Quiz 2

Section 2.1

5. a) They are equal, as $\forall x (x \in A \leftrightarrow x \in B)$

b) They are not equal.

c) they are not equal, empty set vs. Singleton

24 a) Yes

b) Yes

c) no

d) Yes

Section 2.2

18 c

$\{x \mid x \in A \wedge x \notin B \wedge x \notin C\} \subseteq \{x \mid x \in A \wedge x \notin B\}$

if $A - B = A$: $(A-B) - C = A - C$

else: $(A-B) - C \in A - C$

Therefore $(A-B) - C \subseteq A - C$

18 d

$(A - C) \cap (C - B) = \emptyset$

$A \cap \overline{C} \cap C \cap B = \emptyset$

$A \cap B \cap (C \cap \overline{C}) = \emptyset$

$A \cap B \cap (\emptyset) = \emptyset$

$\emptyset = \emptyset$

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$$|A \cup B \cup C| = |(A \cup B) \cup C|$$
$$= |A \cup B| + |C| - |(A \cup B) \cap C|$$
$$= |A| + |B| + |C| - |A \cap B| - |(A \cup B) \cap C|$$
$$= |A| + |B| + |C| - |A \cap B| - |(A \cap C) \cup (B \cap C)|$$
$$= |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Section 2.3

12 a) One-to-one

b) not one to one

c) one-to-one

d) not one-to-one

38

$f(x) = ax + b$
$g(x) = cx + d$

$f \circ g = acx + ad + b$

$g \circ f = acx + bc + d$

$f \circ g = g \circ f$

$ad + b = bc + d$

$d(a-1) = b(c-1)$ These conditions must be met for $f \circ g = g \circ f$

42

a) $f^{-1}(\{1\}) = \{-1, 1\}$

b) $f^{-1}(\{x \mid 0 < x < 1\}) = \{x \mid (0 < x < 1) \cup (-1 < x < 0)\}$

c) $f^{-1}(\{x \mid x > 4\}) = \{x \mid (x > 2) \cup (x < -2)\}$

Section 2.4

12 c

$$a_n = -3a_{n-1} + 4a_{n-2}$$

$$(-4)^n = -3(-4)^{n-1} + 4(-4)^{n-2}$$

$$(-4)^n = -3(-4)^{n-1} - (-4)^{n-1}$$

$$(-4)^n = -4(-4)^{n-1}$$

$$(-4)^n = (-4)^n$$

33 d) $\sum_{i=0}^{2}\sum_{j=1}^{3} ij = \sum_{i=0}^{2}(i+2i+3i)$

$= \sum (6i)$

$= 6 + 12 = 18$

Section 2.5

16 A subset has at most the same cardinality of its superset, therefore if the cardinality of the superset is such that it is countable, then so is its subset.