Recommendation systems

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The recommendation problem

At time t

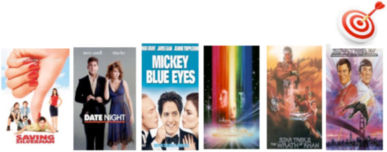
1. A customer $\boldsymbol{x}_t$ appears.
2. We present a choice a_t.
3. The customer chooses y_t.
4. We obtain a reward $r_t = \rho(a_t, y_t) \in \mathbb{R}$.

The two problems in recommendation systems

- The modelling (or prediction) problem.
- The recommendation problem.

How to predict user preferences?

Example: Item-based CF



	2			4	5	2.94*
	5		4			1
			5		2	2.48*
		1		5		4
			4			2
	4	5		1		1.12*
sim(i,j)	-1	-1	0.86	1	NA	

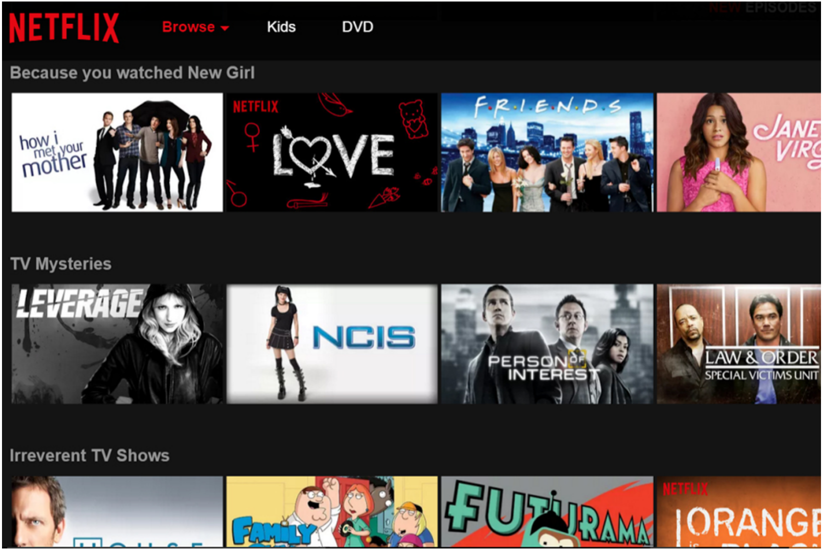


Figure: What to recommend?

Predictions based on similarity

Content-based filtering.

- Users typically like similar items.
- That means we can one user's ratings and item information to predict their ratings for other items.

Collaborative filtering

- Similar users have similar tastes.
- That means we can use similar user's ratings to predict the ratings for other users.

k-NN for similarity

Exercise 1

- Define a distance $d : \mathcal{X}^M \times \mathcal{X}^M \to \mathbb{R}_+$ between user ratings.
- Apply a k-NN-like algorithm to prediction of user ratings from the dataset.

Similarity between users

$$\sum_{j \neq i} w_{i,j} = 1, \qquad w_{i,j}^m \triangleq w_{i,j} \, \mathbb{I}\left\{x_{j,m}\right\} / \sum_k w_{i,k} \, \mathbb{I}\left\{x_{k,m}\right\}.$$

Example 1 (k-nearest neighbours)

$w_{i,j} = 1/k$ for the k nearest neighbours with respect to d.

Example 2 (Weighted distance)

$$w_{i,j} = \frac{\exp[-d(i,j)]}{\sum_{k \neq i} \exp[-d(i,j)]}$$

Inferred ratings

$$\hat{x}_{u.m} = \sum_{j \neq u} w_{u,j}^m x_{u,m}.$$

A naive distance metric

$$d(i, j) \triangleq \|\boldsymbol{x}_i - \boldsymbol{x}_j\|_1.$$

Ignoring movies which are not shared.

$$d(i, j) \triangleq \sum_m \mathbb{I}\left\{x_{i,m} \wedge x_{j,m}\right\} |x_{i,m} - x_{j,m}|$$

Using side-information

Social network data

Inferring a latent representation

$$d(i, j) \triangleq f(\boldsymbol{x}_i, \boldsymbol{x}_j, \theta)$$

Latent representation

The predictive model

- x_{um} rating of user u for movie m.
- $r_{um} = \mathbb{I}\left\{x_{um} > 0\right\}$ indicates which movies are rated.
- $\boldsymbol{z}_m \in \mathbb{R}^n$: an n-dimensional representation of a movie.
- $\boldsymbol{c}_u \in \mathbb{R}^n$: an n-dimensional representation of a user.

