Optimal Constant-Envelope Jamming Waveform Design Against M-QAM Modulation

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***Abstract*—Recently, unmanned aerial vehicles and Internet-of-Things devices have significantly facilitated daily life, but they can also pose serious threats to public security if maliciously used. An effective way to counteract these threats is to transmit interference signals to disrupt their wireless communication links. However, excessively complex jamming waveforms are difficult to be realized. In addition, due to the limited jamming power budget and the long distance between jammer and receiver, the jamming-to-noise-ratio (JNR) at the receiver may be low. Thus, the efficiency of jamming power is vital. The constant-envelope jamming waveform appears to be a promising solution to both problems. Therefore, we consider the commonly-used quadrature amplitude modulation (QAM) wireless system, and optimize the constant-envelope jamming waveform to maximize the error probability when JNR $<$ signal-to-noise-ratio (SNR). This is a complicated non-convex problem that is difficult to solve directly. As a compromise, we seek a closed-form solution for a simplified version of it, which demonstrates that the simple binary phase shift keying (BPSK) is optimal. Numerical results confirm its superiority.**

***Index Terms*—Jamming waveform design, QAM, constant envelope, BPSK, error probability.**

I. INTRODUCTION

RECENTLY unmanned aerial vehicles (UAVs) and Internet-of-Things (IoT) devices have become popular in express delivery, surveillance, and photography, but they can also pose serious threats to public security if maliciously used. In recent years, some airlines have been involved in collisions with uncensored UAVs [1], and some terrorists may use them to carry out attacks [2]. Well-designed jamming signals can prevent those threats by disrupting the communication link between attackers and devices. Jamming techniques have emerged as efficient approaches to ensure public security.

Jamming techniques have long intrigued scientists. Recent research on optimal jamming signal design in physical layer can be divided into two categories according to the type of jamming signal: correlated jamming [3], [4] and uncorrelated jamming. Contrary to correlated jamming techniques, uncorrelated jamming techniques are more practical as they assume that the jamming signal is independent of the communication signal. The uncorrelated jamming power (energy) allocation problems are discussed in [5]–[9], and the performance of several common jamming waveforms, e.g., quadrature phase-shift keying (QPSK) and white Gaussian noise, is analyzed in [10]–[12]. Some works about optimal jamming waveform are investigated in [13]–[16]. In particular, [13] demonstrated that QPSK is the optimal jamming signaling scheme against quadrature amplitude modulation (QAM) when the jamming-to-noise ratio (JNR) is greater than the signal-to-noise ratio (SNR). Moreover, an effective jamming strategy of decreasing the pulsing duration for low jamming power was also proposed in this letter. Following that, these jamming signaling schemes were extended to an orthogonal frequency division multiplexing (OFDM) system [14]. In [15], an optimal jamming signaling scheme based on the particle swarm optimization algorithm was proposed to cope with constellation distortion. Those works provide a lot of insights into jamming waveform design, however, some problems still exist. For instance, these jamming techniques are difficult to be applied in practice because the hardware to generate such complex jamming waveforms is quite expensive. On the other hand, malicious devices may be far from jammer, leading to a considerable attenuation in jamming signal propagation. This jamming-power-limited scenario is very practical and universal. In this case, the efficiency of jamming power becomes extremely critical. When JNR $<$ SNR, what is the optimal and efficient jamming waveform needs to be carefully addressed.

Due to the constant envelop (CE) characteristics, the nonlinear amplifiers in CE transmitter can be used with high power efficiency [17], and the CE waveform is easy to be deployed in hardware. Therefore, we adopt the CE jamming waveform to tackle the aforementioned two problems. However, there are numerous CE waveform options. To the best of our knowledge, the optimal CE jamming waveform against commonly-used M-QAM modulation when JNR $<$ SNR is unknown. In this letter, we will address this problem. We optimize the CE jamming waveform to maximize the symbol error probability (SEP). It is quite challenging since the SEP is non-closed-form and non-convex with respect to jamming strength. As a compromise, we seek a closed-form solution of a simplified version of it. Interestingly, we demonstrate that binary phase shift keying (BPSK), although it is simple, is the optimal jamming waveform. Numerical simulations confirm its superiority.

