Advanced Digital Signal Processing

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Problem 2.
Two complex numbers $x = a + ib$ and $y = c + id$ are multiplied as follows:

$$\begin{aligned} xy &= (a+ib)(c+id) \\ &= ac + ibc + iad - bd \\ &= (ac - bd) + i(ad + bc) \end{aligned} \tag{1}$$

Complex multiplication can be carried out using only three real multiplications, ac, bd, and $(a+b)(c+d)$ as:

$$\begin{aligned} \Re[(a+ib)(c+id)] &= ac - bd \\ \Im[(a+ib)(c+id)] &= (a+b)(c+d) - ac - bd \end{aligned} \tag{2}$$

Problem 3.
(a) $y_1 = bx_1 + ax_2 + bx_1$, $y_2 = ax_1 + bx_2 + ax_3$ and $y_3 = bx_1 + ax_2 + bx_3$. We can set $x_4 = \frac{b+a}{2}(x_1 + x_2 + x_3)$ and $x_5 = \frac{b-a}{2}(x_1 - x_2 + x_3)$. Thus, $y_1 = y_3 = x_4 + x_5$ and $y_2 = x_4 - x_5$, we only need 2 multiplications.
(b)

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ b & -d & -a & -c \\ c & -a & d & b \\ d & -c & b & -a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

And we can exchange the rows to make the matrix be the form below:

$$\begin{aligned} \begin{bmatrix} y_1 \\ y_4 \\ y_3 \\ y_2 \end{bmatrix} &= \begin{bmatrix} a & d & c & b \\ d & -a & b & -c \\ c & b & d & -a \\ b & -c & -a & -d \end{bmatrix} \begin{bmatrix} x_1 \\ x_4 \\ x_3 \\ x_2 \end{bmatrix} \\ &= \begin{bmatrix} c & b & c & b \\ b & -c & b & -c \\ c & b & c & b \\ b & -c & b & -c \end{bmatrix} \begin{bmatrix} x_1 \\ x_4 \\ x_3 \\ x_2 \end{bmatrix} + \begin{bmatrix} a-c & d-b & 0 & 0 \\ d-b & c-a & 0 & 0 \\ 0 & 0 & d-c & -a-b \\ 0 & 0 & -a-b & c-d \end{bmatrix} \begin{bmatrix} x_1 \\ x_4 \\ x_3 \\ x_2 \end{bmatrix} \end{aligned}$$

Thus, we can split the whole matrix into two part:

$$\begin{bmatrix} y_1 \\ y_4 \end{bmatrix} = \begin{bmatrix} c & b \\ b & -c \end{bmatrix} \begin{bmatrix} x_1 + x_3 \\ x_4 + x_2 \end{bmatrix} + \begin{bmatrix} a-c & d-b \\ d-b & c-a \end{bmatrix} \begin{bmatrix} x_1 \\ x_4 \end{bmatrix}$$
$$\begin{bmatrix} y_3 \\ y_2 \end{bmatrix} = \begin{bmatrix} c & b \\ b & -c \end{bmatrix} \begin{bmatrix} x_1 + x_3 \\ x_4 + x_2 \end{bmatrix} + \begin{bmatrix} d-c & -a-b \\ -a-b & c-d \end{bmatrix} \begin{bmatrix} x_3 \\ x_2 \end{bmatrix}$$

And

$$\begin{bmatrix} c & b \\ b & -c \end{bmatrix} \begin{bmatrix} x_1 + x_3 \\ x_4 + x_2 \end{bmatrix} = \begin{bmatrix} c & c \\ -c & -c \end{bmatrix} \begin{bmatrix} x_1 + x_3 \\ x_4 + x_2 \end{bmatrix} + \begin{bmatrix} 0 & b-c \\ c+b & 0 \end{bmatrix} \begin{bmatrix} x_1 + x_3 \\ x_4 + x_2 \end{bmatrix}$$
$$\begin{bmatrix} a-c & d-b \\ d-b & c-a \end{bmatrix} \begin{bmatrix} x_1 \\ x_4 \end{bmatrix} = \begin{bmatrix} a-c & a-c \\ c-a & c-a \end{bmatrix} \begin{bmatrix} x_1 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & (d-b)-(a-c) \\ (d-b)-(c-a) & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_4 \end{bmatrix}$$
$$\begin{bmatrix} d-c & -a-b \\ -a-b & c-d \end{bmatrix} \begin{bmatrix} x_3 \\ x_2 \end{bmatrix} = \begin{bmatrix} d-c & d-c \\ c-d & c-d \end{bmatrix} \begin{bmatrix} x_3 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & (-a-b)-(d-c) \\ (-a-b)-(c-d) & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ x_2 \end{bmatrix}$$

