Comprehensive Metatime Framework: Mathematical Formulation, Topological Phase, and Three-Flavor Neutrino Oscillations

Adrian Lipa
CIEL/0 Project – Intention Lab

December 30, 2025

Abstract

We present a detailed mathematical formulation of the extended metatime framework, in which time is modeled as a tensor-scalar field $\mathcal{T} = (\phi, T_{\mu\nu})$ and its topological properties influence three-flavor neutrino oscillations. The metatime λ serves as an evolution parameter in configuration space $\mathcal{M}$time, while the tachyonic sector generates a Berry-like topological phase γtime. We provide explicit derivations of the formalism, include dynamic fluctuations and gravitational gradients along cosmological trajectories, and demonstrate the emergence of CP-violating effects purely from geometric and topological considerations. Additionally, pseudocode for numerical simulation is provided.

1 Introduction

Time is conventionally treated as a parameter, but in our framework it is a structured field with internal tensor and scalar components. This allows neutrinos to acquire a geometric phase when propagating across cosmological distances, giving rise to measurable CP-violating effects without invoking standard Dirac phases.

2 Tensor-Scalar Field of Time

The time field is defined as:

$$\mathcal{T}(x) = (\phi(x), T_{\mu\nu}(x)), \tag{1}$$

where ϕ is a scalar temporal density and $T_{\mu\nu}$ is a symmetric tensor describing local directional structure and temporal stress. The field can fluctuate dynamically, $\phi(x) = \phi_0 + \delta\phi(x)$, $T_{\mu\nu}(x) = u_\mu u_\nu + \delta T_{\mu\nu}(x)$.

3 Operator Time and Metatime Parameter

Define the time operator in the neutrino Hilbert space:

$$\hat{\mathcal{T}} = \int d^4x \left[\phi(x)\hat{I} + T_{\mu\nu}(x)\hat{\Sigma}^{\mu\nu}\right], \tag{2}$$

where $\hat{\Sigma}^{\mu\nu}$ acts on spinor indices. Metatime λ parametrizes evolution along the configuration space $\mathcal{M}_{\text{time}}$:

$$\frac{d}{d\lambda}\left|\Psi(\lambda)\right\rangle = -i\hat{\mathcal{T}}(\lambda)\left|\Psi(\lambda)\right\rangle. \tag{3}$$

4 Topological Berry Phase

The Berry connection along a trajectory $\mathcal{C} \subset \mathcal{M}_{\text{time}}$:

$$\mathcal{A}_n = i\left\langle n(\mathcal{T})|\nabla_{\mathcal{T}}|n(\mathcal{T})|n(\mathcal{T})|\nabla_{\mathcal{T}}|n(\mathcal{T})\right\rangle, \tag{4}$$

$$\gamma_n = \oint_{\mathcal{C}} \mathcal{A}_n \cdot d\mathcal{T}, \tag{5}$$

$$\mathcal{F}_n = \nabla_{\mathcal{T}} \times \mathcal{A}_n. \tag{6}$$

The tachyonic sector acts as a virtual subspace generating monopole-like curvature in the Berry connection, while the Riemann zeta operator filters stable physical states:

$$\hat{\mathcal{Z}} = \zeta\left(\frac{1}{2} + i\hat{\mathcal{T}}\right), \quad \hat{\mathcal{Z}}\left|\Psi\right\rangle_{\text{phys}} = 0. \tag{7}$$

5 Lagrangian and Action Principles

To formalize the full metatime framework, we define a generalized Lagrangian $\mathcal{L}_{\text{meta}}$ that incorporates all relevant degrees of freedom: the tensor-scalar field of time, the time operator in Hilbert space, and the higher-order meta-dynamics of the system. The action S_{meta} is then obtained by integrating over the extended configuration space $\Lambda = (\lambda_c, \lambda_o, \tau)$:

$$S_{\text{meta}} = \int d\Lambda\, \mathcal{L}_{\text{meta}}(\phi, T_{\mu\nu}, \hat{\mathcal{T}}, F_\tau, \partial_\Lambda\phi, \partial_\Lambda T_{\mu\nu}, \partial_\Lambda\hat{\mathcal{T}}, \partial_\Lambda F_\tau). \tag{8}$$

