Defaults

II. Circumscription

One way to express exceptional cases where a default should not apply is by using a predicate Ab (from abnormal):

$$\forall x.[Bird(x) \land \neg Ab(x) \supset Flies(x)].$$

Assuming that in KB we have the additional facts:

Bird(chilly)

Bird(tweety)

(tweety≠chilly)

¬Flies(chilly)

we would like to conclude that Tweety flies, whereas Chilly does not.

But KB $\not\models$ Flies(tweety) because there are interpretations that satisfy KB where Flies(tweety) is false. In these interpretations, the denotation of Tweety is included in the interpretation of Ab.

Defaults

The strategy for minimizing abnormality: we consider the interpretations of the KB where the interpretation of Ab is (a set) as small as possible.

The default conclusions are true in models of the KB where as few of the individuals as possible are abnormal.

In the previous example, we know that Chilly is an abnormal bird, but we don't know anything about Tweety. The interpretation of Ab must include Chilly, but excludes Tweety (because nothing dictates otherwise). This technique is called circumscribing the predicate Ab.

In general, a family of predicates Ab_i is used to describe various aspects of individuals. Chilly may be in the interpretation of Ab_1, but not in that of Ab_2 and so on.

Defaults

Minimal entailment

A new form of entailment is characterized in terms of properties of interpretations.

Let P be a fixed set of unary predicates Ab. Let $\mathcal{I}_1$=<D,I_1> and $\mathcal{I}_2$=<D,I_2> be interpretations over the same domain such that every constant and function is interpreted the same.

We define the relationship ≤ as following:

$$\mathcal{I}_1 \leq \mathcal{I}_2 \text{ iff for every } Pr \in P \text{ then } I_1[Pr] \subseteq I_2[Pr].$$

We say that $\mathcal{I}_1 < \mathcal{I}_2$ iff $\mathcal{I}_1 \leq \mathcal{I}_2$ and $\mathcal{I}_2 \nleq \mathcal{I}_1$.

$\mathcal{I}_1$ makes the interpretation of all Ab predicates smaller than $\mathcal{I}_2$. In other words, $\mathcal{I}_1$ is more normal than $\mathcal{I}_2$.

Defaults

Def. The minimal entailment $\models_{\leq}$ is defined as follows:

KB$\models_{\leq}$ α iff for every interpretation $\mathcal{I}$ such that $\mathcal{I}\models$KB, either $\mathcal{I}\models$ α or there is an $\mathcal{I}'$ such that $\mathcal{I}' < \mathcal{I}$ and $\mathcal{I}'\models$KB.

In our example,

$\forall$x.[Bird(x) ∧¬Ab(x) ⊃Flies(x)]

Bird(chilly)

Bird(tweety)

(tweety≠chilly)

¬Flies(chilly)

we have that KB$\not\models$ Flies(tweety) but KB$\models_{\leq}$ Flies(tweety).

If $\mathcal{I}\models$KB but $\mathcal{I} \not\models$ Flies(tweety) then $\mathcal{I}\models$Ab(tweety).

We take $\mathcal{I}'$ to be exactly $\mathcal{I}$, except that we remove the denotation of tweety from the interpretation of Ab.

Assuming that P={Ab}, we have that $\mathcal{I}' < \mathcal{I}$ and $\mathcal{I}'\models$KB.

But $\mathcal{I}'\not\models$ Ab(tweety), so $\mathcal{I}'\models$Flies(tweety).

Defaults

Thus, in the minimal models of KB, Tweety is a normal bird

$$KB \vDash_{\leq} \neg Ab(tweety),$$

therefore $KB \vDash_{\leq} Flies(tweety)$.

Instead, in all the models of KB, Chilly is an abnormal bird.

In this reasoning, the only default step was to conclude that Tweety was a normal bird; the rest was ordinary deductive reasoning.

Obs. The “most normal” models of the KB may not all satisfy exactly the same sentences.

Defaults

For example, suppose that the KB contains:

$$\text{Bird}(c)$$
$$\text{Bird}(d)$$
$$\neg\text{Flies}(c) \lor \neg\text{Flies}(d)$$
$$\forall x.[\text{Bird}(x) \land \neg\text{Ab}(x) \supset \text{Flies}(x)].$$

In any model of the KB, the interpretation of Ab must contain either the denotation of c or the denotation of d. Any model containing other abnormal individuals would not be minimal (e.g. the model including both c and d).

So, in any minimal model $\mathcal{I} \models \text{KB}$, we have either $\mathcal{I} \models \text{Ab}(c)$ or $\mathcal{I} \models \text{Ab}(d)$.

If $\mathcal{I} \models \text{Ab}(c)$ then $\text{KB} \not\models_{\leq} \text{Flies}(c)$. Similarly, if $\mathcal{I} \models \text{Ab}(d)$ then $\text{KB} \not\models_{\leq} \text{Flies}(d)$.

