Assignment 1

Deadline: Sunday, 25th of April 2021

Upload your solutions at: https://tinyurl.com/AML-2021-ASSIGNMENT1

1. **(0.5 points)** Give an example of a finite hypothesis class $\mathcal{H}$ with VCdim($\mathcal{H}$) = 2021. Justify your choice.

2. **(0.5 points)** Consider $\mathcal{H}_{\text{balls}}$ to be the set of all balls in $\mathbf{R}^2$:
   $\mathcal{H}_{\text{balls}} = \{B(x,r),\ x \in \mathbb{R}^2,\ r \geq 0\}$, where $B(x,r) = \{y \in \mathbb{R}^2 \mid \| y - x \|_2 \leq r\}$
   As mentioned in the lecture, we can also view $\mathcal{H}_{\text{balls}}$ as the set of indicator functions of the balls B(x,r) in the plane: $\mathcal{H}_{\text{balls}} = \{ h_{x.r}: \mathbb{R}^2 \to \{0,1\}, h_{x.r} = \mathbf{1}_{B(x,r)}, x \in \mathbb{R}^2, r > 0\}$.
   Can you give an example of a set A in $\mathbf{R}^2$ of size 4 that is shattered by $\mathcal{H}_{\text{balls}}$? Give such an example or justify why you cannot find a set A of size 4 shattered by $\mathcal{H}_{\text{balls}}$.

3. **(1 point)** Let $\mathcal{X} = \mathbf{R}^2$ and consider $\mathcal{H}_\alpha$ the set of concepts defined by the area inside a right triangle ABC with the two catheti AB and AC parallel to the axes (Ox and Oy) and with AB/AC = α (fixed constant > 0). Consider the realizability assumption. Show that the class $\mathcal{H}_\alpha$ can be $(\epsilon, \delta) -$ PAC learned by giving an algorithm A and determining an upper bound on the sample complexity $m_H(\epsilon, \delta)$ such that the definition of PAC-learnability is satisfied.

4. **(1 point)** Consider $\mathcal{H}$ to be the class of all centered in origin sphere classifiers in the 3D space. A centered in origin sphere classifier in the 3D space is a classifier h_r that assigns the value 1 to a point if and only if it is inside the sphere with radius r > 0 and center given by the origin **O**(0,0,0). Consider the realizability assumption.
   a. show that the class $\mathcal{H}$ can be $(\epsilon, \delta) -$ PAC learned by giving an algorithm A and determining an upper bound on the sample complexity $m_H(\epsilon, \delta)$ such that the definition of PAC-learnability is satisfied. **(0.5 points)**
   b. compute VCdim($\mathcal{H}$). **(0.5 points)**

5. **(1 point)** Let $\mathcal{H} = \{h_\theta: \mathbb{R} \to \{0,1\}, h_\theta(x) = \mathbf{1}_{[\theta,\theta+1]\cup[\theta+2,+\infty)}(x), \theta \in \mathbb{R}\}$. Compute VCdim($\mathcal{H}$).

6. **(1 point)** Let $\mathcal{X}$ be an instance space and consider $\mathcal{H} \subseteq \{0,1\}^{\mathcal{X}}$ a hypothesis space with finite VC dimension. For each $x \in \mathcal{X}$, we consider the function z_x: $\mathcal{H} \to \{0,1\}$ such that $z_x(h) = h(x)$ for each $h \in \mathcal{H}$. Let $Z = \{z_x: \mathcal{H} \to \{0,1\}, x \in \mathcal{X}\}$. Prove that $\text{VCdim}(Z) < 2^{\text{VCdim(H)}+1}$.

Ex-officio: 0.5 points