Three algorithms from control engineering that you should know about.

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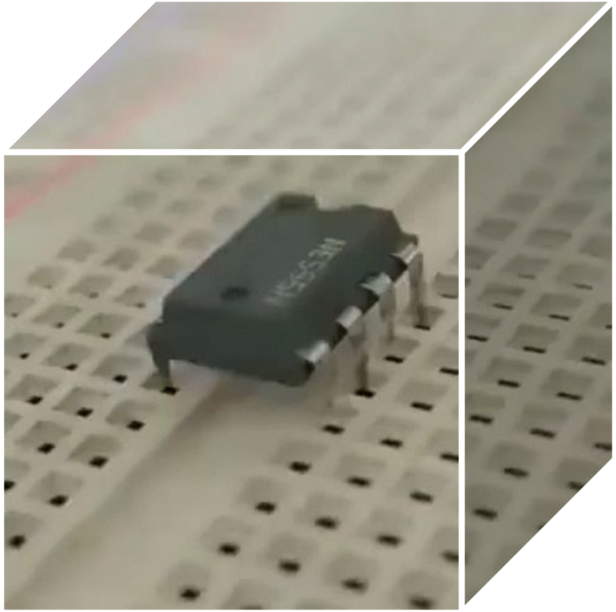
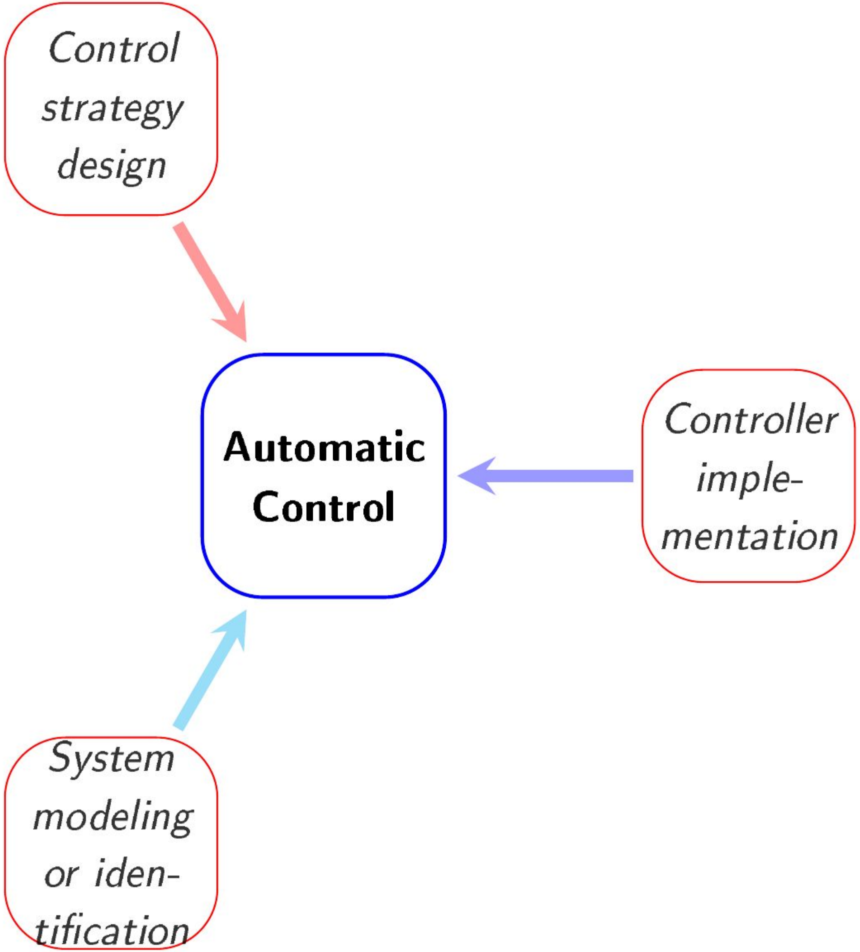
What is control engineering?





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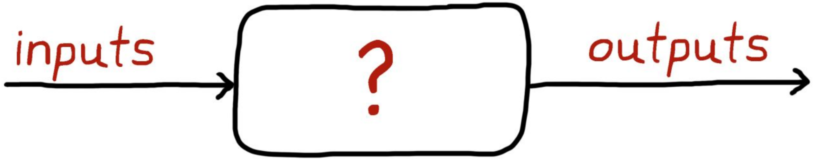
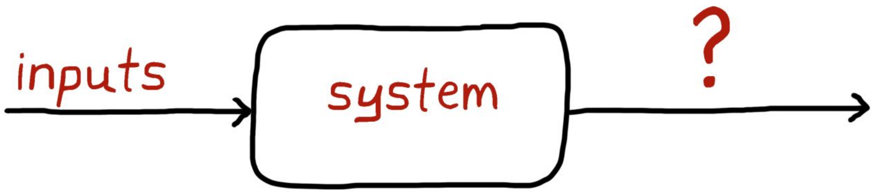
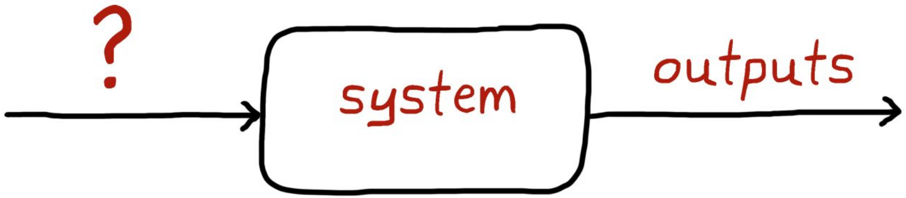
We have three problems



