Capa L

$$\frac{\partial C}{\partial w_{11}^2} = \frac{\partial z_1^2}{\partial w_{11}^2} * \frac{\partial a_1^2}{\partial z_1^2} * \frac{\partial C}{\partial a_1^2}$$

$$\frac{\partial C}{\partial a_1^2} = 2 * (a_1^2 - y_1)$$

$$\frac{\partial a_1^2}{\partial z_1^2} = sig(z_1^2) * \left(1 - sig(z_1^2)\right) = sig'(z_1^2)$$

$$\frac{\partial z_1^2}{\partial w_{11}^2} = a_1^1$$

$$\frac{\partial C}{\partial w_{11}^2} = \frac{\partial z_1^2}{\partial w_{11}^2} * \frac{\partial a_1^2}{\partial z_1^2} * \frac{\partial C}{\partial a_1^2} = a_1^1 * sig'(z_1^2) * 2 * (a_1^2 - y_1)$$

Pesos en general L

$$\frac{\partial C}{\partial w_{11}^2} = \frac{\partial z_1^2}{\partial w_{11}^2} * \frac{\partial a_1^2}{\partial z_1^2} * \frac{\partial C}{\partial a_1^2} = a_1^1 * sig'(z_1^2) * 2 * (a_1^2 - y_1)$$

$$\frac{\partial C}{\partial w_{12}^2} = \frac{\partial z_1^2}{\partial w_{12}^2} * \frac{\partial a_1^2}{\partial z_1^2} * \frac{\partial C}{\partial a_1^2} = a_2^1 * sig'(z_1^2) * 2 * (a_1^2 - y_1)$$

$$\frac{\partial C}{\partial w_{13}^2} = \frac{\partial z_1^2}{\partial w_{13}^2} * \frac{\partial a_1^2}{\partial z_1^2} * \frac{\partial C}{\partial a_1^2} = a_3^1 * sig'(z_1^2) * 2 * (a_1^2 - y_1)$$

$$\frac{\partial a_1^2}{\partial z_1^2} * \frac{\partial C}{\partial a_1^2} = sig'(z_1^2) * 2 * (a_1^2 - y_1) = \delta_1^2$$

$$\frac{\partial C}{\partial w_{21}^2} = \frac{\partial z_1^2}{\partial w_{21}^2} * \frac{\partial a_2^2}{\partial z_2^2} * \frac{\partial C}{\partial a_2^2} = a_1^1 * sig'(z_2^2) * 2 * (a_2^2 - y_2)$$

$$\frac{\partial C}{\partial w_{22}^2} = \frac{\partial z_1^2}{\partial w_{22}^2} * \frac{\partial a_2^2}{\partial z_2^2} * \frac{\partial C}{\partial a_2^2} = a_2^1 * sig'(z_2^2) * 2 * (a_2^2 - y_2)$$

$$\frac{\partial C}{\partial w_{23}^2} = \frac{\partial z_1^2}{\partial w_{23}^2} * \frac{\partial a_2^2}{\partial z_2^2} * \frac{\partial C}{\partial a_2^2} = a_3^1 * sig'(z_2^2) * 2 * (a_2^2 - y_2)$$

$$\frac{\partial a_2^2}{\partial z_2^2} * \frac{\partial C}{\partial a_2^2} = sig'(z_2^2) * 2 * (a_2^2 - y_2) = \delta_2^2$$

$$w_{2,3}^2 = \begin{bmatrix} \frac{\partial C}{\partial w_{11}^2} & \frac{\partial C}{\partial w_{12}^2} & \frac{\partial C}{\partial w_{13}^2} \\ \frac{\partial C}{\partial w_{21}^2} & \frac{\partial C}{\partial w_{22}^2} & \frac{\partial C}{\partial w_{23}^2} \end{bmatrix} = \begin{bmatrix} a_1^1 * \delta_1^2 & a_2^1 * \delta_1^2 & a_3^1 * \delta_1^2 \\ a_1^1 * \delta_2^2 & a_2^1 * \delta_2^2 & a_3^1 * \delta_2^2 \end{bmatrix} = \begin{bmatrix} \delta_1^2 \\ \delta_2^2 \end{bmatrix} * \begin{bmatrix} a_1^1 & a_2^1 & a_3^1 \end{bmatrix} = [\delta^2] * [a^1]$$

Sesgo en general L

$$\frac{\partial C}{\partial b_1^2} = \frac{\partial z_1^2}{\partial b_1^2} * \frac{\partial a_1^2}{\partial z_1^2} * \frac{\partial C}{\partial a_1^2} = 1 * sig'(z_1^2) * 2 * (a_1^2 - y_1)$$

$$\frac{\partial C}{\partial b_2^2} = \frac{\partial z_1^2}{\partial b_2^2} * \frac{\partial a_2^2}{\partial z_2^2} * \frac{\partial C}{\partial a_2^2} = 1 * sig'(z_2^2) * 2 * (a_2^2 - y_2)$$

$$b_{2,1}^2 = \begin{bmatrix} \frac{\partial C}{\partial b_1^2} \\ \frac{\partial C}{\partial b_2^2} \end{bmatrix} = \begin{bmatrix} sig'(z_1^2) * 2 * (a_1^2 - y_1) \\ sig'(z_2^2) * 2 * (a_2^2 - y_2) \end{bmatrix} = \begin{bmatrix} \delta_1^2 \\ \delta_2^2 \end{bmatrix} = [\delta^2]$$

