Design of a sliding mode controller for trajectory tracking problem of marine vessels

J. Cheng, J. Yi and D. Zhao

Abstract: Trajectory tracking is an issue of vital practical importance for manoeuvreing of marine vessels. Because of the nonlinearities of kinematics and dynamics of motion, conventional control designs, based on the assumption that the kinematics and dynamics can be linearised, are not competent for tracking application of marine vessels. A multivariable sliding mode control law is proposed for the trajectory tracking problem on the basis of nonlinear horizontal motion dynamics of a class of marine vessels, in which three degrees of freedom are concerned. Ship positions and yaw angle are simultaneously tracked. Lyapunov theory is used to prove the stability of the proposed control law. Simulation results show the validity of this method for ship trajectory tracking problem.

1 Introduction

The trajectory tracking problem is a field of high interest in the manoeuvreing of marine vessels. Although dynamics of marine vessels is accurately described by complex nonlinear hydrodynamic models, the models for control purposes are still simple linear models developed decades ago. Examples are the linear steering model of Davidson and Schiff [1] or the course-keeping models of Nomoto *et al.* [2]. The contribution of Holzhuter [3] exploits the simplicity in practical applications.

Most conventional ship control systems are designed under the assumption that the kinematics and dynamic equations of motion can be linearised such that gain-scheduling techniques and optimal control theory can be applied [4]. This is not a good assumption for tracking applications where the surge and sway positions and yaw angle must be controlled simultaneously. The main reason is that the rotation matrix for coordinate transformation between the Earth-fixed space and the body-fixed space is practically impossible to be linearised with adequate accuracy. In addition, assumptions like linear damping and negligible Coriolis and centripetal forces are only good for low-speed applications that are station-keeping and dynamic positioning [5].

These limitations clearly motivate a nonlinear design for ship trajectory tracking problem, and the developments in the last decade in nonlinear control theory greatly extends the possibilities [6–9]. Zhang *et al.* [10], Encarnacao *et al.* [11] and Pettersen and Lefeber [12] address the problem of following straight lines, which is a system with relative two degrees of freedom (DOF) controlled by input–output linearisation and robustly by sliding mode control (SMC). However, the authors use a very simplified dynamic model.

doi:10.1049/iet-cta:20050357

Paper first received 21st September 2005 and in revised form 24th April 2006

The authors are with the Key Laboratory of Complex Systems and Intelligence Science, Institute of Automation, Chinese Academy of Sciences, Beijing 100080, People's Republic of China

E-mail: jin.cheng@ia.ac.cn

This paper proposes a nonlinear SMC law for the trajectory tracking problem on the basis of the dynamic model of marine vessels for horizontal motion, in which three DOF are concerned. The stability analysis based on Lyapunov stability theory proves that the proposed control system is globally stable and realises the simultaneous tracking of positions and heading angle. The proposed control law is simulated on a supply vessel.

2 Ship model

The nonlinear dynamics and kinematics model of marine vessels are derived from the robot model, which was modified by Fossen [13, 14] and written in a compact vector form. Fossen introduced a parallel reference frame to eliminate the nonlinearities associated with the kinematical transformation between body- and Earth-fixed velocities, which high simplifies the model [15]. The six DOF marine vessel equations of motion are written as

$$\boldsymbol{M}\dot{\boldsymbol{\nu}} + \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{g}(\boldsymbol{\eta}) = \boldsymbol{\tau} + \boldsymbol{\tau}_{\mathrm{b}}\,\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\boldsymbol{\eta})\boldsymbol{\nu} \quad (1)$$

where $\boldsymbol{\nu} \in \mathbf{R}^{n_1}$, $\boldsymbol{\eta} \in \mathbf{R}^{n_2}$, $n_1 = n_2 = 6$. The vector $\boldsymbol{\nu}$ denotes the linear velocities in surge, sway and heave, and the angular velocities in roll, pitch and yaw, decomposed in the body-fixed frame. The vector $\boldsymbol{\eta}$ denotes the position and orientation decomposed in the Earth-fixed frame. $\boldsymbol{\tau} \in \mathbf{R}^{m}$ is the vector of control inputs, which denotes the control forces and moments decomposed in the body-fixed frame, and m (>2) is the number of the control inputs that depends on the configuration of the marine vessel. $\boldsymbol{\tau}_{\mathrm{b}}$ is accounting for the unmodelled external forces and moment mainly imposed by wind and second-order wave [16]. $\boldsymbol{M}$ is a rigid-body inertia matrix including added mass, which is constant, non-singular and positive definite. $\boldsymbol{C}(\boldsymbol{\nu})$ is a skew-symmetric parameterisation of the rigid-body Coriolis and centripetal matrix including added mass. The total hydrodynamic damping matrix $\boldsymbol{D}(\boldsymbol{\nu})$ is symmetric and positive definite. The coordinate transformation matrix $\boldsymbol{J}(\boldsymbol{\eta})$ has full rank. $\boldsymbol{g}(\boldsymbol{\eta})$ is the vector of gravitational and buoyant forces and moments.