Identification problem

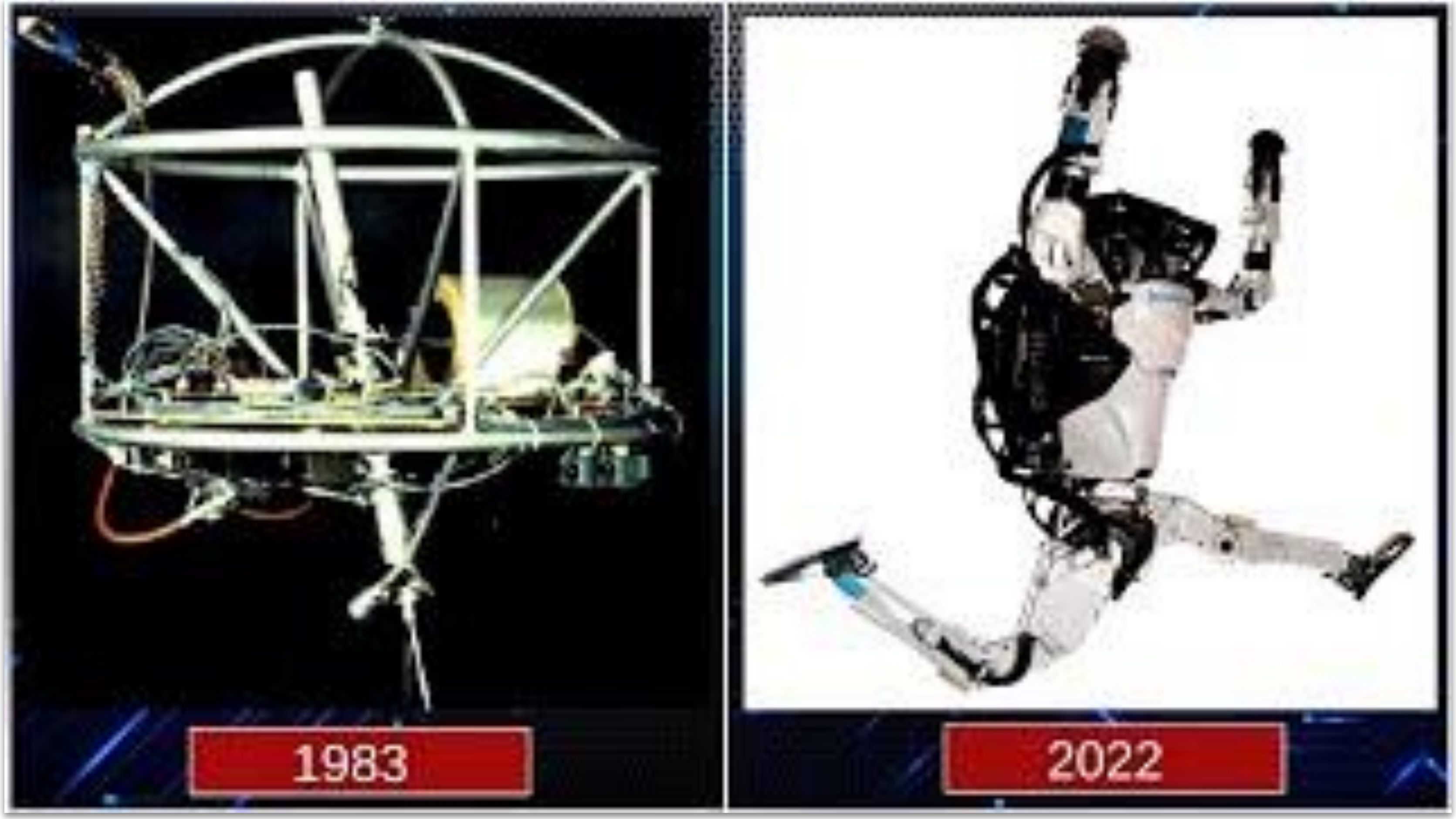


Simulation problem

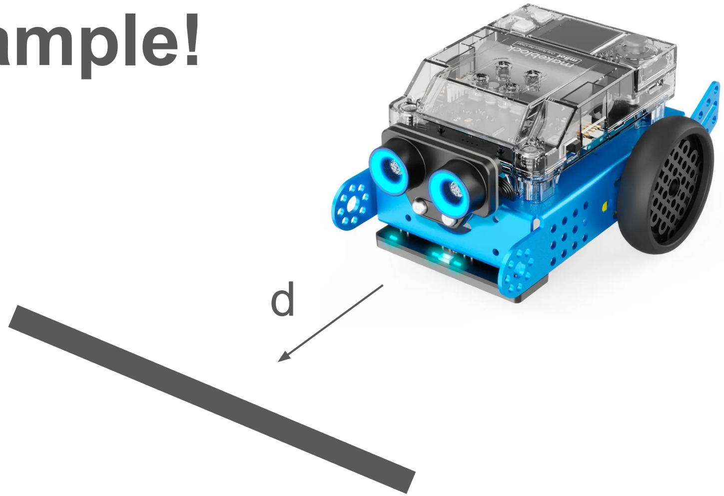


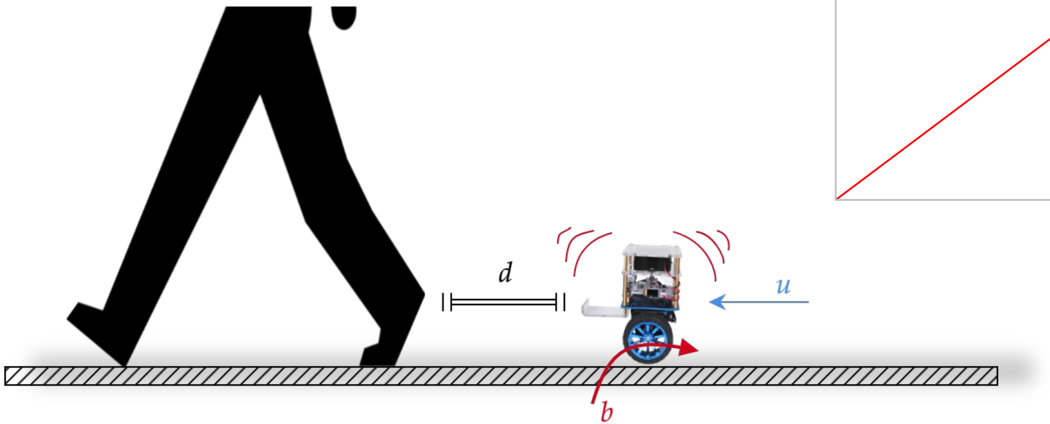
Control problem

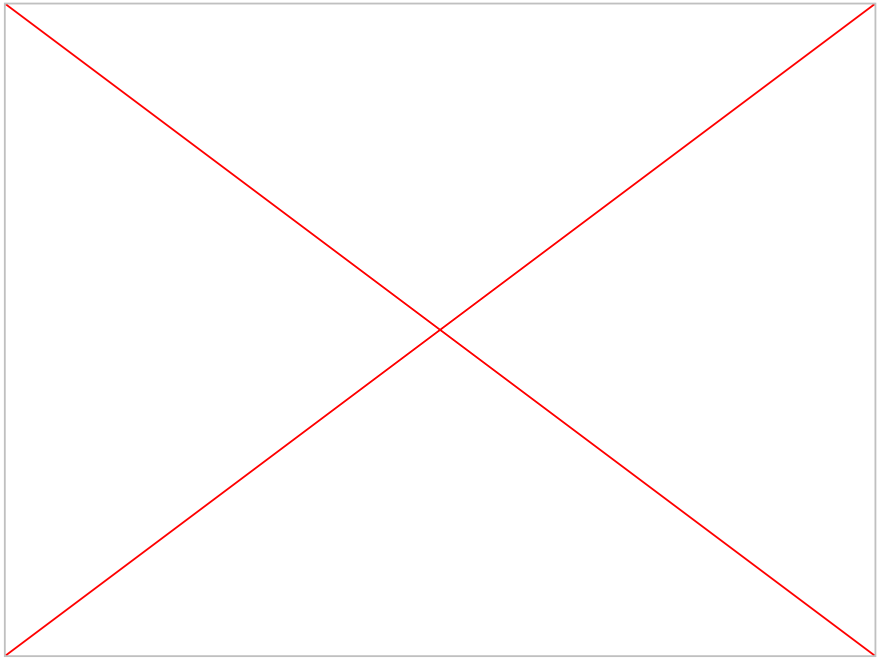
Ref: Douglas, B. (2017). The fundamentals of control theory.


