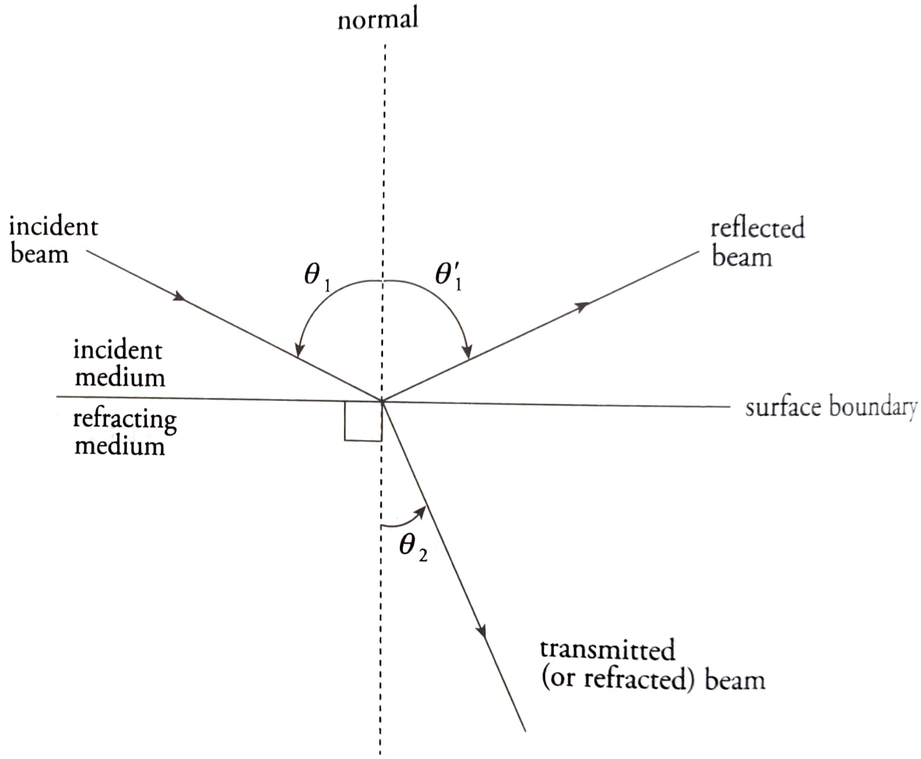
## REFLECTION AND REFRACTION

Imagine a beam of light directed toward a smooth transparent surface. When it hits this surface, some of its energy will be reflected off the surface and some will be transmitted into the new medium. Some of the transmitted light will be absorbed, which is the source of heating from radiation discussed in Thermodynamics, while some of the light will emerge from the new medium as a transmitted beam. We can figure out the directions of the reflected and transmitted beams by calculating the angles that the beams make with the normal to the interface. In the following figure, an incident beam strikes the boundary of another medium; it could be a beam of light in air striking a piece of glass. Notice all angles are measured from the normal.

**Tricky Problems!**
Remember that the normal is always perpendicular to the surface. When a question asks for the angle of incidence or reflection, it wants the angle formed with the normal, not the angle formed with the surface. Make sure you know if you're solving for the angle of incidence, the angle of reflection, or the angle of refraction.




The angle that the **incident beam** makes with the normal is called the **angle of incidence**, or $\theta_1$. The angle that the **reflected beam** makes with the normal is called the **angle of reflection**, $\theta'_1$, and the angle that the **transmitted beam** makes with the normal is called the **angle of refraction**, $\theta_2$. The incident, reflected, and transmitted beams of light all lie in the same plane.

The relationship between $\theta_1$ and $\theta'_1$ is pretty easy; it is called the **Law of Reflection**:

$$\theta_1 = \theta'_1$$

The Law of Reflection basically states the Angle of Reflection is equal to the Angle of Incidence. In order to describe how $\theta_1$ and $\theta_2$ are related, we first need to talk about a medium's index of refraction.

When light travels through empty space (vacuum), its speed is $c = 3.00 \times 10^8$ m/s; this is one of the fundamental constants of nature. But when light travels through a medium (such as water or glass), it's constantly being absorbed and re-emitted by the atoms that compose the material and, as a result, its apparent speed, $v$, is some fraction of $c$. The reciprocal of this fraction,

$$n = \frac{c}{v}$$

Equation Sheet

is called the medium's **index of refraction,** $n$. For example, since the speed of light in water is $v = 2.25 \times 10^8$ m/s, the index of refraction of water is

$$n = \frac{3.00 \times 10^8 \text{ m/s}}{2.25 \times 10^8 \text{ m/s}} = 1.33$$

Note that $n$ has no units; it's also never less than 1, since light always travels slower in a medium than in a vacuum.

***n* of air**
The index of refraction of air is 1.00029, which is so close to 1 that the norm is to consider it to just be 1.

The equation that relates $\theta_1$ and $\theta_2$ involves the index of refraction of the incident medium ($n_1$) and the index of refraction of the refracting medium ($n_2$); it's called **Snell's Law**:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Equation Sheet

If $n_2 > n_1$, then Snell's Law tells us that $\theta_2 < \theta_1$; that is, the beam will bend (**refract**) *toward* the normal as it enters the medium. On the other hand, if $n_2 < n_1$, then $\theta_2 > \theta_1$, and the beam will bend *away* from the normal.

**Example 1** A beam of light in air is incident upon a piece of glass, striking the surface at an angle of 30°. If the index of refraction of the glass is 1.5, what are the angles of reflection and refraction?

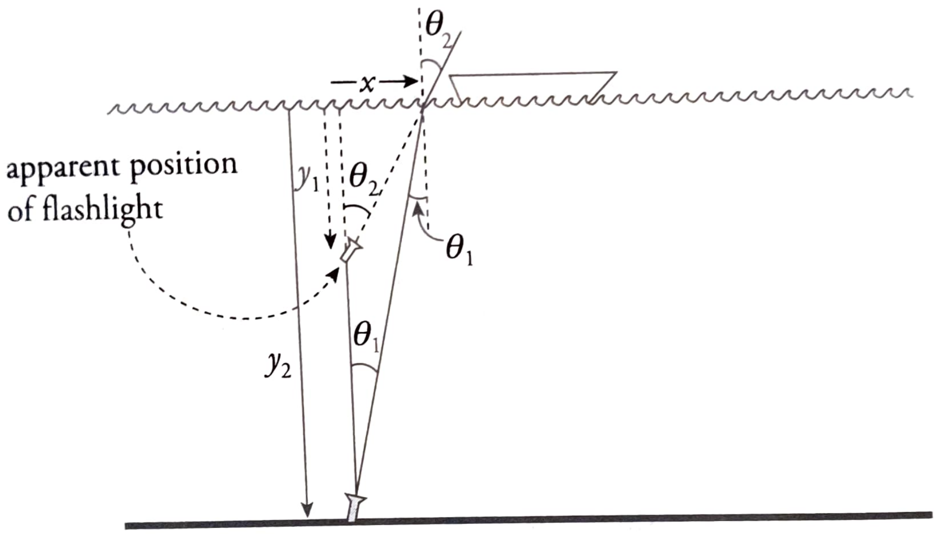
**Solution.** If the light beam makes an angle of 30° with the surface, then it makes an angle of 60° with the normal; this is the angle of incidence. By the Law of Reflection, then, the angle of reflection is also 60°. We use Snell's Law to find the angle of refraction. The index of refraction of air is close to 1, so we can say that $n = 1$ for air.

$$\begin{aligned} n_1 \sin \theta_1 &= n_2 \sin \theta_2 \\ (1) \sin 60° &= 1.5 \sin \theta_2 \\ \sin \theta_2 &= 0.5774 \\ \theta_2 &= 35° \end{aligned}$$

Note that $\theta_2 < \theta_1$, as we would expect since the refracting medium (glass) has a greater index than does the incident medium (air).

> **Example 2** A fisherman drops a flashlight into a lake that's 10 m deep. The flashlight sinks to the bottom where its emerging light beam is directed almost vertically upward toward the surface of the lake, making a small angle ($\theta_1$) with the normal. How deep will the flashlight appear to be to the fisherman? (Use the fact that tan $\theta$ is almost equal to sin $\theta$ if $\theta$ is small.)