As the trajectory tracking problem for marine vessels is to find a control law that asymptotically stabilises both the positions and the orientation, shown in Fig. 1, which are mainly associated with the horizontal motions, we will restrict the six-dimensional dynamics to the horizontal plane by making the following assumptions.

Assumption 1: The dynamics associated with the motion in heave, roll and pitch is negligible.

This is a well known assumption used in all industrial ship control systems because the magnitude of the heave, roll and pitch variables are very small (second-order damped oscillators), and therefore their influence on the motion in horizontal plane can be neglected.

Assumption 2: The vector of restoring forces $\boldsymbol{g}(\boldsymbol{\eta})$ (gravitation and buoyancy) is equal to zero.

As the direction of the gravity and buoyancy is perpendicular to horizontal plane, the forces of gravitation and buoyancy have no effect on the dynamics in the horizontal motion of marine vessels.

Under these assumptions, the dynamic model of marine vessels for motion in horizontal plane is given by the following 3 DOF model

$$\boldsymbol{M}\dot{\boldsymbol{\nu}} + \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu} = \boldsymbol{\tau} + \boldsymbol{\tau}_{\mathrm{b}} \tag{2}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\boldsymbol{\eta})\boldsymbol{\nu} \tag{3}$$

where

$$\boldsymbol{M} = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & mx_g - Y_{\dot{r}} \\ 0 & mx_g - N_{\dot{v}} & I_z - N_{\dot{r}} \end{bmatrix} \tag{4}$$

$$\boldsymbol{C}(\boldsymbol{\nu}) = \begin{bmatrix} 0 & 0 & c_{13} \\ 0 & 0 & c_{23} \\ c_{31} & c_{32} & 0 \end{bmatrix} \tag{5}$$

with $c_{13} = -(m - Y_{\dot{v}})v - (mx_g - Y_{\dot{r}})r$, $c_{23} = (m - X_{\dot{u}})u$, $c_{31} = (m - Y_{\dot{v}})v + (mx_g - Y_{\dot{r}})r$, $c_{32} = -(m - X_{\dot{u}})u$ and

$$\boldsymbol{D}(\boldsymbol{\nu}) = \begin{bmatrix} -X_u & 0 & 0 \\ 0 & -Y_v & -Y_r \\ 0 & -N_v & -N_r \end{bmatrix} \tag{6}$$

$$\boldsymbol{J}(\boldsymbol{\eta}) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

where $I_z, X_u, X_{\dot{u}}, Y_v, Y_{\dot{v}}, Y_r, Y_{\dot{r}}, N_r, N_{\dot{r}}, N_v, N_{\dot{v}}$ are hydrodynamic parameters of a ship. The vector $\boldsymbol{\eta} = [x, y, \psi]^{\mathrm{T}}$ denotes the position and orientation of the ship in the Earth-fixed coordinate system. The vector $\boldsymbol{\nu} = [u, v, r]^{\mathrm{T}}$ denotes the linear velocities in surge and sway, and the angular velocity in yaw. $\boldsymbol{\tau} = [\tau_1, \tau_2, \tau_3]^{\mathrm{T}}$ denotes the control inputs, respectively, the surge force, the sway force, and the yaw moment. $\boldsymbol{\tau}_{\mathrm{b}}$ is an unknown bias term which is slowly varying depending on the sea state [17]. The estimation of $\boldsymbol{\tau}_{\mathrm{b}}$ is presented in detail by Perez [16] and is omitted here.

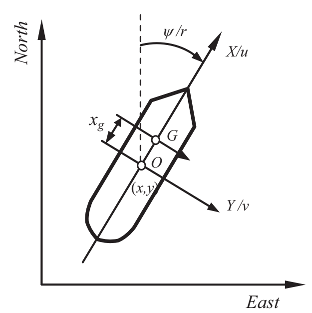

Fig. 1 *Horizontal motion of marine vessels*

System defined by (2)–(7) satisfies the following properties:

1. $\boldsymbol{M}$ is positive, that is $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{x} > 0$, $\forall \boldsymbol{x} \neq 0$.
2. $\mathbf{C}(\boldsymbol{\nu})$ is skew-symmetric, that is $\boldsymbol{C}(\boldsymbol{\nu}) = -\boldsymbol{C}^{\mathrm{T}}(\boldsymbol{\nu})$ $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{x} = 0$, $\forall \boldsymbol{x}$.
3. $\mathbf{D}(\boldsymbol{\nu})$ is positive, that is $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{x} = (1/2)\boldsymbol{x}^{\mathrm{T}}(\boldsymbol{D}(\boldsymbol{\nu}) + \boldsymbol{D}^{\mathrm{T}}(\boldsymbol{\nu}))\boldsymbol{x} > 0$, $\forall \boldsymbol{x} \neq 0$.
4. $\boldsymbol{J}(\boldsymbol{\eta})$ is non-singular, and satisfies $\boldsymbol{J}^{-1}(\boldsymbol{\eta}) = \boldsymbol{J}^{\mathrm{T}}(\boldsymbol{\eta})$, $\forall \boldsymbol{\eta}$, $\dot{\boldsymbol{J}}(\boldsymbol{\eta}) = \boldsymbol{J}(\boldsymbol{\eta})\boldsymbol{S}$, $\forall \boldsymbol{\eta}$, where $\boldsymbol{S} = \begin{bmatrix} 0 & -\dot{\psi} & 0 \\ \dot{\psi} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -\boldsymbol{S}^{\mathrm{T}}$.