1983
2022

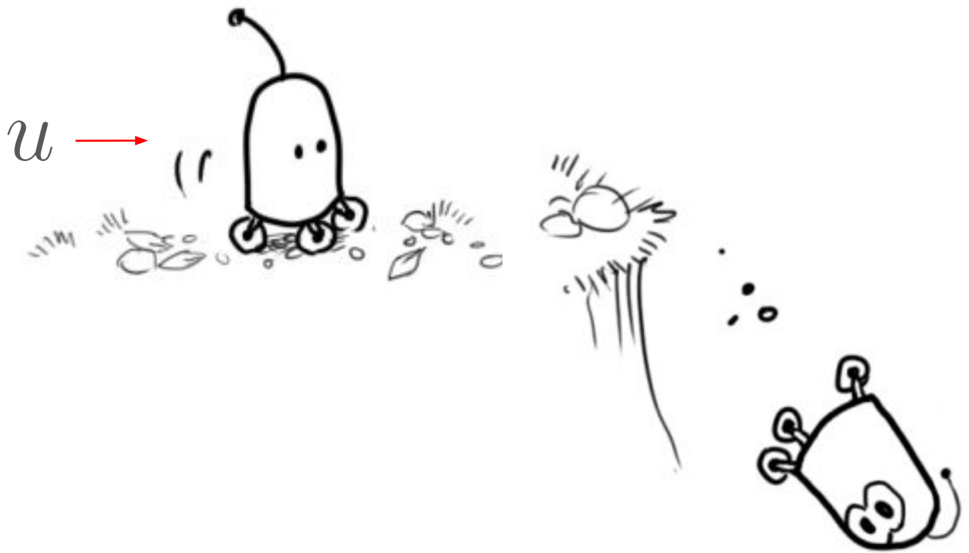
Example!

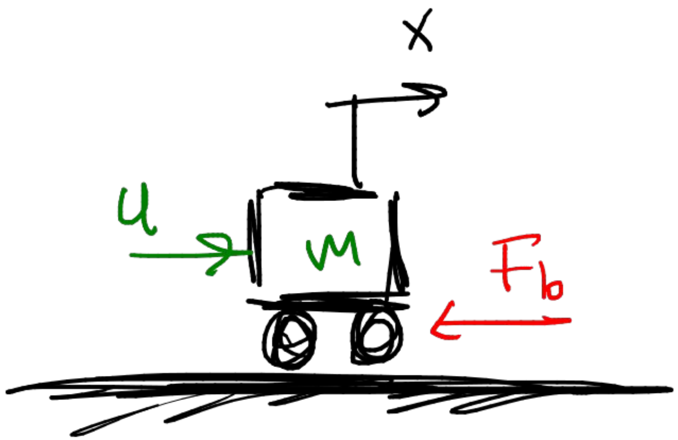






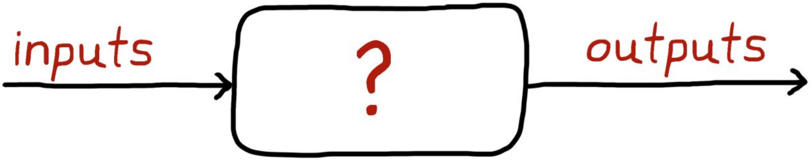
[ELEGOO Tumblller Self-Balancing Robot Car V1.1/V1.0 Tutorial – ELEGOO Official]

