Lead-lag compensator $K\frac{s+a}{s+b}$ design for $G(s)H(s)=\frac{1}{(s+1)(s+2)(s+3)}$

Set the design parameters:

```
clc
clearvars
close all
Mp= 0.02; %overshoot
ts = 5; %settling time
z = log(Mp)/sqrt((log(Mp))^2+pi^2); %damping ratio
wn = 4/(ts*z); %natural frequency
pd = roots([1 2*z*wn wn^2]) %desired poles
```

```
pd = 2×1 complex
  -0.8000 + 0.6424i
  -0.8000 - 0.6424i
```

We now compute the angles of the open-loop poles against the desired poles. This values will permit us to obtain the zero/pole location of the lead-lag compensato through the root locus rules (I followed https://www.ece.mcmaster.ca/~davidson/EE3CL4/slides/Lead_Lag_handout.pdf).

```
t1 = atan(imag(pd(1))/(0.2))*180/pi
```

```
t1 = 72.7082
```

```
t2 = atan(imag(pd(1))/(1.2))*180/pi
```

```
t2 = 28.1634
```

```
t3 = atan(imag(pd(1))/(2.2))*180/pi
```

```
t3 = 16.2790
```

We impose the compensator pole at the origin. i.e. we will tune a classical PI control. PI control allow us zero steady state error.

```
t4 = atan(abs(real(pd(1))/imag(pd(1))))*180/pi+90
```

```
t4 = 141.2334
```

```
angp = t1+t2+t3+t4
```

```
angp = 258.3840
```

According to root locus $\angle Z - \angle P = 180^{\circ}(2\ell + 1)$, therefore the angle of the zero in the lead-lag compensator is

```
angz= -180+angp
```

```
angz = 78.3840
```

Its location:

```
d = imag(pd(1))/tan((angz*pi/180))
```

```
d = 0.1321
```

```
zc = -d+real(pd(1))
```

```
zc = -0.9321
```

```
pc = 0 % pole of the compensator located by design at the origin
```

