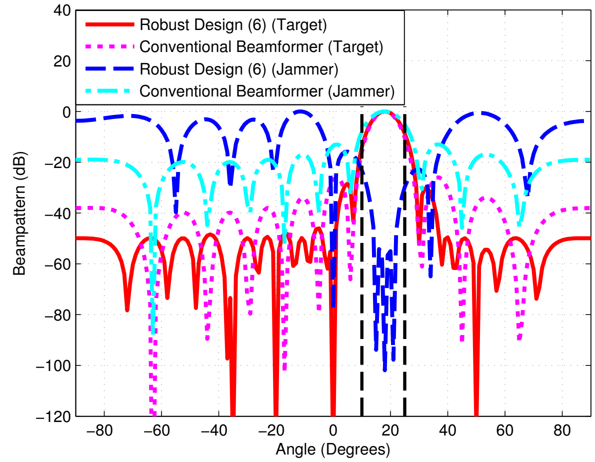

Fig. 3. Beampatterns versus angle for the second example.

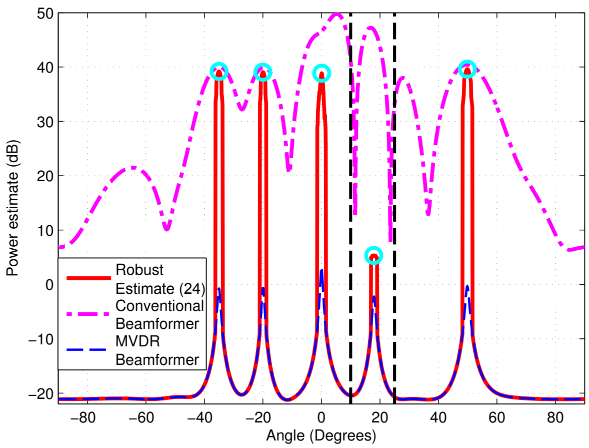

Fig. 2. Power estimates versus angle for the first example.

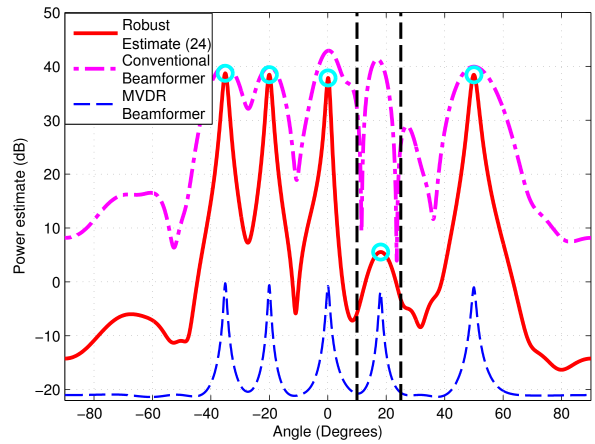

Fig. 4. Power estimates versus angle for the second example.

vectors of each incident source (the desired or interfering one) with a normalized vector whose elements are random independent variables that obey complex Gaussian distribution with mean 0 and variance 1. For each in-sector jammer, we add perturbations of the same Gaussian distribution to the elements of its corresponding receive steering vector. The obtained source power estimates are averaged over 100 independent simulation runs. The in-sector area is identified by two vertical dashed lines, and the power estimate peaks are marked by circles. The CVX toolbox [20] is used to solve the convex optimization problems (5) and (6).

In the first example, we assume that uniform distributed jammers are present in the SOI spaced 1° apart from each other. Fig. 1 shows the ideal beampatterns of both the target and the jammer using the MVDR beamforming design (5) and the conventional beamformer (Bartlett method) [21]. It can be seen that the design (5) is capable of suppressing the known jammers by imposing deep in-sector nulls towards their spatial directions, while the conventional beamforming method is invalid for the powerful in-sector jammers suppression. The desired target is not mitigated by the jammer suppression conducted in its direction. It can also be seen from Fig. 1 that the interfering sources located outside the SOI are adaptively suppressed by the designed beamformer.