3 SMC design

The control objective is to find a nonlinear control law that makes the ship asymptotically tracking a designed trajectory. In this section, an SMC law will be developed.

The SMC method is developed to design systems that have the desired dynamic behaviour and are robust with respect to perturbations. As a nonlinear control approach, SMC method has been successfully applied to the systematic control design for the advantage of the reduction of the system order. The most intriguing aspect of SMC is the discontinuous nature of the control action. Otherwise, it behaves with some important advantages such as insensitivity to parameter variations, and complete rejection of disturbances, which makes SMC approach effective in control under conditions of uncertainty.

For the bias term $\boldsymbol{\tau}_{\mathrm{b}}$ can be dealt as uncertain disturbances of extra states, it is not concerned in SMC design. Suppose the desired position and orientation in the Earth-fixed coordinate system are described by a time-varying reference trajectory, that is defined by vector $\boldsymbol{\eta}_d = [x_d, y_d, \psi_d]^{\mathrm{T}}$. Without any loss of generality, it is assumed that the reference trajectory $\boldsymbol{\eta}_d$, $\dot{\boldsymbol{\eta}}_d$, $\ddot{\boldsymbol{\eta}}_d$ is smooth and bounded.

Definition 1: The measure of tracking is defined as

$$\boldsymbol{s} = \dot{\tilde{\boldsymbol{\eta}}} + \boldsymbol{\Lambda}\tilde{\boldsymbol{\eta}} \tag{8}$$

where $\tilde{\boldsymbol{\eta}} = \boldsymbol{\eta} - \boldsymbol{\eta}_d$ is the Earth-fixed tracking error and $\boldsymbol{\Lambda}$ is a diagonal positive design matrix.

It can be seen from (8) that the convergence of $\boldsymbol{s}$ to zero implies that the tracking error $\tilde{\boldsymbol{\eta}}$ converges to zero. Hence $\boldsymbol{s}$ is defined as the sliding mode in the designed control system.

Definition 2: The virtual reference trajectory and virtual reference velocities vector in body-fixed and Earth-fixed coordinates are defined as

$$\dot{\boldsymbol{\eta}}_{\mathrm{r}} = \dot{\boldsymbol{\eta}}_d - \boldsymbol{\Lambda}\tilde{\boldsymbol{\eta}} \tag{9}$$

$$\boldsymbol{\nu}_{\mathrm{r}} = \boldsymbol{J}^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{\eta}}_{\mathrm{r}} \tag{10}$$

According to (8) and (9), $\boldsymbol{s}$ can be rewritten as

$$\boldsymbol{s} = \dot{\tilde{\boldsymbol{\eta}}} + \boldsymbol{\Lambda}\tilde{\boldsymbol{\eta}} = \dot{\boldsymbol{\eta}} - \dot{\boldsymbol{\eta}}_{\mathrm{r}} \tag{11}$$

On the basis of this definition, the design of the SMC law is developed by the following steps.

Step 1: In order to construct the sliding mode $\boldsymbol{s}$, the differential equation of the dynamics containing $\boldsymbol{\eta}$ should be derived first.

From (3) and (7), we have

$$\boldsymbol{\nu} = \boldsymbol{J}^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{\eta}} \tag{12}$$

and

$$\begin{aligned}\frac{\partial \boldsymbol{J}^{-1}(\boldsymbol{\eta})}{\partial t} &= \frac{\partial \boldsymbol{J}^{\mathrm{T}}(\boldsymbol{\eta})}{\partial t} = (\dot{\boldsymbol{J}}(\boldsymbol{\eta}))^{\mathrm{T}} = (\boldsymbol{J}(\boldsymbol{\eta})\boldsymbol{S})^{\mathrm{T}} \\ &= \boldsymbol{S}^{\mathrm{T}}\boldsymbol{J}^{\mathrm{T}}(\boldsymbol{\eta}) = -\boldsymbol{S}\boldsymbol{J}^{\mathrm{T}}(\boldsymbol{\eta}) \\ &= -\boldsymbol{J}^{-1}(\boldsymbol{\eta})\boldsymbol{J}(\boldsymbol{\eta})\boldsymbol{S}\boldsymbol{J}^{-1}(\boldsymbol{\eta}) \\ &= -\boldsymbol{J}^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{J}}(\boldsymbol{\eta})\boldsymbol{J}^{-1}(\boldsymbol{\eta})\end{aligned} \tag{13}$$