Given $\boldsymbol{C}, \boldsymbol{Z}$, our predicted movie rating can be written as

$$\hat{x}_{u,m} \triangleq \boldsymbol{c}_u^\top \boldsymbol{z}_m, \qquad \hat{\boldsymbol{X}} \triangleq \boldsymbol{C}^\top \boldsymbol{Z}.$$

$$f(\boldsymbol{C}, \boldsymbol{Z}) = \|(\boldsymbol{R} \circ \hat{\boldsymbol{X}} - \boldsymbol{R} \circ \boldsymbol{X})^\top (\boldsymbol{R} \circ \hat{\boldsymbol{X}} - \boldsymbol{R} \circ \boldsymbol{X})\|_1$$

A simple preference model

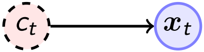


Figure: Basic preference model

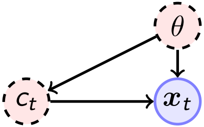
Example 3 (Discrete preference model)

- User type $c \in \mathcal{C}$.
- User ratings $\boldsymbol{x}$ with $x_m \in \mathcal{X} = \{0, 1\}$ rating for movie m.
- Preference distribution

$$P_\theta(\boldsymbol{x}|\boldsymbol{c}) = \prod_{m=1}^{M} \theta_{m,c}^{x_m}(1 - \theta_{m,c})^{(1-x_m)}.$$

- $P_\theta(c) = \theta_c$, $\sum_c \theta_c = 1$.

A simple preference model



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A more complex preference model

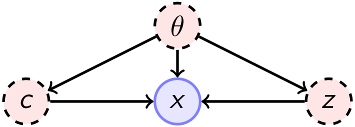

Figure: Preference model

Preference model

- User type $\boldsymbol{c} \in \mathcal{C}$.
- Movie type $\boldsymbol{z} \in \mathcal{Z}$.
- Preference distribution

$$P_\theta(x|\boldsymbol{c}, \boldsymbol{z}) = \mathcal{N}(\boldsymbol{c}^\top \boldsymbol{z}, \sigma_\theta)$$

- Feature prior

$$P_\theta(\boldsymbol{c}) = \mathcal{N}(0, \lambda_\theta)$$

What to recommend

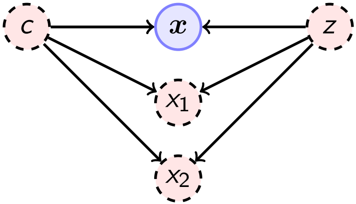

Figure: Preference model

The recommendation problem for a given θ

What to recommend

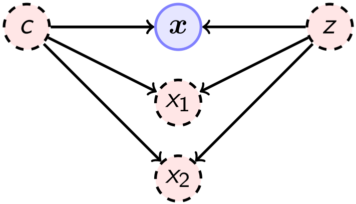

Figure: Preference model

The recommendation problem for a given θ

$$\max_\pi \mathbb{E}^\pi_\theta(U \mid \boldsymbol{x}) = \max_a \sum_{c,z} U(a, y)\, \mathbb{P}(y \mid a, c, z) P_\theta(c, z \mid \boldsymbol{x}) \tag{1.1}$$

$$= \max_a \sum_{c,z} U(a, y) \sum_{x_a} \mathbb{P}(y \mid a, x_a) P_\theta(x_a \mid c, z) P_\theta(c, z \mid \boldsymbol{x}) \tag{1.2}$$

Two ways to model the effect of actions

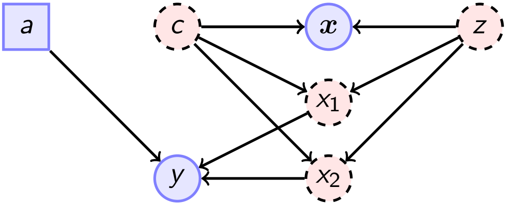

Figure: Preference model

$$\mathbb{E}_\theta(U \mid a, \boldsymbol{x}) = \sum_{c,z} U(a, y) \sum_{x_a} \mathbb{P}(y \mid a, x_a) P_\theta(x_a \mid c, z) P_\theta(c, z \mid \boldsymbol{x}) \quad (1.3)$$

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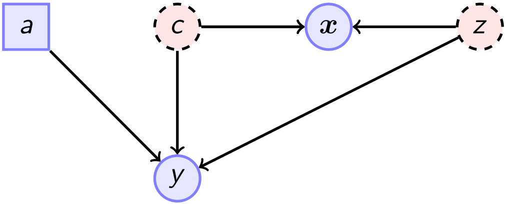

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Recommendation systems

More fun with latent variable models

Social networks

Sequential structures

Clusters as latent variables

$$c_t \longrightarrow x_t$$

Figure: Graphical model for independent data from a cluster distribution.

