# TDT4171: Assignment 4

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## Problem 1: Utility

### Classify the four persons as either risk-seeking, risk-neutral, or risk-averse. Explain your reasoning for the classification.

#### Gabriel

Gabriel is risk-seeking in the sense where a doubling of apples will more than double the utility. In a lottery where one can win n apples 50% of the time and get 0 apples if lost, Gabriel will take the wager rather than a offer of $\frac{n}{2} + 1$ apples.

#### Gustav

Gustav is risk-neutral. He does not consider the variance of the wager any more or less enticing than simply taking a E(apples) way out instead of the wager itself. Even more than that, he does not care the amount of apples at all, as his utility is flat. He's just happy to be along for the trip to the orchard.

#### Maria

Maria is risk-averse in the sense that every new apple picked increses the utility by less than the picking of the last apple did. In a lottery where one can win n apples 50% of the time and get 0 apples if lost, Maria will rather accept the cashout of E(apples) than taking the wager. She will even, with a sufficiently large n, accept $\frac{n}{2} - 1$ apples rather than a wager with expected apples of $\frac{n}{2}$.

### Sonia

Sonia is risk-neutral. When offered a coin-flip wager of n apples or 0 apples, she will not consider the option of accepting a wager-free $\frac{n}{2}$ apples any better or worse than taking the wager, however, she would prefer the wager if offered $\frac{n}{2} - 1$ or less, and she would prefer the wager-free cashout of $\frac{n}{2} + 1$ if offered.

## Show that $U(x) = x^3$ is sometimes risk-averse and sometimes risk-seeking

### Risk-averse

A coin-flip wager of either losing 10 dollars or gaining 2 dollars, the utility function would prefer a wager free sure thing of losing 7 dollars, despite E(wager) = -4. This is characterization of risk-averse behaviour.

### Risk-seeking

In the opposite wager of a coin-flip wager of either winning 10 dollars or losing 2 dollars, the utility function would prefer the wager to the option of a wager free sure thing of winning 7 dollars, despite the expected winnings is only 4. This is characterization of risk-seeking behaviour.

### Risk-neutral

In the very specific example of when the wager is 50/50 and the potential winnings is equal to the potential loss, the function is risk-neutral, having no preference to the wager or the sure thing, but chosing the appropriate one when the offered sure thing is not equal 0.

| | Risk-Seeking | Risk-Averse |
|---|---|---|
| Lottery | [(0.5, 10), (0.5, -2)] | [(0.5, -10), (0.5, 2)] |
| Expected utility of lottery | 496 | -496 |
| Utility of expected monetary value | 64 | -64 |

## Problem 2: Decision Network

The decision network is drawn and shown in figure 1.

$$\begin{aligned}
P(p|b) &= \sum_m P(p|b,m) * P(m|b) \\
&= 0.9 * 0.9 + 0.5 * 0.1 \\
&= 0.86 \\
P(p|\neg b) &= \sum_m P(p|\neg b,m) * P(m|\neg b) \\
&= 0.8 * 0.7 + 0.3 * 0.3 \\
&= 0.65
\end{aligned}$$

This gives the following expected utilities:

$$\begin{aligned}
E(U[b]) &= \sum_p P(p|b) * U(p,b) \\
&= 0.86 * 1900 + 0.14 * (-100) \\
&= \underline{\underline{1620}} \\
E(U[\neg b]) &= \sum_p P(p|\neg b) * U(p,\neg b) \\
&= 0.65 * 2000 + 0.35 * 0 \\
&= \underline{\underline{1300}}
\end{aligned}$$

This means that Sam should buy the book as it give the higher expected utility, given he does not have some sort of insane risk-averse mentality where the 0.14 chance of losing 100 dollars for naught outweighs the high probability of gaining 1900.

