Quiz 7.

Session 5.1

4. $P(1) = \left(\frac{1(1+1)}{2}\right)^2$

$1^3 = \left(\frac{1(2)}{2}\right)^2$

$1 = 1^2$

$\downarrow$

T

Inductive hypothesis: $\sum_{n=1}^{n=k} n^3 = \left(\frac{k(k+1)}{2}\right)^2$
for an arbitrary for an arbitrary
~~k~~ positive integer ~~k~~ k.

To prove in the inductive step, we need to prove $P(k+1)$ for ~~the~~ the assumption $p(k)$

$P(k) = \left(\frac{k(k+1)}{2}\right)^2$ ← inductive hypothesis

$$P(k+1) = P(k) + (k+1)^3$$

$$= \frac{k^2(k+1)^2}{4} + (k+1)^3$$

$$= \frac{[k^2 + 4(k+1)](k+1)^2}{4}$$

$$= \frac{(k+2)^2 (k+1)^2}{4}$$

$$= \frac{(k+1)^2((k+1)+1)^2}{4}$$

$$= \left(\frac{(k+1)((k+1)+1)}{2}\right)^2$$

d) Since $P(k)$ is true for $P(1)$ and for $P(k+1)$, we know $P(k)$ is true for all k, where k is a positive integer, as since it is true for $P(1)$ and $P(1+1)$, and $P(2+1)$ and so on.

6) $$\sum_{k=1}^{k=n} k! \cdot k = (n+1)! - 1$$

$$P(1) = 2! - 1 = \underline{1}$$

$$1! \cdot 1 = \underline{1}$$

$P(1)$ is true

$$P(k) + (k+1)! \cdot (k+1) = ((k+1)+1)! - 1$$

$\cancel{(n+1)! - 1 + k + 1}$

$$(k+1)! - 1 + (k+1)! \cdot (k+1) = (k+2)! - 1$$

$$(k+1)!\,(1 + (k+1)) - 1 =$$

$$(k+2)(k+1)! - 1 =$$

$$(k+2)! - 1 = (k+2)! - 1$$

True

14) $\sum_{k=1}^{n} k2^k = (n-1)2^{n+1} + 2$

$P(1): 1 \cdot 2^1 = 0 + 2$

$2 = 2$ True

$P(n) = (n-1)2^{n+1} + 2$

~~P(n+1) : ... P(n~~

$P(n+1) = P(n) + (n+1)2^{n+1}$

$((n+1)-1)2^{(n+1)+1} + 2 = (n-1)2^{n+1} + 2 + (n+1)2^{n+1}$

$n \cdot 2^{(n+2)} + 2 = 2^{n+1}\left[(n-1) + (n+1)\right] + 2$

$n \cdot 2^{n+2} + 2 = 2n \cdot 2^{n+1} + 2$

$n \cdot 2^{n+2} + 2 = n \cdot 2^{n+2} + 2$ -> True

Session 8.1

11) $Y_1 = 1$, $Y_2 = 2$

$Y_n = Y_{n-1} + Y_{n-2}$

c)

$Y_3 = 3$
$Y_4 = 3 + 2 = 5$
$Y_5 = 8$
$Y_6 = 13$
$Y_7 = 21$
$Y_8 = 34$

20. $a_{10} = 2$, $a_5 = 1$

$Y_n = Y_{n-10} + Y_{n-5}$

Seksjon 8.2

3 a.

$a_n = 5a_{n-1} - 6a_{n-2}$

$r^2 - 5r + 6 = 0$

$(r - \cancel{5}2)(r-3) = 0$

$r = 2$, $r = 3$

$a_n = \alpha \cdot r_1^n + \beta \cdot r_2^n$

$a_n = \alpha\, 2^n + \beta\, 3^n$

$a_0 = 1 : 1 = \alpha + \beta$

$a_1 = 0 = 2\alpha + 3\beta$

$\beta = -2$, $\alpha = 3$

$a_n = 3 \cdot 3^n - 2 \cdot 2^n$

$a_n = 3^{n+1} - 2^{n+1}$

3d)

