Quing 8

Session 5.2

4) $P(18) = 7 \cdot 2 + 4 \cdot 1 \rightarrow$ True

$P(19) = 7 \cdot 1 + 4 \cdot 3 \rightarrow$ True

$P(20) = 4 \cdot 5 \rightarrow$ True

$P(21) = 7 \cdot 3 \rightarrow$ True

4 ~~$P(k-4) \rightarrow P(k)$~~

$P(k-3) \Rightarrow P(k+1)$

This is because we can add a 4-cent to the solution of $P(k-3)$ to acquire the solution of $P(k-1)$.

This gives us a proof telling us

$[P(k-3) \wedge P(k-2) \wedge P(k-1) \wedge P(k)] \Rightarrow P(k+1)$

since if we have proved four amounts in a row, we know that one of them can be added a multiple of 4 to reach any amount of cents.

7) Basis Step:

$P(0)$ = True because $0 = 0 \cdot 1 + 0 \cdot 5$

$P(1)$ = True because $1 = 1 \cdot 1 + 0 \cdot 5$

$P(2)$ = True because $2 = 2 \cdot 1 + 0 \cdot 5$

Inductive Hypothesis:

$[P(k) \rightarrow P(k+5)]$

Inductive Step:

As we can add as many 5-notes as we'd like, it follows that if we can represent K, we can represent K+5. ~~Therefore we can represent all variations of~~

Continues →

~~Therefore~~

Since $P(0)$, $P(1)$ and $P(2)$ are True, we know $P(0+k\cdot 5)$, $P(1+5\cdot k)$ and $P(2+k\cdot 5)$ all true. We can form amounts of $5\cdot k$, $1+5k$ and $2+5\cdot k$.

$\boxed{\text{Section 5.3}}$

17. We need to prove $\sum_{k=1}^{n} f_k^2 = f_n \cdot f_{n+1}$

we will use induction for this

$$\sum_{k=1}^{1} f_k^2 = f_1 \cdot f_2$$

$$\downarrow$$

$$0^2 = 0 \cdot 1 \quad \checkmark \text{ True}$$

We assume the statement is true for n, and prove for $n+1$:

$$\sum_{k=1}^{n} f_k^2 + f_{n+1}^2 = f_{n+1} \cdot f_{n+2}$$

$$f_n \cdot f_{n+1} + f_{n+1}^2 = f_{n+1} \cdot f_{n+2}$$

$$f_{n+1} \underbrace{(f_n + f_{n+1})}_{f_{n+2}} = f_{n+1} \cdot f_{n+2}$$

$$f_{n+1} \cdot f_{n+2} = f_{n+1} \cdot f_{n+2}$$

Tautology

Proof complete

18) Induction : $P(n): \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n = \begin{bmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{bmatrix}$

$P(1): A^1 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} f_2 & f_1 \\ f_1 & f_0 \end{bmatrix}$ ← True

$P(k): A^k = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^k = \begin{bmatrix} f_{k+1} & f_k \\ f_k & f_{k-1} \end{bmatrix}$ ← Assumption

$P(k+1): A^k \cdot A^1 = \begin{bmatrix} f_{k+1} & f_k \\ f_k & f_{k-1} \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} f_{k+1} + f_k & f_{k+1} \\ f_k + f_{k-1} & f_k \end{bmatrix}$

$= \begin{bmatrix} f_{k+2} & f_{k+1} \\ f_{k+1} & f_k \end{bmatrix}$

Proof

Section 5.4

3) gcd(8,13):

gcd(5, 8)

gcd (3, 5)

gcd (2, 3)

gcd (1, 2)

gcd (0, 1)

return 1

Section 9.1

7) a) Symmetric, ~~transitive~~

b) Symmetric, transitive

c) Symmetric

d) reflexive, Symmetric

e) reflexive, Asymmetric, transitive

f) reflexive, Symmetric, transitive

g) Asymmetric,

h) transitive, Asymmetric

40 a) $R_1 \cup R_2 = \{(a,b), (b,a) \mid a | b\}$

c) $R_1 - R_2 = \{(a,b) \mid a | b \ \& \ a \neq b\}$

Session 9.3

10 a) $\frac{1}{2} \cdot 1000 \cdot (1000+1) = \underline{500\,500}$

b) $2 \cdot (998) + 1 \cdot 2 = \underline{1998}$

c) $\underline{999}$

d) $\frac{1}{2} \cdot 1000 \cdot (1000+1) = \underline{500\,500}$

e) $1000 \cdot 1000 = \underline{1\,000\,000}$

14 a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$