

Team Contest Reference

ChaosKITs
Karlsruhe Institute of Technology

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1 Datenstrukturen

1.1 Union-Find

```

1 vector<int> parent, rank2; //manche compiler verbieten Variable mit Namen rank
2
3 int findSet(int n) { //Pfadkompression
4     if (parent[n] != n) parent[n] = findSet(parent[n]);
5     return parent[n];
6 }
7
8 void linkSets(int a, int b) { //union by rank
9     if (rank2[a] < rank2[b]) parent[a] = b;
10    else if (rank2[b] < rank2[a]) parent[b] = a;
11    else {
12        parent[a] = b;
13        rank2[b]++;
14    }
15 }
16
17 void unionSets(int a, int b) {
18     if (findSet(a) != findSet(b)) linkSets(findSet(a), findSet(b));
19 }

```

1.2 Segmentbaum

```

1 int a[MAX_N], m[4 * MAX_N];
2
3 int query(int x, int y, int k = 0, int X = 0, int Y = MAX_N - 1) {
4     if (x <= X && Y <= y) return m[k];
5     if (y < X || Y < x) return -1000000000; //ein "neutrales" Element
6     int M = (X + Y) / 2;
7     return max(query(x, y, 2 * k + 1, X, M), query(x, y, 2 * k + 2, M + 1, Y));
8 }
9
10 void update(int i, int v, int k = 0, int X = 0, int Y = MAX_N - 1) {
11     if (i < X || Y < i) return;
12     if (X == Y) {
13         m[k] = v;
14         a[i] = v;
15         return;
16     }
17     int M = (X + Y) / 2;
18     update(i, v, 2 * k + 1, X, M);
19     update(i, v, 2 * k + 2, M + 1, Y);
20     m[k] = max(m[2 * k + 1], m[2 * k + 2]);
21 }
22
23 void init(int k = 0, int X = 0, int Y = MAX_N - 1) {
24     if (X == Y) {
25         m[k] = a[X];
26         return;
27     }
28     int M = (X + Y) / 2;
29     init(2 * k + 1, X, M);
30     init(2 * k + 2, M + 1, Y);
31     m[k] = max(m[2 * k + 1], m[2 * k + 2]);
32 }

```

1.3 Range Minimum Query

```

1 vector<int> data(RMQ_SIZE);
2 vector<vector<int>> rmq(floor(log2(RMQ_SIZE)) + 1, vector<int>(RMQ_SIZE));
3
4 void initRMQ() {
5     for(int i = 0, s = 1, ss = 1; s <= RMQ_SIZE; ss=s, s*=2, i++) {
6         for(int l = 0; l + s <= RMQ_SIZE; l++) {
7             if(i == 0) rmq[0][l] = 1;
8             else {
9                 int r = l + ss;

```

```

10         rmq[i][l] = (data[rmq[i-1][l]] <= data[rmq[i-1][r]] ? rmq[i-1][l] : rmq[i-1][r]);
11     }
12 }
13 }
14 }
15 //returns index of minimum! [a, b)
16 int queryRMQ(int l, int r) {
17     if(l >= r) return l;
18     int s = floor(log2(r-l)); r = r - (1 << s);
19     return (data[rmq[s][l]] <= data[rmq[s][r]] ? rmq[s][l] : rmq[s][r]);
20 }

```

2 Graphen

2.1 Lowest Common Ancestor

```

1 //RMQ muss hinzugefuegt werden!
2 vector<int> visited(2*MAX_N), first(MAX_N, 2*MAX_N), depth(2*MAX_N);
3 vector<vector<int>> graph(MAX_N);
4
5 void initLCA(int gi, int d, int &c) {
6     visited[c] = gi, depth[c] = d, first[gi] = min(c, first[gi]), c++;
7     for(int gn : graph[gi]) {
8         initLCA(gn, d+1, c);
9     }
10 }
11 }
12 // [a, b]
13 int getLCA(int a, int b) {
14     return visited[queryRMQ(min(first[a], first[b]), max(first[a], first[b]))];
15 }
16 //=> int c = 0; initLCA(0,0,c); initRMQ(); done!