Capa L-1

Pesos en general L-1

$$\frac{\partial C}{\partial w_{11}^{1}} = \boldsymbol{\frac{\partial z_1^1}{\partial w_{11}^1}} * \boldsymbol{\frac{\partial a_1^1}{\partial z_1^1}} * \frac{\partial C}{\partial a_1^1}$$

$$\frac{\partial C}{\partial a_1^1} = \frac{\partial C_1}{\partial a_1^1} + \frac{\partial C_2}{\partial a_1^1}$$

Debido que la activación de la neurona afecta a ambas salidas para este caso, se debe sumar esa diferencia para cada una

$$\frac{\partial C}{\partial a_1^1} = \boldsymbol{\frac{\partial z_1^2}{\partial a_1^1}} * \boldsymbol{\frac{\partial a_1^2}{\partial z_1^2}} * \boldsymbol{\frac{\partial C}{\partial a_1^2}} + \boldsymbol{\frac{\partial z_2^2}{\partial a_1^1}} * \boldsymbol{\frac{\partial a_2^2}{\partial z_2^2}} * \boldsymbol{\frac{\partial C}{\partial a_2^2}}$$

Lo de color negrito ya se obtuvo anteriormente

$$\frac{\partial C}{\partial w_{13}^{2}} = \frac{\partial z_1^2}{\partial w_{13}^2} * \frac{\partial a_1^2}{\partial z_1^2} * \frac{\partial C}{\partial a_1^2} = a_3^1 * sig'(z_1^2) * 2 * (a_1^2 - y_1)$$

$$\frac{\partial C}{\partial w_{21}^{2}} = \frac{\partial z_1^2}{\partial w_{21}^2} * \frac{\partial a_2^2}{\partial z_2^2} * \frac{\partial C}{\partial a_2^2} = a_1^1 * sig'(z_2^2) * 2 * (a_2^2 - y_2)$$

$$\boldsymbol{\frac{\partial a_1^2}{\partial z_1^2}} * \boldsymbol{\frac{\partial C}{\partial a_1^2}} = sig'(z_1^2) * 2 * (a_1^2 - y_1)$$

$$\boldsymbol{\frac{\partial a_2^2}{\partial z_2^2}} * \boldsymbol{\frac{\partial C}{\partial a_2^2}} = sig'(z_2^2) * 2 * (a_2^2 - y_2)$$

Lo nuevo es esto

$$\boldsymbol{\frac{\partial z_1^2}{\partial a_1^1}} = w_{11}^2$$

$$\boldsymbol{\frac{\partial z_2^2}{\partial a_1^1}} = w_{21}^2$$

Y para

$$\frac{\partial C}{\partial w_{11}^{1}} = \boldsymbol{\frac{\partial z_1^1}{\partial w_{11}^1}} * \boldsymbol{\frac{\partial a_1^1}{\partial z_1^1}} * \frac{\partial C}{\partial a_1^1}$$

$$\frac{\partial C}{\partial w_{11}^1}=\frac{\partial \boldsymbol{z_1^1}}{\partial \boldsymbol{w_{11}^1}}*\frac{\partial \boldsymbol{a_1^1}}{\partial \boldsymbol{z_1^1}}*\left(\frac{\partial z_1^2}{\partial a_1^1}*\frac{\partial \boldsymbol{a_1^2}}{\partial \boldsymbol{z_1^2}}*\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{a_1^2}}+\frac{\partial z_2^2}{\partial a_1^1}*\frac{\partial \boldsymbol{a_2^2}}{\partial \boldsymbol{z_2^2}}*\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{a_2^2}}\right)$$

$$\frac{\partial C}{\partial w_{11}^1}=\frac{\partial \boldsymbol{z_1^1}}{\partial \boldsymbol{w_{11}^1}}*\frac{\partial \boldsymbol{a_1^1}}{\partial \boldsymbol{z_1^1}}*\left(\boldsymbol{w_{11}^2}*\frac{\partial \boldsymbol{a_1^2}}{\partial \boldsymbol{z_1^2}}*\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{a_1^2}}+\boldsymbol{w_{21}^2}*\frac{\partial \boldsymbol{a_2^2}}{\partial \boldsymbol{z_2^2}}*\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{a_2^2}}\right)$$

Lo de color azul ya se calculo antes solo cambian los parámetros, pero la fórmula es la misma

$$\frac{\partial a_1^1}{\partial z_1^1}=sig(z_1^1)*\left(1-sig(z_1^1)\right)=sig'(z_1^1)$$

$$\frac{\partial z_1^1}{\partial w_{11}^1}=a_1^0$$

$$\frac{\partial C}{\partial w_{11}^1}=a_1^0*sig'(z_1^1)*\left(\boldsymbol{w_{11}^2}*\frac{\partial \boldsymbol{a_1^2}}{\partial \boldsymbol{z_1^2}}*\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{a_1^2}}+\boldsymbol{w_{21}^2}*\frac{\partial \boldsymbol{a_2^2}}{\partial \boldsymbol{z_2^2}}*\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{a_2^2}}\right)$$