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II. System Model

Consider the scenario in which the attacker Alice conducts a malicious attack by controlling the wireless device Bob. The control messages from Alice to Bob are in the M-QAM wireless communication signals s_l. The security defender, jammer Jason, attempts to disrupt this wireless communication link by sending the jamming signal j_l to Bob. The received signal y_l at Bob is given by

$$y_l = \sqrt{P_s} s_l + \sqrt{P_j} j_l + n_l, \quad l = 1, 2, 3, \ldots \tag{1}$$

where n_l denotes the additive white Gaussian noise with power σ^2; P_s and P_j denote the transmitting power of Alice and Jason, respectively. Define SNR $= P_s/\sigma^2$ and JNR $= P_j/\sigma^2$. We assume that Jason knows the value of P_s so that he can design his jamming signal j_l. In this letter, we consider the more practical scenario in which Jason does not know the value of s_l before jamming. When there is no ambiguity, we remove the subscript l to simplify the notation.

The jamming signal j can be decomposed into the in-phase component j_I and the quadrature component j_Q. Similarly, the communication signal s can be decomposed into s_I and s_Q. The SEP of an M-QAM signal can be approximated by that of two independent $\sqrt{\mathrm{M}}$-pulse amplitude modulation (PAM) signals [18]. Given $j_I = x_1$, the SEP of s_I is equal to the SEP of $\sqrt{\mathrm{M}}$-PAM signal in the presence of jamming:

$$f(x_1) \triangleq p_{e_{I|j_I=x_1}} = \frac{1}{2}(1-\frac{1}{\sqrt{M}})[\mathrm{erfc}(a+bx_1)+\mathrm{erfc}(a-bx_1)] \tag{2}$$

where $a = \sqrt{\frac{SNR}{2}} \frac{d_{min}}{2}$; $b = \sqrt{\frac{JNR}{2}}$; d_{min} is the normalized minimum Euclidean distance of M-QAM constellation points; $\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-\eta^2} d\eta$. Similarly, when $j_Q = x_2$, the SEP of s_Q is given by

$$f(x_2) \triangleq p_{e_{Q|j_Q=x_2}} = \frac{1}{2}(1-\frac{1}{\sqrt{M}})[\mathrm{erfc}(a+bx_2)+\mathrm{erfc}(a-bx_2)]. \tag{3}$$

Thus, the SEP of M-QAM in the presence of jamming j is expressed as

$$p_e(x_1, x_2) \triangleq p_{e_{|j=x_1+ix_2}} \approx f(x_1) + f(x_2) \tag{4}$$

where $i^2 = -1$.

III. Optimal Jamming Waveform Design

In this section, we will discuss the optimal jamming signal against M-QAM when JNR $<$ SNR. The task of Jason is to maximize the SEP p_e by selecting the jamming distribution. This problem can be formulated as follows:

$$\begin{aligned} \max_{g(x_1,x_2),c} \quad & \mathbf{E}_{x_1,x_2} p_e(x_1, x_2) \\ s.t. \quad & \begin{cases} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (x_1^2 + x_2^2) g(x_1, x_2) dx_1 dx_2 \le 1, \\ x_1^2 + x_2^2 = c \le 1, \end{cases} \end{aligned} \tag{5}$$

where $\mathbf{E}$ denotes the expectation; $g(x_1, x_2)$ denotes the probability distribution of the jamming signal j, and c denotes the envelop of it. The first constraint denotes that the total power of Jason is limited, and the second constraint denotes that the envelope of the jamming signal is constant.

This problem is solved in two steps: a) reformulating it into a manageable form; b) solving the reformulated problem.

A. The Reformulation of Problem (5)

Before the reformulation, two properties and a lemma are introduced:

Property 1: $f(x_1)$ is symmetric. Specifically, $f(x_1) = f(-x_1)$.

