19.9

Øving 2

$\boxed{1}$

$X(t_0) = 3 \text{ km}$

$Y(t_0) = 4 \text{ km}$

$Z(t_0) = 5 \text{ km}$

$X'(t_0) = -50 \text{ km/t}$

$Y'(t_0) = 80 \text{ km/t}$

Ved bruk av Pythagoras teorem får vi at:

$$Z(t_0) = \sqrt{x^2(t_0) + y^2(t_0)}$$

$$Z(t_0) = \sqrt{9 + 16} = \underline{5 \text{ km}}$$

Hvis vi deriverer Pythagoras teorem får vi:

$$\left(x^2(t_0) + y^2(t_0)\right)\frac{d}{dx} = \left(z^2(t_0)\right)\frac{d}{dx}$$

$$2x(t_0) \cdot x'(t_0) + 2y(t_0) \cdot y'(t_0) = 2 \cdot z(t_0) \cdot z'(t_0)$$

$$z'(t_0) = \frac{x(t_0) \cdot x'(t_0) + y(t_0) \cdot y'(t_0)}{z(t_0)}$$

$$\cancel{z'(t_0) = \frac{3 \cdot (80) + 4 \cdot (-50)}{5}} \quad \cancel{\frac{3 \cdot (-50) + 4 \cdot 80}{5}}$$

$$\cancel{=} \quad \cancel{8 \text{ km/h}} \quad \cancel{34 \text{ km/h}}$$

~~Siden denne er positiv, så fjerner båtene seg fra hverandre med en fart på [illegible] km/s.~~

$$Z'(t) = \frac{3 \cdot 80 + 4 \cdot (-50)}{5} = 8 \text{ km/h}$$

De fjerner seg fra hverandre med en fart på 8 km/h

Øving 2

$\boxed{2}$ $S_n = \sum_{i=1}^{n} \frac{1}{n(2+\frac{i}{n}) \cdot \ln(2+\frac{i}{n})}$

$P = \Delta x_j = \frac{1}{n}$

$[a,b] = [0,1]$

$C = S_i$

$f(x) = \frac{1}{(2+\frac{i}{n}) \cdot \ln(2+\frac{i}{n})}$

$x =$

$\lim_{n \to \infty} R(f, P, C) = \boxed{\int_0^1 \frac{1}{x \cdot \ln x}}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{(2+\frac{i}{n}) \cdot \ln(2+\frac{i}{n})} = \int_0^1 \frac{1}{(2+x) \cdot \ln(2+x)} = \int_{x=0}^{x=1} \frac{1}{x \cdot u} \cdot \frac{dx}{\frac{du}{2+x}}$$

$$= \int_{x=0}^{x=1} \frac{1}{u}$$

$$= \Big[\ln u\Big]_{x=0}^{x=1}$$

$$= \Big[\ln(\ln(2+x))\Big]_0^1$$

$$= \ln(\ln(3)) - \ln(\ln(2))$$

$$= \underline{\underline{0,46}}$$

$\boxed{3}$

$$\int_2^{\infty} \frac{x^2+9}{x^4+3x^2-4} = \lim_{R\to\infty} \int_2^{R} \frac{x^2+9}{x^4+3x^2-4}$$

$$\frac{A}{x^2+4} + \frac{B}{x+1} + \frac{C}{x-1} = \frac{x^2+9}{x^4+3x^2-4}$$

$$A(x^2-1) + B(x^3+3x^2-4) + C(x^3+5x^2+4) = x^2+9$$

$$B = -C$$

$$A+3B+5C = 1$$

$$A+2C = 1$$

$$4c + (-4b) + (-A) = 9$$

$$8c - A = 9$$

$$10c - 1 = 9$$

$$C = 1$$
$$b = -1$$
$$A = -1$$

$$\lim_{R\to\infty} \int_2^R \left(\frac{1}{x-1} - \frac{1}{x+1} - \frac{1}{x^2+4} \right) dx$$

$$= \lim_{R\to\infty} \left[\ln(x-1) - \ln(x+1) \right] - \int \frac{1}{x^2+4}$$

$$= \lim_{R\to\infty} \left[\ln(x-1) - \ln(x+1) - \frac{1}{2}\arctan\left(\frac{x}{2}\right) \right]_2^R$$

$$= \lim_{R\to\infty} \left[\ln\left(\frac{x-1}{x+1}\right) - \frac{1}{2}\arctan\frac{x}{2} \right]_2^R$$

$$= \lim_{R\to\infty} \left[\ln\left(\frac{R-1}{R+1}\right) - \frac{1}{2}\arctan\left(\frac{R}{2}\right) - \ln\left(\frac{1}{3}\right) + \cancel{\frac{1}{2}\arctan} \right.$$
$$\left. + \frac{1}{2}\arctan(1) \right]$$

$$= 0 - \frac{\pi}{4} + \ln(3) + \frac{\pi}{4}\cdot\frac{1}{2}$$

$$= \ln(3) - \frac{\pi}{8} \approx \underline{\underline{0{,}71}}$$

$\boxed{4}$

Buelengde er gitt ved:

$$S = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

$$[a,b] = [0,1]$$

$$f(x) = \frac{2}{3}x^{\frac{3}{2}}$$

$$f'(x) = \sqrt{x}$$

$$S = \int_0^1 \sqrt{1 + x}$$

$$= \left[\frac{2}{3}(x+1)^{\frac{3}{2}}\right]_0^1$$

$$= \frac{2}{3}(2)^{\frac{3}{2}} - \frac{2}{3} \quad = \frac{2}{3}\left(\sqrt[2]{2^3} - 1\right) \approx \underline{\underline{1,22}}$$

Overflateareal:

$$A = 2\pi \int_a^b |x| \cdot \sqrt{1 + f'(x)^2}$$

$$A = 2\pi \int_0^1 x \cdot \sqrt{1 + x}$$

$$A = 2\pi \left[x \cdot \sqrt{(1+x)^3} \cdot \frac{2}{3} - \int \sqrt{(1+x)^3} \cdot \frac{2}{3} dx\right]_0^1$$

$$A = \frac{4\pi}{3}\left[x \cdot \sqrt{(1+x)^3} - \frac{2}{5}(x+1)^{\frac{5}{2}}\right]_0^1$$

$$A = \frac{4\pi}{3}\left(\sqrt{8} - \frac{2}{5}\sqrt{32}\right)$$

$$A = \frac{4\pi}{3}\left(\sqrt{8}\left(1 - \frac{4}{5}\right)\right)$$

$$A = \frac{4\pi \cdot \sqrt{8}}{15}$$