```

2.2 Kürzeste Wege

2.2.1 Algorithmus von DIJKSTRA

Kürzeste Pfade in Graphen ohne negative Kanten.

```

1 priority_queue<ii, vector<ii>, greater<ii> > pq;
2 vector<int> dist;
3 dist.assign(NUM_VERTICES, INF);
4 dist[0] = 0;
5 pq.push(ii(0, 0));
6
7 while (!pq.empty()) {
8     di front = pq.top(); pq.pop();
9     int curNode = front.second, curDist = front.first;
10
11     if (curDist > dist[curNode]) continue;
12
13     for (i = 0; i < (int)adjlist[curNode].size(); i++) {
14         int nextNode = adjlist[curNode][i].first, nextDist = curDist + adjlist[curNode][i].second;
15
16         if (nextDist < dist[nextNode]) {
17             dist[nextNode] = nextDist; pq.push(ii(nextDist, nextNode));
18         }
19     }
20 }

```

2.2.2 BELLMANN-FORD-Algorithmus

Kürzestes Pfade in Graphen mit negativen Kanten. Erkennt negative Zyklen.

```

1 //n = number of vertices, edges is vector of edges
2 dist.assign(n, INF); dist[0] = 0;
3 parent.assign(n, -1);
4 for (i = 0; i < n - 1; i++) {
5     for (j = 0; j < (int)edges.size(); j++) {
6         if (dist[edges[j].from] + edges[j].cost < dist[edges[j].to]) {

```

```

7         dist[edges[j].to] = dist[edges[j].from] + edges[j].cost;
8         parent[edges[j].to] = edges[j].from;
9     }
10 }
11 }
12 //now dist and parent are correct shortest paths
13 //next lines check for negative cycles
14 for (j = 0; j < (int)edges.size(); j++) {
15     if (dist[edges[j].from] + edges[j].cost < dist[edges[j].to]) {
16         //NEGATIVE CYCLE found
17     }
18 }

```

2.2.3 FLOYD-WARSHALL-Algorithmus

Alle kürzesten Pfade im Graphen.

```

1 //initialize adjmat, adjmat[i][i] = 0, adjmat[i][j] = INF if no edge is between i and j
2 for (k = 0; k < MAX_V; k++) {
3     for (i = 0; i < MAX_V; i++) {
4         for (j = 0; j < MAX_V; j++) {
5             if (adjmat[i][k] + adjmat[k][j] < adjmat[i][j]) adjmat[i][j] = adjmat[i][k] + adjmat[k][j];
6         }
7     }
8 }

```

2.3 Strongly Connected Components (TARJANS-Algorithmus)

```

1 int counter, sccCounter, n; //n == number of vertices
2 vector<bool> visited, inStack;
3 vector< vector<int> > adjlist;
4 vector<int> d, low, sccs;
5 stack<int> s;
6
7 void visit(int v) {
8     visited[v] = true;
9     d[v] = counter;
10    low[v] = counter;
11    counter++;
12    inStack[v] = true;
13    s.push(v);
14
15    for (int i = 0; i < (int)adjlist[v].size(); i++) {
16        int u = adjlist[v][i];
17        if (!visited[u]) {
18            visit(u);
19            low[v] = min(low[v], low[u]);
20        } else if (inStack[u]) {
21            low[v] = min(low[v], low[u]);
22        }
23    }
24
25    if (d[v] == low[v]) {
26        int u;
27        do {
28            u = s.top();
29            s.pop();
30            inStack[u] = false;
31            sccs[u] = sccCounter;
32        } while (u != v);
33        sccCounter++;
34    }
35 }
36
37 void scc() {
38     //read adjlist
39
40     visited.clear(); visited.assign(n, false);
41     d.clear(); d.resize(n);

```

```

42 low.clear(); low.resize(n);
43 inStack.clear(); inStack.assign(n, false);
44 sccs.clear(); sccs.resize(n);
45
46 counter = 0;
47 sccCounter = 0;
48 for (i = 0; i < n; i++) {
49     if (!visited[i]) {
50         visit(i);
51     }
52 }
53 //sccs has the component for each vertex
54 }

```

2.4 Artikulationspunkte und Brücken

```

1 vector< vector<int> > adjlist;
2 vector<int> low;
3 vector<int> d;
4 vector<bool> isArtPoint;
5 vector< vector<int> > bridges; //nur fuer Bruecken
6 int counter = 0;
7
8 void visit(int v, int parent) {
9     d[v] = low[v] = ++counter;
10    int numVisits = 0, maxlow = 0;
11
12    for (vector<int>::iterator vit = adjlist[v].begin(); vit != adjlist[v].end(); vit++) {
13        if (d[*vit] == 0) {
14            numVisits++;
15            visit(*vit, v);
16            if (low[*vit] > maxlow) {
17                maxlow = low[*vit];
18            }
19
20            if (low[*vit] > d[v]) { //nur fuer Bruecken
21                bridges[v].push_back(*vit); bridges[*vit].push_back(v);
22            }
23
24            low[v] = min(low[v], low[*vit]);
25        } else {
26            if (d[*vit] < low[v]) {
27                low[v] = d[*vit];
28            }
29        }
30    }
31
32    if (parent == -1) {
33        if (numVisits > 1) isArtPoint[v] = true;
34    } else {
35        if (maxlow >= d[v]) isArtPoint[v] = true;
36    }
37 }
38
39 void findArticulationPoints() {
40     low.clear(); low.resize(adjlist.size());
41     d.clear(); d.assign(adjlist.size(), 0);
42     isArtPoint.clear(); isArtPoint.assign(adjlist.size(), false);
43     bridges.clear(); isBridge.resize(adjlist.size()); //nur fuer Bruecken
44     for (int v = 0; v < (int)adjlist.size(); v++) {
45         if (d[v] == 0) visit(v, -1);
46     }
47 }

