Seminar class 6

Exercise 1

Fix $\varepsilon \in (0, \frac{1}{2})$. Let the training sample be denoted by m points in the plane with $\frac{m}{4}$ negative points all at coordinate (+1,+1), another $\frac{m}{4}$ negative points all at coordinate (-1,-1), $\frac{m(1+\varepsilon)}{4}$ positive points all at coordinate (-1,+1), $\frac{m(1-\varepsilon)}{4}$ positive points all at coordinate (+1,-1).

a. Describe the behavior of AdaBoost when run on this sample using boosting stumps for the first two rounds.

b. What is the error of the optimal classifier chosen at round 1 in the second round?

AdaBoost

- construct distribution $\mathbf{D}^{(t)}$ on $\{1,\ldots, m\}$:
  - $\mathbf{D}^{(1)}(i) = 1/m$
  - given $\mathbf{D}^{(t)}$ and h_t: $D^{(t+1)}(i) = \dfrac{D^{(t)}(i) \times e^{-w_t h_t(x_i) y_i}}{Z_{t+1}}$

where Z_{t+1} normalization factor ($\mathbf{D}^{(t+1)}$ is a distribution): $Z_{t+1} = \sum_{i=1}^{m} D^{(t)}(i) \times e^{-w_t h_t(x_i) y_i}$

w_t is a weight: $w_t = \frac{1}{2}\ln(\frac{1}{\varepsilon_t} - 1) > 0$ as the error $\varepsilon_t < 0.5$

ε_t is the error of h_t on $\mathbf{D}^{(t)}$: $\varepsilon_t = \Pr_{i \sim D^{(t)}}[h_t(x_i) \neq y_i] = \sum_{i=1}^{m} D^{(t)}(i) \times 1_{[h_t(x_i) \neq y_i]}$

If example $\mathbf{x}_i$ is correctly classified then $h_t(\mathbf{x}_i) = y_i$ so at the next iteration t+1 its importance (probability distribution) will be decreased to:

$$D^{(t+1)}(i) = \frac{D^{(t)}(i) \times e^{-w_t}}{Z_{t+1}} = \frac{D^{(t)}(i) \times e^{-\frac{1}{2}\ln(\frac{1}{\varepsilon_t} - 1)}}{Z_{t+1}} = \frac{D^{(t)}(i) \times (\frac{1}{\varepsilon_t} - 1)^{-\frac{1}{2}}}{Z_{t+1}} = \frac{D^{(t)}(i) \times \sqrt{\frac{\varepsilon_t}{1-\varepsilon_t}}}{Z_{t+1}}$$

If example x_i is misclassified then $h_t(x_i) \neq y_i$ so at the next iteration t+1 its importance (probability distribution) will be increased to:

$$D^{(t+1)}(i) = \frac{D^{(t)}(i) \times e^{w_t}}{Z_{t+1}} = \frac{D^{(t)}(i) \times e^{\frac{1}{2}\ln(\frac{1}{\varepsilon_t} - 1)}}{Z_{t+1}} = \frac{D^{(t)}(i) \times (\frac{1}{\varepsilon_t} - 1)^{\frac{1}{2}}}{Z_{t+1}} = \frac{D^{(t)}(i) \times \sqrt{\frac{1-\varepsilon_t}{\varepsilon_t}}}{Z_{t+1}}$$