Property 2: $f'(x_1) = (1 - \frac{1}{\sqrt{M}})\frac{b}{\sqrt{\pi}}(e^{-(a-bx_1)^2} - e^{-(a+bx_1)^2}) \ge 0, \forall\, x_1 \ge 0, a > b.$

Lemma 1: The optimal distribution of j_I^2 or j_Q^2 is discrete and binary [13].

By property 1, p_e is symmetric. By lemma 1, $g(x_1, x_2)$ and the feasible set of problem (5) are also symmetric. Consequently, this problem can be discussed in $[0, +\infty) \times [0, +\infty)$.

According to property 2, p_e monotonically increases in $[0, +\infty) \times [0, +\infty)$. As a result, $x_1^2 + x_2^2 = c = 1$.

Furthermore, by lemma 1, without loss of generality, we assume that j_I^2 is equal to x_{11}^2 with probability η and x_{12}^2 with probability $1 - \eta$.

Thus, problem (5) can be convert into:

$$\begin{aligned} \max_{x_{11},x_{12},\eta} \quad & \eta p_e(x_{11}, \sqrt{1 - x_{11}^2}) + (1 - \eta) p_e(x_{12}, \sqrt{1 - x_{12}^2}) \\ s.t. \quad & \begin{cases} 0 \le \eta \le 1, \\ 0 \le x_{11} \le 1, \\ 0 \le x_{12} \le 1. \end{cases} \end{aligned} \tag{6}$$

Since the constraints are independent, maximizing the objective function of problem (6) is equivalent to maximizing it over them separately. Optimizing x_{11} or x_{12} individually leads to $x_{11} = x_{12}$, so the objective function of problem (6) is independent of η. Problem (6) is transformed into

$$\begin{aligned} \max_{x_{11},c} \quad & p_e(x_{11}, \sqrt{1 - x_{11}^2}) \\ s.t. \quad & 0 \le x_{11} \le 1. \end{aligned} \tag{7}$$

For notation simplicity, we remove the subscript of x_{11} and define $\tilde{p}_e(x) \triangleq p_e(x, \sqrt{1 - x^2})$. Then, we have

$$\begin{aligned} \max\nolimits_x \quad & \tilde{p}_e(x) \\ s.t. \quad & 0 \le x \le 1. \end{aligned} \tag{8}$$

B. Solving Problem (8)

The optimal jamming waveform is found by solving problem (8), and BPSK is optimal if 0 or 1 is one of the solutions. $\tilde{p}_e$ almost cannot be monotonous since $\tilde{p}_e(0) = \tilde{p}_e(1)$. Then, 0 or 1 is the optimal solution if $\tilde{p}_e$ is decreasing then increasing in (0, 1). Thus,

Claim 1: BPSK is optimal if a) the root of $\tilde{p}_e' = 0$ is unique and b) $\tilde{p}_e$ is decreasing near $x = 0$.

Proof: At the beginning of the proof, two auxiliary functions are defined for the notation simplicity:

$$\mu(x) \triangleq \frac{e^{a^2} x}{2\sqrt{1 - x^2}}(e^{-(a-b\sqrt{1-x^2})^2} - e^{-(a+b\sqrt{1-x^2})^2}) \tag{9}$$

