Innlevering 5

$\boxed{1}$

a) $$Z = \frac{X - \mu}{\sigma}$$

Sannsynlighet for $X > 40 = 1 - Z$

$\mu = 35$

$\sigma = 5$

$$Z = \frac{40 - \mu}{5} = 1$$

$$n(1;0,1) = 0,8413$$

$$\underline{P(X > 40) = 0,8413}$$

$$P(30 < X < 40) = P(x < 40) - P(x < 30)$$
$$= P(Z < 1) - P(x < -1)$$
$$= 0,8413 - 0,1587$$
$$= \underline{\underline{0,6826}}$$

$$P(\text{Sum av målinger} > 80) = 1 - P(\text{Sum av målinger} < 80)$$
$$= 1 - P(Z < \sqrt{2})$$
$$= 1 - 0,9207$$
$$= 0,0793$$
$$= \underline{7,93\%}$$

$$\boxed{Z = \frac{80 - 70}{\sqrt{50}} = \sqrt{2}}$$

b) En god estimator bør være:

- Konsistent
- forventningsrett
- Robust mot grove feil
- Minst mulig spredning

~~$\hat{p} = \frac{1}{n}\sum_{i=1}^{n} X_i = \frac{1420}{5} = 284 = \hat{\mu}$~~

~~$\hat{\sigma}^2 = \frac{\sum (X_i - \bar{X})^2}{n} = \frac{2342}{5} = 468{,}4$~~

$$\hat{\mu} = \bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$E(\hat{\mu}) = E(\bar{X}) = E\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)$$

$$Var(\hat{\mu}) = Var\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) = \frac{1}{n^2}\cdot\sum_{i=1}^{n} Var(x_i) = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

$$S^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

$$Var(\hat{\mu}) = \frac{S^2}{n} = \frac{1}{n^2}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

c) Vi bruker t-distribusjon for å finne konf. int.

$$T = \frac{\bar{\mu} - \mu}{\frac{S}{\sqrt{n}}}$$

$$P(-t_{\frac{\alpha}{2}} < T < t_{\frac{\alpha}{2}}) = 1-\alpha$$

$$P\left(\hat{\mu} - \frac{S}{\sqrt{n}} t_{\frac{\alpha}{2}} < \mu < \hat{\mu} + \frac{S}{\sqrt{n}} t_{\frac{\alpha}{2}}\right) = 1-\alpha$$

$\hat{\mu} = \frac{1420}{5} = 284$ $\qquad S^2 = \frac{2342}{5} = 468{,}4$

$$t_{0{,}025} = 2{,}776$$

$$\frac{S}{\sqrt{n}} t_{\frac{\alpha}{2}} = \frac{\sqrt{468{,}4}}{\sqrt{5}} \cdot 2{,}776 = 26{,}87$$

Grensene er da $[248 \pm 26{,}87]$

Oppgave 2

a) $\gamma = 2$ $\qquad \tau = 2$

$n = 6$

$$P\left(\bar{T} > \frac{15}{6}\right) = 1 - P(\bar{T} < 2{,}5)$$

$$Z = \frac{\bar{T} - \gamma}{\frac{\sigma}{\sqrt{n}}}$$

$$1 - P(\bar{T} < 2{,}5) = 1 - P\left(Z < \frac{2{,}5 - 2}{2/\sqrt{6}}\right) = \underline{0{,}2709}$$

$n = 60$

$$P(\bar{T} > 128/60) = 1 - P(\bar{T} < 2{,}133)$$
$$= 1 - P\left(Z < \frac{2{,}133 - 2}{2/\sqrt{60}}\right) = \underline{0{,}3015}$$

b)

$$1 - F(15; 6, 2) = 1 - 1 + \sum_{i=1}^{k-1} \frac{1}{i!} e^{-\frac{15}{2}} = 0{,}2414$$

$$1 - F(128; 60, 2) = \ldots = 0{,}2917$$ ← På samme måte som

Det tilnærmede svaret er mye nærmere de eksakte tallet når det er flere personer.

Oppg. 4

$$\hat{\mu} = \bar{x} = \frac{1}{n_1} \sum_{i=1}^{n_1} x_i = 13{,}18$$

$$T = \frac{\bar{x} - \mu}{S_x / \sqrt{n_1}} = \frac{13{,}18 - \mu}{4{,}44/\sqrt{10}}$$

$$\alpha = 0{,}1$$

$$t_{\frac{\alpha}{2}} = 1{,}833$$

$$P\left(-t_{\frac{\alpha}{2}} < T < t_{\frac{\alpha}{2}}\right) = P\left(\bar{x} - \frac{S_x}{\sqrt{n_1}} t_{\frac{\alpha}{2}} < \mu < \bar{x} + \frac{S_x}{\sqrt{n_1}} t_{\frac{\alpha}{2}}\right)$$
$$= P(10{,}606 < \mu < 15{,}754)$$

b) $$\hat{\eta} = \bar{y} = \frac{1}{n_2} \sum_{i=1}^{n_2} x_i = 8.00$$

$$T = \frac{\bar{y} - \eta}{S_y / \sqrt{n_2}} = \frac{8 - \mu}{2{,}77/\sqrt{10}}$$

$$\alpha = 0{,}01$$

$$t_{\frac{\alpha}{2}} = 0{,}325$$

$$P\left(-t_{\frac{\alpha}{2}} < T < t_{\frac{\alpha}{2}}\right) = P\left(\bar{y} - \frac{S_y}{\sqrt{n_2}} t_{\frac{\alpha}{2}} < \eta < \bar{y} + \frac{S_y}{\sqrt{n_2}} t_{\frac{\alpha}{2}}\right)$$

$$= P(5{,}153 < \eta < 10{,}847)$$

Et 90% konf. int. er smalere enn et 99% dersom om man vil være sikrere på å havne innenfor må intervallet bli større.

c) $\delta = \mu - \eta$

$\hat{\delta} = \hat{\mu} - \hat{\eta} = 13{,}18 - 8{,}00 = 5{,}18$

$$T = \frac{\hat{\delta} - \delta}{\sqrt{S_x^2/n_1 + S_y^2/n_2}} = \frac{5{,}18 - \delta}{\sqrt{4{,}44^2/10 + 2{,}97^2/10}} = \frac{5{,}18 - \delta}{1{,}655}$$

$\alpha = 0{,}05$

$t_{\frac{\alpha}{2}, 18} = 2{,}101$

$$P(-t_{\frac{\alpha}{2}} < T < t_{\frac{\alpha}{2}}) = P(5{,}18 - 1{,}655 \cdot 2{,}101 < \delta < 5{,}18 + 1{,}655 \cdot 2{,}101)$$
$$= P(1{,}703 < \delta < 8{,}657)$$