**Solution.** Take a look at the figure below.




Since the refracting medium (air) has a lower index than the incident medium (water), the beam of light will bend away from the normal as it emerges from the water. As a result, the fisherman will think that the flashlight is at a depth of only $y_1$, rather than its actual depth of $y_2$ = 10 m. By simple trigonometry, we know that

$$\tan\theta_1 = \frac{x}{y_2} \qquad \text{and} \qquad \tan\theta_2 = \frac{x}{y_1}$$

So,

$$\frac{y_1}{y_2} = \frac{\tan\theta_1}{\tan\theta_2}$$

Snell's Law tells us that $n_1 \sin\theta_1 = \sin\theta_2$ (since $n_2 = n_{air} = 1$), so

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{1}{n_1}$$

Using the approximations $\sin\theta_1 \approx \tan\theta_1$ and $\sin\theta_2 \approx \tan\theta_2$, we can write

$$\frac{\tan\theta_1}{\tan\theta_2} \approx \frac{\sin\theta_1}{\sin\theta_2}$$

which means

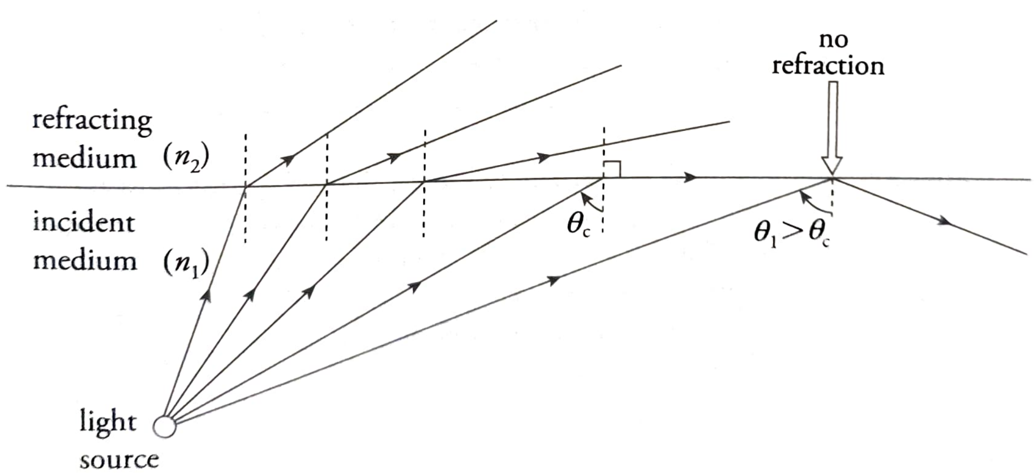
$$\frac{y_1}{y_2} = \frac{1}{n_1}$$

Because $y_2 = 10$ m and $n_1 = n_{water} = 1.33$,

$$y_1 = \frac{y_2}{n_1} = \frac{10 \text{ m}}{1.33} = 7.5 \text{ m}$$

## TOTAL INTERNAL REFLECTION

When a beam of light strikes the boundary to a medium that has a lower index of refraction, the beam bends away from the normal. As the angle of incidence increases, the angle of refraction becomes larger. At some point, when the angle of incidence reaches a **critical angle**, $\theta_c$, the angle of refraction becomes 90°, which means the refracted beam is directed along the surface.




For angles of incidence that are greater than $\theta_c$, there is *no* angle of refraction; the entire beam is reflected back into the original medium. This phenomenon is called **total internal reflection** (sometimes abbreviated **TIR**).

Total internal reflection occurs when:

1) $n_1 > n_2$

and

2) $\theta_1 > \theta_c$, where $\theta_c = \sin^{-1}(n_2/n_1)$

**Subscripts**
Always think of $n_1$ or $\theta_1$ as the incidence ray, meaning where the light comes from. Treat $n_2$ or $\theta_2$ as the resulting ray.

Notice that total internal reflection cannot occur if $n_1 < n_2$. This is because the largest output of $\sin \theta$ is 1, so the largest input of $\sin^{-1}(x)$ is 1. If $n_1 > n_2$, then total internal reflection is a possibility; it will occur if the angle of incidence is large enough; that is, if it's greater than the critical angle, $\theta_c$.

**Example 3** What is the critical angle for total internal reflection between air and water? In which of these media must light be incident for total internal reflection to occur?

**Solution.** First, total internal reflection can occur only when the light is incident in the medium that has the greater refractive index and strikes the boundary to a medium that has a lower index. So, in this case, total internal reflection can occur only when the light source is in the water and the light is incident upon the water/air surface. The critical angle is found as follows:

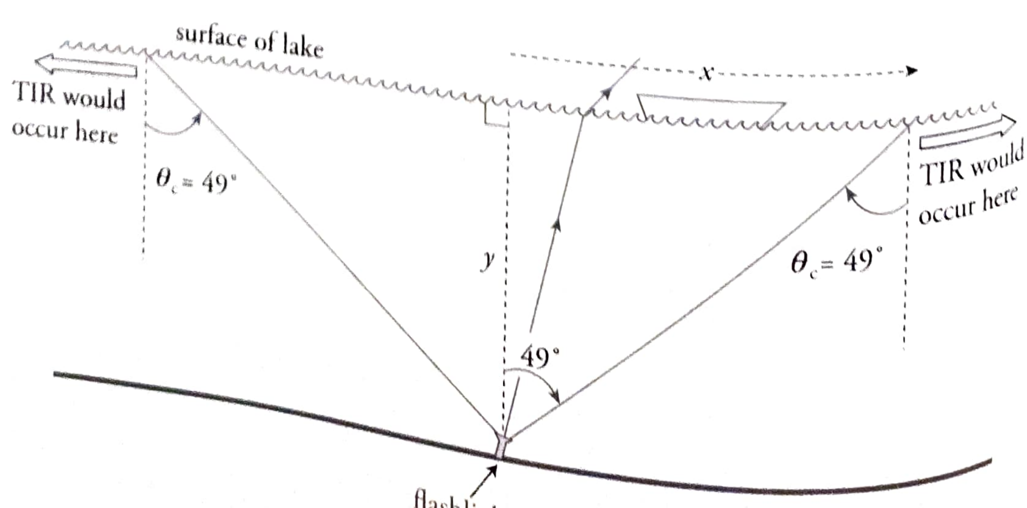
$$\sin \theta_c = \frac{n_2}{n_1} \Rightarrow \sin \theta_c = \frac{n_{\text{air}}}{n_{\text{water}}} = \frac{1}{1.33} \Rightarrow \sin \theta_c = 0.75 \Rightarrow \theta_c = 49°$$

Total internal reflection will occur if the light from the water strikes the water/air boundary at an angle of incidence greater than 49°.

**Example 4** How close must the fisherman be to the flashlight in Example 2 in order to see the light it emits?

**Remember to Check**
For optics problems, you need to always remember to check if the refracted light ray undergoes total internal reflection. If it undergoes total internal reflection, there will be no refracted ray. Hence, if a trick problem asks for an angle of refraction for a situation that undergoes total internal reflection, there is no angle of refraction.

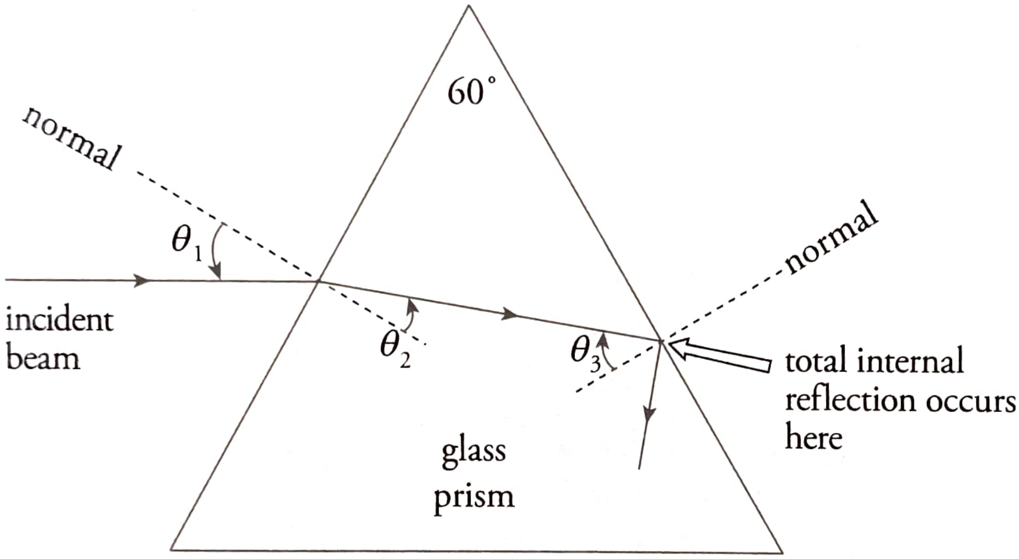
**Solution.** In order for the fisherman to see the light, the light must be transmitted into the air from the water; that is, it cannot undergo total internal reflection. The figure below shows that, within a circle of radius $x$, light from the flashlight will emerge from the water. Outside this circle, the angle of incidence is greater than the critical angle, and the light would be reflected back into the water, rendering it undetectable by the fisherman above.



Because the critical angle for total internal reflection at a water/air interface is 49° (as we found in the preceding example), we can solve for $x$:

$$\tan 49^\circ = \frac{x}{y} \quad \Rightarrow \quad x = y \tan 49^\circ = (10 \text{ m}) \tan 49^\circ = 11.5 \text{ m}$$

**Example 5** The refractive index for the glass prism shown below is 1.55. In order for a beam of light to experience total internal reflection at the right-hand face of the prism, the angle $\theta_1$ must be smaller than what value?




**Solution.** Total internal reflection will occur at the glass/air boundary if $\theta_3$ is greater than the critical angle, $\theta_c$, which we can calculate this way:

$$\sin \theta_c = \frac{n_{\text{air}}}{n_{\text{glass}}} = \frac{1}{1.55} \quad \Rightarrow \quad \theta_c = 40^\circ$$

Because $\theta_3 = 60^\circ - \theta_2$, total internal reflection will take place if $\theta_2$ is smaller than 20°. Now, by Snell's Law, $\theta_2 = 20^\circ$ if

$$\begin{aligned} n_{\text{air}} \sin \theta_1 &= n_{\text{glass}} \sin \theta_2 \\ \sin \theta_1 &= 1.55 \sin 20^\circ \\ \theta_1 &= 32^\circ \end{aligned}$$

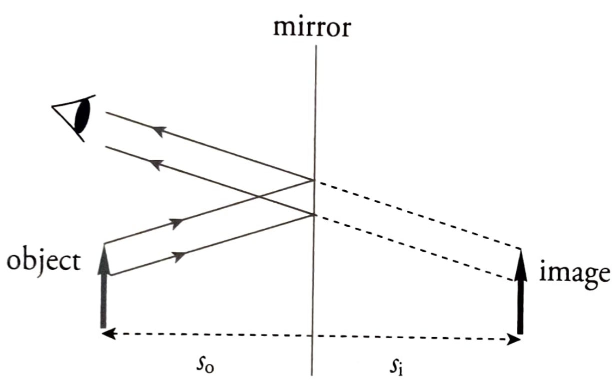
Therefore, total internal reflection will occur at the right-hand face of the prism if $\theta_1$ is smaller than 32°.

