Privacy

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Just because they're the problem,
doesn't mean we aren't.

Privacy in statitical disclosure.

- Public analysis of sensitive data.
- Publication of "anonymised" data.

Not about cryptography

- Secure communication and computation.
- Authentication and verification.

An issue of trust

- Who to trust and how much.
- With what data to trust them.
- What you want out of the service.

Databases

Example 1 (Typical relational database in a tax office)

ID	Name	Salary	Deposits	Age	Postcode	Prof
1959060783	Li Pu	150,000	1e6	60	1001	Polit
1946061408	Sara Lee	300,000	-1e9	72	1001	Rent
2100010101	A. B. Student	10,000	100,000	40	1001	Time

Database access

- When owning the database: Direct look-up.
- When accessing a server etc: Query model.

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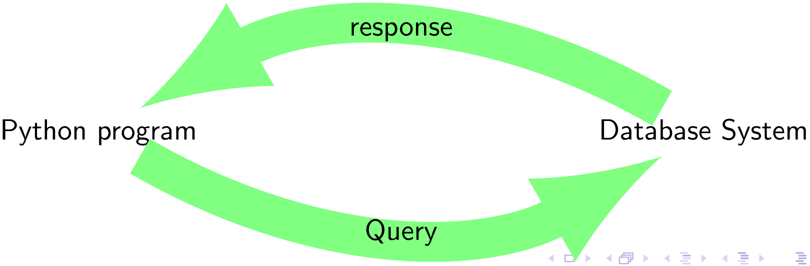

SQL: A language for database access

Creating and filling tables

- `CREATE TABLE table-name (column1, column2)`
- `INSERT INTO table-name VALUES ('value1', 'value2')`
- `INSERT INTO table-name VALUES (?, ?), variable`

Example 2

Database creation `src/privacy/database-creation.py`
`src/privacy/database-access.py`

Queries in SQL

The SELECT statement

- `SELECT column1, column2 FROM table;`
- `SELECT * FROM table;`

Selecting rows

```
SELECT * FROM table WHERE column = value;
```

Arithmetic queries

- `SELECT COUNT(column) FROM table WHERE condition;`
- `SELECT AVG(column) FROM table WHERE condition;`
- `SELECT SUM(column) FROM table WHERE condition;`

Anonymisation

Example 3 (Typical relational database in Tinder)

Birthday	Name	Height	Weight	Age	Postcode	Professio
06/07	Li Pu	190	80	60-70	1001	Politiciar
06/14	Sara Lee	185	110	70+	1001	Rentier
01/01	A. B. Student	170	70	40-60	6732	Time Tra

Anonymisation

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Birthday	Name	Height	Weight	Age	Postcode	Profession
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01/01		170	70	40-60	6732	Time Traveller

The simple act of hiding or using random identifiers is called anonymisation.

Record linkage

Ethnicity
Date
Diagnosis
Procedure
Medication
Charge

Postcode
Birthdate
Sex

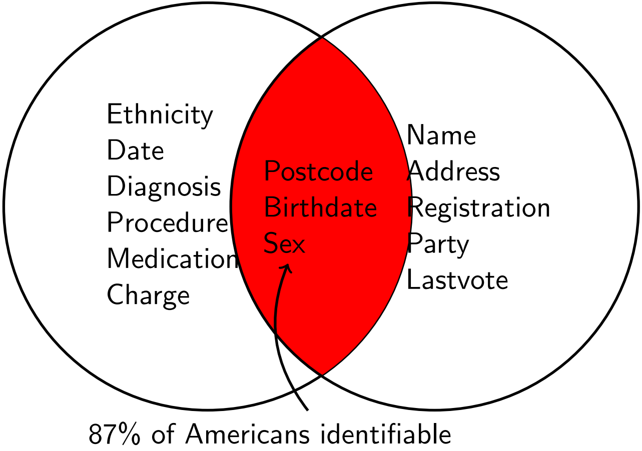
Bill Weld, R-MA

Record linkage

	Name
Postcode	Address
Birthdate	Registration
Sex	Party
	Lastvote

Bill Weld, R-MA

Record linkage



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Example 4 (Typical relational database in a tax office)

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k-anonymity

(a) Samarati

(b) Sweeney

Definition 6 (*k*-anonymity)

A database provides k-anonymity if for every person in the database is indistinguishable from $k - 1$ persons with respect to *quasi-identifiers*.