Substituting (13) into (12) yields

$$\begin{aligned}\dot{\boldsymbol{\nu}} &= \frac{\partial \boldsymbol{J}^{-1}(\boldsymbol{\eta})}{\partial t}\dot{\boldsymbol{\eta}} + \boldsymbol{J}^{-1}(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}} \\ &= -\boldsymbol{J}^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{J}}(\boldsymbol{\eta})\boldsymbol{J}^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \boldsymbol{J}^{-1}(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}}\end{aligned} \tag{14}$$

Substituting (14) into (2), then the differential equation containing $\boldsymbol{\eta}$ is achieved as

$$\begin{aligned}&\boldsymbol{M}\boldsymbol{J}^{-1}(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}} - \boldsymbol{M}\boldsymbol{J}^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{J}}(\boldsymbol{\eta})\boldsymbol{J}^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{\eta}} \\ &\quad + \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{J}^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{J}^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{\eta}} \\ &\quad = \boldsymbol{M}\boldsymbol{J}^{-1}(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}} + [\boldsymbol{C}(\boldsymbol{\nu}) - \boldsymbol{M}\boldsymbol{J}^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{J}}(\boldsymbol{\eta})\boldsymbol{J}^{-1}(\boldsymbol{\eta})] \\ &\quad\quad \times \boldsymbol{J}^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{J}^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{\eta}} = \boldsymbol{\tau}\end{aligned} \tag{15}$$

Define

$$\boldsymbol{M}_{\eta}(\boldsymbol{\eta}) = \boldsymbol{M}\boldsymbol{J}^{-1}(\boldsymbol{\eta}) \tag{16}$$

$$\boldsymbol{C}_{\eta}(\boldsymbol{\nu}, \boldsymbol{\eta}) = [\boldsymbol{C}(\boldsymbol{\nu}) - \boldsymbol{M}\boldsymbol{J}^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{J}}(\boldsymbol{\eta})]\boldsymbol{J}^{-1}(\boldsymbol{\eta}) \tag{17}$$

$$\boldsymbol{D}_{\eta}(\boldsymbol{\nu}, \boldsymbol{\eta}) = \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{J}^{-1}(\boldsymbol{\eta}) \tag{18}$$

then (15) can be rewritten as

$$\boldsymbol{M}_{\eta}(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}} + \boldsymbol{C}_{\eta}(\boldsymbol{\nu}, \boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \boldsymbol{D}_{\eta}(\boldsymbol{\nu}, \boldsymbol{\eta})\dot{\boldsymbol{\eta}} = \boldsymbol{\tau} \tag{19}$$

Step 2: The differential equation of the sliding mode is derived at this step. Equations (11) and (19) yield

$$\begin{aligned}\boldsymbol{M}_{\eta}(\boldsymbol{\eta})\dot{\boldsymbol{s}} &= -\boldsymbol{C}_{\eta}(\boldsymbol{\nu}, \boldsymbol{\eta})\boldsymbol{s} - \boldsymbol{D}_{\eta}(\boldsymbol{\nu}, \boldsymbol{\eta})\boldsymbol{s} + \boldsymbol{\tau} - \boldsymbol{M}_{\eta}(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}}_{\mathrm{r}} \\ &\quad - \boldsymbol{C}_{\eta}(\boldsymbol{\nu}, \boldsymbol{\eta})\dot{\boldsymbol{\eta}}_{\mathrm{r}} - \boldsymbol{D}_{\eta}(\boldsymbol{\nu}, \boldsymbol{\eta})\dot{\boldsymbol{\eta}}_{\mathrm{r}}\end{aligned} \tag{20}$$

From (10) and (14), we can also obtain

$$\begin{aligned}\dot{\boldsymbol{\nu}}_{\mathrm{r}} &= \frac{\partial \boldsymbol{J}^{-1}(\boldsymbol{\eta})}{\partial t}\dot{\boldsymbol{\eta}}_{\mathrm{r}} + \boldsymbol{J}^{-1}(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}}_{\mathrm{r}} \\ &= -\boldsymbol{J}^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{J}}(\boldsymbol{\eta})\boldsymbol{J}^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{\eta}}_{\mathrm{r}} + \boldsymbol{J}^{-1}(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}}_{\mathrm{r}}\end{aligned} \tag{21}$$

With (16)–(18) and (21), (20) can be rewritten as

$$\begin{aligned}\boldsymbol{M}_{\eta}(\boldsymbol{\eta})\dot{\boldsymbol{s}} &= -\boldsymbol{C}_{\eta}(\boldsymbol{\nu}, \boldsymbol{\eta})\boldsymbol{s} - \boldsymbol{D}_{\eta}(\boldsymbol{\nu}, \boldsymbol{\eta})\boldsymbol{s} \\ &\quad + [\boldsymbol{\tau} - \boldsymbol{M}\dot{\boldsymbol{\nu}}_{\mathrm{r}} - \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu}_{\mathrm{r}} - \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu}_{\mathrm{r}}]\end{aligned} \tag{22}$$