```

2.5 Eulertouren

- Zyklus existiert, wenn jeder Knoten geraden Grad hat (ungerichtet), bzw. bei jedem Knoten Ein- und Ausgangsgrad übereinstimmen (gerichtet).
- Pfad existiert, wenn alle bis auf (maximal) zwei Knoten geraden Grad haben (ungerichtet), bzw. bei allen Knoten bis auf zwei Ein- und Ausgangsgrad übereinstimmen, wobei einer eine Ausgangskante mehr hat (Startknoten) und

einer eine Eingangskante mehr hat (Endknoten).

- Je nach Aufgabenstellung überprüfen, wie isolierte Punkte interpretiert werden sollen.
- Der Code unten läuft in Linearzeit. Wenn das nicht notwendig ist (oder bestimmte Sortierungen verlangt werden), gehts mit einem `set` einfacher.

```

1 VISIT(v):
2   forall e=(v,w) in E
3     delete e from E
4   VISIT(w)
5   print e

```

Abbildung 1: Idee für Eulerzyklen

```

1 vector< vector<int> > adjlist;
2 vector< vector<int> > otherIdx;
3 vector<int> cycle;
4 vector<int> validIdx;
5
6 void swapEdges(int n, int a, int b) { // Vertauscht Kanten mit Indizes a und b von Knoten n.
7     int neighA = adjlist[n][a];
8     int neighB = adjlist[n][b];
9     int idxNeighA = otherIdx[n][a];
10    int idxNeighB = otherIdx[n][b];
11    swap(adjlist[n][a], adjlist[n][b]);
12    swap(otherIdx[n][a], otherIdx[n][b]);
13    otherIdx[neighA][idxNeighA] = b;
14    otherIdx[neighB][idxNeighB] = a;
15 }
16
17 void removeEdge(int n, int i) { // Entfernt Kante i von Knoten n (und die zugehoerige Rueckwaerts-kante)
18     int other = adjlist[n][i];
19     if (other == n) { //Schlingen
20         validIdx[n]++;
21         return;
22     }
23     int otherIndex = otherIdx[n][i];
24     validIdx[n]++;
25     if (otherIndex != validIdx[other]) {
26         swapEdges(other, otherIndex, validIdx[other]);
27     }
28     validIdx[other]++;
29 }
30
31 //findet Eulerzyklus an Knoten n startend
32 //teste vorher, dass Graph zusammenhaengend ist! (isolierte Punkte sind ok)
33 //teste vorher, ob Eulerzyklus ueberhaupt existiert!
34 void euler(int n) {
35     while (validIdx[n] < (int)adjlist[n].size()) {
36         int nn = adjlist[n][validIdx[n]];
37         removeEdge(n, validIdx[n]);
38         euler(nn);
39     }
40     cycle.push_back(n); //Zyklus am Ende in cycle
41 }

```

2.6 Max-Flow (EDMONDS-KARP-Algorithmus)

```

1 int s, t, f; //source, target, single flow
2 int res[MAX_V][MAX_V]; //adj-matrix
3 vector< vector<int> > adjList;
4 int p[MAX_V]; //bfs spanning tree
5
6 void augment(int v, int minEdge) {
7     if (v == s) { f = minEdge; return; }
8     else if (p[v] != -1) {

```

```

9      augment(p[v], min(minEdge, res[p[v]][v]));
10     res[p[v]][v] -= f; res[v][p[v]] += f;
11 }}
12
13 int maxFlow() { //first initialize res, adjList, s and t
14     int mf = 0;
15     while (true) {
16         f = 0;
17         bitset<MAX_V> vis; vis[s] = true;
18         queue<int> q; q.push(s);
19         memset(p, -1, sizeof(p));
20         while (!q.empty()) { //BFS
21             int u = q.front(); q.pop();
22             if (u == t) break;
23             for (int j = 0; j < (int)adjList[u].size(); j++) {
24                 int v = adjList[u][j];
25                 if (res[u][v] > 0 && !vis[v]) {
26                     vis[v] = true; q.push(v); p[v] = u;
27                 }
28             }
29             augment(t, INF); //add found path to max flow
30             if (f == 0) break;
31             mf += f;
32         }
33         return mf;
34     }