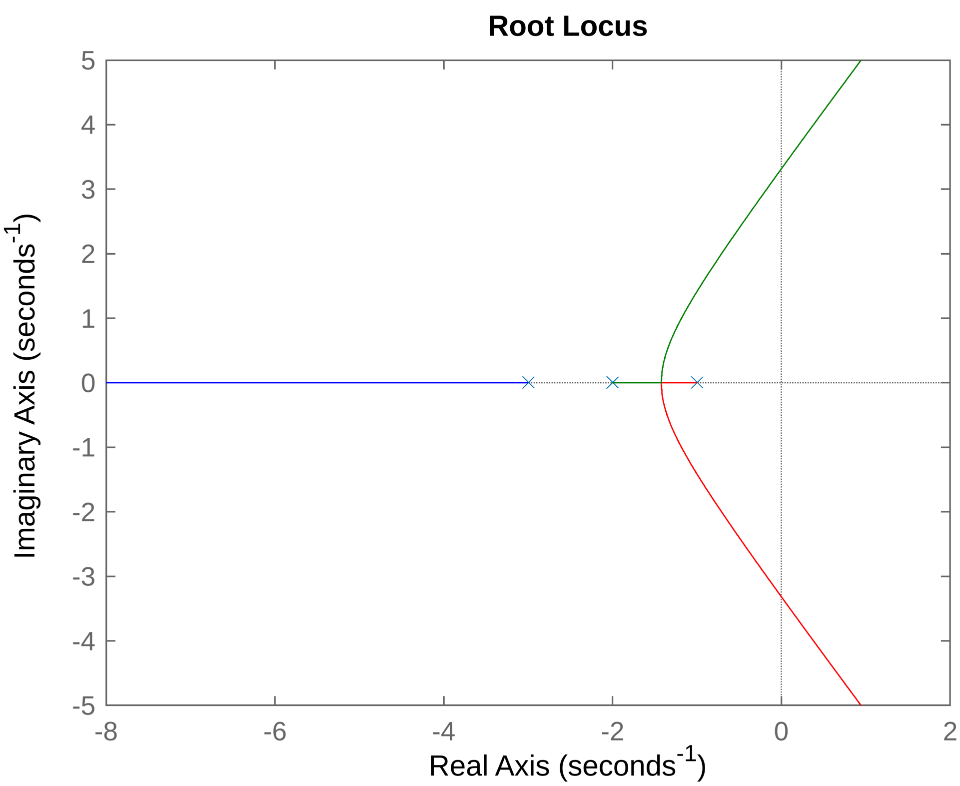
```
pc = 0
```

Open loop root locus (NOTE that I assumed H(s)=1 for simplicity)

```
s = tf('s');
G= 1/((s+1)*(s+2)*(s+3))
```

```
G =
 
            1
  ----------------------
  s^3 + 6 s^2 + 11 s + 6
 
Continuous-time transfer function.
Model Properties
```

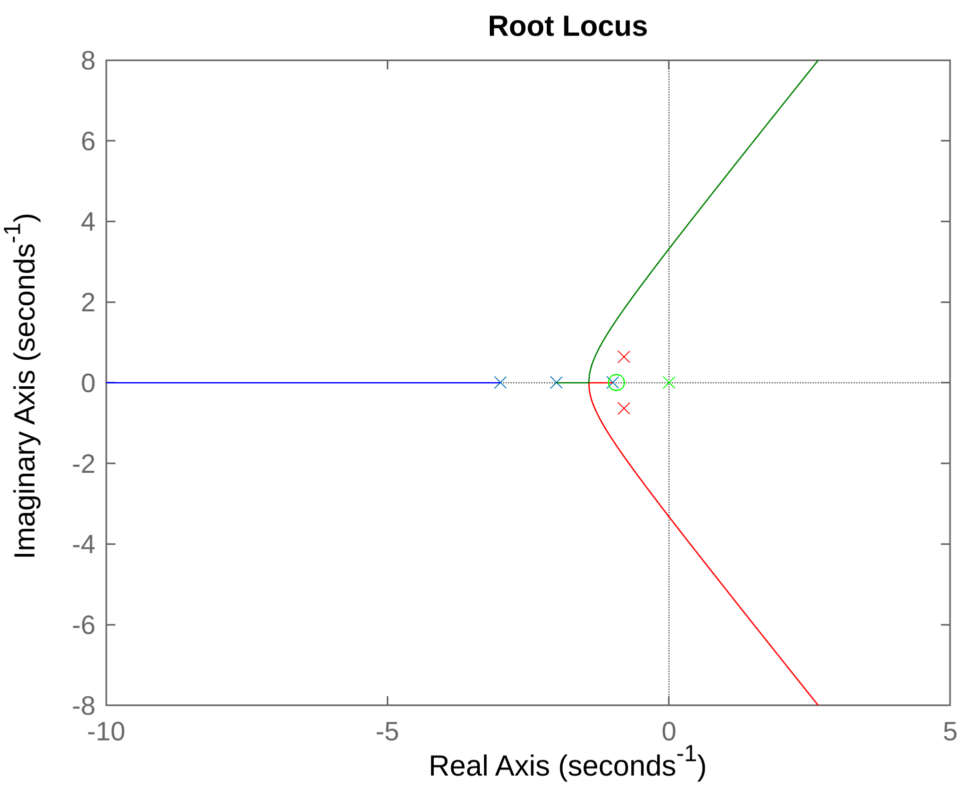
```
figure(1)
rlocus(G)
```



Plotting the location of the desired poles

```
figure(3)
```

```
rlocus(G)
hold on
xlim([-10 5])
ylim([-8 8])
plot(pd,'rx')
plot(zc,0,'go')
plot(pc,0,'gx')
hold off
```

