Work Sheet 9

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C.1.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \mathbf{x} = \lambda \mathbf{x}$$

$$\begin{aligned} ax_1 + bx_2 &= \lambda x_1 \\ cx_1 + dx_2 &= \lambda x_2 \end{aligned}$$

$$\begin{aligned} (a-\lambda)x_1 + bx_2 &= 0 \\ cx_1 + (d-\lambda)x_2 &= 0 \end{aligned}$$

This new system can be represented as $\mathbf{Bx} = \mathbf{0}, where \mathbf{B} = \begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix}$ The system has a unique solution if the determinant of the coefficient matrix does not equal zero.

$$\begin{aligned} det(B) &\neq 0 \\ (a-\lambda) * (d-\lambda) - b * c &\neq 0 \\ \lambda^2 - (a+d)\lambda + ad - bc &\neq 0 \\ \lambda &\neq \frac{a + d \pm \sqrt{(a+d)^2 - 4(ad-bc)}}{2} \\ \lambda &\neq \frac{a + d \pm \sqrt{a^2 + d^2 - 2ad + 4bc}}{2} \\ \lambda &\neq \frac{a + d \pm \sqrt{(a-d)^2 + 4bc}}{2} \end{aligned}$$

C.2.

Multiplying the matrix **A** by r will be the same as multiplying each of the n rows of **A** by r. Multiplying one row by r will increase the determinant by a factor of r. Hence:

$$det(r\mathbf{A}) = r^n det(A)$$

C.3

The determinant of the product of two matrices is equal to the product of their determinants. Therefore:

$$det(\mathbf{A}^k) = (det(\mathbf{A}))^k$$