Let $\dot{\boldsymbol{s}} = 0$, then the equivalent control can be obtained from (22) as

$$\begin{aligned}\boldsymbol{\tau}_{\mathrm{eq}} &= \boldsymbol{M}\dot{\boldsymbol{\nu}}_{\mathrm{r}} + \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu}_{\mathrm{r}} + \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu}_{\mathrm{r}} + [\boldsymbol{C}_{\eta}(\boldsymbol{\nu}, \boldsymbol{\eta}) \\ &\quad + \boldsymbol{D}_{\eta}(\boldsymbol{\nu}, \boldsymbol{\eta})]\boldsymbol{s}\end{aligned} \tag{23}$$

Define the control input as

$$\boldsymbol{\tau} = \boldsymbol{\tau}_{\mathrm{eq}} + \boldsymbol{\tau}_{\mathrm{sw}} \tag{24}$$

where $\boldsymbol{\tau}_{\mathrm{sw}}$ is the switch control part of the SMC input.

To guarantee that the sliding mode $\boldsymbol{s}$ tends to zero in finite time and also with desired convergence rate, the dynamics of the sliding mode is required to have the following form

$$\dot{\boldsymbol{s}} = -\boldsymbol{W}\boldsymbol{s} - \boldsymbol{K}\,\mathrm{sgn}(\boldsymbol{s}) \tag{25}$$

where $\boldsymbol{W} \in \mathrm{diag}\{w_i\}$, $\boldsymbol{K} \in \mathrm{diag}\{k_i\}$, $w_i > 0$, $k_i > 0$, $i = 1, 2, 3$. Substituting (25) into the left section of (22) and (24) into the right section of (22) yield

$$\boldsymbol{\tau}_{\mathrm{sw}} = -\boldsymbol{M}_{\eta}(\boldsymbol{\eta})[\boldsymbol{W}\boldsymbol{s} + \boldsymbol{K}\,\mathrm{sgn}(\boldsymbol{s})] \tag{26}$$

So the control law is designed as

$$\begin{aligned}\boldsymbol{\tau} &= \boldsymbol{M}\dot{\boldsymbol{\nu}}_{\mathrm{r}} + \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu}_{\mathrm{r}} + \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu}_{\mathrm{r}} + [\boldsymbol{C}_{\eta}(\boldsymbol{\nu}, \boldsymbol{\eta}) + \boldsymbol{D}_{\eta}(\boldsymbol{\nu}, \boldsymbol{\eta})]\boldsymbol{s} \\ &\quad - \boldsymbol{M}_{\eta}(\boldsymbol{\eta})[\boldsymbol{W}\boldsymbol{s} + \boldsymbol{K}\,\mathrm{sgn}(\boldsymbol{s})]\end{aligned} \tag{27}$$

4 Stability analysis

The SMC method provides an effective nonlinear control approach for systems difficult to be linearised. When the system reaches the sliding mode, it has very good adaptiveness, that is robustness to disturbances and parameter variations. In this section, the stability of the control system will be analysed.

Theorem 1: Consider the system defined by (2) and (3) and construct the sliding mode as (8). If the control law is designed as (27), then the sliding mode is asymptotically stable.

Proof: To investigate stability of this control system, consider the Lyapunov function candidate

$$V = \frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{s} \tag{28}$$

For $\boldsymbol{M}$ is positive and according to (16), $\boldsymbol{M}_{\eta}(\boldsymbol{\eta})$ is invertible. Then the derivative of V along the trajectory of system (20) is

$$\begin{aligned}\dot{V} &= \boldsymbol{s}^{\mathrm{T}}\dot{\boldsymbol{s}} \\ &= \boldsymbol{s}^{\mathrm{T}}\boldsymbol{M}_{\eta}^{-1}(\boldsymbol{\eta})\{-\boldsymbol{C}_{\eta}(\boldsymbol{\nu}, \boldsymbol{\eta})\boldsymbol{s} - \boldsymbol{D}_{\eta}(\boldsymbol{\nu}, \boldsymbol{\eta})\boldsymbol{s} \\ &\quad + [\boldsymbol{\tau} - \boldsymbol{M}\boldsymbol{\nu}_{\mathrm{r}} - \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu}_{\mathrm{r}} - \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu}_{\mathrm{r}}]\}\end{aligned} \tag{29}$$

Substituting the control input $\boldsymbol{\tau}$ given in (27) into (29) yields

$$\dot{V} = -\boldsymbol{s}^{\mathrm{T}}\boldsymbol{W}\boldsymbol{s} - \boldsymbol{s}^{\mathrm{T}}\boldsymbol{K}\,\mathrm{sgn}(\boldsymbol{s}) \le 0 \tag{30}$$

because $\boldsymbol{W}$ and $\boldsymbol{K}$ are both positive matrix.