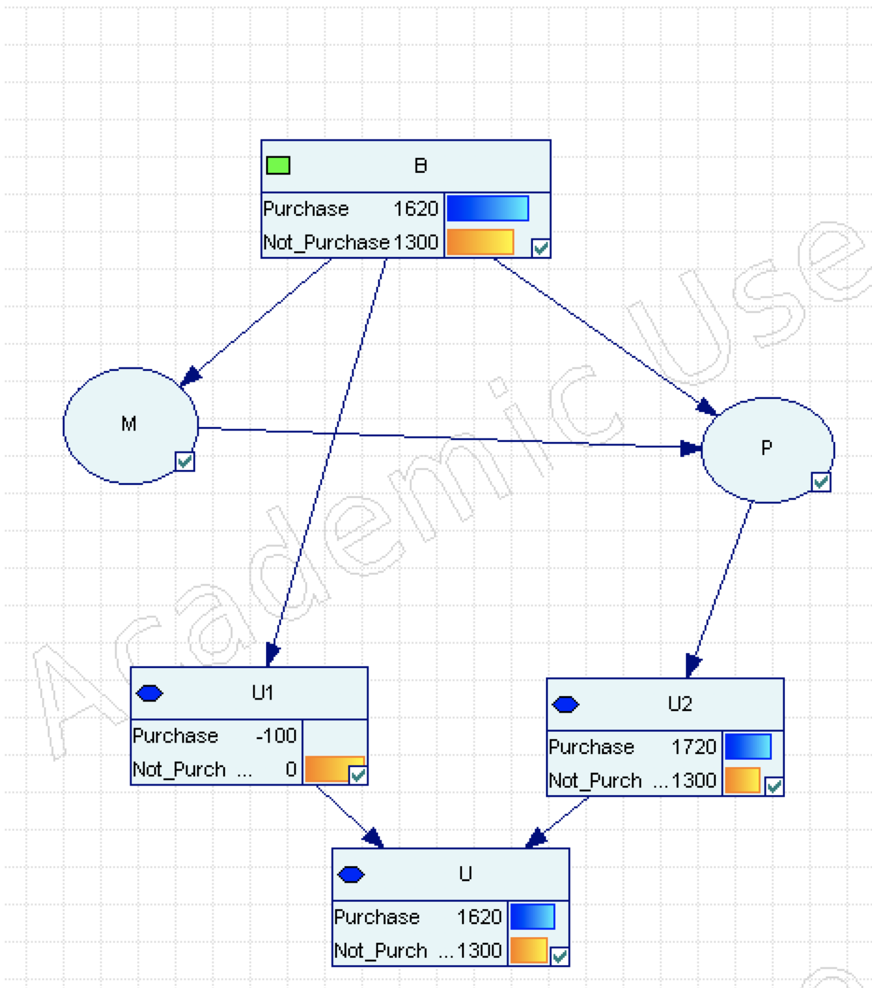


Figure 1: Decision Network for purchase of textbook.

| | | | |
|---|---|---|---|
| 1.1 | 0.5 | 1.5 | 0.2 |
| 1.5 | -10.0 | 2.1 | -10.0 |
| 3.3 | 5.8 | 5.5 | -10.0 |
| -10.0 | 7.1 | 8.3 | 10.0 |

Figure 2: utility values for described environment

# Problem 3: Markov decision process

By following the algorithm from the lecture slides and by using the kept utility table I completed two iterations and recieved the following results.

| | | | |
|---|---|---|---|
| Initial | U[1] = 0 | U[2] = 0 | U[3] = 0 |
| Iteration 1 | U[1] = 0 | U[2] = 1 | U[3] = 0 |
| Iteration 2 | U[1] = 3/8 | U[2] = 1 | U[3] = 3/8 |

The best action to take given the approximately optimal values of the utilities, is R. This results in a utility of $U[R] = \frac{3}{4} * 1.25 + \frac{1}{4} * 0.5 = \frac{17}{16}$. The utility of the other option, L, is $U[L] = \frac{1}{4} * 1.25 + \frac{3}{4} * 0.5 = \frac{11}{16}$.

# Problem 4: Markov decision process

By implementing the remaining functions of the skeleton file, the game board with the utility values for each space becomes as shown in figure 2. The algorithm uses 346 iterations. From these we find the optimal policy which is shown in figure 3. Here, D symbolizes that the optimal choice is downwards, R symbolizes right, L symbolizes left, and U symbolizes up.

```
+-----+-----+-----+-----+
|    D|    R|    D|    L|
+-----+-----+-----+-----+
|    D|    D|    D|    L|
+-----+-----+-----+-----+
|    R|    D|    D|    D|
+-----+-----+-----+-----+
|    R|    R|    R|    L|
+-----+-----+-----+-----+
```

Figure 3: The Optimal Policy