Assignment 1

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Problem 1: Probability

1.1: The probabilities that a person will eat 0, 1, 2, 3, 4, or 5 or more bananas during a certain 30-minute period are 0.03, 0.18, 0.24, 0.28, 0.10, and 0.17, respectively. Find the probability that in this 30-minute period

a) more than 2 bananas are eaten?

$$\begin{aligned} P(X > 2) &= P(X = 3) + P(X = 4) + P(X = 5) \\ &= 0.28 + 0.10 + 0.17 \\ &= \underline{\underline{0.55}} \end{aligned}$$

b) at most 4 bananas are eaten?

$$\begin{aligned} P(X \leq 4) &= 1 - P(X = 5) \\ &= 1 - 0.17 \\ &= \underline{\underline{0.83}} \end{aligned}$$

c) 4 or more bananas are eaten?

$$\begin{aligned} P(X \geq 4) &= P(X = 4) + P(X = 5) \\ &= 0.10 + 0.17 \\ &= \underline{\underline{0.27}} \end{aligned}$$

1.2 Assume that there is an apple supplier that ships apples to grocery stores in batches of size 20. Suppose that 60% of all such batches contain no rotten apples, 30% contain one rotten apple, and 10% contain two rotten apples. A batch is picked, two apples from the batch are randomly selected and inspected, and neither is rotten.

The probability of drawing 2 ripe apples from a random batch is given by the weighted sum of probabilities of picking this from each of the categories:

$$\begin{aligned} P(2ripe) &= P(2ripe|0Rot) * P(0Rot) + P(2ripe|1Rot) * P(1Rot) + P(2ripe|2Rot) * P(2Rot) \\ &= 1 * 0.6 + 0.9 * 0.3 + \frac{18 * 17}{20 * 19} * 0.1 \\ &= \underline{0.9505} \end{aligned}$$

a) What is the probability that zero rotten apples exist in the batch?

$$\begin{aligned} P(0Rotten|2ripedrawn) &= \frac{P(2ripedrawn|0Rotten) * P(0Rotten)}{P(2ripedrawn)} \\ &= \frac{1 * 0.6}{0.9505} \\ &= \underline{\underline{63.1\%}} \end{aligned}$$

b) What is the probability that one rotten apple exists in the batch?

$$\begin{aligned} P(1Rotten|2ripedrawn) &= \frac{P(2ripedrawn|1Rotten) * P(1Rotten)}{P(2ripedrawn)} \\ &= \frac{0.9 * 0.3}{0.9505} \\ &= \underline{\underline{28.4\%}} \end{aligned}$$

c) What is the probability that two rotten apple exists in the batch?

$$\begin{aligned} P(2Rotten|2ripedrawn) &= \frac{P(2ripedrawn|2Rotten) * P(2Rotten)}{P(2ripedrawn)} \\ &= \frac{0.805 * 0.1}{0.9505} \\ &= \underline{\underline{8.5\%}} \end{aligned}$$

1.3 A form of common cold is known to be found in men over 50 with probability 0.07. A blood test exists for the detection of the disease, but the test is not infallible. In fact, it is known that 10% of the time the test gives a false negative (i.e., the test incorrectly gives a negative result) and 5% of the time the test gives a false positive (i.e., incorrectly gives a positive result).

From the text we extract the following useful facts:

P(cold) = 0.07 P(healthy) = 0.93 P(neg—cold) = 0.1 P(neg—healthy) = 0.95

and from this we find:

$$\begin{aligned} P(neg) &= P(neg|cold) * P(cold) + P(neg|healthy) * P(healthy) \\ &= 0.1 * 0.07 + 0.95 * 0.93 \\ &= \underline{0.8905} \end{aligned}$$

a) If a man over 50 is known to have taken the test and received a favorable (i.e., negative) result, what is the probability that he has the disease?

$$\begin{aligned} P(cold|neg) &= \frac{P(neg|cold) * P(cold)}{P(neg)} \\ &= \frac{0.1 * 0.07}{0.8905} \\ &= \underline{\underline{0.79\%}} \end{aligned}$$

1.4: A box of 12 computer sets contains 3 defective sets. In how many ways can a person purchase 5 of these sets and receive at least 2 of the defective sets?

We divide into two cases: the case where the person receives 2 defective, and the case of receiving 3.

