C.I A set of vectors in $\mathbb{C}^n$ is linearly independent if an only if the linear equation $[V_1 \ V_2 \ \dots \ V_m] X = 0$ has only the trivial solution $X = 0$

The span of a set $\{V_1, V_2, V_3, \dots, V_m\}$ of vectors in $\mathbb{C}^n$ is every vector $\vec{b} \in \mathbb{C}^n$ such that the lin. eq. $[V_1, V_2 \dots V_m] x = b$ has a valid solution for b

A transformation $T: \mathbb{C}^m \to \mathbb{C}^n$ is linear if and only if $T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$ and $T(c\vec{u}) = c \cdot T(\vec{u})$ for any vectors $\vec{u}$ and $\vec{v}$ in $\mathbb{C}^n$ and any scalar c.

A transformation $T: \mathbb{C}^m \to \mathbb{C}^n$ is ~~linear~~ onto if and only if for each b in $\mathbb{C}^n$ there is at least one $X \in \mathbb{C}^m$ so that $T(X) = b$.

A transformation $T: \mathbb{C}^m \to \mathbb{C}^n$ is one-to-one if and only if for each $b \in \mathbb{C}^n$ there is at most one X in $\mathbb{C}^m$ such that $T(x) = b$

(C.3) • One to one & onto:

$f(x) = x$

• one-to-one & NOT onto:

$f(x) = e^x$

• Not one-to-one & onto:

$f(x) = \tan x$

• NOT one-to-one & NOT onto:

$f(x) = x^2$

(C.2)

• $m < n$

  ◦ Cannot be onto. True, or else a space of lower dimensionality would have to span one of higher dimensionality.

  ◦ Cannot be one-to-one. False, $\mathbb{C}^m$ can map onto a n-dimensional slice of $\mathbb{C}^n$.

• $m = n$

  ◦ Can be both onto and one-to-one if it maps every vector to itself.

• $m > n$

  ◦ Cannot be onto. False.

  ◦ Cannot be one-to-one. ~~False~~ True. There are more vectors in $\mathbb{C}^m$ than in $\mathbb{C}^n$.