```

3 Geometrie

3.1 Closest Pair

```

1 double squaredDist(point a, point b) {
2     return (a.first-b.first) * (a.first-b.first) + (a.second-b.second) * (a.second-b.second);
3 }
4
5 bool compY(point a, point b) {
6     if (a.second == b.second) return a.first < b.first;
7     return a.second < b.second;
8 }
9
10 double shortestDist(vector<point> &points) {
11     //check that points.size() > 1 and that ALL POINTS ARE DIFFERENT
12     set<point, bool(*)>(point, point)> status(compY);
13     sort(points.begin(), points.end());
14     double opt = 1e30, sqrtOpt = 1e15;
15     auto left = points.begin(), right = points.begin();
16     status.insert(*right); right++;
17
18     while (right != points.end()) {
19         if (fabs(left->first - right->first) >= sqrtOpt) {
20             status.erase(*(left++));
21         } else {
22             auto lower = status.lower_bound(point(-1e20, right->second - sqrtOpt));
23             auto upper = status.upper_bound(point(-1e20, right->second + sqrtOpt));
24             while (lower != upper) {
25                 double cand = squaredDist(*right, *lower);
26                 if (cand < opt) {
27                     opt = cand;
28                     sqrtOpt = sqrt(opt);
29                 }
30                 ++lower;
31             }
32             status.insert(*(right++));
33         }
34     }
35     return sqrtOpt;
36 }

```

3.2 Geraden

```

1 struct pt { //complex<double> does not work here, because we need to set pt.x and pt.y
2     double x, y;
3     pt() {};}
4 pt(double x, double y) : x(x), y(y) {};
5 };
6
7 struct line {
8     double a, b, c; //a*x+b*y+c, b=0 <=> vertical line, b=1 <=> otherwise
9 };
10
11 line pointsToLine(pt p1, pt p2) {
12     line l;
13     if (fabs(p1.x - p2.x) < EPSILON) {
14         l.a = 1; l.b = 0.0; l.c = -p1.x;
15     } else {
16         l.a = -(double)(p1.y - p2.y) / (p1.x - p2.x);
17         l.b = 1.0;
18         l.c = -(double)(l.a * p1.x) - p1.y;
19     }
20     return l;
21 }
22
23 bool areParallel(line l1, line l2) {
24     return (fabs(l1.a - l2.a) < EPSILON) && (fabs(l1.b - l2.b) < EPSILON);
25 }
26
27 bool areSame(line l1, line l2) {
28     return areParallel(l1, l2) && (fabs(l1.c - l2.c) < EPSILON);
29 }
30
31 bool areIntersect(line l1, line l2, pt &p) {
32     if (areParallel(l1, l2)) return false;
33     p.x = (l2.b * l1.c - l1.b * l2.c) / (l2.a * l1.b - l1.a * l2.b);
34     if (fabs(l1.b) > EPSILON) p.y = -(l1.a * p.x + l1.c);
35     else p.y = -(l2.a * p.x + l2.c);
36     return true;
37 }

```

3.3 Konvexe Hülle

```

1 #include <algorithm>
2 #include <iostream>
3 #include <sstream>
4 #include <string>
5 #include <vector>
6 using namespace std;
7
8 struct point {
9     double x, y;
10     point(){} point(double x, double y) : x(x), y(y) {}
11     bool operator <(const point &p) const {
12         return x < p.x || (x == p.x && y < p.y);
13     }
14 };
15
16 // 2D cross product.
17 // Return a positive value, if OAB makes a counter-clockwise turn,
18 // negative for clockwise turn, and zero if the points are collinear.
19 double cross(const point &O, const point &A, const point &B){
20     double d = (A.x - O.x) * (B.y - O.y) - (A.y - O.y) * (B.x - O.x);
21     if (fabs(d) < 1e-9) return 0.0;
22     return d;
23 }
24
25 // Returns a list of points on the convex hull in counter-clockwise order.
26 // Colinear points are not in the convex hull, if you want colinear points in the hull remove "=" in
27 // the CCW-Test
28 // Note: the last point in the returned list is the same as the first one.
29 vector<point> convexHull(vector<point> P){
30     int n = P.size(), k = 0;
31     vector<point> H(2*n);

```



```

31
32 // Sort points lexicographically
33 sort(P.begin(), P.end());
34
35 // Build lower hull
36 for (int i = 0; i < n; i++) {
37     while (k >= 2 && cross(H[k-2], H[k-1], P[i]) <= 0.0) k--;
38     H[k++] = P[i];
39 }
40
41 // Build upper hull
42 for (int i = n-2; i >= 0; i--) {
43     while (k >= 2 && cross(H[k-2], H[k-1], P[i]) <= 0.0) k--;
44     H[k++] = P[i];
45 }
46
47 H.resize(k);
48 return H;
49 }