It's the analyst's job to define quasi-identifiers

Birthday	Name	Height	Weight	Age	Postcode	Pr
06/07	Li Pu	190	80	60+	1001	Po
06/14	Sara Lee	185	110	60+	1001	Re
06/12	Nikos Papadopoulos	180	82	60+	1243	Po
01/01	A. B. Student	170	70	40-60	6732	Ti
05/08	Li Yang	175	72	30-40	6910	Ti

Table: 1-anonymity.

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01/01		170	70	40-60	6732	Time Traveller
05/08		175	72	30-40	6910	Policeman

1-anonymity

Birthday	Name	Height	Weight	Age	Postcode	Profession
06/07		180-190	80+	60+	1*	
06/14		180-190	80+	60+	1*	
06/12		180-190	80+	60+	1*	
01/01		170-180	60-80	20-60	6*	
05/08		170-180	60-80	20-60	6*	

1-anonymity

Birthday	Name	Height	Weight	Age	Postcode	Profession
		180-190	80+	60+	1*	
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		180-190	80+	60+	1*	
		170-180	60-80	20-60	6*	
		170-180	60-80	20-60	6*	

Table: 2-anonymity: the database can be partitioned in sets of at least 2 records

x_1

x

Figure: If two people contribute their data $x = (x_1, x_2)$ to a medical database, and an algorithm π computes some public output a from x, then it should be hard infer anything about the data from the public output.

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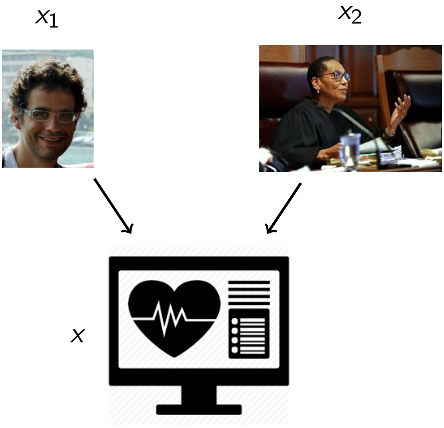

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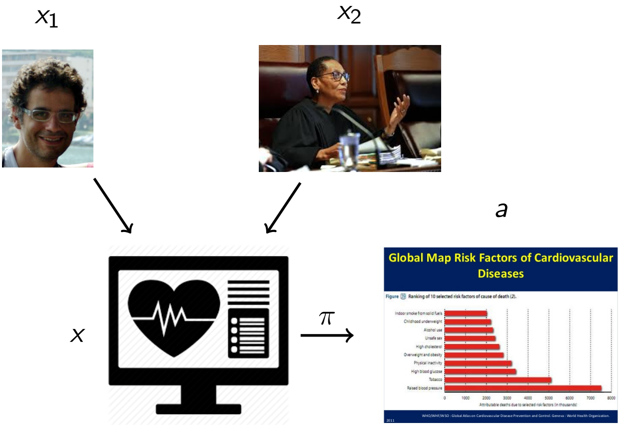

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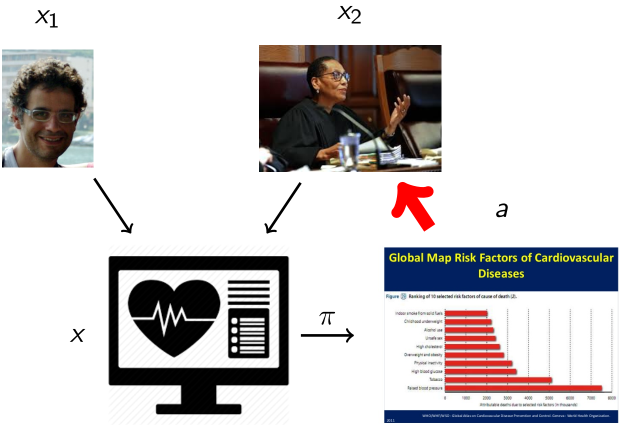