Each of them needs 3 multiplications, so we need 9 multiplications to complete the whole matrix operation.

Problem 4.
(a) $100 = 4 \times 25$, let $P_1 = 4$ and $P_2 = 25$. Since P_1 is prime to P_2, the total number of real multiplications N is

$$N = P_2 B_1 + P_1 B_2 = 25 \times 0 + 4 \times 148 = 592.$$

(b) $176 = 11 \times 16$, let $P_1 = 11$ and $P_2 = 16$. Since P_1 is prime to P_2, the total number of real multiplications N is

$$N = P_2 B_1 + P_1 B_2 = 16 \times 46 + 11 \times 20 = 956.$$

(c) $338 = 2 \times 169$, let $P_1 = 2$ and $P_2 = 169$. Since P_1 is prime to P_2, the total number of real multiplications N is

$$N = P_2 B_1 + P_1 B_2 = 169 \times 0 + 2 \times B_2.$$

And $169 = 13^2$, so $B_2 = 13 \times 52 + 13 \times 52 + 3 \times 12 \times 12 = 1496$. Thus, $N = 2 \times B_2 = 2992$.

Problem 5.
h[n] is symmetric, so

$$\begin{aligned} x_s[n] &= -0.075x[n+3] - 0.125x[n+2] - 0.3x[n+1] + x[n] - 0.3x[n-1] \\ &\quad - 0.125x[n-2] - 0.075x[n-3] \\ &= -0.075(x[n+3] + x[n-3]) - 0.125(x[n+2] + x[n-2]) - 0.3(x[n+1] + x[n-1]) + x[n] \end{aligned}$$

Thus, we can reduce the computational time to the half.

Problem 6.
(a) Since 600 and 1200 are in the same order, using FFT directly is more profitable. In this case, we need 1260 points FFT and it needs 7640 multiplications. Totally, we need $7640 \times 2 + 1260 \times 3 = 19060$ times multiplications.
(b) We should consider using FFT, sectioned convolution and direct computation in this case. According to (a), we know using FFT needs 19060 multiplications. If we use sectioned convolution, we can compute the optimal $L \approx 324$.

P-point FFT	P-point FFT MUL	L	S	Total MUL
312	1324	263	4.56	16343
360	1540	311	3.85	16016
336	1412	289	4.15	15903

Thus, we know that using sectioned convolution only needs 15903 multiplications. If we use direct computation, it needs $3 \times 50 \times 1200 = 180000$ multiplications. In the end, we should use sectioned convolution.
(c) Since $1200 >> 9$, we cannot use FFT. Consider sectioned convolution case, the optimal $L = 36$ and

P-point FFT	P-point FFT MUL	L	S	Total MUL
40	100	32	38	12160
42	124	34	36	13464
44	160	36	33	14916
48	92	40	30	9840

Using sectioned convolution only needs 9840 multiplications. If we use direct computation, it needs $3 \times 1200 \times 9 = 32400$ multiplications. In the end, we should use sectioned convolution.
(d) Since $1200 >> 3$, we cannot use FFT. If we use sectioned convolution, the optimal $L = 5$ and

P-point FFT	P-point FFT MUL	L	S	Total MUL
4	0	2	600	7200
6	4	4	300	7800
7	16	5	240	12720
8	4	6	200	6400

Thus, we need 6400 multiplications if we use sectioned convolution. If we consider direct computation, it needs $3 \times 1200 \times 3 = 10800$ multiplications. Thus, we should use sectioned convolution.