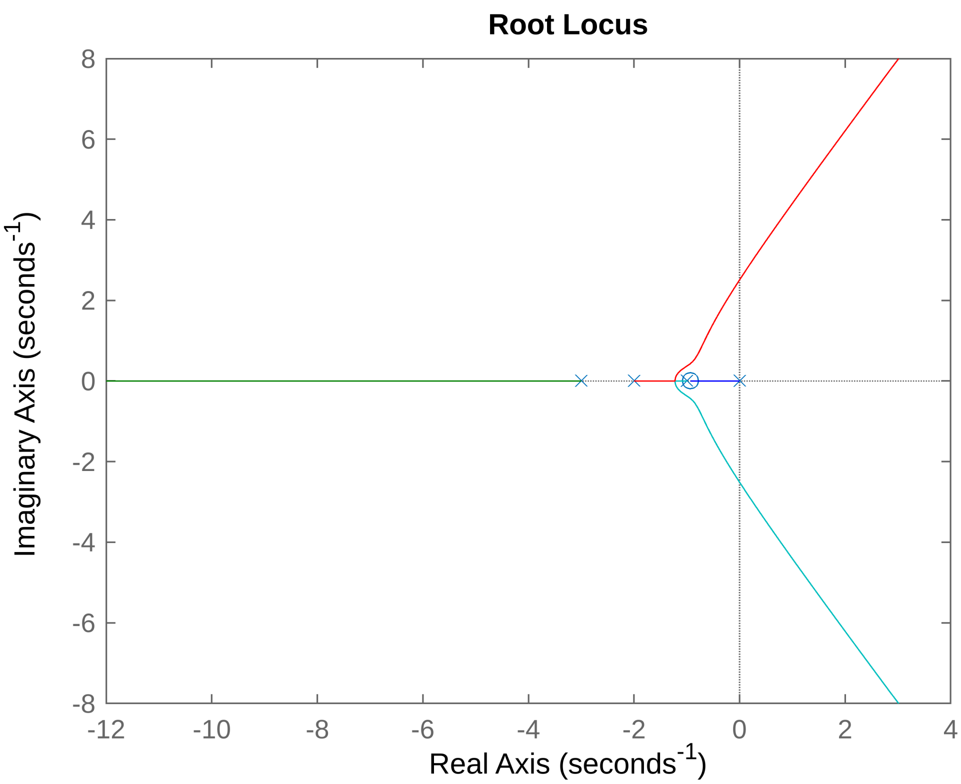


closed loop transfer function root locus

```
Gn = ((s-zc))/((s+1)*(s+2)*(s+3)*(s-pc))
```

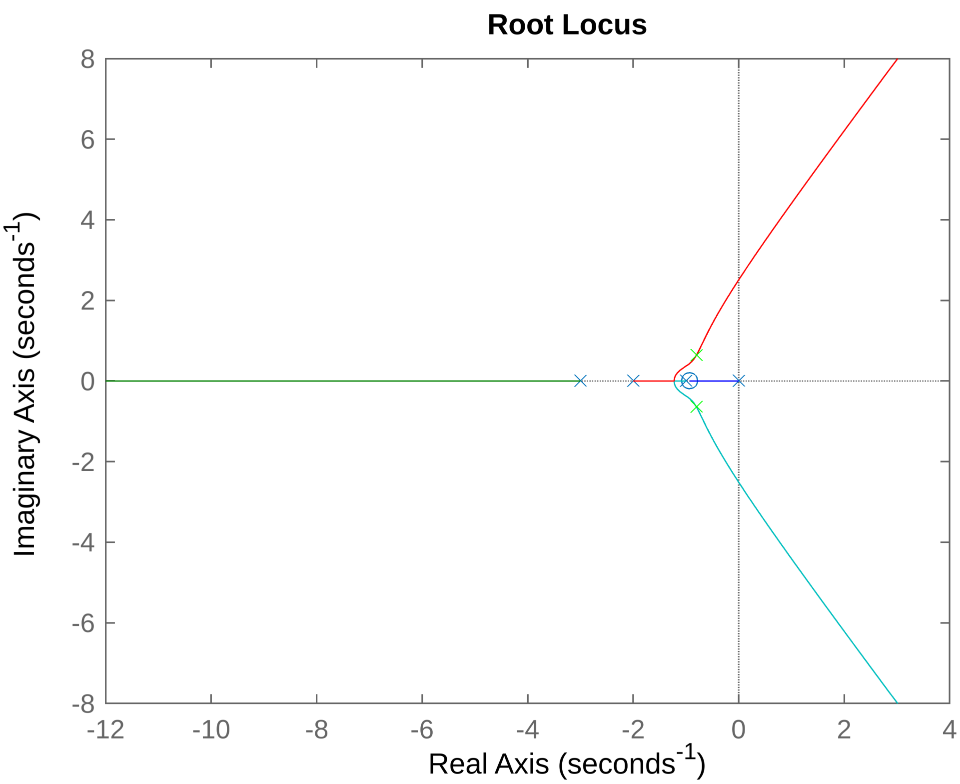
```
Gn =
 
          s + 0.9321
  --------------------------
  s^4 + 6 s^3 + 11 s^2 + 6 s
 
Continuous-time transfer function.
Model Properties
```

```
figure(4)
rlocus(Gn)
```