On the basis of Lyapunov stability theory and LaSalle's principle about invariance set [18], the control system is asymptotically stable therefore it guarantees that the sliding mode can be reached and converge to zero in finite time. □

Remark 1: The dynamic characteristics of the sliding mode are only dependent on the matrix $\boldsymbol{W}$ and $\boldsymbol{K}$. According to (25), for any $s_i \in \boldsymbol{s}$ and $s_i(t_0) \neq 0$, it satisfies

$$\dot{s}_i = -w_i s - k_i\,\mathrm{sgn}(s_i) \tag{31}$$

and its solution is

$$s_i(t) = [|s_i(t_0)| + w_i^{-1}k_i]\mathrm{e}^{-w_i(t-t_0)} - w_i^{-1}k_i \tag{32}$$

hence, when

$$t = t_0 - \ln \frac{k_i/(k_i + w_i|s_i(t_0)|)}{w_i} \tag{33}$$

it satisfies $s_i(t) = 0$. When

$$t \geq t_0 - \ln \frac{k_i/(k_i + w_i|s_i(t_0)|)}{w_i}$$

the sliding mode will be reached.

According to (32), short time to reach sliding mode can be obtained by proper selection of large positive value for w_i. The value of k_i is the key factor of determining the robust stability of the SMC law. In order to have balance property of the sliding control law between the anti-jamming and high frequency chattering, the value of k_i should also be chosen carefully.

Remark 2: According to (3), (9) and (10), the following equation is satisfied at stable tracking stage $\tilde{\boldsymbol{\eta}} = 0$

$$\boldsymbol{\nu} = \boldsymbol{J}^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{\eta}} = \boldsymbol{J}^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{\eta}}_d = \boldsymbol{J}^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{\eta}}_\mathrm{r} = \boldsymbol{\nu}_\mathrm{r} \tag{34}$$

Hence the velocities of the ship in horizontal plane are determined by the velocities of reference trajectory. By programming the reference trajectory, the SMC law can also control the ship with desired velocities in surge, sway and the heading angular velocity.

5 Numerical simulation

Numerical simulation is performed to show the effectiveness of the presented control law. Consider a supply marine vessel with mass $m = 4 \times 10^6$ kg and length $L = 76.2$ m (assuming $x_g = 0$). The ship is assumed to be equipped with a single screw propeller, a rudder and a tunnel thruster mounted at the bow.

The dynamical model parameters of the supply ship has been identified by Fossen *et al.* [19], which are derived from sea trials performed in calm water. The hydrodynamic parameters needed for simulation are listed in Table 1. Applying values from Table 1 to (4)–(6) yields

$$\boldsymbol{M} = 10^6 \times \begin{bmatrix} 4.5096 & 0 & 0 \\ 0 & 7.5608 & -22.68 \\ 0 & -22.68 & 2968.3 \end{bmatrix}$$

$$\boldsymbol{C}(\boldsymbol{\nu}) = \begin{bmatrix} 0 & 0 & c_{13} \\ 0 & 0 & c_{23} \\ c_{31} & c_{32} & 0 \end{bmatrix} \quad \text{with}$$

$$c_{13} = -7.5608v + 22.68r, \quad c_{23} = 4.5096u,$$
$$c_{31} = 7.5608v - 22.68r, \quad c_{32} = -4.5096u$$

and

$$\boldsymbol{D} = 10^6 \times \begin{bmatrix} 0.05138 & 0 & 0 \\ 0 & 0.1698 & -1.5081 \\ 0 & -1.5081 & 253 \end{bmatrix}$$

Define the reference trajectory as (35) with $u_d = 8$ m/s, $r_d = 0.0033\pi$ rad/s, which is a circle

$$\dot{\boldsymbol{\eta}}_d = \begin{bmatrix} \cos(\psi_d) & 0 & 0 \\ 0 & \sin(\psi_d) & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} u_d \\ u_d \\ r_d \end{bmatrix} \tag{35}$$

Table 1: Hydrodynamic parameters

Parameter	Value	SI unit
I_z	2.0903×10^9	kgm^2
X_u	-0.05138×10^6	kg/s
$X_{\dot{u}}$	-0.5096×10^6	kg
Y_v	-0.1698×10^6	kg/s
$Y_{\dot{v}}$	-3.5608×10^6	kg
Y_r	1.5081×10^6	kgm/s
$Y_{\dot{r}}$	-0.02268×10^9	kgm
N_v	1.5081×10^6	kgm/s
$N_{\dot{v}}$	-0.02268×10^9	kgm
N_r	-0.2530×10^9	kgm^2/s
$N_{\dot{r}}$	-0.8780×10^9	kgm^2

The initial state of the reference trajectory and ship course is defined as

$$\begin{aligned} \boldsymbol{\eta}_{d0} &= [\,0(\mathrm{m}) \quad 0(\mathrm{m}) \quad 0(\mathrm{deg})\,] \\ \boldsymbol{\eta}_0 &= [\,-100(\mathrm{m}) \quad 800(\mathrm{m}) \quad 90(\mathrm{deg})\,] \end{aligned} \tag{36}$$

The SMC law is simulated with $\boldsymbol{\Lambda} = \mathrm{diag}\{1, 1, 1\}$, $\boldsymbol{W} = \mathrm{diag}\{1, 2, 8\} \times 10^{-3}$, $\boldsymbol{K} = \mathrm{diag}\{1, 1, 1\} \times 10^{-3}$. Simulation results are shown in the following figures.

