(cont...)

## Goodness-of-fit for linear regression

"All models are wrong, but some are useful."

George Box (1919-2013), pioneering statistician

How well does a model fit our data? In many cases, answers are based on analysis of the residuals:

- -normality of residuals;
- -homoscedasticity: constant residual variance throughout the range of the predicted values.

Another measure is that of "variance explained" R<sup>2</sup> (see C11 slide 14).

```
# example adapted from [1] p. 157
# house sales.csv taken from
#https://drive.google.com/drive/folders/0B98qpkK5EJemYnJ1ajA1ZVJwMzg
house <- read.csv(file.path('C:\\Users\\Marina\\Desktop\\curs Statistics for Data
Science\\house sales.csv'), sep='\t')
house 98105 <- house[house$ZipCode == 98105, ] #select the houses in an
                                                  #area
sample house 98105<- house 98105[sample(nrow(house 98105),30), ]
model <- Im(AdjSalePrice ~ SqFtTotLiving + SqFtLot + Bathrooms +
           Bedrooms + BldgGrade, data=sample house 98105)
summary(model)
#Multiple R-squared: 0.8218
#that is R<sup>2</sup> -- we say that 82.18% of variability can be explained through the
#linear model – by this measure, the model fits well
shapiro.test(residuals(model))
                 Shapiro-Wilk normality test
```

## $\label{eq:data:model} \begin{array}{l} \text{data: residuals(model)} \\ W = 0.96923, \ p\text{-value} = 0.5184 \\ \text{\#normality assumption is met for our model} \end{array}$

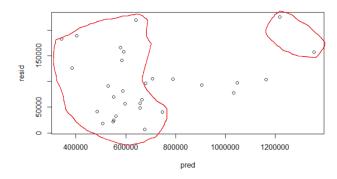
#Obs. Models are robust to non-normality to a certain extent

```
# homoscedasticity
```

resid <- abs(residuals(model)) #interested in the magnitude of the residuals #names(resid)<-NULL #to remove the names of the resid vector

pred<-predict(model)</pre>

plot(pred,resid)



We see a greater variation of residuals for lower-valued homes (left) and for the higher-valued homes (right) and much less variation for the mid-valued homes -- homoscedasticity assumption not met. It may indicate a "cause" missing from the linear equation (causes ->effect) -- worth investigating other (linear) models.

Repeat the same steps for the goodness-of-fit for the example below (taken from Lab6):

```
longley X \leftarrow [n] = 100 # number of people employed Y \leftarrow [n] = 100 # population Y \leftarrow [n] = 100 # pop
```

What is your conclusion after this analysis?

## **Nonlinear regression**

!!Please read C12

```
# we use house sales.csv to generate the data for the nonlinear regression
#examples
house <- read.csv(file.path('C:\\Users\\Marina\\Desktop\\curs Statistics for Data S
cience\\house sales.csv'), sep='\t')
house 98105 <- house[house$ZipCode == 98105, ]
sample house 98105<- house 98105[sample(nrow(house 98105),30), ]
#random selection of 30 rows from the dataset for a clearer visualization of the
#points
Im 30 <- Im(AdjSalePrice ~ SqFtTotLiving + SqFtLot + Bathrooms +</pre>
            Bedrooms + BldgGrade, data=sample house 98105)
resid <- abs(residuals(lm 30))
pred<-predict(Im 30)</pre>
#we consider (pred,resid) as (cause,effect) data for nonlinear regression model
plot(pred, resid, ylim = c(-100000, 300000))
library(splines)
#interpolation using splines
func = splinefun(x=pred, y=resid, method="natural",ties=mean)
#method= the type of the spline used
#ties = the name of a function specifying how to handle duplicate x values. The y
#values corresponding to the same x value are passed to the function, which
#return a single number (mean in our case)
xmin=min(pred)
xmax=max(pred) # [xmin,xmax] is the interval for regression
new points=seg(xmin,xmax,length=50)
allpoints=c(new points, pred) #include the knots among the points for prediction
allpoints ordered = allpoints [order(allpoints)] #necessary to visualize well the
                                                  #regression curve
```

```
#lines(pred,func(pred), type="p", col=2)
lines(allpoints_ordered,func(allpoints_ordered), type="l", col=2)
```

## #smoothing using splines

fit <- smooth.spline(pred, resid, all.knots=T, spar=0.7)
#try for different values of spar in [0,1]
#spar is alpha parameter in SS(g) – see C12 slide 3
#if alpha=0 the regression curve goes through all the knots – like interpolation
#if alpha=1 the regression curbe becomes smoother and it doesn't go through
#(all) the knots

res <- stats:::predict.smooth.spline(fit, allpoints\_ordered)\$y lines(allpoints\_ordered,res, type="l", col=4) # plot the fitted spline