Fig. 2 shows the corresponding source power estimates. The parameter $\tilde{\delta}$ is selected to be equal to 0.0001. We compare the performance of the proposed source power estimate (24) with that of the standard MVDR beamformer associated with (8) and the conventional beamformer which gives the source power estimate as [21] $\bar{\sigma}_0^2 = \bar{\mathbf{v}}\left(\theta_t\right)^H \mathbf{R}\bar{\mathbf{v}}\left(\theta_t\right)/M^2N^2$. It can be seen that the power estimates of the in-sector target and the out-of-sector interfering sources given by (24) are close to the accurate values. The target can still be well estimated even if its power is small (5 dB SNR versus 40 dB INR and 50 dB JNR). However, the standard MVDR beamforming and the conventional beamforming methods fail to estimate the power accurately in the background of powerful jamming signals and non-ideal factors. The former underestimates the power of all the sources, while the latter gives wrong power estimate peaks.

In the second example, we assume that there are three jamming signals with the DOAs $\theta_j = 15^\circ$, 18°, and 21°, respectively. Fig. 3 shows the ideal beampatterns of both the target and the jammer using the MVDR beamforming design (6) and the conventional beamformer used also in the first example. We select the in-sector jammer suppression and out-of-sector interference attenuation parameters as $\delta = 0.0001$

and $\gamma = 0.0032$, respectively. It is shown that three deep nulls towards the directions of the in-sector jammers are formed adaptively using the proposed beamformer, and the out-of-sector sidelobe levels are suppressed below -50 dB. However, the conventional beamformer still has no effect on the in-sector jammers suppression and gives worse sidelobe levels.

The corresponding source power estimates are shown in Fig. 4. It is demonstrated again that the proposed source power estimate method shows much better performance than its two counterparts. The power estimates of all the sources that are above 0 dB (noise power) are denoted by the peaks with small-circle marks. It is shown that the presence of the in-sector jammer located at the same direction as the target does not affect the target power estimate obtained using the proposed estimate method. Moreover, this estimate method shows good robustness against the array calibration errors and target steering vector mismatches.

It is worth noting that the jammer present at the spatial angle $\theta_j = 0°$ has the same virtual steering vector as the target with the same spatial angle, making it impossible to distinguish from the target using only beamforming technique when both are present in this direction. In our simulations, we show the case that the jammer is not present at $\theta_j = 0°$. Successful suppression for this case can be achieved if the information of Doppler is considered, and it is similar to what has been done in STAP. Nevertheless, the MVDR beamforming designs are generally much easier than STAP techniques.

It is also worth noting that the PA radar is unable to suppress such jammers from the same sector while MIMO radar with the proposed beamforming techniques provide this capability. The reason lies in the fact that MIMO radar is capable of utilizing the spatial signature difference between the echoes of the targets and the jammers due to its waveform diversity. In this sense, the jammer suppression capability of MIMO radar is unique. This capability is of great significance since it enables the MIMO radar to easily suppress the jammers with robustness only in spatial domain, which immensely facilitates the application in practice.

VI. Conclusion

We have studied the problem of powerful jammers suppression for MIMO radar with colocated antennas, and have provided two MVDR type beamforming designs with robustness/adaptiveness against the powerful jammers in the SOI. We first have designed an MVDR beamformer for known jammers by maintaining distortionless response towards the direction of the target while imposing nulls towards the directions of the jammers located in the SOI. Then we have designed another MVDR beamformer for the general case that the in-sector jammers are unknown and, if needed, the out-of-sector interfering sources attenuation is also conducted. Both of the robust beamforming designs are cast as convex optimization problems. Furthermore, we have derived the closed-form solution to the simplified second design. Based on this solution, we have provided source power estimates in the background of powerful jamming signals and non-ideal factors such as array calibration errors and target steering vector mismatches. It has been verified by the ideal beampatterns and the source power estimates that the robust MVDR beamforming designs are efficient, and they can serve as easier strategies for powerful jammers suppression compared to STAP techniques. Finally, we have demonstrated that the capability of efficient jammers suppression using these designs is unique in MIMO radar.

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