```

3.4 Formeln - std::complex

```

1 //komplexe Zahlen als Darstellung fuer Punkte
2 typedef pt complex<double>;
3 //Winkel zwischen Punkt und x-Achse in [0, 2 * PI), Winkel zwischen a und b
4 double angle = arg (a), angle_a_b = arg (a - b);
5 //Punkt rotiert um Winkel theta
6 pt a_rotated = a * exp (pt (0, theta));
7 //Mittelpunkt des Dreiecks abc
8 pt centroid = (a + b + c) / 3;
9 //Skalarprodukt
10 double dot(pt a, pt b) {
11     return real(conj(a) * b);
12 }
13 //Kreuzprodukt, 0, falls kollinear
14 double cross(pt a, pt b) {
15     return imag(conj(a) * b);
16 }
17 //wenn Eckpunkte bekannt
18 double areaOfTriangle(pt a, pt b, pt c) {
19     return abs(cross(b - a, c - a)) / 2.0;
20 }
21 //wenn Seitenlaengen bekannt
22 double areaOfTriangle(double a, double b, double c) {
23     double s = (a + b + c) / 2;
24     return sqrt(s * (s-a) * (s-b) * (s-c));
25 }
26 // Sind die Dreiecke a1, b1, c1, and a2, b2, c2 aehnlich?
27 // Erste Zeile testet Aehnlichkeit mit gleicher Orientierung,
28 // zweite Zeile testst Aehnlichkeit mit unterschiedlicher Orientierung
29 bool similar (pt a1, pt b1, pt c1, pt a2, pt b2, pt c2) {
30     return (
31         (b2 - a2) * (c1 - a1) == (b1 - a1) * (c2 - a2) ||
32         (b2 - a2) * (conj (c1) - conj (a1)) == (conj (b1) - conj (a1)) * (c2 - a2)
33     );
34 }
35 //Linksknick von a->b nach a->c
36 double ccw(pt a, pt b, pt c) {
37     return cross(b - a, c - a); //<0 => falls Rechtsknick, 0 => kollinear, >0 => Linksknick
38 }
39 //Streckenschnitt, Strecken a-b und c-d
40 bool lineSegmentIntersection(pt a, pt b, pt c, pt d) {
41     if (ccw(a, b, c) == 0 && ccw(a, b, d) == 0) { //kollinear
42         double dist = abs(a - b);
43         return (abs(a - c) <= dist && abs(b - c) <= dist) || (abs(a - d) <= dist && abs(b - d) <= dist);
44     }
45     return ccw(a, b, c) * ccw(a, b, d) <= 0 && ccw(c, d, a) * ccw(c, d, b) <= 0;
46 }
47 //Entfernung von p zu a-b
48 double distToLine(pt a, pt b, pt p) {
49     return abs(cross(p - a, b - a)) / abs(b - a);

```

```

50 }
51 //liegt p auf a-b
52 bool pointOnLine(pt a, pt b, pt p) {
53     return abs(distToLine(a, b, p)) < EPSILON;
54 }
55 //testet, ob d in der gleichen Ebene liegt wie a, b, und c
56 bool isCoplanar(pt a, pt b, pt c, pt d) {
57     return (b - a) * (c - a) * (d - a) == 0;
58 }
59 //berechnet den Flaecheninhalt eines Polygons (nicht selbstschneidend)
60 double areaOfPolygon(vector<pt> &polygon) { //jeder Eckpunkt nur einmal im Vektor
61     double res = 0; int n = polygon.size();
62     for (int i = 0; i < (int)polygon.size(); i++)
63         res += real(polygon[i]) * imag(polygon[(i + 1) % n]) - real(polygon[(i + 1) % n]) * imag(polygon[i]);
64     return 0.5 * abs(res);
65 }
66 //testet, ob sich zwei Rechtecke (p1, p2) und (p3, p4) schneiden (jeweils gegenueberliegende Ecken)
67 bool rectIntersection(pt p1, pt p2, pt p3, pt p4) {
68     double minx12 = min(real(p1), real(p2)), maxx12 = max(real(p1), real(p2));
69     double minx34 = min(real(p3), real(p4)), maxx34 = max(real(p3), real(p4));
70     double miny12 = min(imag(p1), imag(p2)), maxy12 = max(imag(p1), imag(p2));
71     double miny34 = min(imag(p3), imag(p4)), maxy34 = max(imag(p3), imag(p4));
72     return (maxx12 >= minx34) && (maxx34 >= minx12) && (maxy12 >= miny34) && (maxy34 >= miny12);
73 }
74 //testet, ob ein Punkt im Polygon liegt (beliebige Polygone)
75 bool pointInPolygon(pt p, vector<pt> &polygon) { //jeder Eckpunkt nur einmal im Vektor
76     pt rayEnd = p + pt(1, 1000000);
77     int counter = 0, n = polygon.size();
78     for (int i = 0; i < n; i++) {
79         pt start = polygon[i], end = polygon[(i + 1) % n];
80         if (lineSegmentIntersection(p, rayEnd, start, end)) counter++;
81     }
82     return counter & 1;
83 }