## MIRRORS

A **mirror** is an optical device that forms an image by reflecting light. We've all looked into a mirror and seen images of nearby objects, and the purpose of this section is to analyze these images mathematically. We begin with a plane mirror, which is flat, and is the simplest type of mirror. Then we'll examine curved mirrors; we'll have to use geometrical methods or algebraic equations to analyze the patterns of reflection from these.

### Plane Mirrors

The figure below shows an object (denoted by a vertical, bold arrow) in front of a flat mirror. Light that's reflected off the object strikes the mirror and is reflected back to our eyes. The directions of the rays reflected off the mirror determine where we perceive the image to be.




There are four questions we'll answer about the image formed by a mirror:

(1) Where is the image?

(2) Is the image real, or is it virtual?

(3) Is the image upright, or is it inverted?

(4) What is the height of the image (compared with that of the object)?

When we look at ourselves in a mirror, it seems like our image is *behind* the mirror, and, if we take a step back, our image also takes a step back. The Law of Reflection can be used to show that the image seems as far behind the mirror as the object is in front of the mirror. This answers question (1).

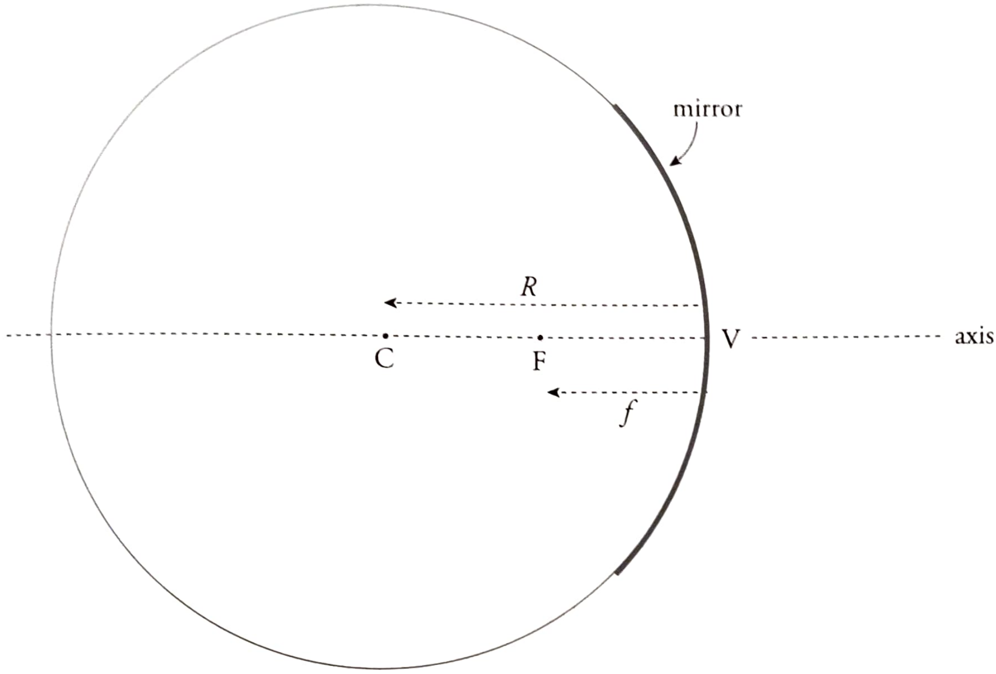
An image is said to be **real** if light rays actually focus at the image. A real image can be projected onto a screen. For a flat mirror, light rays bounce off the front of the mirror; so, of course, no light focuses behind it. Therefore, the images produced by a flat mirror are not real; they are **virtual**. This answers question (2).

When we look into a flat mirror, our image isn't upside down; flat mirrors produce upright images, and question (3) is answered.

Finally, the image formed by a flat mirror is neither magnified nor diminished (minified) relative to the size of the object. This answers question (4).

## Spherical Mirrors

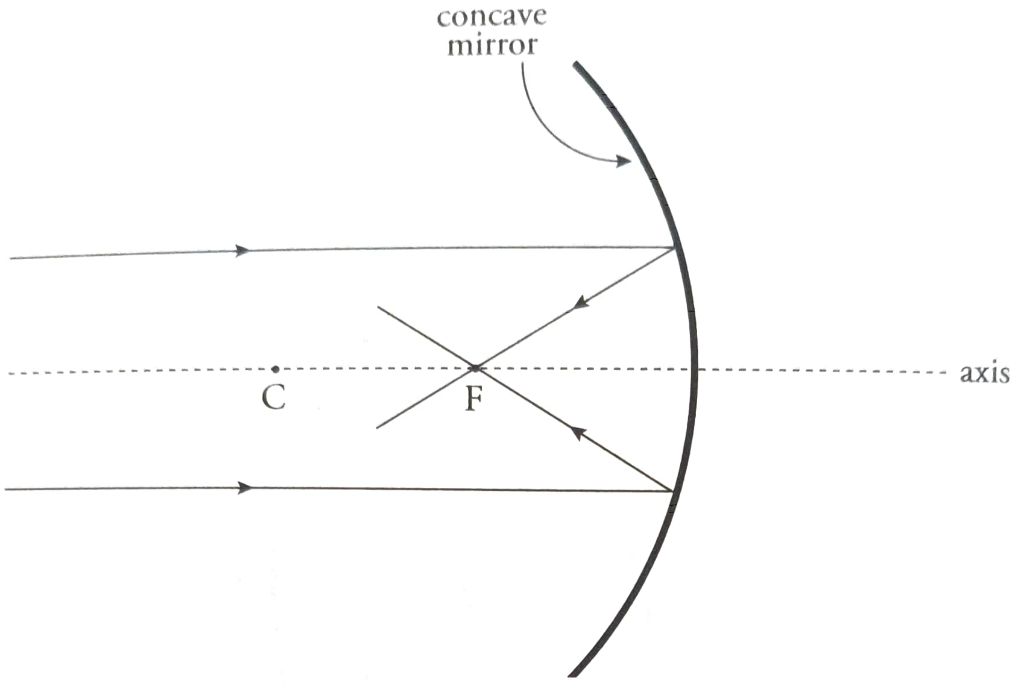
A **spherical mirror** is a mirror that's curved in such a way that its surface forms part of a sphere.



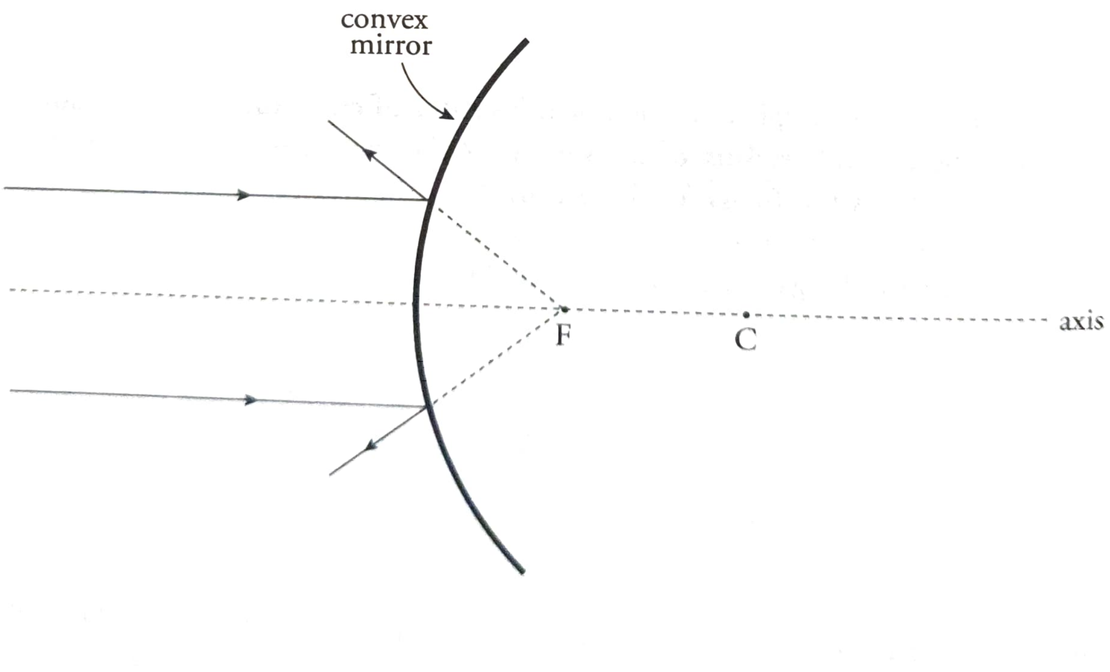

The center of this imaginary sphere is the mirror's **center of curvature**, and the radius of the sphere is called the mirror's **radius of curvature**, $R$. Halfway between the mirror and the center of curvature, C, is the **focus** (or **focal point**), F. The intersection of the mirror's optic **axis** (its axis of symmetry) with the mirror itself is called the **vertex**, V, and the distance from V to F is called the **focal length**, $f$, equal to one-half of the radius of curvature:

$$f = \frac{R}{2}$$

If the mirror had a parabolic cross-section, then any ray parallel to the axis would be reflected by the mirror through the focal point. Spherical mirrors do this for incident light rays near the axis (**paraxial rays**) because, in the region of the mirror that's close to the axis, the shapes of a parabolic mirror and a spherical mirror are nearly identical.