Fig. 2 displays the course of ship trajectory tracking under the proposed SMC law. It is clearly seen that the ship follows the reference trajectory with high accuracy and has fast tracking speed.

Fig. 3 shows the ship position and heading tracking errors in Earth-fixed coordinate. It is shown that the tracking errors are simultaneously convergent to zero with fast

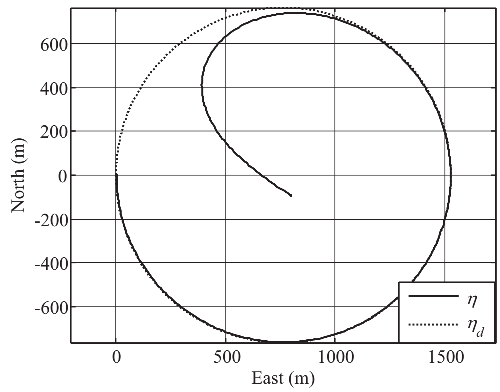

Fig. 2 *Tracking course of marine vessel*

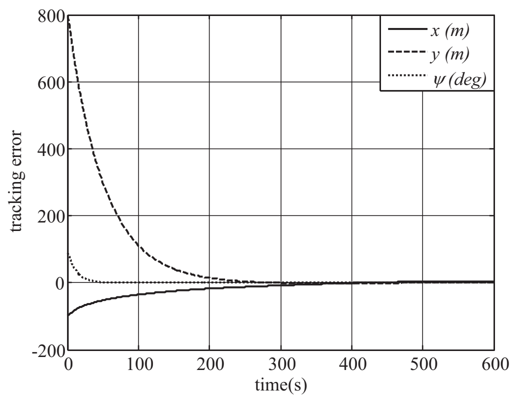

Fig. 3 *Heading tracking error*

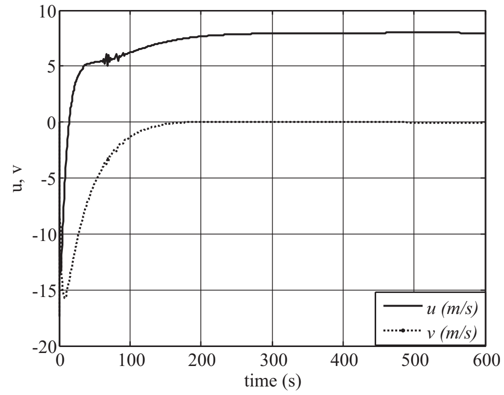

Fig. 4 *Ship velocities in surge and sway*

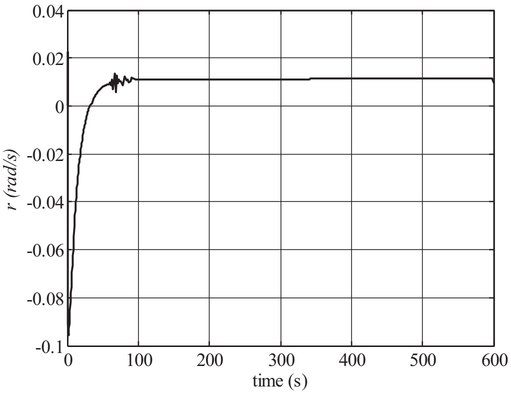

Fig. 5 *Ship angular velocity in yaw*

convergence rate. So the deigned SMC system successfully realises the simultaneous tracking of both the North and East positions together with the orientation of the ship.

Figs. 4 and 5 give the ship velocity in surge, sway and angular velocity in yaw. Note that at stable tracking stage, it satisfies $\boldsymbol{v} = \boldsymbol{v}_r$, so the velocities of the ship can also be controlled by programming velocity of the reference trajectory.

The forces applied to the vessel are shown in Fig. 6. Although restrictions for magnitude and rate were not imposed in the simulation, the applied forces and moment magnitudes are still realistic from a practical viewpoint. Such restrictions and also actuator dynamics will be concerned in the future work.