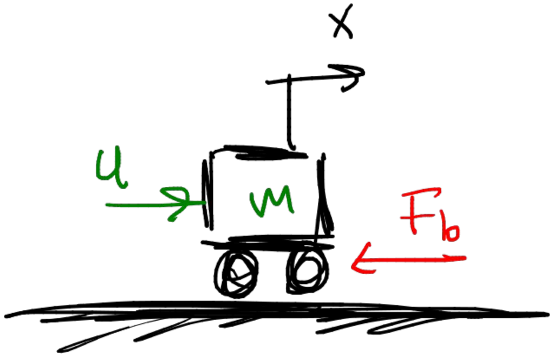




See https://youtu.be/8idcUSEoTAI?si=XTvpKVsV11Zf0muz

Mathematical Modelling





$$\sum F_x = m \cdot a_x$$

$$u(t) - F_b(t) = m \cdot \frac{d^2 x(t)}{dt^2}$$

$$u(t) - b\frac{dx(t)}{dt} = m\frac{d^2 x(t)}{dt^2}$$

$$\Rightarrow \quad m\frac{d^2 x(t)}{dt^2} + b\frac{dx(t)}{dt} = u(t)$$

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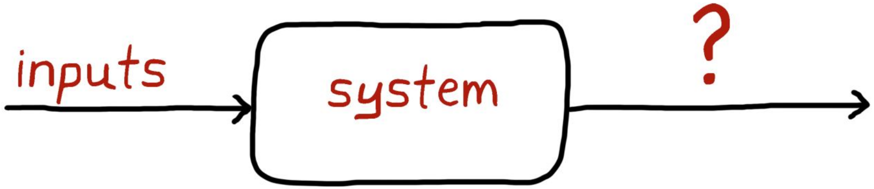
Simulation



$$m \frac{d^2 x(t)}{dt^2} + b \frac{d x(t)}{dt} = u(t) \Rightarrow x(t) = ? \quad \forall\, t \geq 0$$

$$m, b, x(0), \dot{x}(0), u$$

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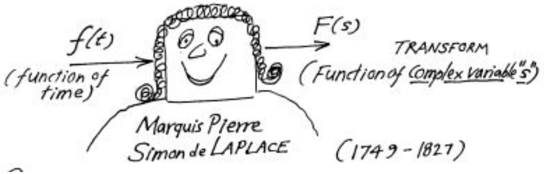
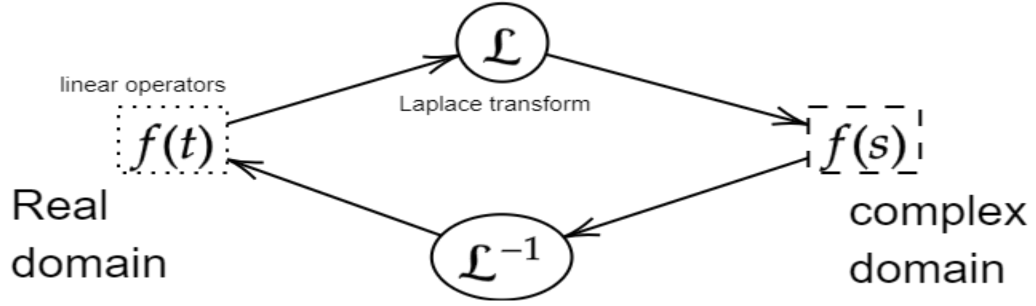
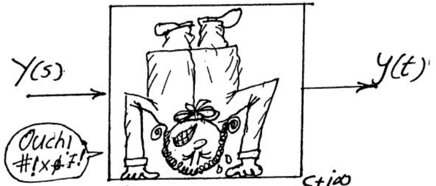
Full solution

LAPLACE TRANSFORM
is a fundamental tool in control systems analysis and design.





INVERSE LAPLACE TRANSFORM recovers the time function y(t) from its transform Y(s).



$$y(t) = \mathcal{L}^{-1}[Y(s)] = \int_{c-j\infty}^{c+j\infty} F(s)e^{st}ds$$

(c = real constant)

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zero initial conditions

$$(ms^2 + bs)\, X(s) = U(s)$$

$$X(s) = \frac{U(s)}{ms^2 + bs}$$

$$u(t) = \delta(t) \Rightarrow X(s) = \frac{1/m}{s^2 + \frac{b}{m}s}$$

$$x(t) = \mathcal{L}^{-1}\{X(s)\}$$

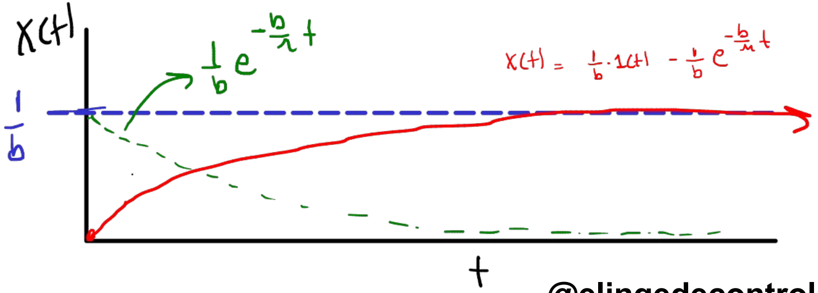
$$\frac{1/m}{s^2 + \frac{b}{m}s} = \frac{A}{s} + \frac{B}{s + b/m}$$

$$A = \lim_{s \to 0} s\,X(s) = \lim_{s \to 0} \frac{1/m}{s + b/m} = \frac{1}{b}$$

$$B = \lim_{s \to -\frac{b}{m}} \left(s + \frac{b}{m}\right) X(s) = \lim_{s \to -\frac{b}{m}} \frac{1/m}{s} = -\frac{1}{b}$$

$$X(s) = \frac{1/b}{s} - \frac{1/b}{s + b/m}$$

$$x(t) = \frac{1}{b} \cdot 1(t) - \frac{1}{b} e^{-\frac{b}{m}t}$$



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Numeric solution

$$m\ddot{x} + b\dot{x} = u$$

$$\Downarrow$$

$$x_1 = x$$
$$x_2 = \dot{x}$$

$$\Rightarrow \quad \dot{x}_1 = x_2$$
$$\dot{x}_2 = -\frac{b}{m} x_2 + \frac{u}{m}$$

$$\dot{x} = Ax + Bu$$

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & -\frac{b}{m} \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}}_{B} u$$