The previous two figures illustrate a **concave mirror**, a mirror whose reflective side is *caved in* toward the center of curvature. The following figure illustrates the **convex mirror**, which has a reflective side curving away from the center of curvature. We will call F the **virtual focus** in this case.




## RAY TRACING FOR MIRRORS

One method of answering the four questions previously listed concerning the image formed by a mirror involves a geometric approach called **ray tracing**. Representative rays of light are sketched in a diagram that depicts the object and the mirror; the point at which the reflected rays intersect (or appear to intersect) is the location of the image.

Some rules governing rays are as follows:

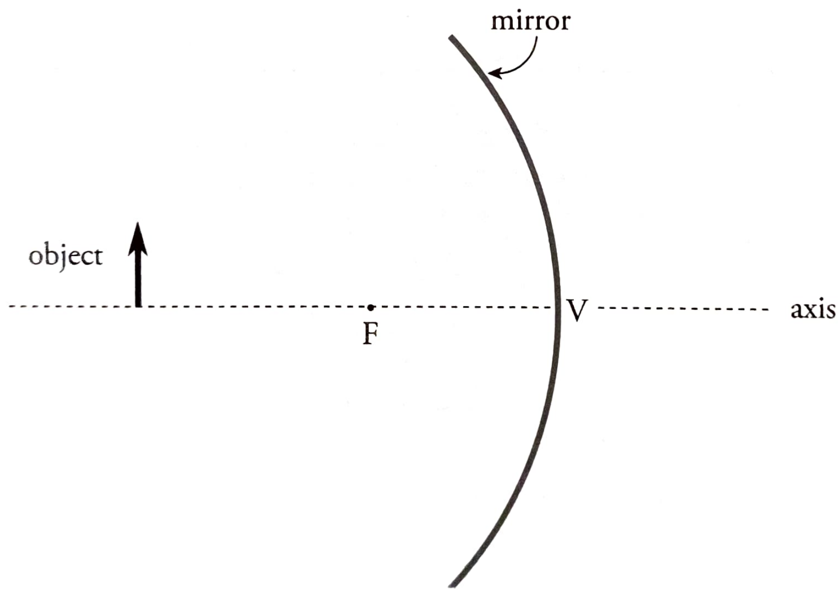
**Concave Mirrors**

- An incident ray parallel to the axis is reflected through the focus.
- An incident ray that passes through the focus is reflected parallel to the axis.
- An incident ray that strikes the vertex is reflected at an equal angle to the axis.

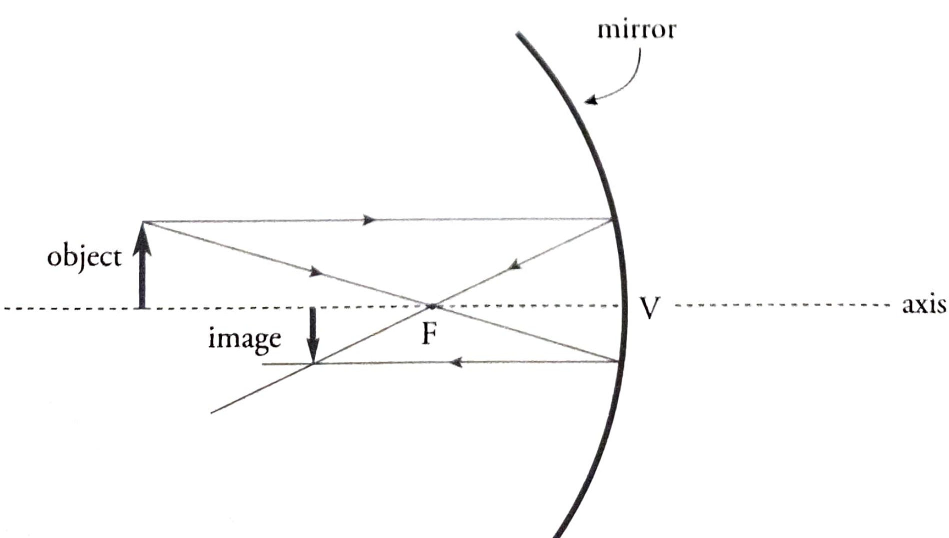
**Convex Mirrors**

- An incident ray parallel to the axis is reflected away from the virtual focus.
- An incident ray directed toward the virtual focus is reflected parallel to the axis.
- An incident ray that strikes the vertex is reflected at an equal angle to the axis.

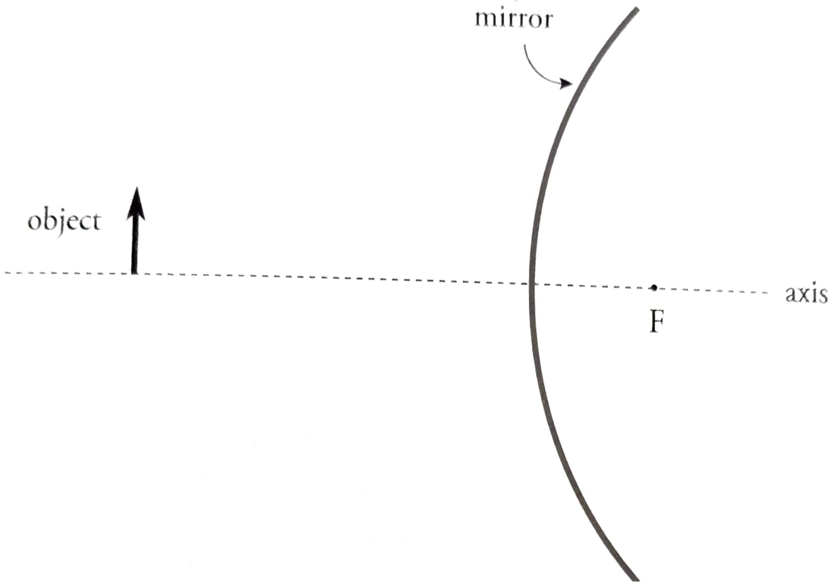
**Example 6** The figure below shows a concave mirror and an object (the bold arrow). Use a ray diagram to locate the image of the object.



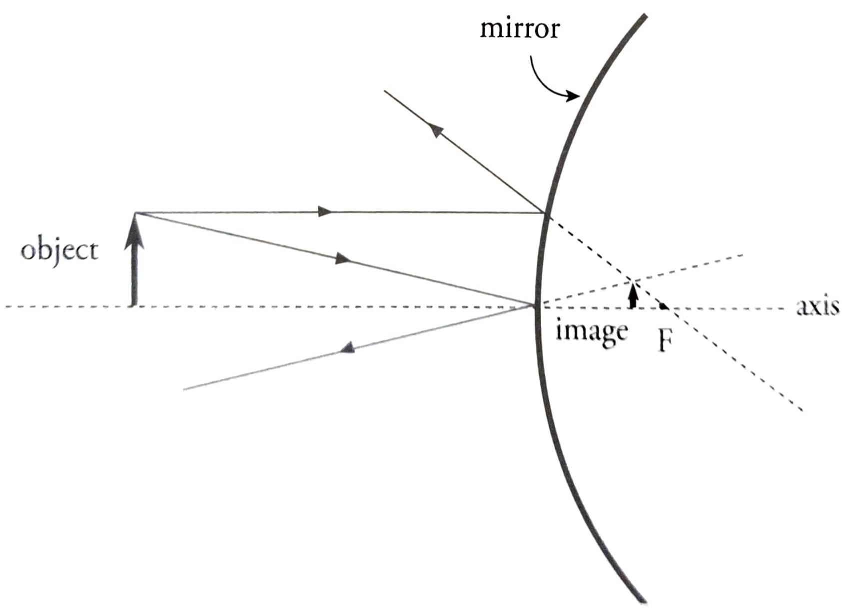

**Solution.** It takes only two distinct rays to locate the image:



**Example 7** The figure below shows a convex mirror and an object (the arrow). Use a ray diagram to locate the image of the object.




**Solution.**




The ray diagrams of the preceding examples can be used to determine the location, orientation, and size of the image. The nature of the image—that is, whether it's real or virtual—can be determined by seeing on which side of the mirror the image is formed. If the image is formed on the same side of the mirror as the object, then the image is real, but if the image is formed on the opposite side of the mirror, it's virtual. Here's another way to look at this: if you had to trace lines back to form an image, that image is virtual. Therefore, the image in Example 6 is real, and the image in Example 7 is virtual.

## Using Equations to Answer Questions About the Image

While ray diagrams can answer our questions about images completely, the fastest and easiest way to get information about an image is to use two equations and some simple conventions. The first equation, called the **mirror equation**, is

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Equation Sheet

where $s_o$ is the object's distance from the mirror, $s_i$ is the image's distance from the mirror, and $f$ is the focal length of the mirror. The value of $s_o$ is *always* positive for a real object, but $s_i$ can be positive or negative. The sign of $s_i$ tells us whether the image is real or virtual: if $s_i$ is positive, the image is real; and if $s_i$ is negative, the image is virtual.