$$\nu(x) \triangleq \frac{e^{a^2}}{2}(e^{-(a-bx)^2} - e^{-(a+bx)^2}) \tag{10}$$

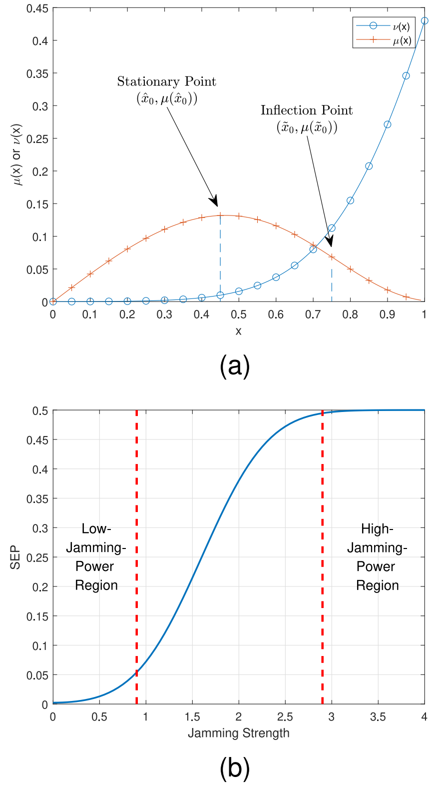

Fig. 1. Illustrations of some functions used in the proofs of four propositions. (a) a typical illustration of $\nu(x)$ and $\mu(x)$. (b) a typical illustration of $f(x_1)$.

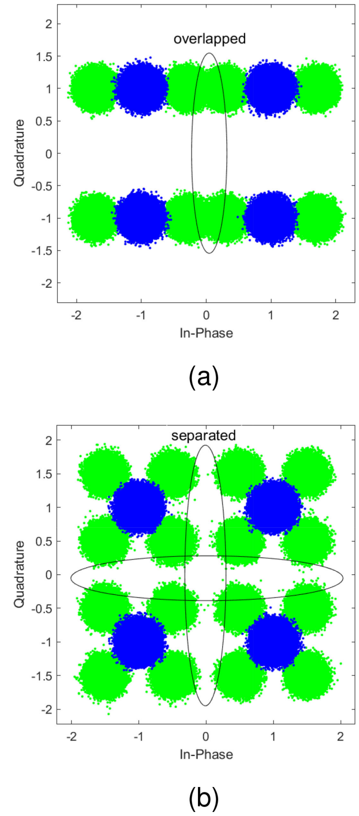

Fig. 2. Constellations under different jamming waveforms when SNR = 20 dB and JNR = 14 dB. (a) Constellations under BPSK jamming. (b) Constellations under QPSK jamming.

Then, the derivative $\tilde{p}_e'(x) = 2(1 - \frac{1}{\sqrt{M}})\frac{b}{\sqrt{\pi}}e^{-a^2}(\nu(x) - \mu(x))$.

This claim is proved in two steps:

1) Step 1: Elucidating the monotonicity and convexity of $\nu(x)$ and $\mu(x)$.

Lemma 2: $2abx > b^2x^2$ and $e^{-(a-bx)^2} \approx e^{-a^2+2abx}$ when $a >> b$, $1 > x > 0$.

Proposition 1: $\nu(x)$ is convex and increasing in [0, 1], as presented in Fig. 1a.

Proposition 2: Suppose that $\tilde{x}_0$ is the inflection point of $\mu(x)$ ($\mu''(\tilde{x}_0) = 0$) and $k \triangleq 2ab \leq 6$, $\mu(x)$ is concave in $[0, \tilde{x}_0]$ and turns to be convex in $[\tilde{x}_0, 1]$, as presented in Fig. 1a.

Proposition 3: Suppose that $\hat{x}_0$ is the stationary point of $\mu(x)$ ($\mu'(\hat{x}_0) = 0$), $\mu(x)$ increases from 0 in $[0, \hat{x}_0]$ and then decreases in $[\hat{x}_0, 1]$, as presented in Fig. 1a.

2) Step 2: Justifying the optimality.

Property 3: $\tilde{p}_e$ is decreasing near $x = 0$ since a) $\nu(0) = \mu(0) = 0$; b) $0 \leq \nu'(0) < \mu'(0)$.

Proposition 4: The root of $\tilde{p}_e'(x) = 0$ is unique.

Combining property 3 and proposition 4, we have claim 1. ∎

Therefore, the optimal jamming waveform against M-QAM is given by the following theorem:

Theorem 1: BPSK is the optimal jamming waveform when $10\log(\frac{d_{min}^2}{4}) + \text{SNR} >> \text{JNR}$ (i.e., $a >> b$) and $\text{SNR} + \text{JNR} + 10\log(d_{min}^2) \leq 21\text{dB}$ (i.e., $k \leq 6$).