```

4 Mathe

4.1 ggT, kgV, erweiterter euklidischer Algorithmus

```

1 ll gcd(ll a, ll b) {
2     return b == 0 ? a : gcd(b, a % b);
3 }
4
5 ll lcm(ll a, ll b) {
6     return a * (b / gcd(a, b)); //Klammern gegen Overflow
7 }
8
9 //Accepted in Aufgabe mit Forderung: |X|+|Y| minimal (primaer) und X<=Y (sekundaer)
10 //hab aber keinen Beweis dafuer :)
11 ll x, y, d; //a * x + b * y = d = ggT(a,b)
12 void extendedEuclid(ll a, ll b) {
13     if (!b) {
14         x = 1; y = 0; d = a; return;
15     }
16     extendedEuclid(b, a % b);
17     ll x1 = y; ll y1 = x - (a / b) * y;
18     x = x1; y = y1;
19 }

```

4.1.1 Multiplikatives Inverses von x in $\mathbb{Z}/n\mathbb{Z}$

Sei $0 \leq x < n$. Definiere $d := \gcd(x, n)$.

Falls $d = 1$:

- Erweiterter euklidischer Algorithmus liefert α und β mit $\alpha x + \beta n = 1$
- Nach Kongruenz gilt $\alpha x + \beta n \equiv \alpha x \equiv 1 \pmod{n}$
- $x^{-1} := \alpha \pmod{n}$

Falls $d \neq 1$: es existiert kein x^{-1}

```

1 ll multInv(ll n, ll p) { //berechnet das multiplikative Inverse von n in F_p
2     extendedEuclid(n, p); //implementierung von oben
3     x += ((x / p) + 1) * p;
4     return x % p;
5 }

```

4.1.2 Faktorisierung

```

1 #include <iostream>
2 #include <vector>
3
4 using namespace std;
5
6 typedef unsigned long long ll;
7
8 const ll PRIME_SIZE = 10000000;
9 vector<int> primes;
10
11 //Call before calculating anything
12 void primeSieve() {
13     vector<int> isPrime(PRIME_SIZE, true);
14     for(ll i = 2; i < PRIME_SIZE; i+=2) {
15         if(isPrime[i]) {
16             primes.push_back(i);
17             if(i*i <= PRIME_SIZE) {
18                 for(ll j = i; i*j < PRIME_SIZE; j+=2) isPrime[i*j] = false;
19             }
20         }
21         if(i == 2)
22             i--;
23     }
24 }
25
26 //Factorize the number n
27 vector<int> factorize(ll n) {
28     vector<int> factor;
29     ll num = n;
30     int pos = 0;
31     while(num != 1) {
32         if(num % primes[pos] == 0) {
33             num /= primes[pos];
34             factor.push_back(primes[pos]);
35         }
36         else
37             pos++;
38         if(primes[pos]*primes[pos] > num)
39             break;
40     }
41     if(num != 1)
42         factor.push_back(num);
43     return factor;
44 }
45 }

```

4.1.3 Mod-Exponent über \mathbb{F}_p

```

1 ll modPow(ll b, ll e, ll p) {
2     if (e == 0) return 1;
3     if (e == 1) return b;
4     ll half = modPow(b, e / 2, p), res = (half * half) % p;
5     if (e & 1) res *= b; res %= p;
6     return res;
7 }

```

4.2 LGS über \mathbb{F}_p

```

1 void normalLine(ll n, ll line, ll p) { //normalisiert Zeile line
2     ll factor = multInv(mat[line][line], p); //Implementierung von oben
3     for (ll i = 0; i <= n; i++) {

```

```

4     mat[line][i] *= factor;
5     mat[line][i] %= p;
6 }
7 }
8
9 void takeAll(ll n, ll line, ll p) { //zieht Vielfaches von line von allen anderen Zeilen ab
10    for (ll i = 0; i < n; i++) {
11        if (i == line) continue;
12        ll diff = mat[i][line]; //abziehen
13        for (ll j = 0; j <= n; j++) {
14            mat[i][j] -= (diff * mat[line][j]) % p;
15            while (mat[i][j] < 0) {
16                mat[i][j] += p;
17            }
18        }
19    }
20 }
21
22 void gauss(ll n, ll p) { //n x n+1-Matrix, Koerper F_p
23    for (ll line = 0; line < n; line++) {
24        normalLine(n, line, p);
25        takeAll(n, line, p);
26    }
27 }