Figure: If two people contribute their data $x = (x_1, x_2)$ to a medical database, and an algorithm π computes some public output a from x, then it should be hard infer anything about the data from the public output.

Privacy desiderata

We wish to calculate something on some private data and publish a privacy-preserving, but useful, version of the result.

- Anonymity: Individual participation remains hidden.
- Secrecy: Individual data x_i is not revealed.
- Side-information: Linkage attacks are not possible.
- Utility: The calculation remains useful.

Example: The prevalence of drug use in sport

- n athletes
- Ask whether they have doped in the past year.
- Aim: calculate % of doping.
- How can we get truthful / accurate results?

Write responses in class

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Algorithm for randomising responses about drug use

1. Flip a coin.
2. If it comes heads, respond truthfully.
3. Otherwise, flip another coin and respond `yes` if it comes heads and `no` otherwise.

Exercise 1

Assume that the observed rate of positive responses in a sample is p, that everybody follows the protocol, and the coin is fair. Then, what is the true rate q of drug use in the population?

Example: The prevalence of drug use in sport

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Solution.

Since the responses are random, we will deal with expectations first

$$\mathbb{E}\, p = \frac{1}{2} \times \frac{1}{2} + q \times \frac{1}{2}$$

$\square$

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□

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$$\mathbb{E}\, p = \frac{1}{2} \times \frac{1}{2} + q \times \frac{1}{2} = \frac{1}{4} + \frac{q}{2}$$
$$q = 2\,\mathbb{E}\, p - \frac{1}{2}.$$

□

The randomised response mechanism

Definition 7 (Randomised response)

The i-th user, whose data is $x_i \in \{0, 1\}$, responds with $a_i \in \{0, 1\}$ with probability

$$\pi(a_i = j \mid x_i = k) = p, \qquad \pi(a_i = k \mid x_i = k) = 1 - p,$$

where $j \neq k$.

The randomised response mechanism

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Given the complete data x, the mechanism's output is $a = (a_1, \ldots, a_n)$. Since the algorithm independently calculates a new value for each data entry, the output is

$$\pi(a \mid x) = \prod_i \pi(a_i \mid x_i)$$

Exercise 1

Let the adversary have a prior $\xi(x = 0) = 1 - \xi(x = 1)$ over the values of the true response of an individual. we use the randomised response mechanism with p and the adversary observes the randomised data $a = 1$ for that individual, then what is $\xi(x = 1 \mid a = 1)$?

The local privacy model

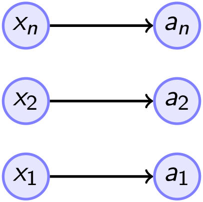
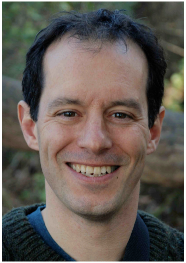


Figure: The local privacy model

Differential privacy.

Definition 8 (ϵ-Differential Privacy)

A stochastic algorithm $\pi : \mathcal{X} \to \mathcal{A}$, where $\mathcal{X}$ is endowed with a neighbourhood relation N, is said to be ϵ-differentially private if

$$\left| \ln \frac{\pi(a \mid x)}{\pi(a \mid x')} \right| \leq \epsilon, \qquad \forall x N x'. \tag{5.1}$$

Defining neighbourhoods

Birthday	Name	Height	Weight
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Table: Data x

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Table: 1-Neighbour x'

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Table: 2-Neighbour x'

The definition of differential privacy

- First rigorous mathematical definition of privacy.
- Relaxations and generalisations possible.
- Connection to learning theory and reproducibility.