$$\frac{\partial a_1^2}{\partial z_1^2}*\frac{\partial C}{\partial a_1^2}=sig'(z_1^2)*2*(a_1^2-y_1)=\delta_1^2$$

$$\frac{\partial a_2^2}{\partial z_2^2}*\frac{\partial C}{\partial a_2^2}=sig'(z_2^2)*2*(a_2^2-y_2)=\delta_2^2$$

$$\frac{\partial C}{\partial w_{11}^1}=a_1^0*sig'(z_1^1)*(\boldsymbol{w_{11}^2}*\boldsymbol{\delta_1^2}+\boldsymbol{w_{21}^2}*\boldsymbol{\delta_2^2})$$

$$\frac{\partial C}{\partial w_{11}^1}=a_1^0*sig'(z_1^1)*\left(\begin{bmatrix}\boldsymbol{w_{11}^2} & \boldsymbol{w_{21}^2}\end{bmatrix}*\begin{bmatrix}\boldsymbol{\delta_1^2}\\ \boldsymbol{\delta_2^2}\end{bmatrix}\right)$$

$$\frac{\partial C}{\partial w_{11}^1}=a_1^0*sig'(z_1^1)*\left(\begin{bmatrix}\boldsymbol{w_{11}^2} & \boldsymbol{w_{21}^2}\end{bmatrix}*\begin{bmatrix}\boldsymbol{\delta^2}\end{bmatrix}\right)$$

Para todos los demás

$$\frac{\partial C}{\partial w_{11}^1}=a_1^0*sig'(z_1^1)*\left(\begin{bmatrix}\boldsymbol{w_{11}^2} & \boldsymbol{w_{21}^2}\end{bmatrix}*\begin{bmatrix}\boldsymbol{\delta^2}\end{bmatrix}\right)$$

$$\frac{\partial C}{\partial w_{12}^1}=a_2^0*sig'(z_1^1)*\left(\begin{bmatrix}\boldsymbol{w_{11}^2} & \boldsymbol{w_{21}^2}\end{bmatrix}*\begin{bmatrix}\boldsymbol{\delta^2}\end{bmatrix}\right)$$

$$\frac{\partial C}{\partial w_{13}^1} = a_3^0 * sig'(z_1^1) * \left(\begin{bmatrix} \boldsymbol{w_{11}^2} & \boldsymbol{w_{21}^2} \end{bmatrix} * \begin{bmatrix} \boldsymbol{\delta^2} \end{bmatrix}\right)$$

$$\frac{\partial C}{\partial w_{14}^1} = a_4^0 * sig'(z_1^1) * \left(\begin{bmatrix} \boldsymbol{w_{11}^2} & \boldsymbol{w_{21}^2} \end{bmatrix} * \begin{bmatrix} \boldsymbol{\delta^2} \end{bmatrix}\right)$$

$$\frac{\partial C}{\partial w_{21}^1} = a_1^0 * sig'(z_2^1) * \left(\begin{bmatrix} \boldsymbol{w_{12}^2} & \boldsymbol{w_{22}^2} \end{bmatrix} * \begin{bmatrix} \boldsymbol{\delta^2} \end{bmatrix}\right)$$

$$\frac{\partial C}{\partial w_{22}^1} = a_2^0 * sig'(z_2^1) * \left(\begin{bmatrix} \boldsymbol{w_{12}^2} & \boldsymbol{w_{22}^2} \end{bmatrix} * \begin{bmatrix} \boldsymbol{\delta^2} \end{bmatrix}\right)$$

$$\frac{\partial C}{\partial w_{23}^1} = a_3^0 * sig'(z_2^1) * \left(\begin{bmatrix} \boldsymbol{w_{12}^2} & \boldsymbol{w_{22}^2} \end{bmatrix} * \begin{bmatrix} \boldsymbol{\delta^2} \end{bmatrix}\right)$$

$$\frac{\partial C}{\partial w_{24}^1} = a_4^0 * sig'(z_2^1) * \left(\begin{bmatrix} \boldsymbol{w_{12}^2} & \boldsymbol{w_{22}^2} \end{bmatrix} * \begin{bmatrix} \boldsymbol{\delta^2} \end{bmatrix}\right)$$

$$\frac{\partial C}{\partial w_{31}^1} = a_1^0 * sig'(z_3^1) * \left(\begin{bmatrix} \boldsymbol{w_{13}^2} & \boldsymbol{w_{23}^2} \end{bmatrix} * \begin{bmatrix} \boldsymbol{\delta^2} \end{bmatrix}\right)$$

$$\frac{\partial C}{\partial w_{32}^1} = a_2^0 * sig'(z_3^1) * \left(\begin{bmatrix} \boldsymbol{w_{13}^2} & \boldsymbol{w_{23}^2} \end{bmatrix} * \begin{bmatrix} \boldsymbol{\delta^2} \end{bmatrix}\right)$$

$$\frac{\partial C}{\partial w_{33}^1} = a_3^0 * sig'(z_3^1) * \left(\begin{bmatrix} \boldsymbol{w_{13}^2} & \boldsymbol{w_{23}^2} \end{bmatrix} * \begin{bmatrix} \boldsymbol{\delta^2} \end{bmatrix}\right)$$

$$\frac{\partial C}{\partial w_{34}^1} = a_4^0 * sig'(z_3^1) * \left(\begin{bmatrix} \boldsymbol{w_{13}^2} & \boldsymbol{w_{23}^2} \end{bmatrix} * \begin{bmatrix} \boldsymbol{\delta^2} \end{bmatrix}\right)$$