Remark 1: As shown in equation (4), both jamming signals j_I and j_Q identically contribute to p_e, so we first focus on $f(x_1)$. As shown in Fig. 1b, $f(x_1)$ is convex and increasing when x_1 is small, and tends to be stable when x_1 is large. When x_1 is large, $f(x_1)$ is nearly invariant as x_1 decreases. Since the total power is fixed, x_2 will increase when x_1 decreases, causing p_e to increase. Accordingly, Jason tends to allocate power to both the in-phase and the quadrature components. However, when the jamming power is small, $f(x_1)$ is convex and increasing. The greater the x_1 is, the greater the marginal benefit $f'(x_1)$ is. As a result, Jason is prone to allocate power to one dimension to maximize p_e. Fig. 2 presents an illustration for this remark. Original constellations (blue circles) are disturbed by BPSK (in Fig. 2a) and QPSK (in Fig. 2b) jamming signals, respectively. The jammed constellations (green circles) overlap under BPSK jamming, whereas they are separate under QPSK jamming. Overlapping circles manifest that communication symbols may incorrectly decode in high probability. In other words, BPSK causes higher p_e.

IV. Numerical Results

We have already demonstrated that the BPSK is the optimal CE jamming waveform against M-QAM theoretically when $10\log(\frac{d_{min}^2}{4}) + \text{SNR} >> \text{JNR}$ and $\text{SNR} + \text{JNR} + 10\log(d_{min}^2) \leq 21$ dB in previous sections. In this section, we present the performance of the proposed jamming waveform. We confirm its superiority by comparing several common CE jamming waveforms against the popular 4-QAM (QPSK) and 16-QAM communication signals. All of the results are presented in Fig. 3 and Fig. 4.

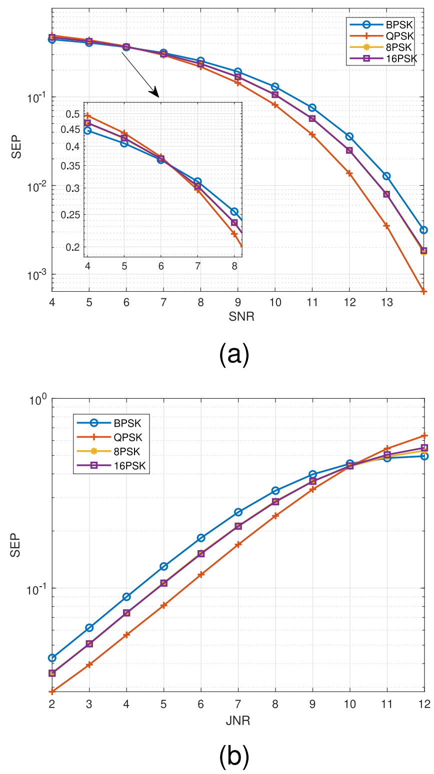

Fig. 3. Comparison of various jamming methods against a QPSK communication signal. (a) SEP versus SNR when JNR $= 5$ dB. (b) SEP versus JNR when SNR $= 10$ dB.

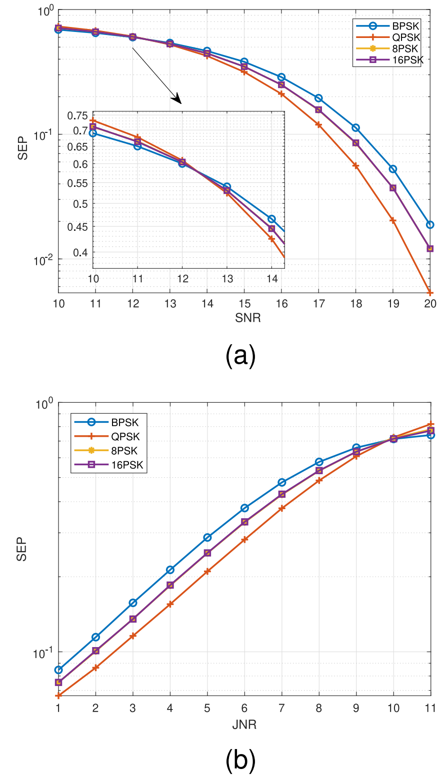

Fig. 4. Comparison of various jamming methods against a 16-QAM communication signal. (a) SEP versus SNR when JNR $= 5$ dB. (b) SEP versus JNR when SNR $= 16$ dB.