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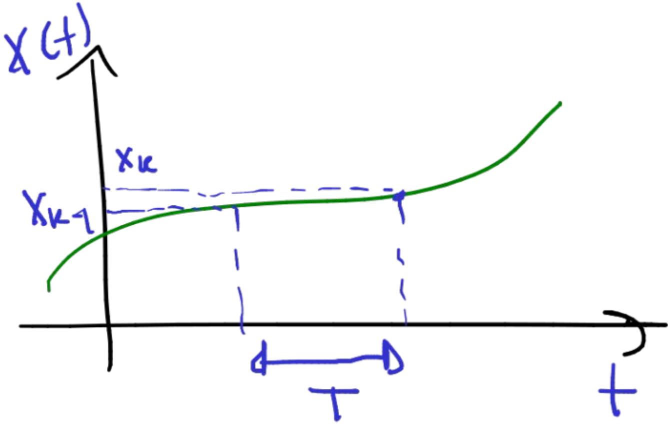
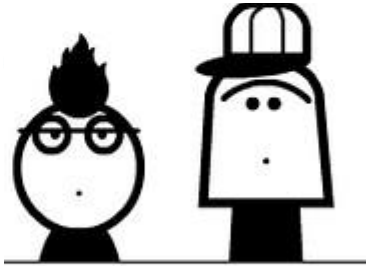
$$\dot{x} \approx \frac{x_k - x_{k-1}}{T}$$





https://openprocessing.org/sketch/1623835

$$\dot{x} = Ax + Bu \Rightarrow \frac{x_k - x_{k-1}}{T} = Ax_{k-1} + Bu_{k-1}$$

$$\Rightarrow x_k = x_{k-1} + T\{Ax_{k-1} + Bu_{k-1}\} \quad \text{if} \quad \boxed{\begin{aligned} A_d &= I + TA \\ B_d &= TB \end{aligned}}$$

$$x_k = A_d x_{k-1} + B_d u_{k-1}$$

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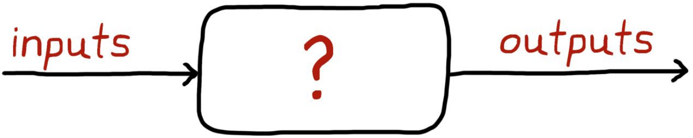
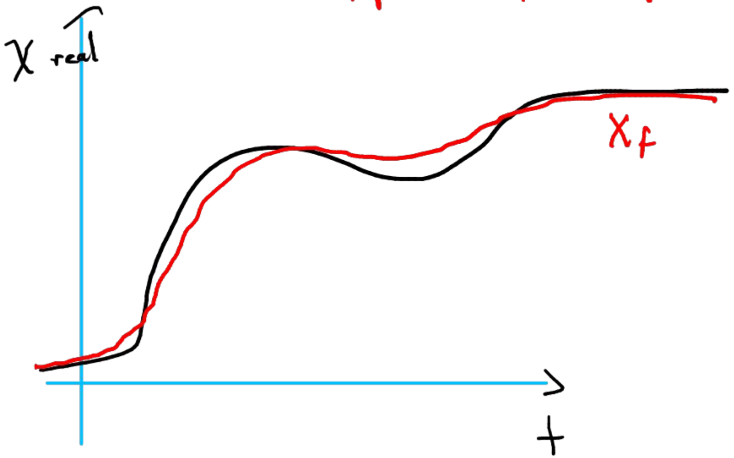
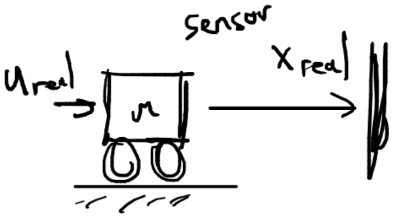
Identification problem



$$m\ddot{x}_f + b\dot{x}_f = u_f$$



t	x_{real}
t_0	x_0
t_1	x_1
$\vdots$	
t_N	x_N



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An optimisation problem:

