Lab 2

Time Series in R[1]

1. White Noise and Moving Average Example

$W_t \sim \text{iid } N(0,\sigma_w^2)$

$V_t=\frac{1}{3}(W_{t-1}+W_t+W_{t+1})$

```
w = rnorm(500,0,1)
v = filter(w, sides=2, filter=rep(1/3,3))  # moving average
 #  filter(w, sides=1, filter=c(1, phi))  # MA(1) with parameter phi
#   https://www.rdocumentation.org/packages/stats/versions/3.6.2/topics/filter

par(mfrow=c(2,1))
plot.ts(w, main="white noise")
plot.ts(v, ylim=c(-3,3), main="moving average")
```

2. Autoregression Example

$X_t = X_{t-1} - 0.9X_{t-2} + W_t$

```
w = rnorm(550,0,1)     # 50 extra to avoid startup problems
x = filter(w, filter=c(1,-.9), method="recursive")[-(1:50)]        # remove first 50
plot.ts(x, main="autoregression")
```

3. Random Walk with Drift

$X_t = \delta + X_{t-1} + W_t$

```
set.seed(154)             # so you can reproduce the results
w = rnorm(200); x = cumsum(w)
```

[1] Robert H. Shumway, David S. Stoffer. Time Series Analysis and Its Applications – with R examples, Springer 2017

```
wd = w +.2; xd = cumsum(wd)
plot.ts(xd, ylim=c(-5,55), main="random walk", ylab='')
lines(x, col=4)            #adds a line to a plot
abline(h=0, col=4, lty=2, lwd=3); abline(a=0, b=.2, lty=2)    #adds a line to a plot
```

4. Signal in Noise

$X_t = 2 \cos (2\pi(t+15)/50) + W_t$

```
cs = 2*cos(2*pi*1:500/50 + .6*pi);
w = rnorm(500,0,1)
par(mfrow=c(3,1), mar=c(3,2,2,1), cex.main=1.5)
# cex.main to magnify the titles; mar sets the margins
plot.ts(cs, main=expression(2*cos(2*pi*t/50+.6*pi)))
plot.ts(cs+w, main=expression(2*cos(2*pi*t/50+.6*pi) + N(0,1))
plot.ts(cs+5*w, main=expression(2*cos(2*pi*t/50+.6*pi) + N(0,25)))
```

The ACF – the autocovariance/autocorrelation function

```
w = rnorm(500);
acf(w, lag.max=20, na.action = na.pass, plot=TRUR)  #by default type='correlation'

acf(w, type='covariance', lag.max=20, na.action = na.pass, plot=T,
main=expression('white noise'))
```

Exercise. Apply the ACF for the all the time series above. Make comments for each case, especially for no. 3

Exercise. Consider a signal-plus-noise model of the general form:

$$X_t = S_t + W_t,$$

where W_t ~ iid N(0,1). Simulate and plot n = 200 observations from the model, knowing that

$$S_t = \begin{cases} 0, & t = 1, \ldots, 100 \\ 10 \exp\left(-\dfrac{t-100}{100}\right) \cos\left(\dfrac{2\pi t}{4}\right), & t = 101, \ldots, 200 \end{cases}$$

Compare the general appearance of the time series with the earthquake series EQ5 from the R package "astsa".

```
> install.packages('astsa')
> library(astsa)
```

Exercise. Generate n = 100 observations from the autoregression

$$X_t = -0.9X_{t-2} + W_t$$

with σ_w^2 = 1. Next, apply the moving average filter

$$V_t = \frac{1}{4}(X_t + X_{t-1} + X_{t-2} + X_{t-3})$$

to X_t, the data you generated. Now plot X_t as a line and superimpose V_t as a dashed line. Comment on the behavior of X_t and how applying the moving average filter changes that behavior.

Show a time plot of two AR(1) processes, one with Φ = 0.9 and the other one with Φ=-0.9; in both cases, σ_w^2 = 1.

```
par(mfrow=c(2,1))
plot(arima.sim(list(order=c(1,0,0), ar=.9), n=100),
ylab="x",main=(expression(AR(1)~~~phi==+.9)))
plot(arima.sim(list(order=c(1,0,0), ar=-.9), n=100), ylab="x",
main=(expression(AR(1)~~~phi==-.9)))
```

Using "filter", simulate and plot n=60 observations of an AR(1) process with Φ = 1.9.

Show a time plot of two MA(1) processes, one with Θ = 0.9 and the other one with Θ=-0.9; in both cases, σ_w^2 = 1.

```
par(mfrow = c(2,1))
plot(arima.sim(list(order=c(0,0,1), ma=.9), n=100),
ylab="x",main=(expression(MA(1)~~~theta==+.9)))
plot(arima.sim(list(order=c(0,0,1), ma=-.9), n=100),
ylab="x",main=(expression(MA(1)~~~theta==-.9)))

# by default WN is N(0,1); it can be changed using rand.gen
# x<-arima.sim(list(order=c(0,0,1), ma=-.9),n=100, rand.gen = function(n, ...)
rnorm(n,sd=5))
```

Using "filter", simulate and plot n=60 observations of an MA(1) process with Θ = 1.9.