$a_n = 4a_{n-1} - 4a_{n-2}$

$r^2 - 4r + 4$

$(r-2)^2 = 0$

$a_n = \alpha r^n + \beta n r^n$

$a_0 = 6 = \alpha$

$a_1 = 8 = 6 \cdot 2^1 + \beta \cdot 1 \cdot 2^1 = 12 + 2\beta$

$\beta = -2$

$a_n = 6 \cdot 2^n - 2^{n+1}$

3 e) $r^2 + 4r + 4 = 0$

$\beta = \frac{4}{3}$

$\alpha = -\frac{1}{3}$

$a_n = \alpha \cdot (-2)^n + \beta \cdot n \cdot (-2)^n$

$a_0 = \underline{0 = \alpha}$

$a_1 = 1 = \beta \cdot 1 \cdot -2$

$\beta = -\frac{1}{2}$

$a_n = -\frac{1}{2} \cdot n \cdot (-2)^n$

3 g) $a_n = \frac{a_{n-2}}{4}$

$r^2 - \frac{1}{4} = 0$

$(r + \frac{1}{2})(r - \frac{1}{2}) = 0$

$a_n = \alpha(-\frac{1}{2})^n + \beta(\frac{1}{2})^n$

$a_0 = 1 = \alpha + \beta$

$a_1 = \frac{\beta - \alpha}{2} = 0$

$\alpha = \beta = \frac{1}{2}$

6) $Y_n = Y_{n-1} + 2Y_{n-2}$

$(r+1)(r-2)$

$a_n = \alpha(-1)^n + \beta(2)^n$

$a_0 = 1 = \alpha + \beta$

$a_1 = 3 = \alpha \cdot (-1) + \beta \cdot 2 \rightarrow 3 = 2\beta - \alpha$

$\alpha = -\frac{1}{3}, \beta = \frac{4}{3}$

$$\underline{\underline{a_n = \frac{4}{3} \cdot 2^n - \frac{1}{3} \cdot (-1)^n}}$$

ii) $\mathcal{L}_n = L_{n-1} + L_{n-2}$

$r^2 - r - 1 = 0$ $\qquad$ $r_1 = \cancel{\varphi}\ \varphi$, $\quad r_2 = -(\varphi + 1)$

$a_n = \alpha r_1^{n} + \beta r_2^{n}$

$L: \alpha + \beta = 2$ $\qquad$ $f: \alpha + \beta = 0$

$L: r_1 \cdot \alpha + r_2 \cdot \beta = 1$ $\qquad$ $f: r_1 \cdot \alpha + r_2 \cdot \beta = 1$

$f_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$

$\mathcal{L}_n = \alpha \varphi^n + \beta\left(-(\varphi - 1)\right)^n$

$\mathcal{L}_0 = 2 = \alpha + \beta$

$\mathcal{L}_1 = 1 = \alpha\left(\frac{1+\sqrt{5}}{2}\right) + \beta\left(\frac{1-\sqrt{5}}{2}\right) = \frac{\alpha+\beta}{2} + \frac{(\alpha-\beta)\sqrt{5}}{2}$

$\alpha = 1 \qquad \beta = 1$

$\mathcal{L}_n = \varphi^n + (1-\varphi)^n$ $\qquad$ $f_n = \frac{1}{\sqrt{5}}\left(\varphi^n - (1-\varphi)^n\right)$

$\mathcal{L}_n = f_{n-1} + f_{n+1}$

$= \frac{1}{\sqrt{5}}\left(\varphi^{n-1} - (1-\varphi)^{n-1}\right) + \frac{1}{\sqrt{5}}\left(\varphi^{n+1} - (1-\varphi)^{n+1}\right)$

$= \frac{1}{\sqrt{5}}\left(\varphi^{n-1}(1+\varphi^2) - (1-\varphi)^{n-1}(1+\varphi^2)\right)$

$= \frac{1}{\sqrt{5}}\left[\underbrace{(1+\varphi^2)}\left(\varphi^{n-1} - (1-\varphi)^{n-1}\right)\right]$

$= \frac{1+\sqrt{5}}{2}\left(\varphi^{n-1} - (1-\varphi)^{n-1}\right)$

$= \varphi^n - \varphi(1-\varphi)^{n-1}$

$= \varphi^n + (1-\varphi)^n$

$= \underline{\underline{\mathcal{L}_n}}$