Sustituyendo los vectores de pesos

$$\begin{bmatrix} \boldsymbol{w_{11}^2} & \boldsymbol{w_{21}^2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{W_1^2} \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{w_{12}^2} & \boldsymbol{w_{22}^2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{W_2^2} \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{w_{13}^2} & \boldsymbol{w_{23}^2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{W_3^2} \end{bmatrix}$$

$$\frac{\partial C}{\partial w_{11}^{1}} = a_1^0 * sig'(z_1^1) * [\boldsymbol{W_1^2}] * [\boldsymbol{\delta^2}]$$

$$\frac{\partial C}{\partial w_{12}^{1}} = a_2^0 * sig'(z_1^1) * [\boldsymbol{W_1^2}] * [\boldsymbol{\delta^2}]$$

$$\frac{\partial C}{\partial w_{13}^{1}} = a_3^0 * sig'(z_1^1) * [\boldsymbol{W_1^2}] * [\boldsymbol{\delta^2}]$$

$$\frac{\partial C}{\partial w_{14}^{1}} = a_4^0 * sig'(z_1^1) * [\boldsymbol{W_1^2}] * [\boldsymbol{\delta^2}]$$

$$\delta_1^1 = sig'(z_1^1) * [\boldsymbol{W_1^2}] * [\boldsymbol{\delta^2}]$$

$$\frac{\partial C}{\partial w_{21}^{1}} = a_1^0 * sig'(z_2^1) * [\boldsymbol{W_2^2}] * [\boldsymbol{\delta^2}]$$

$$\frac{\partial C}{\partial w_{22}^{1}} = a_2^0 * sig'(z_2^1) * [\boldsymbol{W_2^2}] * [\boldsymbol{\delta^2}]$$

$$\frac{\partial C}{\partial w_{23}^{1}} = a_3^0 * sig'(z_2^1) * [\boldsymbol{W_2^2}] * [\boldsymbol{\delta^2}]$$

$$\frac{\partial C}{\partial w_{24}^{1}} = a_4^0 * sig'(z_2^1) * [\boldsymbol{W_2^2}] * [\boldsymbol{\delta^2}]$$

$$\delta_2^1 = sig'(z_2^1) * [\boldsymbol{W_2^2}] * [\boldsymbol{\delta^2}]$$

$$\frac{\partial C}{\partial w_{31}^{1}} = a_1^0 * sig'(z_3^1) * [\boldsymbol{W_3^2}] * [\boldsymbol{\delta^2}]$$

$$\frac{\partial C}{\partial w_{32}^{1}} = a_2^0 * sig'(z_3^1) * [\boldsymbol{W_3^2}] * [\boldsymbol{\delta^2}]$$

$$\frac{\partial C}{\partial w_{33}^{1}} = a_3^0 * sig'(z_3^1) * [\boldsymbol{W_3^2}] * [\boldsymbol{\delta^2}]$$

$$\frac{\partial C}{\partial w_{34}^{1}} = a_4^0 * sig'(z_3^1) * [\boldsymbol{W_3^2}] * [\boldsymbol{\delta^2}]$$