We cannot conclude by default that c is a normal bird, nor that d is. But we can conclude by default that one of them is:

$$\text{KB} \models_{\leq} \text{Flies}(c) \lor \text{Flies}(d)$$

Defaults

<u>Obs</u>. CWA and GCWA have a different behavior.

$KB^{+}=KB \cup \{\neg p \mid p$ is atomic and $KB \not\models p\}$

$KB^{*}= KB \cup \{\neg p \mid$ for all collections of atoms $q_1,\ldots q_n$, if $KB \models (p \vee q_1 \vee \ldots \vee q_n)$ then $KB \models (q_1 \vee \ldots \vee q_n)\}$.

Because neither $KB \not\models Ab(c)$ nor $KB \not\models Ab(d)$, it results that

$$KB^{+} \supset KB \cup \{\neg Ab(c), \neg Ab(d)\}.$$

So, $KB \models_c (Flies(c) \wedge Flies(d))$, that is KB^{+} is not consistent.

On the other hand, under GCWA, $\neg Ab(c) \notin KB^{*}$ and $\neg Ab(d) \notin KB^{*}$.

$$[KB \models Ab(c) \vee \neg Bird(c) \vee Flies(c) \text{ but } KB \not\models \neg Bird(c) \vee Flies(c)]$$

So, under GCWA we cannot conclude anything about Flies(c) or Flies(d) (or their disjunction).

Defaults

In the circumscription case, one model of the KB is preferred to another one if it has less abnormal individuals.

Assuming that we have the statements: Richard Nixon was both quaker (thus implicitly pacifist) and republican (thus implicitly not pacifist), we have the following KB:

$$Republican(nixon) \wedge Quaker(nixon)$$

$$\forall x.[Republican(x) \wedge \neg Ab_2(x) \supset \neg Pacifist(x)]$$

$$\forall x.[Quaker(x) \wedge \neg Ab_3(x) \supset Pacifist(x)]$$

Defaults

In the circumscription case, one model of the KB is preferred to another one if it has less abnormal individuals.

Assuming that we have the statements: Richard Nixon was both quaker (thus implicitly pacifist) and republican (thus implicitly not pacifist), we have the following KB:

$$Republican(nixon) \land Quaker(nixon)$$

$$\forall x.[Republican(x) \land \neg Ab_2(x) \supset \neg Pacifist(x)]$$

$$\forall x.[Quaker(x) \land \neg Ab_3(x) \supset Pacifist(x)]$$

If we circumscribe the predicates Ab_2 and Ab_3, we have two minimal models:

$$\mathcal{I}_1 \models Ab_2(nixon) \text{ and } \mathcal{I}_1 \models Pacifist(nixon)$$

$$\mathcal{I}_2 \models Ab_3(nixon) \text{ and } \mathcal{I}_2 \models \neg Pacifist(nixon)$$

So, $KB \not\models_{\leq} Pacifist(nixon)$ and $KB \not\models_{\leq} \neg Pacifist(nixon)$.

If, for example, we give priority to religious convictions rather than to political ones, we can express it by prioritized circumscription, where we prefer the (minimal) model that minimizes Ab_3.

Defaults

III. Default Logic

It provides a mechanism that explicitly specifies which sentences should be added to the KB, maintaining consistency.

For example, if Bird(t) is entailed by the KB, we might want to add the default assumption Flies(t), if it is consistent to do so.

In default logic, a KB consists of two parts: a set $\mathcal{F}$ of first-order sentences and a set $\mathcal{D}$ of default rules, which are specifications of what assumptions can be made and when.

The role of the default logic is to specify the following:

- -the appropriate set of implicit beliefs that incorporate the facts in $\mathcal{F}$;
- -as many default assumptions as possible, given the default rules in $\mathcal{D}$;
- -the logical entailments inferred from the implicit beliefs and the default assumptions.

Defaults

Default rules

A default rule is written in the form <α:β/δ >, where α is the prerequisite, β is the justification and δ is the conclusion. δ is considered to be true if α is true and it is consistent to believe β (that is $\neg\beta$ is not true)

<Bird(tweety):Flies(tweety)/Flies(tweety)>

A rule where the justification and conclusion are the same is called a normal default rule and it is written as Bird(tweety) $\Rightarrow$ Flies(tweety).

A rule can be formulated using free variables:

<Bird(x):Flies(x)/Flies(x)>

represents the set of all its instances, formed by replacing x by a ground term.

Defaults

Default extensions

Given a default theory KB=($\mathcal{F}$,$\mathcal{D}$), what are the sentences that should be believed?

We define an extension of the theory as a reasonable set of beliefs given a default theory.