Confirming the desired poles are over the root locus.

```
figure(5)
rlocus(Gn)
hold on
plot(pd,'gx')
hold off
```



Finally the compensator gain K is obtained solving $|\lim_{s \to p_d}[KC(s)G(s)]| = 1$ where pd is the desired pole.

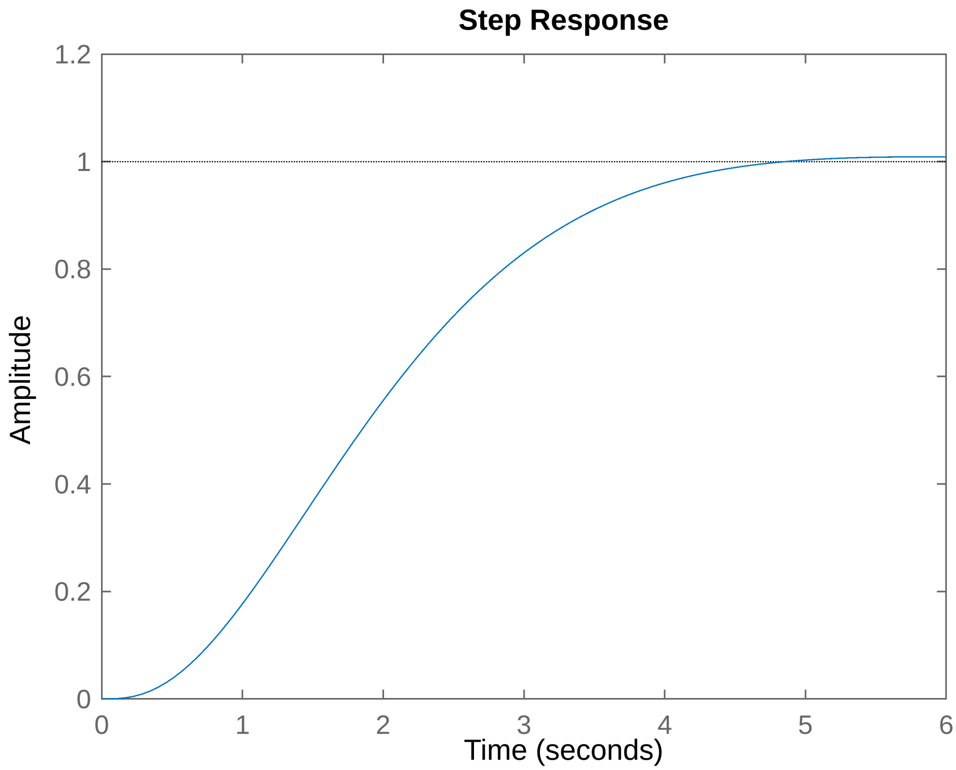
```
kc= 3.28; %obtained value
Gc = kc*((s-zc)/(s-pc))
```

```
Gc =

  3.28 s + 3.057
  --------------
        s

Continuous-time transfer function.
Model Properties
```

```
%confirming the desired performance specifications
L=Gc*G;
LC = feedback(L,1);
stepinfo(LC)
```

```
ans = struct with fields:
         RiseTime: 2.6610
    TransientTime: 4.3060
     SettlingTime: 4.3060
      SettlingMin: 0.9041
      SettlingMax: 1.0088
        Overshoot: 0.8787
       Undershoot: 0
             Peak: 1.0088
         PeakTime: 5.8110
```

```
step(LC)
```


Step Response
Amplitude
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6