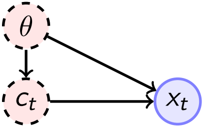
The clustering distribution

- Cluster c_t
- Observation x_t
- Parameter θ.

$$x_t \mid c_t = c, \theta \sim P_\theta(x|c), \qquad c_t \mid \theta \sim P_\theta(c), \qquad \theta \sim \xi(\theta)$$

$$P_\theta(c_t \mid x_t) =$$

Clusters as latent variables



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$$P_\theta(c_t \mid x_t) = \frac{P_\theta(x_t \mid c_t) P_\theta(c_t)}{\sum_{c'} P_\theta(x_t \mid c_t = c') P_\theta(c_t = c')}$$

Bayesian formulation of the clustering problem

- Prior ξ on parameter space Θ.
- Data $x^T = x_1, \ldots, x_T$. Cluster assignments c^T unknown.
- Posterior $\xi(\cdot \mid x^T)$.

Posterior distribution

$$\xi(\theta \mid x^T) = \frac{P_\theta(x^T)\xi(\theta)}{\sum_{\theta \in \Theta} P_{\theta'}(x^T)\xi(\theta')}, \quad P_\theta(x^T) = \sum_{c^T \in \mathcal{C}^T} \overbrace{P_\theta(x^T \mid c^T)}^{\text{Cluster Density}} \underbrace{P_\theta(c^T)}_{\text{Cluster prior}} \tag{2.1}$$

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Marginal posterior prediction

$$P_\xi(c_t \mid x_t, x^T) = \sum_{\theta \in \Theta} P_\theta(c_t \mid x_t) \xi(\theta \mid x^T)$$

Example 4 (Preference clustering)

$$\mathcal{C} = \{1, \ldots, C\}, \qquad x_{t.m} \in \{0, 1\}.$$

$\theta = (\theta_1, \theta_2)$.

Model family

$$P_{\theta_1}(c_t = c) = \theta_{1,c}, \qquad c_t \sim \mathcal{Multinomial}(\theta_1) \tag{2.2}$$

$$P_{\theta_2}(x_{t,m} = 1 \mid c_t = c) = \theta_{2,m,c} \qquad x_{t,m} \mid c_t = c \sim \mathcal{Bernoulli}(\theta_{2,m,c}) \tag{2.3}$$

Prior

$$\theta_1 \sim \mathcal{Dirichlet}(\gamma), \qquad \theta_2 \sim \mathcal{Beta}(\alpha, \beta) \tag{2.4}$$

See `src/pymc/beta_bernoulli_clustering.py`

Supervised learning

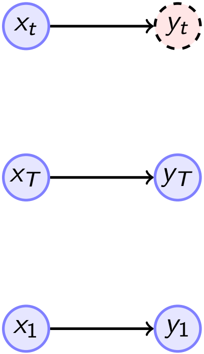


Figure: Graphical model for a classical supervised learning problem.

Semi-supervised learning

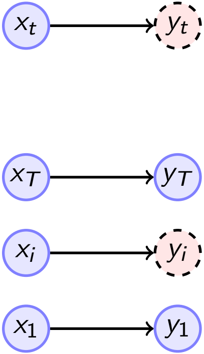


Figure: Graphical model for a classical semi-supervised learning problem.