Current uses

- Apple.
- Google.
- Uber.
- US 2020 Census.

Open problems

- Complexity of differential privacy.
- Verification of implementations and queries.

Remark 1

The randomised response mechanism with $p \leq 1/2$ is $(\ln \frac{1-p}{p})$-DP.

Proof.

Consider $x = (x_1, \ldots, x_j, \ldots, x_n)$, $x' = (x_1, \ldots, x'_j, \ldots, x_n)$. Then

$$\pi(a \mid x)$$

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&= \pi(a_j \mid x_j) \prod_{i \neq j} \pi(a_i \mid x_i)
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&\leq \frac{1-p}{p} \pi(a_j \mid x'_j) \prod_{i \neq j} \pi(a_i \mid x_i)
\end{aligned}$$

$\pi(a_j = k \mid x_j = k) = 1 - p$ so the ratio is $\max\{(1-p)/p, p/(1-p)\} \leq (1-p)/p$ for $p \leq 1/2$. □

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&= \pi(a_j \mid x_j) \prod_{i \neq j} \pi(a_i \mid x_i) \\
&\leq \frac{1-p}{p} \pi(a_j \mid x'_j) \prod_{i \neq j} \pi(a_i \mid x_i) \\
&= \frac{1-p}{p} \pi(a \mid x')
\end{aligned}$$

$\square$

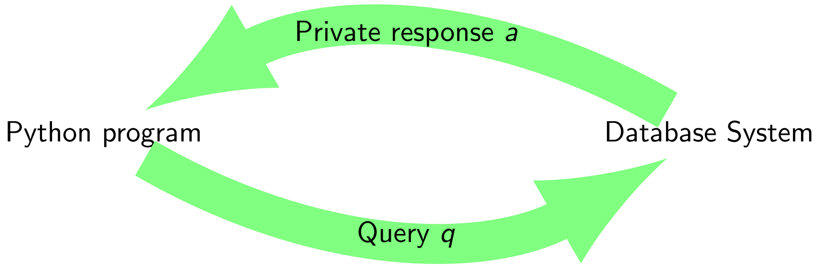

Figure: Private database access model

Response policy

The policy defines a distribution over responses a given the data x and the query q.

$$\pi(a \mid x, q)$$

Differentially private queries

The DP-SELECT statement

- `DP-SELECT` ϵ `column1, column2 FROM table;`
- `DP-SELECT` ϵ `* FROM table;`

Selecting rows

`DP-SELECT` ϵ `* FROM table WHERE column = value;`

Arithmetic queries

- `DP-SELECT` ϵ `COUNT(column) FROM table WHERE condition;`
- `DP-SELECT` ϵ `AVG(column) FROM table WHERE condition;`
- `DP-SELECT` ϵ `SUM(column) FROM table WHERE condition;`

Composition

If we answer T queries with an ϵ-DP mechanism, then our cumulative privacy loss is ϵT.

Exercise 2

Adversary knowledge

$$\boldsymbol{x} = (x_1, \ldots, x_j = 0, \ldots, x_n)$$

$$\boldsymbol{x}' = (x_1, \ldots, x_j = 1, \ldots, x_n).$$

$$\xi(\boldsymbol{x}) = 1 - \xi(\boldsymbol{x}')$$

What can we say about the posterior distribution of the adversary $\xi(\boldsymbol{x} \mid a, \pi)$ after having seen the output, if π is ϵ-DP?

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Adversary knowledge

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$$\boldsymbol{x}' = (x_1, \ldots, x_j = 1, \ldots, x_n).$$

$$\xi(\boldsymbol{x}) = 1 - \xi(\boldsymbol{x}')$$

$$a_t, \qquad \pi(a_t \mid \boldsymbol{x}_t) \Rightarrow \begin{cases} \pi(a_t \mid \boldsymbol{x}_t = \boldsymbol{x}) \\ \pi(a_t \mid \boldsymbol{x}_t = \boldsymbol{x}') \end{cases}$$

What can we say about the posterior distribution of the adversary $\xi(\boldsymbol{x} \mid a, \pi)$ after having seen the output, if π is ϵ-DP?

