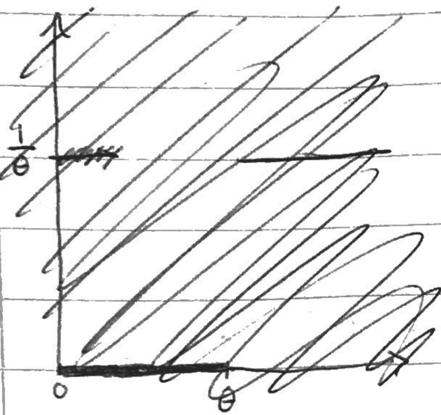
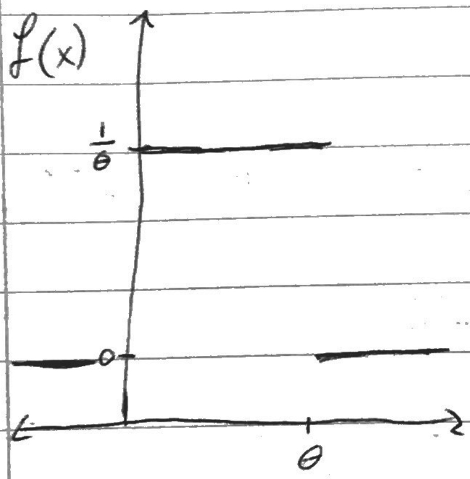
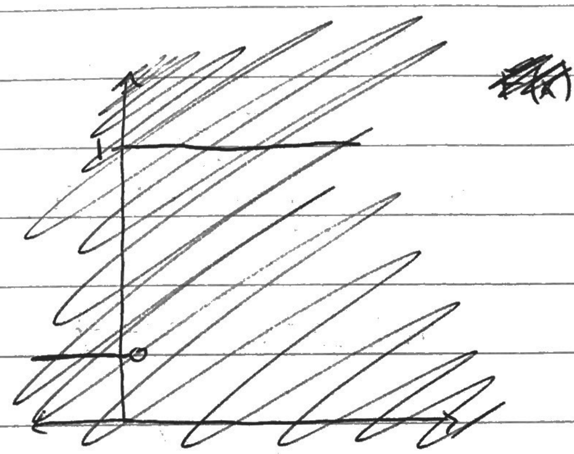
Innlevering 2

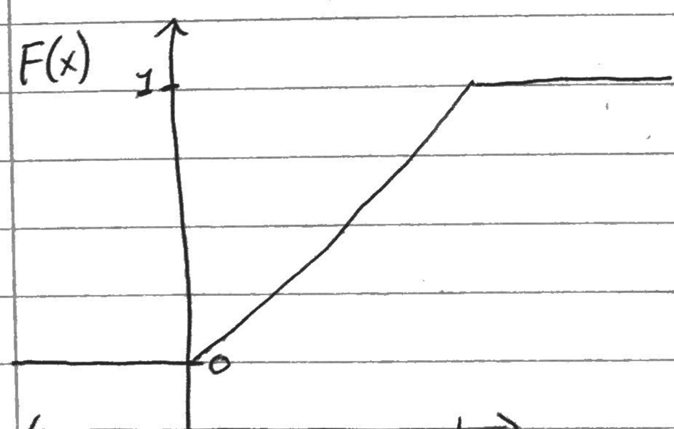
1) $$f(x)\begin{cases}\frac{1}{\theta}, & 0\le x\le\theta\\ 0, & \text{ellers}\end{cases}$$

$$F(x)\begin{cases}0, & x<0\\ \frac{1}{\theta}x, & 0\le x\le\theta\\ 1, & x>\theta\end{cases}$$









$P(X \leq 0{,}4) = F(0{,}4) = \frac{0{,}4}{2} = 0{,}2$

2)

$\int_0^\infty 4xe^{-2x} = 1$

$4\left(\frac{1!}{2^{1+1}}\right) = 1$

$\frac{4}{4} = 1$

$E(x) = \int_{-\infty}^{\infty} x \cdot g(x)\, dx$

$= \int_0^\infty 4x^2 e^{-2x}\, dx = 4 \cdot \frac{2!}{2^{2+1}} = 4 \cdot \frac{2}{8} = 1 \text{ min}$

$P(x > 2) = 1 - P(x < 2)$

$= 1 - G(2)$

$G(x) = 4\int_0^x xe^{-2x}\, dx = 4\left(-\frac{1}{2}xe^{-2x} + \frac{1}{2}\int_0^x e^{-2x}\, dx\right) = -e^{-2x}(2x+1)$

$\cancel{1 - G(2) = 1 + e^{-2x}(2x+1) = 1 + e^{-4}}$

$1 - G(2) = 1 - (-e^{-2\cdot 2}(2\cdot 2+1) + e^{-2\cdot 0}(0+1))$

$= 1 - (-e^{-4}(5) + 1)$

$= 1 - 1 + 5e^{-4}$

$= 5e^{-4}$

$= 0{,}0916 \approx 0{,}09$

3) $F(x) = \int_{-\infty}^{x} f(x) = \int_{1}^{x} \beta x^{-\beta-1}$

$= \left[\frac{\beta}{-\beta} x^{-\beta}\right]_1^x$

$= -x^{-\beta} - (-1)$

$= \underline{\underline{1 - x^{-\beta}}}$

$P(X>2) = \int_2^{\infty} f(x)\,dx = F(\infty) - F(2) = 1 - (1 - 2^{-3})$

$= \frac{1}{8}$

$P(X < 3{,}5 \mid X > 2) = \frac{P(X < 3{,}5 \cap X > 2)}{P(X>2)}$

$= \frac{P(3{,}5 > X > 2)}{P(X>2}$

$= \frac{F(3{,}5) - F(2)}{F(2)}$

$= \frac{1 - 3{,}5^{-3} - 1 + 2^{-3}}{2^{-3}}$

$= \frac{\frac{1}{8} - \frac{8}{343}}{\frac{1}{8}} = \underline{\underline{0{,}813}}$

4) Marginal $g(x)$:

x	-1	0	1
$g(x)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Marginal $h(y)$:

Y	0	1	2
$h(y)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

$$E(X) = -1 \cdot \frac{1}{3} + 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} = 0$$

$$E(Y) = 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 2 \cdot \frac{1}{3} = 1$$

$$Var(X) = \frac{(-1)^2}{3} + \frac{0^2}{3} + \frac{(1)^2}{3} = \frac{2}{3}$$

$$Var(Y) = \frac{(0-1)^2}{3} + \frac{(1-1)^2}{3} + \frac{(2-1)^2}{3} = \frac{2}{3}$$