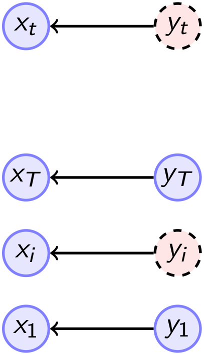


Figure: Generative version of the semi-supervised model

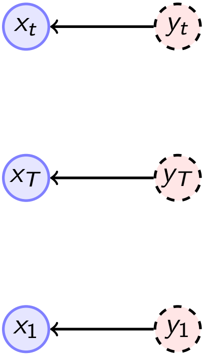


Figure: Basic unsupervised learning model

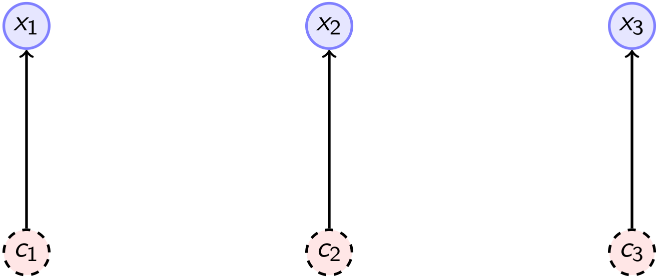
Applications

- Clustering
- Compression

Network model

Figure: Graphical model for data from a social network.

Network model



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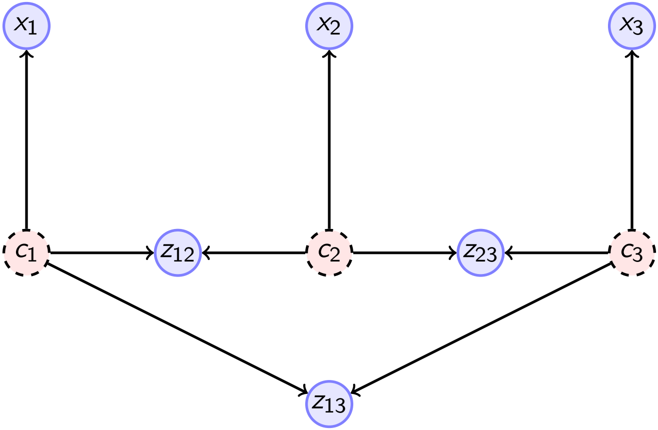

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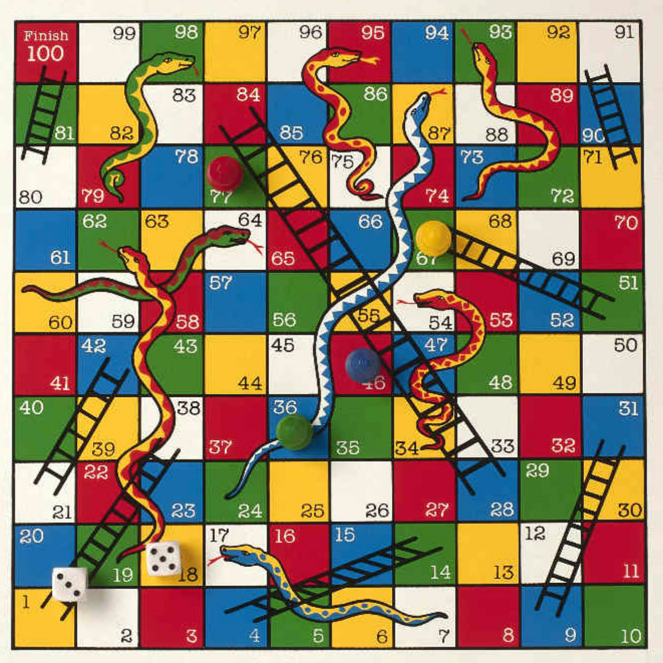
Finish
100

Markov process

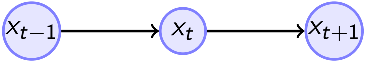

Figure: Graphical model for a Markov process.

Definition 5 (Markov process)

A Markov process is a sequence of variables $x_t : \Omega \to \mathcal{X}$ such that $x_{t+1} \mid x_t \perp\!\!\!\perp x_{t-k} \forall k \leq 1$.

Application

- Sequence compression (especially with variable order Models).
- Web-search (Page-Rank)
- Hidden Markov Models.
- MCMC.

Hidden Markov model

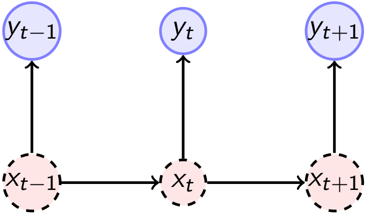

Figure: Graphical model for a hidden Markov model.

$$P_\theta(x_{t+1} \mid x_t) \qquad \text{(transition distribution)}$$
$$P_\theta(y_t \mid x_t) \qquad \text{(emission distribution)}$$

Application

- Speech recognition.
- Filtering (Kalman Filter).
- DNA analysis