Dealing with multiple attributes.

Independent release of multiple attributes.

For n users and k attributes, if the release of each attribute i is ϵ-DP then the data release is $k\epsilon$-DP. Thus to get ϵ-DP overall, we need ϵ/k-DP per attribute.

The Laplace mechanism.

Definition 9 (The Laplace mechanism)

For any function $f : \mathcal{X} \to \mathbb{R}$,

$$\pi(a \mid x) = \mathit{Laplace}(f(x), \lambda), \tag{5.2}$$

where the Laplace density is defined as

$$p(\omega \mid \mu, \lambda) = \frac{1}{2\lambda} \exp\left(-\frac{|\omega - \mu|}{\lambda}\right).$$

and has mean μ and variance $2\lambda^2$.

.

Example 10 (Calculating the average salary)

- The i-th person receives salary x_i
- We wish to calculate the average salary in a private manner.

Local privacy model

- Obtain $y_i = x_i + \omega$, where $\omega \sim \mathcal{Laplace}(\lambda)$.
- Return $a = n^{-1} \sum_{i=1}^{n} y_i$.

Centralised privacy model

Return $a = n^{-1} \sum_{i=1}^{n} x_i + \omega$, where $\omega \sim \mathcal{Laplace}(\lambda')$.

How should we add noise in order to guarantee privacy?

The centralised privacy model

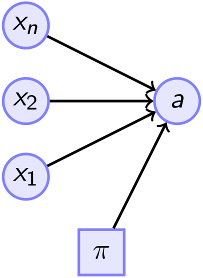

Figure: The centralised privacy model

Assumption 1

The data x is collected and the result a is published by a trusted curator

DP properties of the Laplace mechanism

Definition 11 (Sensitivity)

The sensitivity of a function f is

$$\mathbb{L}(f) \triangleq \sup_{xNx'} |f(x) - f(x')|$$

Example 12

If $f : \mathcal{X} \to [0, B]$, e.g. $\mathcal{X} = \mathbb{R}$ and $f(x) = \min\{B, \max\{0, x\}\}$, then

DP properties of the Laplace mechanism

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Example 13

If $f : [0, B]^n \to [0, B]$ is $f = \frac{1}{n} \sum_{t=1}^{n} x_t$, then

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Example 13

If $f : [0, B]^n \to [0, B]$ is $f = \frac{1}{n} \sum_{t=1}^{n} x_t$, then $\mathbb{L}(f) = B/n$.

Theorem 14

The Laplace mechanism on a function f with sensitivity $\mathbb{L}(f)$, ran with $\mathcal{Laplace}(\lambda)$ is $\mathbb{L}(f)/\lambda$-DP.

Proof.

$$\frac{\pi(a \mid x)}{\pi(a \mid x')} = \frac{e^{|a-f(x')|/\lambda}}{e^{|a-f(x)|/\lambda}} \leq \frac{e^{|a-f(x)|/\lambda + \mathbb{L}(f)/\lambda}}{e^{|a-f(x)|/\lambda}} = e^{\mathbb{L}(f)/\lambda}$$

□

So we need to use $\lambda = \mathbb{L}(f)/\epsilon$ for ϵ-DP. What is the effect of applying the Laplace mechanism in the local versus centralised model?

Interactive queries

- System has data x.
- User asks query q.
- System responds with a.
- There is a common utility function $U : \mathcal{X}, \mathcal{A}, \mathcal{Q} \to \mathbb{R}$.

We wish to maximisation U with our answers, but are constrained by the fact that we also want to preserve privacy.

The Exponential Mechanism.

