3

$a_1 = 1 \qquad a_{n+1} = \sqrt{1+a_n}$

$a_2 = \sqrt{1+1\cdot 2} = \sqrt{3}$

$a_2 > a_1$ derfor antar vi at $a_{n+1} > a_n$:

$a_{n+1} = \sqrt{2 \cdot a_{n+1} + 1} > \sqrt{2a_n + 1} = a_n$

$a_{n+1} > a_n$

a_n vil derfor alltid være voksende.

$a = \lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \sqrt{2a_n + 1} = \sqrt{2 \lim_{n\to\infty} a_n + 1}$

Dersom følgen konvergerer må:

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1} = a$

$a = \sqrt{2a+1}$

$a^2 = 2a+1$

$a = \frac{2 \pm \sqrt{8}}{2}, \quad \underline{\underline{a = 1+\sqrt{2}}} \lor a = 1-\sqrt{2}$

For øvre skranke bruker vi det positive løsningen.

Siden den er stigende og har en øvre skranke, er den konvergerende.

$\boxed{2}$

$$f(x) = \arctan(x) - x^2$$

$$\lim_{x \to -\infty} f(x) = -\infty$$

$$f(0) = 0$$

$$\lim_{x \to \infty} f(x) = \infty$$

$$f'(x) = \frac{1}{1+x^2} - 2x = \frac{1-2x-2x^3}{1+x^2}$$

Vi setter $g(x) = 1 - 2x - 2x^3$

$$g'(x) = -2 - 6x^2$$

Siden $g'(x)$ er alltid negativt vil $f(x)$ krysse $f(x) = 0$ på den positive siden:

$g(0) = 1$, $g(1) = -3$

dermed kan vi konkludere med at den stiger fra $f(0) = 0$ på den positive siden og krysser punktet $f(r) = 0$ mellom $f(0) = 0$ og $f(1) = \frac{\pi}{4} - 1$ (som er negativt).

b) $P_3(x) = f(0) + f'(0)(x-0) + f''(0)(x-0)^2 + f'''(0)(x-0)^3$

$f(x) = \arctan(x)$, $f'(x) = \frac{1}{1+x^2}$, $f''(x) = -\frac{2x}{(x^2+1)^2}$, $f'''(x) = \frac{6x^2-2}{(x^2+1)^3}$

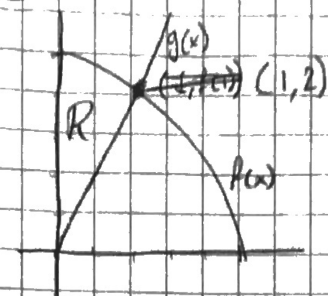
$$P_3(x) = 0 + \frac{1(x-0)}{1} + \frac{\left(\frac{-0}{1}\right)(x+0)^2}{2} + \frac{\left(\frac{-2}{1}\right)(x-0)^3}{6}$$

$$P_3(x) = \underline{\underline{x - \frac{x^3}{3}}}$$

1

Øving 3

T



$$f(x) = \sqrt{5-x^2}$$

$$g(x) = 2x$$

$$R = \int_{y=0}^{1} \sqrt{5-x^2} - \int$$

$$2x = \sqrt{5-x^2}$$

$$4x^2 = 5 - x^2$$

$$5x^2 = \cancel{5}$$

$$x^2 = 1$$

$$\underline{x = 1} \quad \underline{y = 2}$$

$$R = \int_0^2 \frac{x}{2}\,dx + \int_2^{\sqrt{5}} \sqrt{5-x^2}\,dx$$

$$T = \pi \int_0^2 \left(\frac{x}{2}\right)^2 dx + \pi \int_2^{\sqrt{5}} \left(5 - x^2\right) dx$$

$$= \pi\left(\left[\frac{x^3}{12}\right]_0^2 + \left[5x - \frac{x^3}{3}\right]_2^{\sqrt{5}}\right)$$

$$= \pi\left(\frac{2}{3} + \sqrt{5}^3 - \frac{\sqrt{5}^3}{3} - 10 + \frac{8}{3}\right)$$

$$= \pi\left(\frac{2}{3} + \frac{2\sqrt{5}^3}{3} - \frac{22}{3}\right)$$

$$= \frac{2\pi}{3}\left(\sqrt{5}^3 - 10\right)$$

$$= \frac{10\pi}{3}\left(\sqrt{5} - 2\right) = \underline{\underline{2,47}}$$