$$\delta_3^1 = sig'(z_3^1) * [\boldsymbol{W_3^2}] * [\boldsymbol{\delta^2}]$$

Sustituyendo de nuevo

$$\frac{\partial C}{\partial w_{11}^1} = a_1^0 * \delta_1^1$$

$$\frac{\partial C}{\partial w_{12}^1} = a_2^0 * \delta_1^1$$

$$\frac{\partial C}{\partial w_{13}^1} = a_3^0 * \delta_1^1$$

$$\frac{\partial C}{\partial w_{14}^1} = a_4^0 * \delta_1^1$$

$$\delta_1^1 = sig'(z_1^1) * [\boldsymbol{W_1^2}] * [\boldsymbol{\delta^2}]$$

$$\frac{\partial C}{\partial w_{21}^1} = a_1^0 * \delta_2^1$$

$$\frac{\partial C}{\partial w_{22}^1} = a_2^0 * \delta_2^1$$

$$\frac{\partial C}{\partial w_{23}^1} = a_3^0 * \delta_2^1$$

$$\frac{\partial C}{\partial w_{24}^1} = a_4^0 * \delta_2^1$$

$$\delta_2^1 = sig'(z_2^1) * [\boldsymbol{W_2^2}] * [\boldsymbol{\delta^2}]$$

$$\frac{\partial C}{\partial w_{31}^1} = a_1^0 * \delta_3^1$$

$$\frac{\partial C}{\partial w_{32}^1} = a_2^0 * \delta_3^1$$

$$\frac{\partial C}{\partial w_{33}^{1}} = a_3^0 * \delta_3^1$$

$$\frac{\partial C}{\partial w_{34}^{1}} = a_4^0 * \delta_3^1$$

$$\delta_3^1 = sig'(z_3^1) * [\boldsymbol{W_3^2}] * [\boldsymbol{\delta^2}]$$

$$w_{3,4}^1 = \begin{bmatrix} \frac{\partial C}{\partial w_{11}^1} & \frac{\partial C}{\partial w_{12}^1} & \frac{\partial C}{\partial w_{13}^1} & \frac{\partial C}{\partial w_{14}^1} \\ \frac{\partial C}{\partial w_{21}^1} & \frac{\partial C}{\partial w_{22}^1} & \frac{\partial C}{\partial w_{23}^1} & \frac{\partial C}{\partial w_{24}^1} \\ \frac{\partial C}{\partial w_{31}^1} & \frac{\partial C}{\partial w_{32}^1} & \frac{\partial C}{\partial w_{33}^1} & \frac{\partial C}{\partial w_{34}^1} \end{bmatrix} = \begin{bmatrix} a_1^0 * \delta_1^1 & a_2^0 * \delta_1^1 & a_3^0 * \delta_1^1 & a_4^0 * \delta_1^1 \\ a_1^0 * \delta_2^1 & a_2^0 * \delta_2^1 & a_3^0 * \delta_2^1 & a_4^0 * \delta_2^1 \\ a_1^0 * \delta_3^1 & a_2^0 * \delta_3^1 & a_3^0 * \delta_3^1 & a_4^0 * \delta_3^1 \end{bmatrix}$$

$$= \begin{bmatrix} \delta_1^1 \\ \delta_2^1 \\ \delta_3^1 \end{bmatrix} * \begin{bmatrix} a_1^0 & a_2^0 & a_3^0 & a_4^0 \end{bmatrix} = [\boldsymbol{\delta^1}] * [\boldsymbol{a^0}]$$

Sesgo en general L-1

$$\frac{\partial C}{\partial b_1^1} = \frac{\boldsymbol{\partial z_1^1}}{\boldsymbol{\partial b_1^1}} * \frac{\boldsymbol{\partial a_1^1}}{\boldsymbol{\partial z_1^1}} * \frac{\partial C}{\partial a_1^1}$$

$$\frac{\partial C}{\partial b_1^1} = \frac{\boldsymbol{\partial z_1^1}}{\boldsymbol{\partial b_1^1}} * \frac{\boldsymbol{\partial a_1^1}}{\boldsymbol{\partial z_1^1}} * \left(\frac{\partial z_1^2}{\partial a_1^1} * \frac{\boldsymbol{\partial a_1^2}}{\boldsymbol{\partial z_1^2}} * \frac{\boldsymbol{\partial C}}{\boldsymbol{\partial a_1^2}} + \frac{\partial z_2^2}{\partial a_1^1} * \frac{\boldsymbol{\partial a_2^2}}{\boldsymbol{\partial z_2^2}} * \frac{\boldsymbol{\partial C}}{\boldsymbol{\partial a_2^2}} \right)$$

$$\frac{\partial C}{\partial b_1^1} = \frac{\boldsymbol{\partial z_1^1}}{\boldsymbol{\partial b_1^1}} * \frac{\boldsymbol{\partial a_1^1}}{\boldsymbol{\partial z_1^1}} * \left(\boldsymbol{w_{11}^2} * \frac{\boldsymbol{\partial a_1^2}}{\boldsymbol{\partial z_1^2}} * \frac{\boldsymbol{\partial C}}{\boldsymbol{\partial a_1^2}} + \boldsymbol{w_{21}^2} * \frac{\boldsymbol{\partial a_2^2}}{\boldsymbol{\partial z_2^2}} * \frac{\boldsymbol{\partial C}}{\boldsymbol{\partial a_2^2}} \right)$$

$$\frac{\boldsymbol{\partial z_1^1}}{\boldsymbol{\partial b_1^1}} = \boldsymbol{1}$$

$$\delta_1^1 = sig'(z_1^1) * [\boldsymbol{W_1^2}] * [\boldsymbol{\delta^2}]$$

$$\frac{\partial C}{\partial b_1^1} = \delta_1^1$$

$$\frac{\partial C}{\partial b_2^1} = \delta_2^1$$

$$\frac{\partial C}{\partial b_3^1} = \delta_3^1$$

$$b_{3,1}^1 = \begin{bmatrix} \delta_1^1 \\ \delta_2^1 \\ \delta_3^1 \end{bmatrix} = [\boldsymbol{\delta}^{\mathbf{1}}]$$

Ingeniería inversa de la matriz de pesos L-1

$$[\boldsymbol{\delta}^1] * [\boldsymbol{a}^0] = \begin{bmatrix} \delta_1^1 \\ \delta_2^1 \\ \delta_3^1 \end{bmatrix} * \begin{bmatrix} a_1^0 & a_2^0 & a_3^0 & a_4^0 \end{bmatrix} = \begin{bmatrix} \delta_1^1 \\ \delta_2^1 \\ \delta_3^1 \end{bmatrix} * \begin{bmatrix} a_1^0 & a_2^0 & a_3^0 & a_4^0 \end{bmatrix}$$

