Worksheet week 5

C.1. $A = \begin{bmatrix} a & 1 \\ 1 & a \end{bmatrix}$ $\quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$Ax = \begin{bmatrix} a & 1 \\ 1 & a \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= x_1 \begin{bmatrix} a \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ a \end{bmatrix}$$

$$\lambda X = \begin{bmatrix} ax_1 + x_2 \\ x_1 + ax_2 \end{bmatrix}$$

$$\lambda x_1 = ax_1 + x_2$$

$$\lambda x_2 = ax_2 + x_1$$

$$0 = x_1(a-\lambda) + x_2$$

$$0 = x_2(a-\lambda) + x_1$$

$$x_2 = x_1(\lambda - a)$$

$$x_1 = x_2(\lambda - a)$$

$$x_1 = x_1(\lambda - a)(\lambda - a)$$

$$1 = (\lambda - a)^2$$

$$\lambda = a \pm \sqrt{1}$$

$$x_1 = x_2(a \pm \sqrt{1} - a)$$

$$x_1 = \pm x_2$$

When $x_1 = x_2$, $\lambda = a+1$. When $x_1 = -x_2$, $\lambda = a-1$.

When $x_1 = x_2 = 0$, $\lambda \in \mathbb{R}$.

C.3 All solutions to the inhomogeneous linear system is given by $\vec{X}_{inHom} + \vec{X}_{Hom}$

As the solution to the homogenous linear system is $x=0$, the solution will be unique. There will be more solutions if the homogenous solution gives another answer than $x=0$