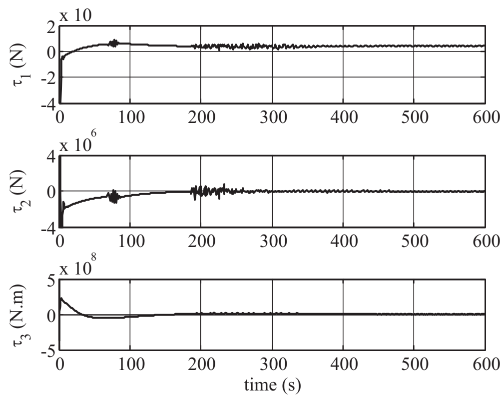

Fig. 6 *Control inputs* $\boldsymbol{\tau}$

6 Conclusions

The paper proposes an SMC law for trajectory tracking problem of marine vessels. As it is desired to control both the North and East positions together with the orientation of the ship, the nonlinear dynamic model for ship horizontal motion with three DOF is constructed first. Design of the SMC law as a nonlinear control method is presented in detail. To prove the effectiveness of the SMC law, the Lyapunov stability analysis has been applied. The proposed control law is simulated on a supply vessel, and simulation results demonstrate that it exhibits an excellent tracking performance.

Future work will be to add actuator dynamics, to consider the estimation of the environmental disturbances because of wind, waves and currents in the analysis of the design, and to get a robust control design. Moreover, most work lies in designing a practical guidance system for marine vessels.

7 References

1 Davidson, K.S.M., and Schiff, L.I.: 'Turning and course keeping qualities', *Trans. Soc. Nav. Archit. Mar. Eng.*, 1946, **54**

2 Nomoto, K.T., Taguchi, K.H., and Hirano, S.: 'On the steering qualities of ships'. Technical report 4(35), Int. Shipbuilding Progress, 1957, pp. 354–370

3 Holzhuter, T.: 'LQG approach for the high precision track control of ships', *IEE Proc. Control Theory Appl.*, 1997, **144**, (2), pp. 121–127

4 Fossen, T.I., and Grøvlen, Å.: 'Nonlinear output feedback control of dynamically positioned ships using vectorial observer backstepping', *IEEE Trans. Control Syst. Technol.*, 1998, **1**, (6), pp. 121–128

5 Fossen, T.I., and Fjellstad, O.E.: 'Nonlinear modeling of marine vehicles in six degrees of freedom', *Int. J. Math. Model. Syst.*, 1995, **1**, (1), pp. 17–27

6 Fossen, T.I.: 'A survey on nonlinear ship control: from theory to practice'. Proc. 5th IFAC Conf. Manoeuvering Control Marine Crafts, International Federation of Automatic Control, Aalborg, Denmark, August 2000, pp. 1–16

7 Juang, J.Y., and Chang, B.C.: 'Robust control theory applied to ship maneuvering'. Proc. 38th Conf. on Decision and Control, USA, December 1999, pp. 2186–2190

8 Wang, W., Yi, J., and Liu, D.: 'Design of a stable sliding-mode controller for a class of second-order underactuated systems', *IEE Proc. Control Theory Appl.*, 2004, **151**, (6), pp. 691–698

9 Fossen, T.I., and Berge, S.P.: 'Nonlinear vectorial backstepping design for global exponential tracking of marine vessels in the presence of actuator dynamics'. Proc. 36th Conf. on Decision and Control, 1997, vol. 9, pp. 4237–4242

10 Zhang, R., Chen, Y., Sun, Z., Sun, F., and Xu, H.: 'Path control of a surface ship in restricted waters using sliding mode'. Proc. 37th IEEE Conf. on Decision and Control, Tampa, FL, USA, December 1998, pp. 3195–3200

11 Encarnacao, P., Pascoal, A., and Arcak, M.: 'Path following for autonomous marine craft'. Proc. 5th IFAC Conf. on Manoeuvering Control Marine Craft, Aalborg, Denmark, August 2000, pp. 117–122

12 Pettersen, K.Y., and Lefeber, E.: 'Way-point tracking control of ships'. Proc. 40th IEEE Conf. on Decision and Control, December 2001

13 Fossen, T.I.: 'Guidance and control of ocean vehicles' (Wiley, New York, 1994)

14 Fossen, T.I., and Fjellstad, O.E.: 'Nonlinear modeling of marine vehicles in six degrees of freedom', *Int. J. Math. Model. Syst.*, 1995, **1**, (1), pp. 17–27

15 Fossen, T.I.: 'Marine control systems: guidance, navigation, and control of ships, rigs and underwater vehicles' (Marine Cybernetics, Trondheim, Norway, 2002)

16 Perez, T.: 'Ship motion control: course keeping and roll stabilisation using rudder and fins', Advances in Industrial Control Series, (Springer-Verlag, 2005)

17 Skejtne, R.: 'The manoeuvring problem', PhD Thesis, Department of Marine Cybernetics, NTNU, Norway, 2005

18 La Salle, J., and Lefschetz, S.: 'Stability by Lyapunov: direct method with applications' (Academic, New York, 1961)

19 Fossen, T.I., Sagatun, S.I., and Sørensen, A.J.: 'Identification of dynamically positioned ships', *J. Control Eng. Pract.*, 1996, **4**, (1), pp. 369–376