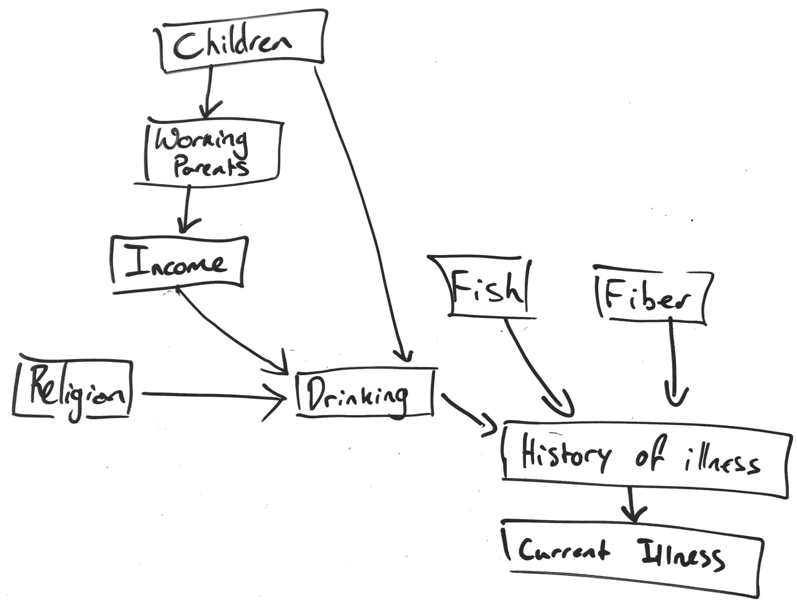
Case of 2: $\binom{3}{2} * \binom{9}{3} = 3 * 84 = 252$

Case of 3: $\binom{3}{3} * \binom{9}{2} = 1 * 36 = 36$

Total:

$\binom{3}{2} * \binom{9}{3} + \binom{3}{3} * \binom{9}{2} = 252 + 36 = \underline{\underline{288}}$

Problem 2: Bayesian Network Construction



A node is conditionally independent of non-descendants given its predecessors. This means we have the following conditional independencies:

note: i will not mention any conditional dependency if it has already been stated in a previous node-case, as we know conditional independence to be a transitive property.

As fish, fiber, religion and children do not have any predecessors, they are conditionally independent of all non-descendants. This is consistent with what we know of the real world. e.g consumption of fish is independent of amount of children.

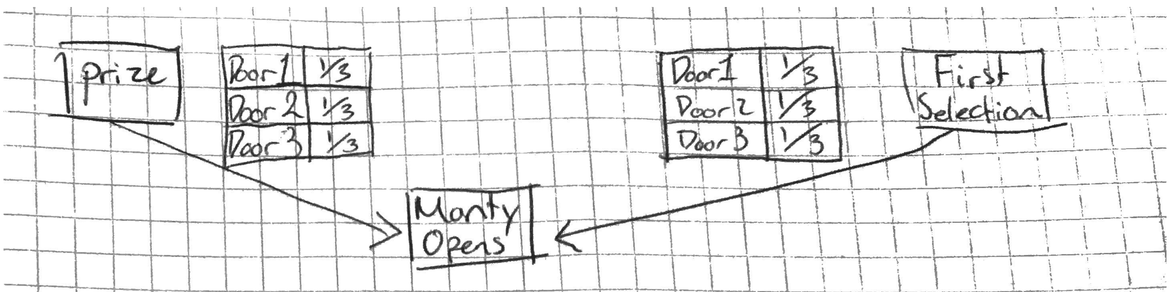
Income is conditionally independent of Children given Working parents. This is consistent with what we know of the real world.

Drinking is conditionally independent of Working Parents given Income, Children and Religion. This is consistent with what we know of the real world.

History of illness is conditionally independent of Children, working parents, Income and religion given Drinking. This is consistent with what we know of the real world.

Current Illness is conditionally independent of all others except history of illness, given History of Illness. This is also consistent with our real world perceptions. e.g. Given a history of illness, a current illness is not correlated with income, amount of children or fiber-eating-habits.

Problem 3: The Monty Hall problem



Prize	First Selection	Monty opens D1	Monty opens D2	Monty Opens D3
Door 1	Door 1	0	0.5	0.5
Door 1	Door 2	0	0 ~~0.5~~	1
Door 1	Door 3	0	1	0
Door 2	Door 1	0	0	1
Door 2	Door 2	0.5	0	0.5
Door 2	Door 3	1	0	0
Door 3	Door 1	0	1	0
Door 3	Door 2	1	0	0
Door 3	Door 3	0.5	0.5	0

We will focus on the case that we choose Door 1 as first choice since we do not lose any specificity by doing this. We will then find the probability of getting the prize by sticking to our choice, in other words that the prize is behind the chosen door, 1.

$$
\begin{aligned}
P(prize = 1) &= P(prize = 1, open = 1) + P(prize = 1, open = 2) + P(prize = 1, open = 3) \\
&= 0 + P(prize = 1|open = 2) * P(open = 2) + P(prize = 1|open = 3) * P(open = 3) \\
&= P(open = 2|prize = 1) * P(prize = 1) + P(open = 3|prize = 1) * P(prize = 1) \\
&= \frac{1}{2} * \frac{1}{3} + \frac{1}{2} * \frac{1}{3} \\
&= \underline{\underline{\frac{1}{3}}}
\end{aligned}
$$

By this we know that by sticking to our choice we have 1/3 chance of receiving the prize, and 2/3 if one chooses to swap for the other door.