$$\delta_1^1 = sig'(z_1^1) * [\boldsymbol{W}_1^2] * [\boldsymbol{\delta}^2]$$

$$\delta_2^1 = sig'(z_2^1) * [\boldsymbol{W}_2^2] * [\boldsymbol{\delta}^2]$$

$$\delta_3^1 = sig'(z_3^1) * [\boldsymbol{W}_3^2] * [\boldsymbol{\delta}^2]$$

$$\begin{bmatrix} \delta_1^1 \\ \delta_2^1 \\ \delta_3^1 \end{bmatrix} * \begin{bmatrix} a_1^0 & a_2^0 & a_3^0 & a_4^0 \end{bmatrix} = \begin{bmatrix} sig'(z_1^1) * [\boldsymbol{W}_1^2] * [\boldsymbol{\delta}^2] \\ sig'(z_2^1) * [\boldsymbol{W}_2^2] * [\boldsymbol{\delta}^2] \\ sig'(z_3^1) * [\boldsymbol{W}_3^2] * [\boldsymbol{\delta}^2] \end{bmatrix} * \begin{bmatrix} a_1^0 & a_2^0 & a_3^0 & a_4^0 \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{w}_{11}^2 & \boldsymbol{w}_{21}^2 \end{bmatrix} = [\boldsymbol{W}_1^2]$$

$$\begin{bmatrix} \boldsymbol{w}_{12}^2 & \boldsymbol{w}_{22}^2 \end{bmatrix} = [\boldsymbol{W}_2^2]$$

$$\begin{bmatrix} \boldsymbol{w}_{13}^2 & \boldsymbol{w}_{23}^2 \end{bmatrix} = [\boldsymbol{W}_3^2]$$

$$\begin{bmatrix} sig'(z_1^1) * [\boldsymbol{W}_1^2] * [\boldsymbol{\delta}^2] \\ sig'(z_2^1) * [\boldsymbol{W}_2^2] * [\boldsymbol{\delta}^2] \\ sig'(z_3^1) * [\boldsymbol{W}_3^2] * [\boldsymbol{\delta}^2] \end{bmatrix} * \begin{bmatrix} a_1^0 & a_2^0 & a_3^0 & a_4^0 \end{bmatrix} = \begin{bmatrix} sig'(z_1^1) * \begin{bmatrix} \boldsymbol{w}_{11}^2 & \boldsymbol{w}_{21}^2 \end{bmatrix} * [\boldsymbol{\delta}^2] \\ sig'(z_2^1) * \begin{bmatrix} \boldsymbol{w}_{12}^2 & \boldsymbol{w}_{22}^2 \end{bmatrix} * [\boldsymbol{\delta}^2] \\ sig'(z_3^1) * \begin{bmatrix} \boldsymbol{w}_{13}^2 & \boldsymbol{w}_{23}^2 \end{bmatrix} * [\boldsymbol{\delta}^2] \end{bmatrix} * \begin{bmatrix} a_1^0 & a_2^0 & a_3^0 & a_4^0 \end{bmatrix}$$

$$[\boldsymbol{\delta}^2] = \begin{bmatrix} \boldsymbol{\delta}_1^2 \\ \boldsymbol{\delta}_2^2 \end{bmatrix}$$

$$\begin{bmatrix} sig'(z_1^1) * \begin{bmatrix} \boldsymbol{w}_{11}^2 & \boldsymbol{w}_{21}^2 \end{bmatrix} * \begin{bmatrix} \boldsymbol{\delta}_1^2 \\ \boldsymbol{\delta}_2^2 \end{bmatrix} \\ sig'(z_2^1) * \begin{bmatrix} \boldsymbol{w}_{12}^2 & \boldsymbol{w}_{22}^2 \end{bmatrix} * \begin{bmatrix} \boldsymbol{\delta}_1^2 \\ \boldsymbol{\delta}_2^2 \end{bmatrix} \\ sig'(z_3^1) * \begin{bmatrix} \boldsymbol{w}_{13}^2 & \boldsymbol{w}_{23}^2 \end{bmatrix} * \begin{bmatrix} \boldsymbol{\delta}_1^2 \\ \boldsymbol{\delta}_2^2 \end{bmatrix} \end{bmatrix} * \begin{bmatrix} a_1^0 & a_2^0 & a_3^0 & a_4^0 \end{bmatrix} = \begin{bmatrix} sig'(z_1^1) \\ sig'(z_2^1) \\ sig'(z_3^1) \end{bmatrix} \cdot \begin{bmatrix} \begin{bmatrix} \boldsymbol{w}_{11}^2 & \boldsymbol{w}_{21}^2 \end{bmatrix} * \begin{bmatrix} \boldsymbol{\delta}_1^2 \\ \boldsymbol{\delta}_2^2 \end{bmatrix} \\ \begin{bmatrix} \boldsymbol{w}_{12}^2 & \boldsymbol{w}_{22}^2 \end{bmatrix} * \begin{bmatrix} \boldsymbol{\delta}_1^2 \\ \boldsymbol{\delta}_2^2 \end{bmatrix} \\ \begin{bmatrix} \boldsymbol{w}_{13}^2 & \boldsymbol{w}_{23}^2 \end{bmatrix} * \begin{bmatrix} \boldsymbol{\delta}_1^2 \\ \boldsymbol{\delta}_2^2 \end{bmatrix} \end{bmatrix} * \begin{bmatrix} a_1^0 & a_2^0 & a_3^0 & a_4^0 \end{bmatrix}$$