least-squares problem

$$J := \min_{\theta} \left\| X_{real} - X_t(\theta) \right\|_2^2$$

$$\text{s.t.} \quad \theta \in S$$
$$t \in [0, t_f]$$

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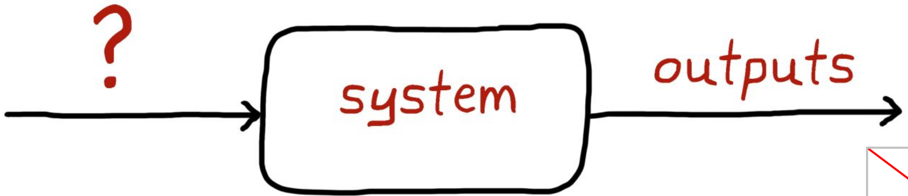
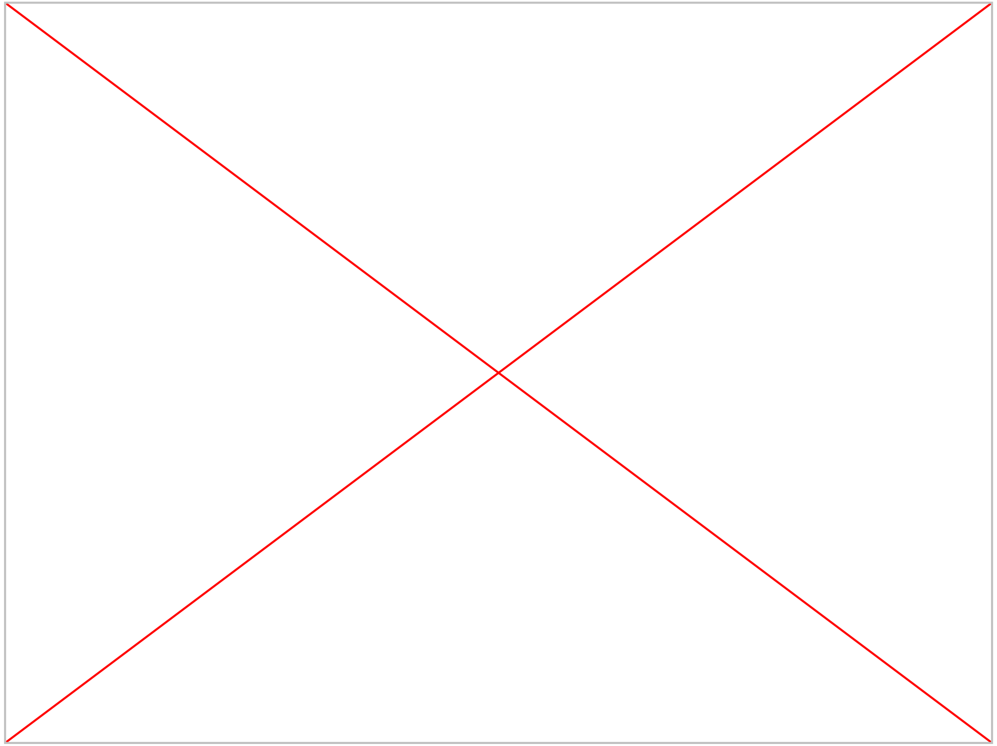
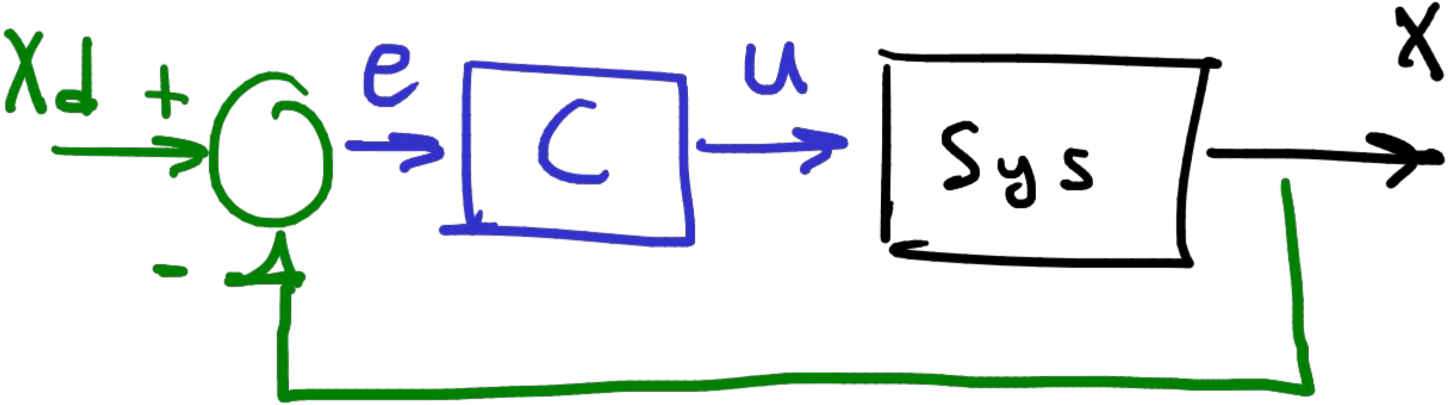
Control



$$u = ?$$



Cybernetics



$$e = X_d - X$$

$$C = ?$$

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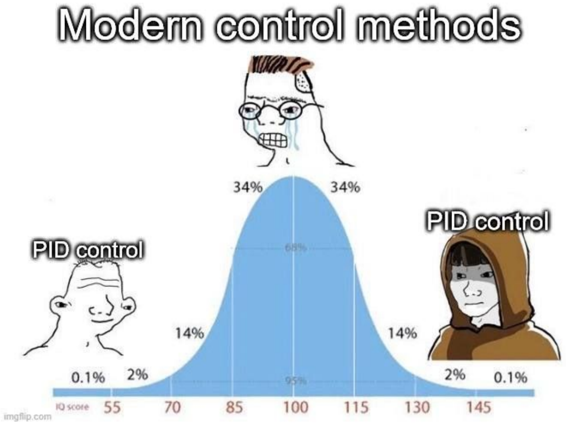
We have many options!

$$u(t) = K_p e(t)$$

$$u(t) = K_p e(t) + K_d \dot{e}(t)$$

$$u(t) = K_p e(t) + K_d \dot{e}(t) + K_i \int_0^t e(\tau) d\tau$$

PID



$$u(t) = K_1 x_1 + K_2 x_2 \rightarrow$$ Full State Feedback

$$u(t) = K_p e(t) + K_\delta e(t-L) \rightarrow$$ delayed

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$$u(t) = K_p e(t) + K_p \frac{d^\alpha e(t)}{dt^\alpha} \rightarrow$$ fractional

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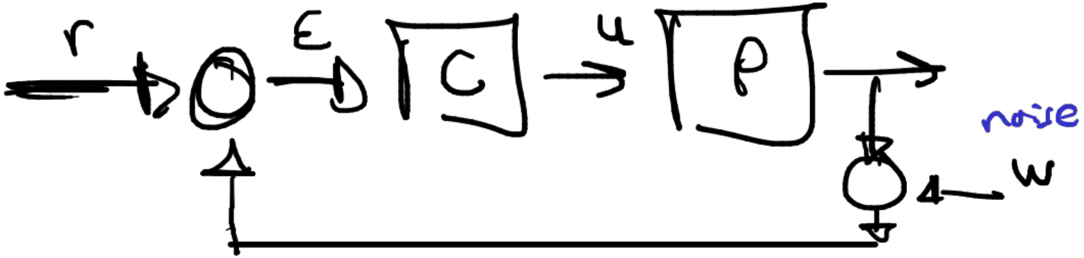
An estimation problem!

What if we cannot measure the states???

