Seminar class 4

Exercise 1 (exercise 6.6 in the book)

. Consider the class $\mathcal{H}^d_{mcon}$ of monotone Boolean conjunctions over $\{0,1\}^d$. Monotonicity here means that the conjunctions do not contain negations. As in $\mathcal{H}^d_{con}$, the empty conjunction is interpreted as the all-positive hypothesis. We augment $\mathcal{H}^d_{mcon}$ with the all-negative hypothesis h^-. Show that $\text{VCdim}(\mathcal{H}^d_{mcon}) = d$.

Exercise 2 (exercise 6.3 in the book)

6.3 Let $\mathcal{X}$ be the Boolean hypercube $\{0,1\}^n$. For a set $I \subseteq \{1,2,\ldots,n\}$ we define a *parity function* h_I as follows. On a binary vector $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in \{0,1\}^n$,

$$h_I(\mathbf{x}) = \left(\sum_{i \in I} x_i\right) \bmod 2 .$$

(That is, h_I computes parity of bits in I.) What is the VC-dimension of the class of all such parity functions, $\mathcal{H}_{n\text{-parity}} = \{h_I : I \subseteq \{1,2,\ldots,n\}\}$?

Exercise 3 (exercise 6.2 in the book)

6.2 Given some finite domain set, $\mathcal{X}$, and a number $k \le |\mathcal{X}|$, figure out the VC-dimension of each of the following classes (and prove your claims):

1. $\mathcal{H}^{\mathcal{X}}_{=k} = \{h \in \{0,1\}^{\mathcal{X}} : |\{x : h(x) = 1\}| = k\}$: that is, the set of all functions that assign the value 1 to exactly k elements of $\mathcal{X}$.
2. $\mathcal{H}_{at-most-k} = \{h \in \{0,1\}^{\mathcal{X}} : |\{x : h(x) = 1\}| \le k \text{ or } |\{x : h(x) = 0\}| \le k\}$.