```

4.3 Binomialkoeffizienten

```

1 ll calc_binom(ll N, ll K) {
2     ll r = 1, d;
3     if (K > N) return 0;
4     for (d = 1; d <= K; d++) {
5         r *= N--;
6         r /= d;
7     }
8     return r;
9 }

```

4.4 Primzahlsieb von Eratosthenes

```

1 #include <iostream>
2 #include <vector>
3
4 using namespace std;
5
6 typedef unsigned long long ll;
7
8 vector<int> primeSieve(ll n) {
9     vector<int> primes;
10    vector<int> isPrime(n, true);
11    for (ll i = 2; i < n; i+=2) {
12        if (isPrime[i]) {
13            primes.push_back(i);
14            if (i*i <= n) {
15                for (ll j = i; i*j < n; j+=2) isPrime[i*j] = false;
16            }
17        }
18        if (i == 2)
19            i--;
20    }
21    return primes;
22 }

```

4.5 Satz von SPRAGUE-GRUNDY

Weise jedem Zustand X wie folgt eine GRUNDY-Zahl $g(X)$ zu:

$$g(X) := \min\{\mathbb{Z}_0^+ \setminus \{g(Y) \mid Y \text{ von } X \text{ aus direkt erreichbar}\}\}$$

X ist genau dann gewonnen, wenn $g(X) > 0$ ist.

Wenn man k Spiele in den Zuständen X_1, \dots, X_k hat, dann ist die GRUNDY-Zahl des Gesamtzustandes $g(X_1) \oplus \dots \oplus g(X_k)$.

4.6 Maximales Teilfeld

```

1 //N := length of field
2 int maxStart = 1, maxLen = 0, curStart = 1, len = 0;
3 double maxValue = 0, sum = 0;
4 for (int pos = 0; pos < N; pos++) {
5     sum += values[pos];
6     len++;
7     if (sum > maxValue) { // neues Maximum
8         maxValue = sum; maxStart = curStart; maxLen = len;
9     }
10    if (sum < 0) { // alles zuruecksetzen
11        curStart = pos + 2; len = 0; sum = 0;
12    }
13 }
14 //maxSum := maximaler Wert, maxStart := Startposition, maxLen := Laenge der Sequenz

```

Obiger Code findet kein maximales Teilfeld, das über das Ende hinausgeht. Dazu:

1. finde maximales Teilfeld, das nicht übers Ende geht
2. berechne minimales Teilfeld, das nicht über den Rand geht (analog)
3. nimm Maximum aus gefundenem Maximalem und Allem\Minimalem

5 Strings

5.1 KNUTH-MORRIS-PRATT-Algorithmus

```

1 #include <iostream>
2 #include <vector>
3
4 using namespace std;
5
6 //Preprocessing Substring sub for KMP-Search
7 vector<int> kmp_preprocessing(string& sub) {
8     vector<int> b(sub.size() + 1);
9     b[0] = -1;
10    int i = 0, j = -1;
11    while(i < sub.size()) {
12        while(j >= 0 && sub[i] != sub[j])
13            j = b[j];
14        i++; j++;
15        b[i] = j;
16    }
17    return b;
18 }
19
20 //Searching after Substring sub in s
21 vector<int> kmp_search(string& s, string& sub) {
22     vector<int> pre = kmp_preprocessing(sub);
23     vector<int> result;
24     int i = 0, j = -1;
25     while(i < s.size()) {
26         while(j >= 0 && s[i] != sub[j])
27             j = pre[j];
28         i++; j++;
29         if(j == sub.size()) {
30             result.push_back(i-j);
31             j = pre[j];
32         }
33     }
34     return result;
35 }

```

5.2 Trie

```

1 //nur fuer kleinbuchstaben!
2 struct node {
3     node *(e)[26];
4     int c = 0; //anzahl der woerter die an dem node enden.

```

```

5   node() { for(int i = 0; i < 26; i++) e[i] = NULL; }
6   };
7
8   void insert(node *root, string *txt, int s) {
9       if(s >= txt->length()) root->c++;
10      else {
11          int idx = (int)((*txt).at(s) - 'a');
12          if(root->e[idx] == NULL) {
13              root->e[idx] = new node();
14          }
15          insert(root->e[idx], txt, s+1);
16      }
17  }
18
19  int contains(node *root, string *txt, int s) {
20      if(s >= txt->length()) return root->c;
21      int idx = (int)((*txt).at(s) - 'a');
22      if(root->e[idx] != NULL) {
23          return contains(root->e[idx], txt, s+1);
24      } else return 0;
25  }