Definition 15 (The Exponential mechanism)

For any utility function $U : \mathcal{Q} \times \mathcal{A} \times \mathcal{X} \to \mathbb{R}$, define the policy

$$\pi(a \mid x) \triangleq \frac{e^{\epsilon U(q,a,x)/\mathbb{L}(U(q))}}{\sum_{a'} e^{\epsilon U(q,a',x)/\mathbb{L}(U(q))}} \tag{5.3}$$

What happens when $\epsilon \to \infty$? What about when $\epsilon \to 0$?

The unfortunate practice of adaptive analysis

Prior

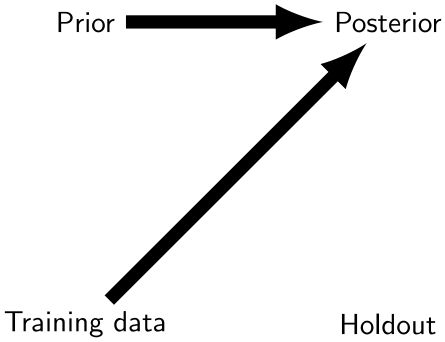
The unfortunate practice of adaptive analysis

Prior

Training data

Holdout

The unfortunate practice of adaptive analysis



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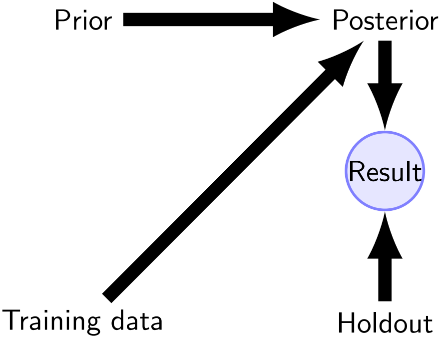

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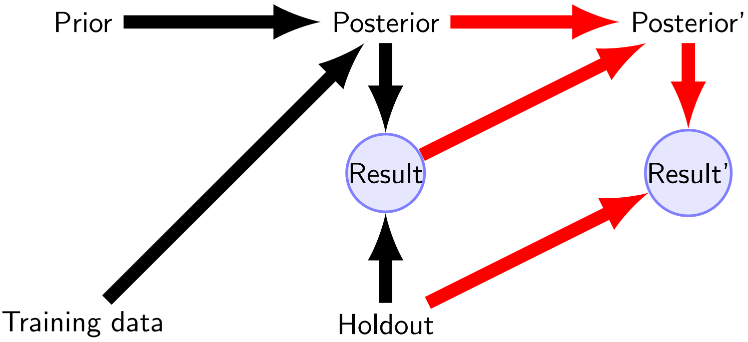

The reusable holdout? [1]

Algorithm parameters

- Performance measure f.
- Threshold τ.
- Noise σ.
- Budget B.

Algorithm idea

Run algorithm λ on data D_T and get e.g. classifier parameters θ.
Run a DP version of the function
$f(\theta, D_H) = \mathbb{I}\left\{U(\theta, D_T) \geq \tau U(\theta, D_H)\right\}$.

[1] Also see
https://ai.googleblog.com/2015/08/the-reusable-holdout-preserving.html

Available privacy toolboxes

k-anonymity

- `https://github.com/qiyuangong/Mondrian` Mondrian k-anonymity

Differential privacy

- `https://github.com/bmcmenamin/thresholdOut-explorations` Threshold out
- `https://github.com/steven7woo/Accuracy-First-Differential-Privacy` Accuracy-constrained DP
- `https://github.com/menisadi/pydp` Various DP algorithms
- `https://github.com/haiphanNJIT/PrivateDeepLearning` Deep learning and DP

Learning outcomes

Understanding

- Linkage attacks and k-anonymity.
- Inferring data from summary statistics.
- The local versus global differential privacy model.
- False discovery rates.

Skills

- Make a dataset satisfy k-anonymity with respect to identifying attributes.
- Apply the randomised response and Laplace mechanism to data.
- Apply the exponential mechanism to simple decision problems.
- Use differential privacy to improve reproducibility.

Reflection