Analizando la parte de los pesos y las diferenciales

$$w_{2,3}^2 = \begin{bmatrix} \frac{\partial C}{\partial w_{11}^2} & \frac{\partial C}{\partial w_{12}^2} & \frac{\partial C}{\partial w_{13}^2} \\ \frac{\partial C}{\partial w_{21}^2} & \frac{\partial C}{\partial w_{22}^2} & \frac{\partial C}{\partial w_{23}^2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{w_{11}^2} & \boldsymbol{w_{12}^2} & \boldsymbol{w_{13}^2} \\ \boldsymbol{w_{21}^2} & \boldsymbol{w_{22}^2} & \boldsymbol{w_{23}^2} \end{bmatrix}$$

$$Trans(w_{2,3}^2) = w_{3,2}^{2T} = \begin{bmatrix} \boldsymbol{w_{11}^2} & \boldsymbol{w_{21}^2} \\ \boldsymbol{w_{12}^2} & \boldsymbol{w_{22}^2} \\ \boldsymbol{w_{13}^2} & \boldsymbol{w_{23}^2} \end{bmatrix}$$

$$\begin{bmatrix} [\boldsymbol{w_{11}^2} \quad \boldsymbol{w_{21}^2}] * \begin{bmatrix} \boldsymbol{\delta_1^2} \\ \boldsymbol{\delta_2^2} \end{bmatrix} \\ [\boldsymbol{w_{12}^2} \quad \boldsymbol{w_{22}^2}] * \begin{bmatrix} \boldsymbol{\delta_1^2} \\ \boldsymbol{\delta_2^2} \end{bmatrix} \\ [\boldsymbol{w_{13}^2} \quad \boldsymbol{w_{23}^2}] * \begin{bmatrix} \boldsymbol{\delta_1^2} \\ \boldsymbol{\delta_2^2} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \boldsymbol{w_{11}^2} & \boldsymbol{w_{21}^2} \\ \boldsymbol{w_{12}^2} & \boldsymbol{w_{22}^2} \\ \boldsymbol{w_{13}^2} & \boldsymbol{w_{23}^2} \end{bmatrix} * \begin{bmatrix} \boldsymbol{\delta_1^2} \\ \boldsymbol{\delta_2^2} \end{bmatrix}$$

Sustituyendo tenemos

$$\begin{bmatrix} sig'(z_1^1) \\ sig'(z_2^1) \\ sig'(z_3^1) \end{bmatrix} \cdot \begin{bmatrix} [\boldsymbol{w_{11}^2} \quad \boldsymbol{w_{21}^2}] * \begin{bmatrix} \boldsymbol{\delta_1^2} \\ \boldsymbol{\delta_2^2} \end{bmatrix} \\ [\boldsymbol{w_{12}^2} \quad \boldsymbol{w_{22}^2}] * \begin{bmatrix} \boldsymbol{\delta_1^2} \\ \boldsymbol{\delta_2^2} \end{bmatrix} \\ [\boldsymbol{w_{13}^2} \quad \boldsymbol{w_{23}^2}] * \begin{bmatrix} \boldsymbol{\delta_1^2} \\ \boldsymbol{\delta_2^2} \end{bmatrix} \end{bmatrix} * [a_1^0 \quad a_2^0 \quad a_3^0 \quad a_4^0] = \begin{bmatrix} sig'(z_1^1) \\ sig'(z_2^1) \\ sig'(z_3^1) \end{bmatrix} \cdot \left(\begin{bmatrix} \boldsymbol{w_{11}^2} & \boldsymbol{w_{21}^2} \\ \boldsymbol{w_{12}^2} & \boldsymbol{w_{22}^2} \\ \boldsymbol{w_{13}^2} & \boldsymbol{w_{23}^2} \end{bmatrix} * \begin{bmatrix} \boldsymbol{\delta_1^2} \\ \boldsymbol{\delta_2^2} \end{bmatrix} \right) * [a_1^0 \quad a_2^0 \quad a_3^0 \quad a_4^0]$$

$$[\boldsymbol{\delta^1}] * [\boldsymbol{a^0}] = \begin{bmatrix} sig'(z_1^1) \\ sig'(z_2^1) \\ sig'(z_3^1) \end{bmatrix} \cdot \begin{bmatrix} \begin{bmatrix} \boldsymbol{w_{11}^2} & \boldsymbol{w_{21}^2} \\ \boldsymbol{w_{12}^2} & \boldsymbol{w_{22}^2} \\ \boldsymbol{w_{13}^2} & \boldsymbol{w_{23}^2} \end{bmatrix} * \begin{bmatrix} \boldsymbol{\delta_1^2} \\ \boldsymbol{\delta_2^2} \end{bmatrix} \end{bmatrix} * [a_1^0 \quad a_2^0 \quad a_3^0 \quad a_4^0]$$