The second equation is called the **magnification equation**:

$$M = \frac{h_i}{h_o} = \frac{-s_i}{s_o}$$

Equation Sheet

This gives the magnification; the height of the image, $h_i$, is $|M|$ times the height of the object, $h_o$. If $M$ is positive, then the image is upright relative to the object; if $M$ is negative, it's inverted relative to the object. Because $s_o$ is always positive, we can come to two important conclusions. If $s_i$ is positive, then $M$ is negative, so real images are always inverted, and, if $s_i$ is negative, then $M$ is positive, so virtual images are always upright.

Finally, to distinguish *mathematically* between concave and convex mirrors, we always write the focal length $f$ as a positive value for concave mirrors and a negative value for convex mirrors. With these two equations and their accompanying conventions, all four questions about an image can be answered.

**Which Is the Positive Side?**

We will always treat the object's position $s_o$ as being the positive side. For mirrors, if the image is produced on the same side as the object, it is considered a positive distance. If an image is produced opposite the side of the object, it is considered a negative distance. Concave mirrors will always have a positive focal length (meaning the focal point is on the same side of the object). Convex mirrors will always have a negative focal length (meaning the focal point is on the opposite side of the object).

## MIRRORS

converging—concave $\Leftrightarrow$ $f$ positive
diverging—convex $\Leftrightarrow$ $f$ negative

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \begin{cases} s_o \text{ always positive (real object)} \\ s_i \text{ positive} \Rightarrow \text{image is real (located on the } \textit{same} \text{ side of mirror as object)} \\ s_i \text{ negative} \Rightarrow \text{image is virtual (located on the } \textit{opposite} \text{ side of mirror from object)} \end{cases}$$

**Real or Virtual?**
Remember that all real images are inverted and all virtual images are upright.

$$M = \frac{h_i}{h_o} = \frac{-s_i}{s_o} \begin{cases} \text{given } h_o \text{ is positive} \\ h_i \text{ and } M \text{ positive} \Rightarrow \text{image is upright} \\ h_i \text{ and } M \text{ negative} \Rightarrow \text{image is inverted} \end{cases}$$

**Example 8** An object of height 4 cm is placed 30 cm in front of a concave mirror whose focal length is 10 cm.

(a) Where's the image?
(b) Is it real or virtual?
(c) Is it upright or inverted?
(d) What's the height of the image?

**Solution.**

(a) With $s_o = 30$ cm and $f = 10$ cm, the mirror equation gives

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \Rightarrow \frac{1}{30 \text{ cm}} + \frac{1}{s_i} = \frac{1}{10 \text{ cm}} \Rightarrow \frac{1}{s_i} = \frac{1}{15 \text{ cm}} \Rightarrow s_i = 15 \text{ cm}$$

The image is located 15 cm in front of the mirror.

(b) Because $s_i$ is positive, the image is real.

(c) Real images are inverted.

(d) $$\frac{h_i}{h_o} = \frac{-s_i}{s_o} \Rightarrow h_i = \frac{-s_i h_o}{s_o} \Rightarrow h_i = \frac{(-15 \text{ cm})(4 \text{ cm})}{30 \text{ cm}} = -2 \text{ cm}$$

The –2 cm also confirms the image is inverted.

**Example 9** An object of height 4 cm is placed 20 cm in front of a convex mirror whose focal length is –30 cm.

(a) Where's the image?

(b) Is it real or virtual?

(c) Is it upright or inverted?

(d) What's the height of the image?

**Solution.**

(a) With $s_o = 20$ cm and $f = -30$ cm, the mirror equation gives us

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \Rightarrow \frac{1}{20\text{ cm}} + \frac{1}{s_i} = \frac{1}{-30\text{ cm}} \Rightarrow \frac{1}{s_i} = -\frac{1}{12\text{ cm}} \Rightarrow s_i = -12\text{ cm}$$

So, the image is located 12 cm behind the mirror.

(b) Because $s_i$ is negative, the image is virtual.

(c) Virtual images are upright.

(d) $$\frac{h_i}{h_o} = \frac{-s_i}{s_o} \Rightarrow h_i = \frac{-s_i h_o}{s_o} \Rightarrow h_i = \frac{-(-12\text{ cm})(4\text{ cm})}{20\text{ cm}} = 2.4\text{ cm}$$

The +2.4 cm also confirms the image is upright.

**What Produces What?**

Only concave mirrors can produce both real images (if $s_o > f$) and virtual images (if $s_o < f$). Convex mirrors and plane mirrors can produce only virtual images.

**Example 10** Show how the statements made earlier about plane mirrors can be derived from the mirror and magnification equations.

**Solution.** A plane mirror can be considered a spherical mirror with an infinite radius of curvature (and an infinite focal length). If $f = \infty$, then $1/f = 0$, so the mirror equation becomes

$$\frac{1}{s_o} + \frac{1}{s_i} = 0 \Rightarrow s_i = -s_o$$

So, the image is as far behind the mirror as the object is in front. Also, since $s_o$ is always positive, $s_i$ is negative, so the image is virtual. The magnification is

$$M = -\frac{s_i}{s_o} = -\frac{-s_o}{s_o} = 1$$

and the image is upright and has the same height as the object. The mirror and magnification equations confirm our description of images formed by plane mirrors.

**Example 11** Show why convex mirrors can form only virtual images.

**Solution.** Because $f$ is negative and $s_o$ is positive, the mirror equation

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f}$$

immediately tells us that $s_i$ cannot be positive (if it were, the left-hand side would be the sum of two positive numbers, while the right-hand side would be negative). Since $s_i$ must be negative, the image must be virtual.

**Example 12** An object placed 60 cm in front of a spherical mirror forms a real image at a distance of 30 cm from the mirror.

(a) Is the mirror concave or convex?

(b) What's the mirror's focal length?

(c) Is the image taller or shorter than the object?

**Solution.**

(a) The fact that the image is real tells us that the mirror cannot be convex, since convex mirrors form only virtual images. The mirror is concave.

(b) With $s_o$ = 60 cm and $s_i$ = 30 cm ($s_i$ is positive since the image is real), the mirror equation tells us that

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f} \quad\Rightarrow\quad \frac{1}{60\text{ cm}}+\frac{1}{30\text{ cm}}=\frac{1}{f} \quad\Rightarrow\quad \frac{1}{f}=\frac{1}{20\text{ cm}} \quad\Rightarrow\quad f=20\text{ cm}$$

Note that $f$ is positive, which confirms that the mirror is concave.

(c) The magnification is

$$M=-\frac{s_i}{s_o}=-\frac{30\text{ cm}}{60\text{ cm}}=-\frac{1}{2}$$

Since the absolute value of $M$ is less than 1, the mirror makes the object look smaller (minifies the height of the object). The image is only half as tall as the object and is inverted, since $M$ is negative.

**Example 13** A concave mirror with a focal length of 25 cm is used to create a real image that has twice the height of the object. How far is the image from the mirror?

**Solution.** Since $h_i$ (the height of the image) is twice $h_o$ (the height of the object), the value of the magnification is either +2 or −2. To figure out which, we just notice that the image is real; real images are inverted, so the magnification, $M$, must be negative. Therefore, $M = -2$, so

$$-\frac{s_i}{s_o} = -2 \quad \Rightarrow \quad s_o = \frac{1}{2}s_i$$

Substituting this into the mirror equation gives us

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{\frac{1}{2}s_i} + \frac{1}{s_i} = \frac{1}{f} \quad \Rightarrow \quad \frac{3}{s_i} = \frac{1}{f} \quad \Rightarrow \quad s_i = 3f = 3(25\text{ cm}) = 75\text{ cm}$$

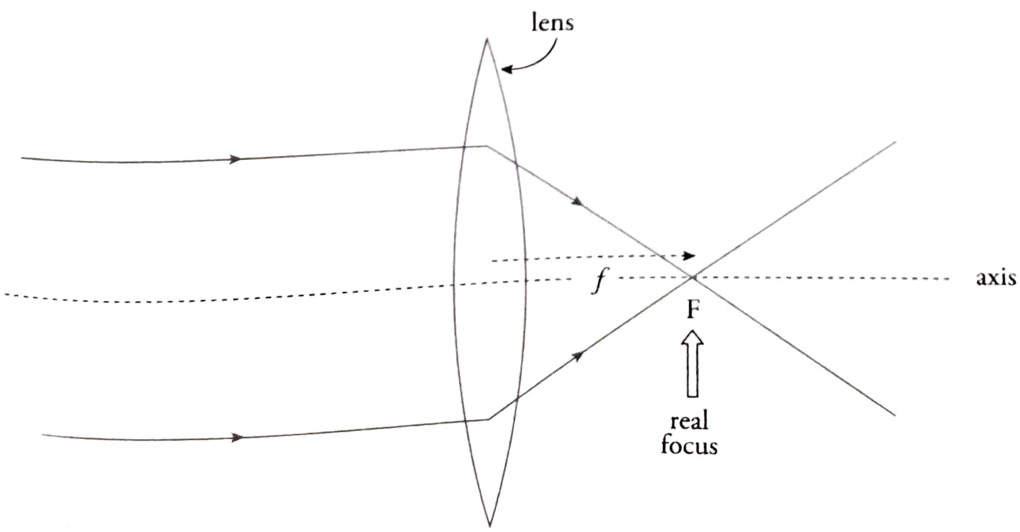
**So Many Equations!**
The purpose of a mirror is to reflect an image. The purpose of a lens is to refract an image (or see through it). Fortunately, the equations for mirrors and lenses will be the same with a minor difference on the focal length.