Def. A set of sentences ε is an extension of a default theory ($\mathcal{F}$,$\mathcal{D}$) if for every sentence π we have:

$$\pi \in \varepsilon \text{ iff } \mathcal{F} \cup \{\delta \mid \langle \alpha : \beta / \delta \rangle \in \mathcal{D}, \alpha \in \varepsilon, \neg\beta \notin \varepsilon\} \models \pi.$$

Thus, an extension is the set of all entailments of $\mathcal{F} \cup \Delta$, where Δ is a suitable set of assumptions.

Obs. The definition of ε does not say how to find an ε, but ε is completely characterized by its set of applicable assumptions Δ.

Defaults

Example:

$\mathcal{F}$={Bird(tweety), Bird(chilly), ¬Flies(chilly)}

$\mathcal{D}$={Bird(x) ⇒ Flies(x)}

Let ε= $\mathcal{F}$∪{Flies(tweety)}.

Flies(tweety) is the only assumption applicable to ε.

$\left.\begin{array}{l}\text{Bird(tweety)} \in \varepsilon \\ \neg\text{Flies(tweety)} \notin \varepsilon\end{array}\right\} \Rightarrow$ Flies(tweety) is applicable.

Flies(t) is not applicable for any other t (in our example t could be only chilly).

Thus, Flies(tweety) is the only applicable assumption, so ε is an extension

(it can be proven that it is the only one).

Obs. An extension ε of a default theory ($\mathcal{F}$,$\mathcal{D}$) is inconsistent iff $\mathcal{F}$ is inconsistent.

Defaults

Multiple extensions

Consider the following default theory:

$\mathcal{F}$={Republican(nixon), Quaker(nixon)}

$\mathcal{D}$={Republican(x) ⇒ ¬Pacifist(x), Quaker(x) ⇒Pacifist(x)}

Let ε_1 be the extension characterized by the assumption Pacifist(nixon) and ε_2 be the extension characterized by the assumption ¬Pacifist(nixon).

ε_1 and ε_2 are extensions because their assumptions are applicable and there are no other applicable ones (for t≠nixon).

The empty set of assumptions does not give an extension, because both Pacifist(Nixon) and ¬Pacifist(Nixon) would be applicable. For any other extensions, assumptions would be of the form Pacifist(t) or ¬Pacifist(t), but none are applicable for t≠nixon.

Thus, ε_1 and ε_2 are the only extensions possible.

Defaults

On the basis of what we know, either Nixon is a pacifist or he is not a pacifist are reasonable beliefs. There are two options:

1. A skeptical reasoner will only believe those sentences that are common to all extensions of the default theory;
2. A credulous reasoner will simply choose an extension as a set of sentences to believe.

In some cases, the existence of multiple extensions is an indication that we have not said enough to make a reasonable decision.

Defaults

In the previous example, we may want to say that the default about Quakers should apply only to individuals that are not politically active.

If we add in $\mathcal{F}$ the fact

$$\forall x.[\text{Republican}(x) \supset \text{Political}(x)],$$

we can replace the rule Quaker(x) ⇒ Pacifist(x) by a non-normal one:

$$\frac{Quaker(x) \colon [Pacifist(x) \wedge \neg \mathrm{Political(x)}]}{Pacifist(x)}$$

$\mathcal{F}$={Republican(nixon), Quaker(nixon), ∀x.[Republican(x) ⊃ Political(x)]}

$\mathcal{D}$={Republican(x) ⇒ ¬Pacifist(x), $\frac{Quaker(x) \colon [Pacifist(x) \wedge \neg \mathrm{Political(x)}]}{Pacifist(x)}$}

For ordinary Quakers, the assumption is that they are pacifists. But for Quaker Republicans like Nixon, we assume that they are not pacifists.

Defaults

If we replace ∀x.[Republican(x) ⊃ Political(x)] by the default rule

Republican(x) ⇒ Political(x),

$\mathcal{F}$={Republican(nixon), Quaker(nixon)}

$\mathcal{D}$={Republican(x) ⇒ ¬Pacifist(x), Republican(x) ⇒ Political(x),

$$\frac{Quaker(x):[Pacifist(x) \wedge \neg \mathrm{Political(x)}]}{Pacifist(x)}\}$$

then we have two extensions:

-one characterized by the assumptions {¬Pacifist(nixon), Political(nixon)}

-one characterized by the assumptions {Pacifist(nixon)}

Defaults

Resolving conflicts among default rules is crucial when we deal with concept hierarchies.

For example, for the following KB

$\mathcal{F}$={∀x.[Penguin(x) ⊃Bird(x)], Penguin(chilly)}

$\mathcal{D}$={Bird(x) ⇒ Flies(x), Penguin(x) ⇒¬Flies(x)}

we have two extensions: one where Chilly is assumed to fly and one where Chilly is assumed not to fly.

The default that penguins do not fly should preempt the more general default that birds fly.

$$\frac{Bird(x) : [Flies(x) \wedge \neg \mathrm{Penguin(x)}]}{Flies(x)}$$

Unlike defaults in an inheritance mechanism, the default logic do not automatically prefer the most specific defaults.