Ingeniería inversa de la matriz de pesos L

$$[\delta^2] * [a^1] = \begin{bmatrix}\delta_1^2\\ \delta_2^2\end{bmatrix} * [a_1^1 \quad a_2^1 \quad a_3^1] = \begin{bmatrix}\boldsymbol{\delta_1^2}\\ \boldsymbol{\delta_2^2}\end{bmatrix} * [a_1^1 \quad a_2^1 \quad a_3^1]$$

$$\delta_1^2 = sig'(z_1^2) * 2 * (a_1^2 - y_1)$$

$$\delta_2^2 = sig'(z_2^2) * 2 * (a_2^2 - y_2)$$

$$\begin{bmatrix}\boldsymbol{\delta_1^2}\\ \boldsymbol{\delta_2^2}\end{bmatrix} * [a_1^1 \quad a_2^1 \quad a_3^1] = \begin{bmatrix}sig'(z_1^2) * 2 * (a_1^2 - y_1)\\ sig'(z_2^2) * 2 * (a_2^2 - y_2)\end{bmatrix} * [a_1^1 \quad a_2^1 \quad a_3^1] = \begin{bmatrix}sig'(z_1^2)\\ sig'(z_2^2)\end{bmatrix} \cdot \begin{bmatrix}2 * (a_1^2 - y_1)\\ 2 * (a_2^2 - y_2)\end{bmatrix} * [a_1^1 \quad a_2^1 \quad a_3^1]$$

Comparaciones

$$[\boldsymbol{\delta^2}] * [\boldsymbol{a^1}] = \begin{bmatrix}a_1^1 * \delta_1^2 & a_2^1 * \delta_1^2 & a_3^1 * \delta_1^2\\ a_1^1 * \delta_2^2 & a_2^1 * \delta_2^2 & a_3^1 * \delta_2^2\end{bmatrix} = \begin{bmatrix}\boldsymbol{\delta_1^2}\\ \boldsymbol{\delta_2^2}\end{bmatrix} * [a_1^1 \quad a_2^1 \quad a_3^1] = \begin{bmatrix}sig'(z_1^2)\\ sig'(z_2^2)\end{bmatrix} \cdot \begin{bmatrix}2 * (a_1^2 - y_1)\\ 2 * (a_2^2 - y_2)\end{bmatrix} * [a_1^1 \quad a_2^1 \quad a_3^1]$$

$$[\boldsymbol{\delta^1}] * [\boldsymbol{a^0}] = \begin{bmatrix}a_1^0 * \delta_1^1 & a_2^0 * \delta_1^1 & a_3^0 * \delta_1^1 & a_4^0 * \delta_1^1\\ a_1^0 * \delta_2^1 & a_2^0 * \delta_2^1 & a_3^0 * \delta_2^1 & a_4^0 * \delta_2^1\\ a_1^0 * \delta_3^1 & a_2^0 * \delta_3^1 & a_3^0 * \delta_3^1 & a_4^0 * \delta_3^1\end{bmatrix} = \begin{bmatrix}\boldsymbol{\delta_1^1}\\ \boldsymbol{\delta_2^1}\\ \boldsymbol{\delta_3^1}\end{bmatrix} * [a_1^0 \quad a_2^0 \quad a_3^0 \quad a_4^0] = \begin{bmatrix}sig'(z_1^1)\\ sig'(z_2^1)\\ sig'(z_3^1)\end{bmatrix} \cdot \left[\begin{bmatrix}\boldsymbol{w_{11}^2} & \boldsymbol{w_{21}^2}\\ \boldsymbol{w_{12}^2} & \boldsymbol{w_{22}^2}\\ \boldsymbol{w_{13}^2} & \boldsymbol{w_{23}^2}\end{bmatrix} * \begin{bmatrix}\boldsymbol{\delta_1^2}\\ \boldsymbol{\delta_2^2}\end{bmatrix}\right] * [a_1^0 \quad a_2^0 \quad a_3^0 \quad a_4^0]$$

En el caso hipotético que hubiera una capa anterior de 2 neuronas entonces la derivada seria

$$[\boldsymbol{\delta^0}] * [\boldsymbol{a^{-1}}] = \begin{bmatrix}a_1^{-1} * \delta_2^0 & a_2^{-1} * \delta_1^0\\ a_1^{-1} * \delta_2^0 & a_2^{-1} * \delta_2^0\\ a_1^{-1} * \delta_3^0 & a_2^{-1} * \delta_3^0\\ a_1^{-1} * \delta_4^0 & a_2^{-1} * \delta_4^0\end{bmatrix} = \begin{bmatrix}\delta_1^0\\ \delta_2^0\\ \delta_3^0\\ \delta_4^0\end{bmatrix} * [a_1^{-1} \quad a_2^{-1}] = \begin{bmatrix}sig'(z_1^0)\\ sig'(z_2^0)\\ sig'(z_3^0)\\ sig'(z_4^0)\end{bmatrix} \cdot \left[\begin{bmatrix}\boldsymbol{w_{11}^1} & \boldsymbol{w_{21}^1} & \boldsymbol{w_{31}^1}\\ \boldsymbol{w_{12}^1} & \boldsymbol{w_{22}^1} & \boldsymbol{w_{32}^1}\\ \boldsymbol{w_{13}^1} & \boldsymbol{w_{23}^1} & \boldsymbol{w_{33}^1}\\ \boldsymbol{w_{14}^1} & \boldsymbol{w_{24}^1} & \boldsymbol{w_{34}^1}\end{bmatrix} * \begin{bmatrix}\boldsymbol{\delta_1^1}\\ \boldsymbol{\delta_2^1}\\ \boldsymbol{\delta_3^1}\end{bmatrix}\right] * [a_1^{-1} \quad a_2^{-1}]$$