## THIN LENSES

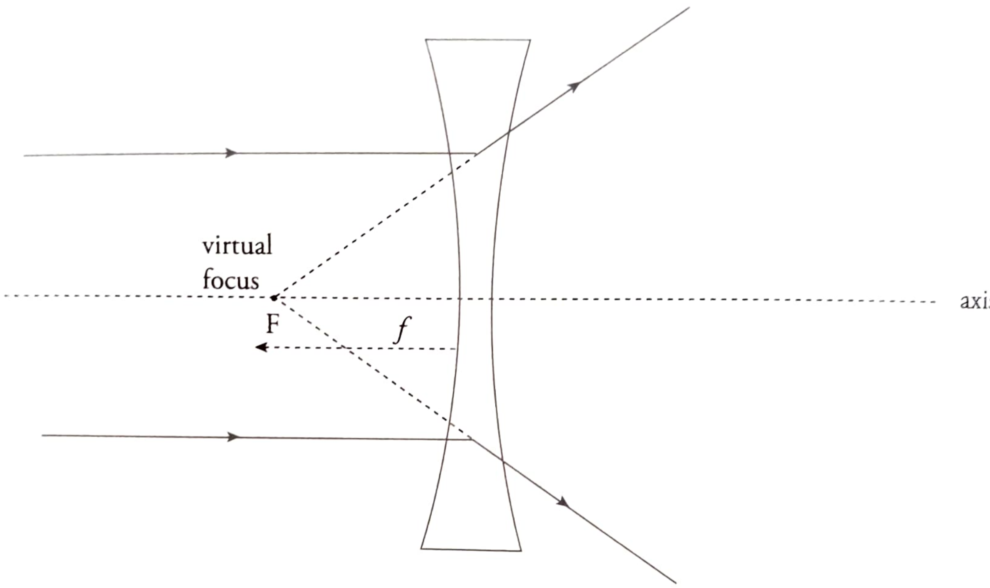
A lens is an optical device that forms an image by *refracting* light. We'll now talk about the equations and conventions that are used to analyze images formed by the two major categories of lenses: converging and diverging.

A **converging lens**—like the bi-convex one shown below—converges parallel paraxial rays of light to a focal point on the far side. (This lens is *bi-convex*; both of its faces are convex. All converging lenses have at least one convex face.) Because parallel light rays actually focus at F, this point is called a **real focus**. Its distance from the lens is the focal length, $f$.

**Mirrors vs. Lenses**
The equations are exactly the same. The only difference is we switch the locations of the focus. A positive focal length lens will have a focus on the opposite side of the lens from the object. If the focal point is located on the same side of the object, then it is a negative focal length.



A **diverging lens**—like the *bi-concave* one shown below—causes parallel paraxial rays of light to diverge away from a **virtual focus**, F, on the same side as the incident rays. (All diverging lenses have at least one concave face.)




**Don't Mix Them Up**
Converging Lens = Bi-Convex Lens
Diverging Lens = Bi-Concave Lens

## RAY TRACING FOR LENSES

Just as is the case with mirrors, representative rays of light can be sketched in a diagram along with the object and the lens; the point at which the reflected rays intersect (or appear to intersect) is the location of the image. The rules that govern these rays are as follows:

**Converging Lenses**

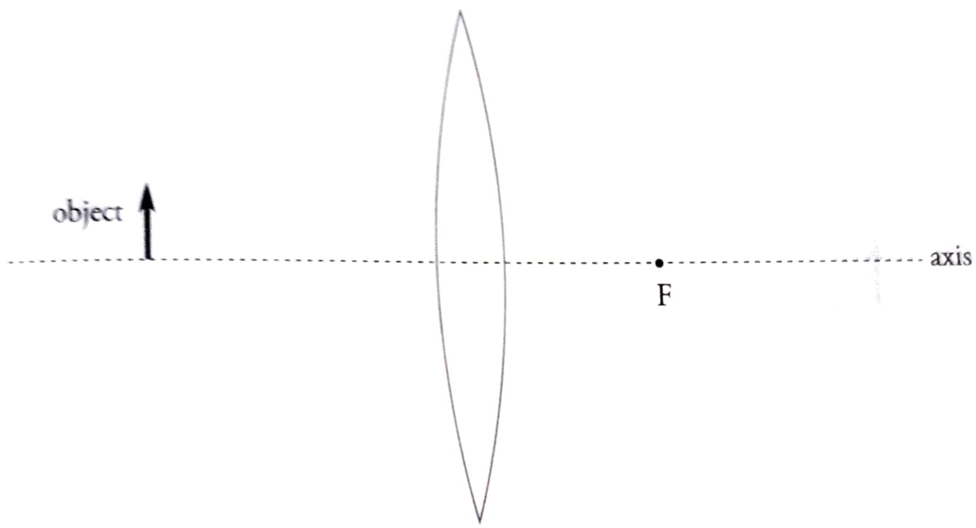
- An incident ray parallel to the axis is refracted through the real focus.
- An incident ray that passes through the focus is refracted parallel to the axis.
- Incident rays pass undeflected through the optical center, O (the central point within the lens where the axis intersects the lens).

Lenses actually have two focal points: a near focal point (on the same side of the lens as the object) and a far focal point (on the opposite side of the lens as the object).

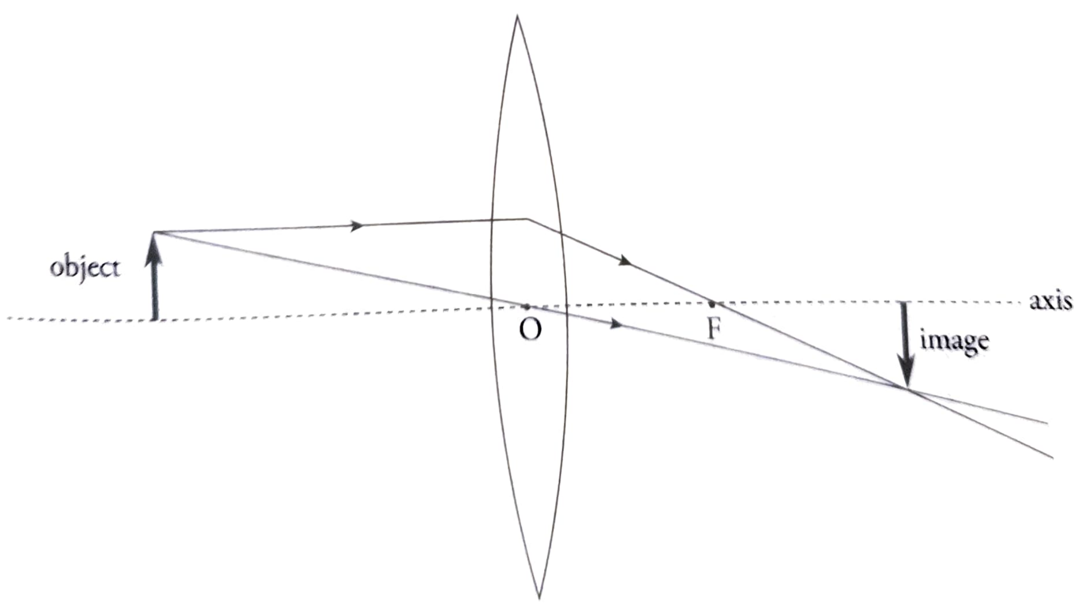
**Diverging Lenses**

- An incident ray parallel to the axis is refracted away from the virtual focus.
- An incident ray heading toward the far focus is refracted parallel to the axis.
- Incident rays pass undeflected through the optical center, O.

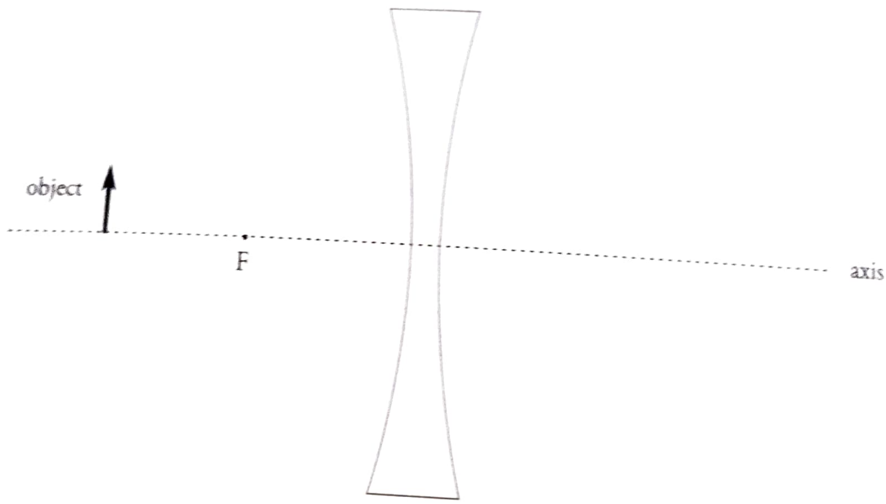
**Example 14** The figure below shows a converging lens and an object (denoted by the bold arrow). Use a ray diagram to locate the image of the object.