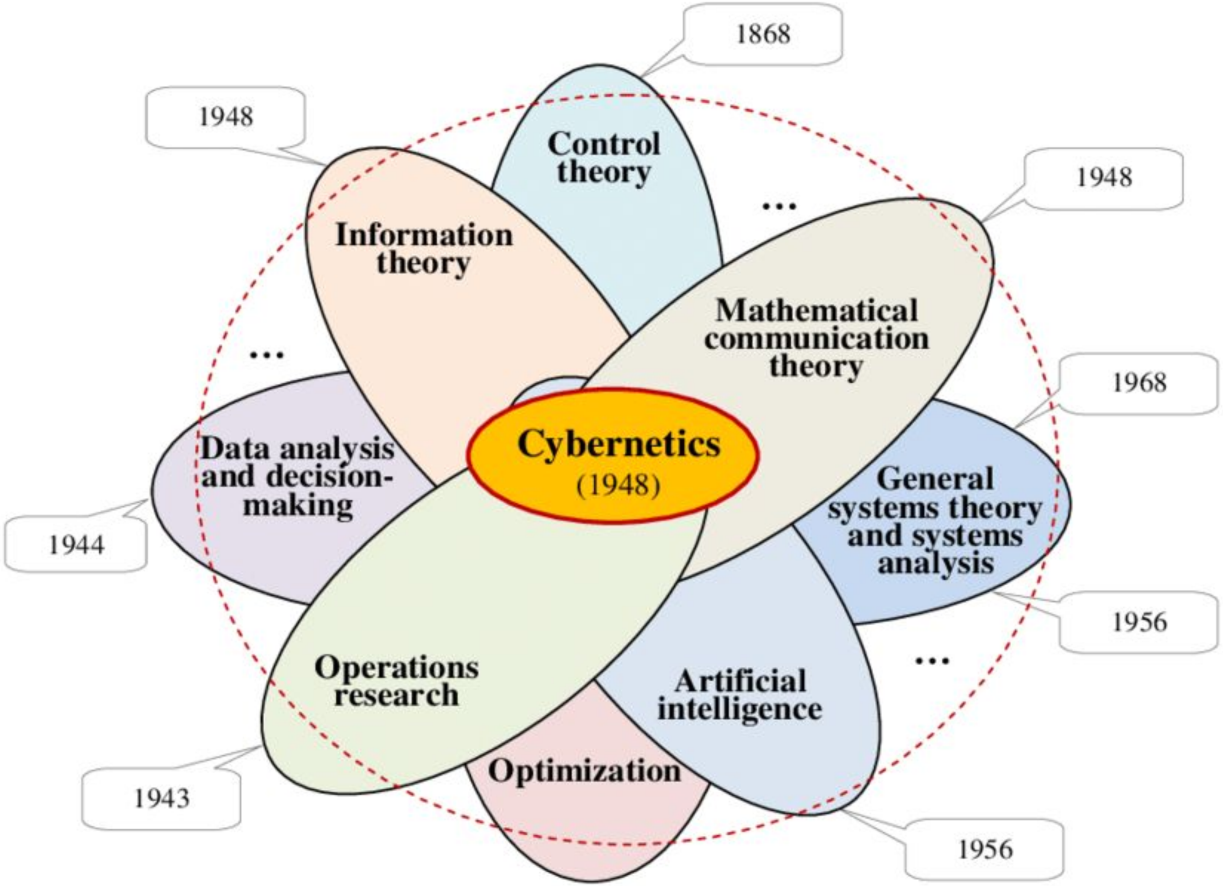


Cartinvertedpendulum - OpenProcessing

Conclusions

- There is a lot more to learn.
- Reality is nonlinear!
- Not only math is important, but also physics!
- Control systems give you a full engineering experience.

CYBERNETICS



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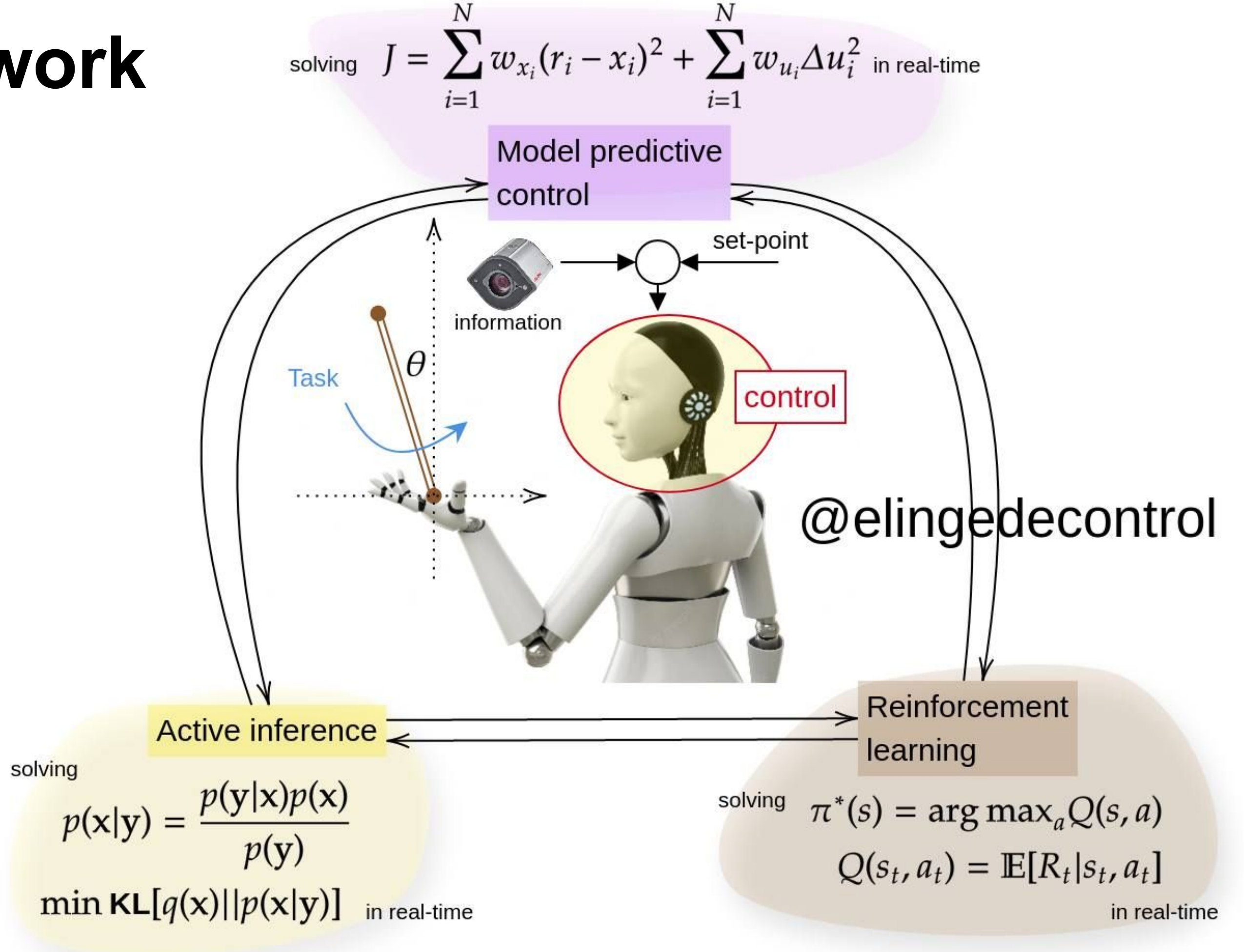
Future work