To begin with, we show the performance of the BPSK jamming signal against QPSK (presented in Fig. 3). In Fig. 3a, all SEPs decrease with the increase of SNR. QPSK is the best option when SNR is less than about 6 dB. However, as SNR increases, Jason could not afford to allocate power to both signaling dimensions to achieve a high payback, so BPSK outperforms the others. It should be noted that the conditions on the proposed method are slightly conservative, because in Fig. 3a BPSK still performs best in two cases: a) when SNR $=$ 7 dB; b) when SNR $=$ 14 dB. Then, we turn to Fig. 3b. As JNR increases, all SEPs increase. When JNR is low, BPSK is optimal. However, as JNR grows, the conditions in Theorem 1 will be violated, so QPSK will become optimal. Finally, when the communication signaling scheme is 16-QAM, similar trends are observed (presented in Fig. 4), except that the SNR threshold for BPSK is higher. This is because the d_{min} of 16-QAM is smaller than that of QPSK.

V. Conclusion

In this letter, we discussed the problem of optimal CE jamming waveform against M-QAM wireless communication system, which is critical to maintaining urban security by invalidating malicious wireless devices. By calculating the SEP of the received signal at malicious devices, we derived that the optimal jamming waveform is BPSK when $10\log(\frac{d_{min}^2}{4}) + \mathrm{SNR} >> \mathrm{JNR}$ and $\mathrm{SNR} + \mathrm{JNR} + 10\log(d_{min}^2) \leq 21$ dB. Then, the performance of the proposed jamming method was compared with that of some traditional jamming methods. Results indicate that the proposed method outperforms traditional methods in jamming malicious wireless attackers.

Appendix

Proof of Proposition 1: By lemma 2,

$$\nu(x) \approx \sinh(2abx) \tag{11}$$

Thus, $\nu(x)$ is increasing and convex in [0, 1].

Proof of Proposition 2: By lemma 2

$$\mu(x) \approx \frac{x}{\sqrt{1-x^2}}\sinh(k\sqrt{1-x^2}) \tag{12}$$

where $k = 2ab$. It is not difficult to know that $\lim_{x\to 0^+}\mu(x) = 0$, $\lim_{x\to 1^-}\mu(x) = k$. Then

$$\mu'(x) = \frac{\sinh(k\sqrt{1-x^2}) - kx^2\sqrt{1-x^2}\cosh(k\sqrt{1-x^2})}{(\sqrt{1-x^2})^3} \tag{13}$$

and

$$\begin{aligned}\mu''(x) &= \frac{[3x + k^2x^3(1-x^2)]\sinh(k\sqrt{1-x^2})}{(\sqrt{1-x^2})^5} \\ &\quad - \frac{3kx\sqrt{1-x^2}\cosh(k\sqrt{1-x^2})}{(\sqrt{1-x^2})^5} \\ &\approx \frac{3x + k^2x^3 - k^2x^5 - 3kx\sqrt{1-x^2}}{(\sqrt{1-x^2})^5}\sinh(k\sqrt{1-x^2})\end{aligned} \tag{14}$$