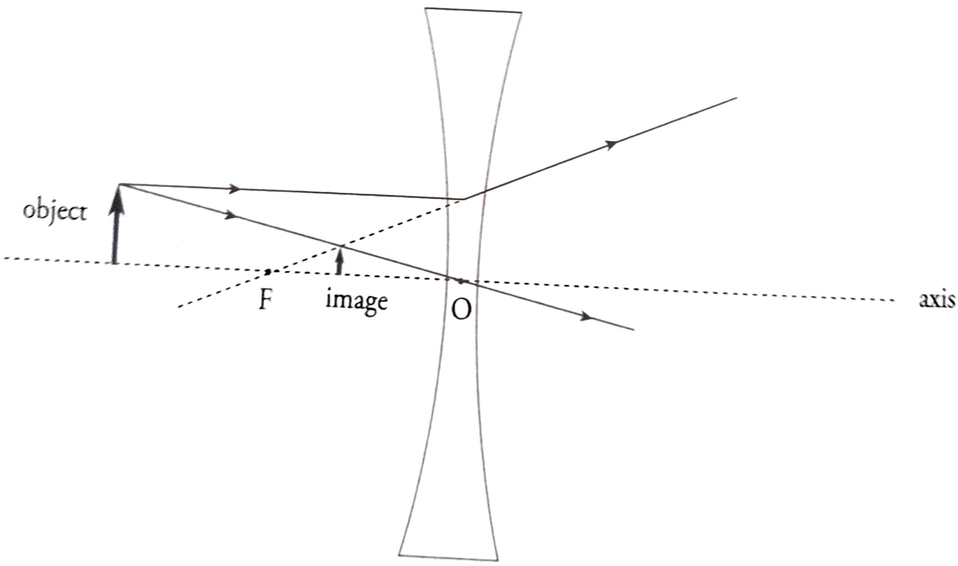

**Solution.**



**Example 15** The figure below shows a diverging lens and an object. Use a ray diagram to locate the image of the object.




**Solution.**




The nature of the image—that is, whether it's real or virtual—is determined by the side of the lens upon which the image is formed. If the image is formed on the side of the lens that's opposite the object, then the image is real, and if the image is formed on the same side of the lens as the object, then it's virtual. Another way of looking at this is if you had to trace lines back to form an image, that image is virtual. Therefore, the image in Example 14 is real, while the image in Example 15 is virtual.

## USING EQUATIONS TO ANSWER QUESTIONS ABOUT THE IMAGE

Lenses and mirrors use the same equations, notation, and sign conventions, with the following note. Converging optical devices ($+f$) are con<u>cave</u> mirrors and con<u>vex</u> lenses. Diverging optical devices ($-f$) are con<u>vex</u> mirrors and con<u>cave</u> lenses.

**LENSES**

convex lens—converging $\Leftrightarrow$ $f$ positive
concave lens—diverging $\Leftrightarrow$ $f$ negative

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f}\begin{cases} s_o \text{ always positive (real object)} \\ s_i \text{ positive} \Rightarrow \text{image is real (located on } \textit{opposite} \text{ side of lens from object)} \\ s_i \text{ negative} \Rightarrow \text{image is virtual (located on } \textit{same} \text{ side of lens as object)} \end{cases}$$

$$M=\frac{h_i}{h_o}=\frac{-s_i}{s_o}\begin{cases} \text{given } h_o \text{ is positive} \\ h_i \text{ and } M \text{ positive} \Rightarrow \text{image is upright} \\ h_i \text{ and } M \text{ negative} \Rightarrow \text{image is inverted} \end{cases}$$

**Example 16** An object of height 11 cm is placed 44 cm in front of a converging lens with a focal length of 24 cm.

(a) Where's the image?
(b) Is it real or virtual?
(c) Is it upright or inverted?
(d) What's the height of the image?

**Solution.**

(a) With $s_o = 44$ cm and $f = 24$ cm, the lens equation gives us

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f} \Rightarrow \frac{1}{44\text{ cm}}+\frac{1}{s_i}=\frac{1}{24\text{ cm}} \Rightarrow \frac{1}{s_i}=0.0189\text{ cm}^{-1} \Rightarrow s_i=53\text{ cm}$$

So, the image is located 53 cm from the lens, on the opposite side from the object.

(b) Because $s_i$ is positive, the image is real.

(c) Real images are inverted.

(d) $$\frac{h_i}{h_o}=\frac{-s_i}{s_o} \Rightarrow h_i=\frac{-s_i h_o}{s_o} \Rightarrow h_i=\frac{(-53\text{ cm})(11\text{ cm})}{44\text{ cm}}=-13\text{ cm}$$

The negative $h_i$ reaffirms that we have an inverted image.

**Example 17** An object of height 11 cm is placed 48 cm in front of a diverging lens with a focal length of –24.5 cm.

(a) Where's the image?

(b) Is it real or virtual?

(c) Is it upright or inverted?

(d) What's the height of the image?

**What Produces What?**

Only converging lenses can produce both real images (if $s_o > f$) and virtual images (if $s_o < f$). Diverging lenses can produce only virtual images.

**Solution.**

(a) With $s_o$ = 48 cm and $f$ = –24.5 cm, the lens equation gives us

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f} \Rightarrow \frac{1}{48\text{ cm}}+\frac{1}{s_i}=\frac{1}{-24.5\text{ cm}} \Rightarrow \frac{1}{s_i}=-0.0616\text{ cm}^{-1} \Rightarrow s_i=-16\text{ cm}$$

The image is 16 cm from the lens, on the same side as the object.

(b) Because $s_i$ is negative, the image is virtual.

(c) Virtual images are upright.

(d) $$\frac{h_i}{h_o}=\frac{-s_i}{s_o} \Rightarrow h_i=\frac{-s_i h_o}{s_o} \Rightarrow h_i=\frac{-(-16\text{ cm})(11\text{ cm})}{48\text{ cm}}=3.7\text{ cm}$$

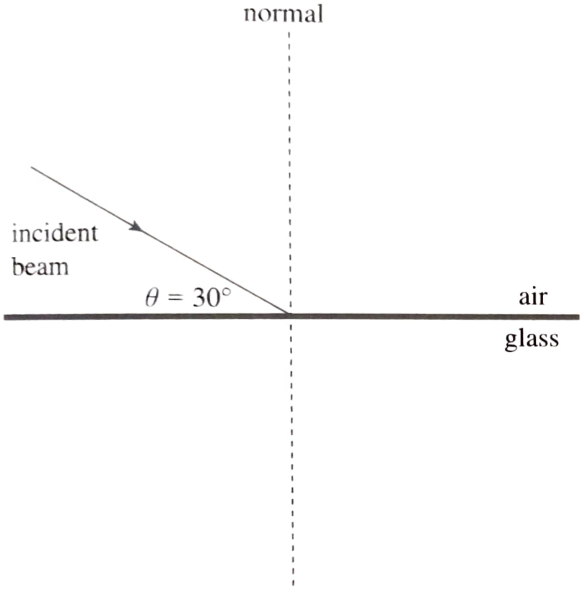
The positive $h_i$ reaffirms that we have an upright image.

# Chapter 8 Review Questions

Solutions can be found in Chapter 11.

## Section I: Multiple Choice

1. A beam of light in air is incident upon the smooth surface of a piece of flint glass, as shown:




As the incident angle is increased toward $\theta = 90°$, what observation is made of the refracted ray? (All angle references are relative to the surface as shown for both rays.)

(A) The refracted ray angle increases as the incident angle increases, but the value of the refracted angle is always smaller than the incident angle.
(B) The refracted ray angle increases as the incident angle increases, but the value of the refracted angle is always larger than the incident angle.
(C) The refracted ray angle increases as the incident angle increases until at some angle total internal reflection begins to occur.
(D) The refracted ray angle decreases as the incident angle increases, but the value of the refracted angle is always smaller than the incident angle.

2. A convex lens constructed of glass makes a real image of an object when it is in air. When the object is located $d_o$ in front of the lens, the image appears in air at a distance $d_i$ behind the lens. What occurs if the object is still at $d_o$, but the object and the lens are submerged in water with an index of refraction between that of air and the glass of the lens?

(A) The image is still at $d_i$ and is still real.
(B) The image is at a position closer to the lens than $d_i$ and is real.
(C) The image is at a position farther from the lens than $d_i$ and is real.
(D) The image becomes virtual.

3. A beam of light traveling in Medium 1 strikes the interface to another transparent medium with a lower index of refraction, Medium 2. If the intensity of light is measured to be less in Medium 2 than in Medium 1, what can be concluded?

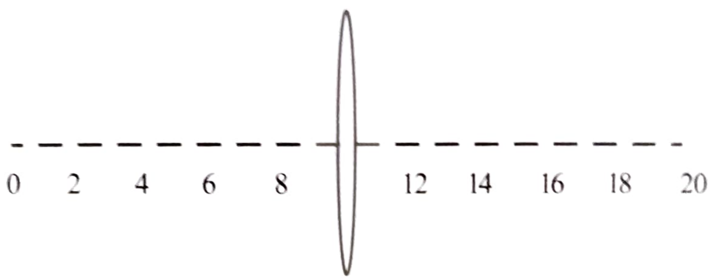
(A) The decrease in intensity was caused by the change in the speed of light in the different media.
(B) Total internal reflection occurred at the interface.
(C) The angle the light travels relative to the normal in Medium 2 will be the same as the angle the light had traveled relative to the normal in Medium 1.
(D) Some part of the light was reflected and/or absorbed at the interface.