solving $J = \sum_{i=1}^{N} w_{x_i}(r_i - x_i)^2 + \sum_{i=1}^{N} w_{u_i}\Delta u_i^2$ in real-time
Model predictive control
set-point
information
Task
θ
control
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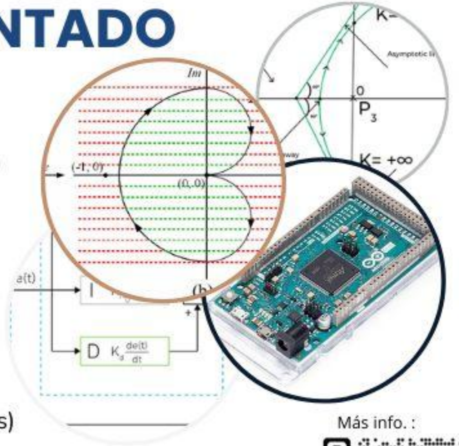
Active inference
Reinforcement learning
solving
$p(\mathrm{x}|\mathrm{y}) = \dfrac{p(\mathrm{y}|\mathrm{x})p(\mathrm{x})}{p(\mathrm{y})}$
$\min \mathbf{KL}[q(\mathrm{x})||p(\mathrm{x}|\mathrm{y})]$ in real-time
solving
$\pi^*(s) = \arg\max_a Q(s,a)$
$Q(s_t, a_t) = \mathbb{E}[R_t|s_t, a_t]$
in real-time

TALLER DE SISTEMAS DE CONTROL RETROALIMENTADO



Temario:

1. Introducción al Control de Retroalimentación (2 hr)
2. Modelado de Sistemas en el Dominio del Tiempo (2 hrs)
3. Respuesta Dinámica (2 hrs)
4. Propiedades Básicas de la Retroalimentación (2 hr)
5. Análisis de Estabilidad (4 hrs)
6. Análisis del Lugar de las Raíces (2 hrs)
7. Diseño de Controladores del Lugar de las Raíces (2 hrs)
8. Análisis de Respuesta en Frecuencia (2 hrs)
9. Diseño de Respuesta en Frecuencia (2 hrs)
10. Implementación de Controlador Digital (Arduino) (2 hrs)

Más info. :

Sábados 11:00 - 13:00 hrs y 15:00 - 17:00 hrs tiempo de la ciudad de México.

Del 22 de Febrero al 29 de Marzo.

120 USD

INSTRUCTORES:

Dr Enrique Diez
Sistemas con retardo de tiempo, electrónica de potencia, variable compleja

Dr. Bryan Rojas R.
Robótica, visión artificial, sistemas con retardo de tiempo, aplicaciones industriales.

Mtro. Juan José Meza G.
Algoritmos de estimación, visión por computadora, robótica

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Thank you!

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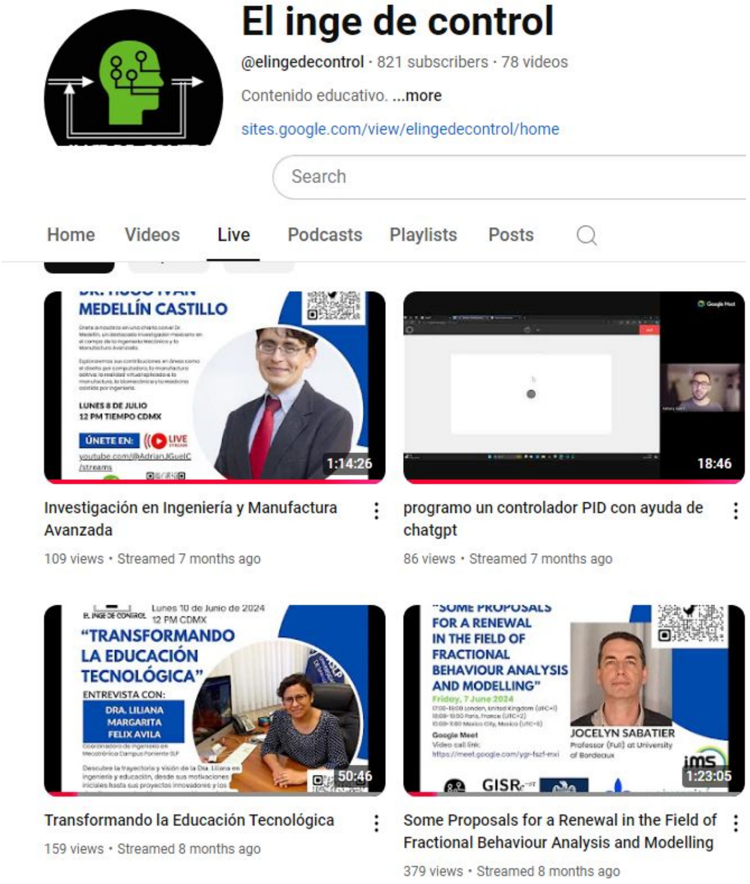

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