Suppose that $y = \sqrt{1-x^2}$, $\mu''(x) \leq 0$ is equivalent to $k^2y^4 - k^2y^2 + 3ky - 3 \geq 0$. For notation simplicity, we define

$$h(y) \triangleq k^2y^4 - k^2y^2 + 3ky - 3 \tag{15}$$

and then

$$h'(y) = 4k^2y^3 - 2k^2y + 3k \tag{16}$$

$$h''(y) = 12ky^2 - 2k \tag{17}$$

Clearly, $h''(\sqrt{1/6}) = 0$, $h'(\sqrt{1/6}) = k(3 - 8k/\sqrt{6^3})$. If $h'(y) \geq 0$ for $0 \leq y \leq 1$ (i.e., $k \leq 3\sqrt{6^3}/8 \approx 6$), $h(y)$ increases. As $h(0) = -3$, $h(1) = 3k$ - 3 > 0, and suppose that $\tilde{x}_0 = \sqrt{1-(y_0')^2}$ and $h(y_0') = 0$, $\mu''(x)$ is non-positive in $[0, \tilde{x}_0]$ and non-negative in $[\tilde{x}_0, 1]$. $\mu(x)$ is concave in $[0, \tilde{x}_0]$ and turns to be convex in $[\tilde{x}_0, 1]$.

Proof of Proposition 3: $\mu'(0) = \frac{e^{2k}-1}{2e^k} > 0$, $\mu'(1) = k - \frac{1}{3}k^3 < 0$. Because $\mu''(x)$ is non-positive in $[0, \tilde{x}_0]$ and non-negative in $[\tilde{x}_0, 1]$, $\mu'(x)$ decreases and then increases in [0, 1]. As a result, $\mu'(x) = 0$ has unique solution $\hat{x}_0$. $\mu(x)$ increases in $[0, \hat{x}_0]$ and decreases in $[\hat{x}_0, 1]$.

Proof of Proposition 4: At the beginning of this proof, we will show some characteristics of $\tilde{p}_e'(x)$. First, there exists at least one intersection point $x_\times$ of $\nu(x)$ and $\mu(x)$ in $(0, 1)$ since $\mu'(0) = \frac{e^{2k}-1}{2e^k} > \nu'(0)$, $\mu(0) = \nu(0)$, $\mu(1) < \nu(1)$. Second, the stationary point $\hat{x}_0$ of $\mu(x)$ must be less than $\tilde{x}_0$. In $[\tilde{x}_0, 1]$, $\mu''(x) \geq 0$ so $\mu'(x)$ increases. If $\mu'(x) \geq 0$, then $\mu(x)$ must increase in $(\hat{x}_0, 1)$. It contradicts to the fact that $\mu(x)$ decreases around 1.

Subsequently, we demonstrate the uniqueness of the root of $\tilde{p}_e'(x) = 0$. More specifically, we discuss this problem in two cases: 1) at least one intersection point $x_\times$ is in $(0, \tilde{x}_0)$, and 2) at least one $x_\times$ is in $[\tilde{x}_0, 1)$.

1) *When* $x_\times \in (0, \tilde{x}_0)$*:* It is not difficult to verify that

$$\nu''(x) - \mu''(x) \geq 0 \quad \forall x \in [0, \tilde{x}_0] \tag{18}$$

$$\nu'(0) \leq \mu'(0) \tag{19}$$

$$\nu'(\tilde{x}_0) \geq \mu'(\tilde{x}_0) \tag{20}$$

$\nu''(x) - \mu''(x) \geq 0$ indicates that $\nu'(x) - \mu'(x)$ continuously increases. By equation (19) and (20), $\nu'(x) - \mu'(x) = 0$ has a unique root in $(0, \tilde{x}_0)$, and $\nu(x) - \mu(x)$ decreases and then increases. Since $\mu(0) = \nu(0)$, $x_\times$ must be unique in $(0, \tilde{x}_0)$. Furthermore, because $\hat{x}_0 < \tilde{x}_0$, $\mu(x)$ decreases in $[\tilde{x}_0, 1]$. There cannot be any root. Thus, $x_\times$ is unique in $(0, 1)$.

2) *When* $x_\times \in [\tilde{x}_0, 1)$*:* $\mu(x)$ must be greater than $\nu(x)$ if $x_\times \in (0, \tilde{x}_0)$, so $x_\times$ cannot be in $(0, \tilde{x}_0)$. Since $\mu(x)$ decreases and $\nu(x)$ increases in $[\tilde{x}_0, 1]$, $x_\times$ is unique.

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