4. If a clear liquid has a refractive index of 1.45 and a transparent solid has an index of 2.90 then, for total internal reflection to occur at the interface between these two media, which of the following must be true?

| | incident beam originates in | at an angle of incidence greater than |
|---|---|---|
| (A) | The solid | 30° |
| (B) | The liquid | 30° |
| (C) | The liquid | 60° |
| (D) | Total internal reflection cannot occur. | |

2. An experiment is set up with an object, a single converging lens, and a screen to make an image appear on the screen. The lens has a focal length of 3 cm. The arrangement is configured so that the image which appears on the screen is the same size as the object. The lens is placed at a position labeled 10 cm on the optic axis.

   (a) Draw a diagram of the experimental setup along the optic axis below. Justify your answer.




   (b) If the converging lens were replaced with a convex mirror of the same focal length, explain how (if at all) your experimental setup would have to change to produce an image of the same height as the object.

   (c) In a clear, coherent paragraph-length response, explain what a virtual image is. Explain what steps would have to be taken to experimentally verify the existence of a virtual image.

# Summary

- Geometric optics always measures angles from the normal line. Topics include the fact that the angle of incidence is equal to the angle of reflection and that these angles lie in the same plane.
- Snell's Law states when light enters a new medium, it changes speed and may change direction. This is stated in the formula

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

  where $n$ is the index of refraction, which is a ratio of the speed of light in a vacuum to the speed of light in the substance ($n = \frac{c}{v}$). The index of refraction is always greater than 1 and has no units. When going from a higher index of refraction to a lower index of refraction, light may experience total internal reflection if the angle of incidence is larger than the critical angle ($\sin\theta_c = \frac{n_2}{n_1}$).

- For both thin lenses and curved mirrors, you can use the mirror equation: $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$.

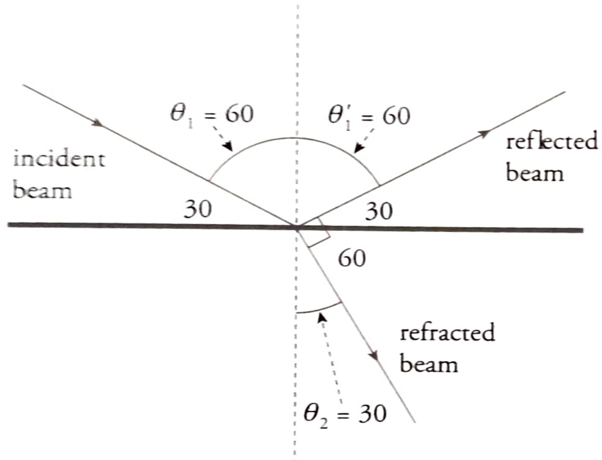
  Note that $f$ is positive for a convex lens or a concave mirror, $f$ is negative for a concave lens or a convex mirror, and $s_o$ is always positive.

- For a lens, if $s_i$ is negative, the lens image is virtual and located on the same side of the lens as the object. If $s_i$ is positive, the lens image is real and located on the opposite side of the lens from the object.
- For a mirror, if $s_i$ is negative, the image is virtual and located on the opposite side of the mirror from the object. If $s_i$ is positive, the image is real and located on the same side of the mirror as the object.
- The magnification equation is given by $M = \frac{h_i}{h_o} = \frac{-s_i}{s_o}$.

# CHAPTER 8: GEOMETRIC OPTICS REVIEW QUESTIONS

## Section I: Multiple Choice

1. **B** As light travels into an optically dense medium, it will refract in toward the normal and away from the surface. So the light in the glass will always be at a greater angle from the surface than the light in the air, so (B) is correct. Note that total internal reflection, (C), will not occur in this situation because it happens only with the initial medium being more optically dense.




2. **C** Any convex lens made of a material with an index of refraction larger than the surroundings will create a real image, so (D) is incorrect. Because the glass-to-air difference is greater than the water-to-air difference, the glass-to-air transition results in a greater angle of refraction, so the light in air bends more when exiting the lens than the light in water. As a result, the point where the rays intersect will be farther from the lens than $d_i$.

3. **D** Choice (A) is false because a change in speed at an interface does not cause intensity to change. Choice (B) is false because had there been total internal reflection, the light would have remained in Medium 1. Choice (C) is false because the problem states that Medium 2 has a lower index of refraction, and from Snell's Law, $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$, if $n_2 < n_1$, then $\sin(\theta_2) > \sin(\theta_1)$ and the angles are not equal. Choice (D) is true because when light arrives at an interface, it must be absorbed, reflected, or transmitted. Therefore, any intensity not detected as transmitted into Medium 2 was either reflected back into Medium 1 or absorbed.

4. **A** The critical angle for total internal reflection is computed as follows:

$$n_1 \sin \theta_c = n_2 \sin 90^\circ$$

$$\sin \theta_c = \frac{n_2}{n_1} = \frac{1.45}{2.90} = \frac{1}{2} \quad \Rightarrow \quad \theta_c = 30^\circ$$

Total internal reflection can happen only if the incident beam originates in the medium with the higher index of refraction, and therefore the beam must originate in the solid. The refracted ray must strike the interface of the other medium at an angle of incidence greater than the critical angle.

5. **C** Every ray, regardless of whether it is a principal ray or not, which originates at the tip of an object and strikes a mirror, will converge at the tip of the image.

6. **A** All of the rays which originate at the tip of the image and travel along paths that remain above the optic axis will be blocked. However, all of the rays that originate at the tip of the image and travel along paths that are below the optic axis at the plane of the block will be unaltered. As a result, fewer rays will converge at the image location, resulting in a less bright image.

7. **D** Diverging lenses always create virtual images.

## Section II: Free Response

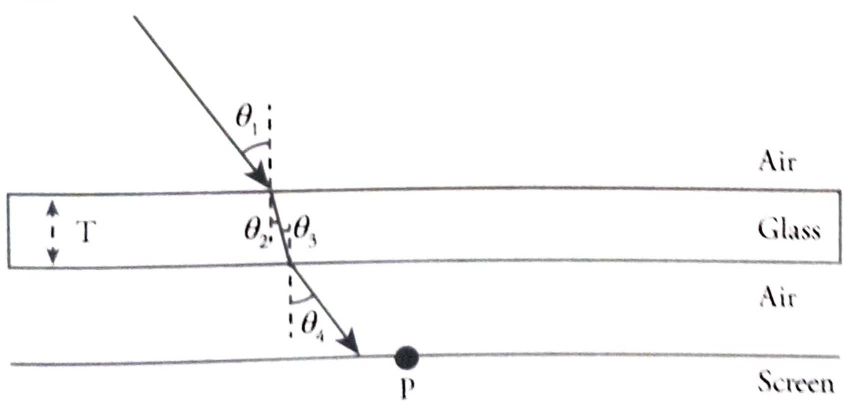
1. (a) i. Student 1 is correct that "According to Snell's Law, the light will bend at the first interface" and also "unbend at the second interface and travel parallel to its original path."

ii. Student 2 is correct that "Snell's Law is $n_{in}\sin(\theta_{in}) = n_{out}\sin(\theta_{out})$" and that "$\theta_{out}$ is different from $\theta_{in}$ at the first interface."

iii. Student 1 is incorrect that "The spot on the screen will still be at point P."

iv. Student 2 is incorrect that "The beam after the glass cannot be parallel to the beam before the glass."

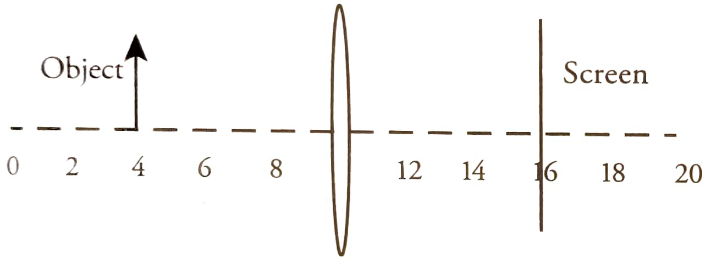
(b) Incident beam




Here, $\theta_1 = \theta_4$ and $\theta_2 = \theta_3$.

(c) If the area between the glass and screen were filled with water, it would still be the case that $\theta_2 = \theta_3$ because there is no change to the glass. However, upon leaving the glass and entering the water, the amount that the light would bend away from the normal would be less than in the case of air because the difference in the indices of refraction is smaller with glass-to-water than glass-to-air. As a result, the beams in air and water would not be parallel and the spot that would appear on the screen would be even farther away from point P than in the original experiment.

2. (a)




In order for the object and image to be the same size, the magnitude of the magnification must be 1, so $|M| = \left|\left(\frac{s_i}{s_o}\right)\right| = 1$ and therefore $|s_o| = |s_i|$. The only way to satisfy the equation $\frac{1}{f} = \frac{1}{s_i} + \frac{1}{s_o}$ when the image distance is equal to the object distance is for both to be $2f$. The object should be placed $2f$ before the lens and the screen should be placed $2f$ after the lens. The image will be below the optical axis in this case, so the part of the screen where the image forms needs to also be placed below the optic axis. Note, switching the positions of the object and screen (so that the object is on the right and the screen is on the left) is also correct.

(b) A converging lens is similar to a convex mirror, but with the mirror, the light always stays on one side of the mirror. Both the object and the screen should be placed at the 4 cm mark. The image will be below the optical axis in this case.

(c) A virtual image is a position in space where the light rays appear to have converged, although no light is actually present at the "image position." Ray tracing can yield a set of non-converging rays. Tracing those rays "backward" to the point where the lines do converge results in the virtual image position. In order to experimentally verify that a virtual image exists, the virtual image location may be used as the object location for a second imaging system. The second imaging system may then produce a real image on a screen of the virtual image that was its object.

# CHAPTER 9: WAVES, SOUND, AND