```

5.3 Suffix-Array

```

1  //longest common substring in one string (overlapping not excluded)
2  //contains suffix array:-----
3  int cmp(string &s, vector<vector<int>> &v, int i, int vi, int u, int l) {
4      int vi2 = (vi + 1) % 2, u2 = u + i / 2, l2 = l + i / 2;
5      if(i == 1) return s[u] - s[l];
6      else if (v[vi2][u] != v[vi2][l]) return (v[vi2][u] - v[vi2][l]);
7      else { //beide groesser tiff nicht mehr ein, da ansonsten vorher schon unterschied in Laenge
8          if(u2 >= s.length()) return -1;
9          else if(l2 >= s.length()) return 1;
10         else return v[vi2][u2] - v[vi2][l2];
11     }
12 }
13
14 string lcsb(string s) {
15     if(s.length() == 0) return "";
16     vector<int> a(s.length());
17     vector<vector<int>> v(2, vector<int>(s.length(), 0));
18     int vi = 0;
19     for(int k = 0; k < a.size(); k++) a[k] = k;
20     for(int i = 1; i <= s.length(); i *= 2, vi = (vi + 1) % 2) {
21         sort(a.begin(), a.end(), [&] (const int &u, const int &l) {
22             return cmp(s, v, i, vi, u, l) < 0;
23         });
24         v[vi][a[0]] = 0;
25         for(int z = 1; z < a.size(); z++) v[vi][a[z]] = v[vi][a[z-1]] + (cmp(s, v, i, vi, a[z], a[z-1]) ==
26             0 ? 0 : 1);
27     }
28     //-----
29     int r = 0, m=0, c=0;
30     for(int i = 0; i < a.size() - 1; i++) {
31         c = 0;
32         while(a[i]+c < s.length() && a[i+1]+c < s.length() && s[a[i]+c] == s[a[i+1]+c]) c++;
33         if(c > m) r=i, m=c;
34     }
35     return m == 0 ? "" : s.substr(a[r], m);
36 }

```

5.4 Longest Common Substring

```

1  //longest common substring.
2  struct lcse {
3      int i = 0, s = 0;
4  };
5  string lcp(string s[2]) {
6      if(s[0].length() == 0 || s[1].length() == 0) return "";
7      vector<lcse> a(s[0].length()+s[1].length());

```

```

8   for(int k = 0; k < a.size(); k++) a[k].i=(k < s[0].length() ? k : k - s[0].length()), a[k].s = (k < s
    [0].length() ? 0 : 1);
9   sort(a.begin(), a.end(), [&] (const lcse &u, const lcse &l) {
10      int ui = u.i, li = l.i;
11      while(ui < s[u.s].length() && li < s[l.s].length()) {
12          if(s[u.s][ui] < s[l.s][li]) return true;
13          else if(s[u.s][ui] > s[l.s][li]) return false;
14          ui++; li++;
15      }
16      return !(ui < s[u.s].length());
17  });
18  int r = 0, m=0, c=0;
19  for(int i = 0; i < a.size() - 1; i++) {
20      if(a[i].s == a[i+1].s) continue;
21      c = 0;
22      while(a[i].i+c < s[a[i].s].length() && a[i+1].i+c < s[a[i+1].s].length() && s[a[i].s][a[i].i+c] ==
    s[a[i+1].s][a[i+1].i+c]) c++;
23      if(c > m) r=i, m=c;
24  }
25  return m == 0 ? "" : s[a[r].s].substr(a[r].i, m);
26 }

```

5.5 Longest Common Subsequence

```

1  string lcsub(string &a, string &b) {
2      int m[a.length() + 1][b.length() + 1], x=0, y=0;
3      memset(m, 0, sizeof(m));
4      for(int y = a.length() - 1; y >= 0; y--) {
5          for(int x = b.length() - 1; x >= 0; x--) {
6              if(a[y] == b[x]) m[y][x] = 1 + m[y+1][x+1];
7              else m[y][x] = max(m[y+1][x], m[y][x+1]);
8          }
9      } //for length only: return m[0][0];
10     string res;
11     while(x < b.length() && y < a.length()) {
12         if(a[y] == b[x]) res += a[y++], x++;
13         else if(m[y][x+1] > m[y+1][x+1]) x++;
14         else y++;
15     }
16     return res;
17 }

```

6 Sonstiges

6.1 2-SAT

1. Bedingungen in 2-CNF formulieren.
2. Implikationsgraph bauen, $(a \vee b)$ wird zu $\neg a \Rightarrow b$ und $\neg b \Rightarrow a$.
3. Finde die starken Zusammenhangskomponenten.
4. Genau dann lösbar, wenn keine Variable mit ihrer Negation in einer SCC liegt.