Exercises.

1 de junio de 2023

1. Exercise 1.2

Suppose that we use a perceptron to detect spam messages. Let's say that each email message is represented by the frequency of occurrence of keywords, and the output is +1 if the message is considered spam.

- Can you think of some keywords that will end up with a large positive weight in the perceptron?

  We can think of some words that occur repeatedly in spam mails, like free, winner, offert, prize, etc.

- How abaut keywords that will get a negative weight?

  Some examples are: deadline, meeting, date, etc.

- What parameter in the perceptron directly affects how many border-line messages end up being classified as spam?

  Since the bias moves the hyperplane keeping its orientation the same, the points near the hyperplane are greatly affected by this value.

2. Exercise 1.3

The weight update rule in (1.3) has the nice interpretation that it moves in the direction of classifying $x(t)$ correctly.

- Show that $y(t)\vec{w}^T(t)\vec{x}(t) < 0$.
  Since $\vec{x}(t)$ is classified in the wrong class, the sign of $\vec{w}^T(t)\vec{x}(t)$ is the opposite to the sign of $y(t)$, therefore, $y(t)\vec{w}^T(t)\vec{x}(t) < 0$ in every case.

- Show that $y(t)\vec{w}^T(t+1)\vec{x}(t) > y(t)\vec{w}^T(t)\vec{x}(t)$.
  Since $\vec{w}(t+1) = \vec{w}(t) + y(t)\vec{x}(t)$, we have

  $$y(t)\vec{w}^T(t+1)\vec{x}(t) = y(t)(\vec{w}(t) + y(t)\vec{x}(t))\vec{x}(t) = y(t)\vec{w}^T(t)\vec{x}(t) + y^2(t)||\vec{x}(t)||^2 > y(t)\vec{w}^T(t)\vec{x}(t)$$

- As far as classifying $x(t)$ is concerned, argue that the move from $\vec{w}(t)$ to $\vec{w}(t+1)$ is a move ïn the right direction".

  Since $y(t)\vec{w}^T\vec{x}(t) < 0$ and $y(t)\vec{w}(t+1)\vec{x}(t)$, then the next step $\vec{w}(t+1) = \vec{w}(t) + y(t)\vec{x}(t)$ increases until it surpasses 0, in which case $\{x(t), y(t)\}$ won't be wrong classified.