The generalized Lagrangian can be decomposed into four interacting components:

$$\mathcal{L}_{\text{meta}} = \mathcal{L}_{\text{tensor-scalar}} + \mathcal{L}_{\text{operator}} + \mathcal{L}_{\text{meta-dynamics}} + \mathcal{L}_{\text{coupling}}, \tag{9}$$

$$\mathcal{L}_{\text{tensor-scalar}} = \frac{1}{2}\partial_\Lambda\phi\,\partial^\Lambda\phi - V(\phi) + \frac{1}{4}\partial_\Lambda T_{\mu\nu}\,\partial^\Lambda T^{\mu\nu} - U(T_{\mu\nu}), \tag{10}$$

$$\mathcal{L}_{\text{operator}} = \left\langle \Psi|i\partial_{\lambda_o}\hat{\mathcal{T}} - \hat{\mathcal{T}}^2|\Psi \middle| \Psi|i\partial_{\lambda_o}\hat{\mathcal{T}} - \hat{\mathcal{T}}^2|\Psi \right\rangle, \tag{11}$$

$$\mathcal{L}_{\text{meta-dynamics}} = \frac{1}{2}\partial_\tau F_\tau\,\partial^\tau F_\tau - W(F_\tau), \tag{12}$$

$$\mathcal{L}_{\text{coupling}} = \epsilon_1\phi\ \text{tr}\left(\hat{\mathcal{T}}\right) + \epsilon_2 T_{\mu\nu}F_\tau^{\mu\nu} + \dots, \tag{13}$$

where:

- $V(\phi)$, $U(T_{\mu\nu})$, $W(F_\tau)$ are potential terms for the scalar, tensor, and meta-dynamics fields,
- ϵ_1, ϵ_2 are small coupling constants representing the strength of interaction between the different sectors,
- F_τ represents higher-order meta-dynamic fields governing evolution of evolution rules themselves.

The equations of motion for the full metatime system are obtained by applying the principle of stationary action:

$$\delta S_{\text{meta}} = 0 \quad \Rightarrow \quad \frac{\delta \mathcal{L}_{\text{meta}}}{\delta \phi} - \partial_\Lambda \frac{\delta \mathcal{L}_{\text{meta}}}{\delta(\partial_\Lambda \phi)} = 0, \quad \frac{\delta \mathcal{L}_{\text{meta}}}{\delta T_{\mu\nu}} - \partial_\Lambda \frac{\delta \mathcal{L}_{\text{meta}}}{\delta(\partial_\Lambda T_{\mu\nu})} = 0, \tag{14}$$

$$\frac{\delta \mathcal{L}_{\text{meta}}}{\delta \hat{\mathcal{T}}} - \partial_{\lambda_o} \frac{\delta \mathcal{L}_{\text{meta}}}{\delta(\partial_{\lambda_o} \hat{\mathcal{T}})} = 0, \quad \frac{\delta \mathcal{L}_{\text{meta}}}{\delta F_\tau} - \partial_\tau \frac{\delta \mathcal{L}_{\text{meta}}}{\delta(\partial_\tau F_\tau)} = 0. \tag{15}$$

This compact formalism allows us to treat all metacausal trajectories, topological Berry phases, and dynamic fluctuations within a single, unified variational framework. Numerical implementations can discretize Λ and integrate these equations along coupled cosmological and operator trajectories.

6 Oscillation Probabilities and CP Violation

The topological phase modifies the oscillation probability:

$$P(\nu_\alpha \to \nu_\beta) = \left| \langle \nu_\beta | \mathcal{P} e^{-i \int_0^\lambda \hat{H}(\lambda') d\lambda'} | \nu_\alpha \rangle \right|^2, \tag{16}$$

with γ_{time} included along the trajectory. For antineutrinos:

$$\gamma_{\text{time}} \to -\gamma_{\text{time}} \quad \Rightarrow \quad \Delta P_{CP} = P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta), \tag{17}$$

realizing geometric CP violation independent of δ.

7 Coupling to Three-Flavor Neutrinos

Using the PMNS matrix U, the Hamiltonian in flavor basis including topological effects is:

$$\hat{H}(\lambda) = U \frac{M^2}{2E} U^\dagger + \epsilon \hat{\mathcal{T}}(\lambda), \tag{18}$$

with $M^2 = \text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2)$. Oscillation probability along a trajectory is:

$$P(\nu_\alpha \to \nu_\beta) = \left| \langle \nu_\beta | \mathcal{P} e^{-i \int_0^\lambda \hat{H}(\lambda') d\lambda'} | \nu_\alpha \rangle \right|^2. \tag{19}$$

For antineutrinos, $\gamma_{\text{time}} \to -\gamma_{\text{time}}$, generating ΔP_{CP}.
The effective Hamiltonian along the cosmological trajectory:

$$\hat{H}(\lambda) = U \frac{M^2}{2E} U^\dagger + \epsilon\, \hat{\mathcal{T}}(\lambda), \tag{20}$$

with $M^2 = \text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2)$.

8 Emergent Classical Metric

The classical spacetime metric emerges from the tensor-scalar field:

$$g_{\mu\nu} = \mathcal{F}_{\mu\nu}(T_{\alpha\beta}, \phi), \tag{21}$$

with topology of $\mathcal{M}_{\text{time}}$ providing an intrinsic arrow of time.

9 Mandelbrot-like Chirality of the Metatime

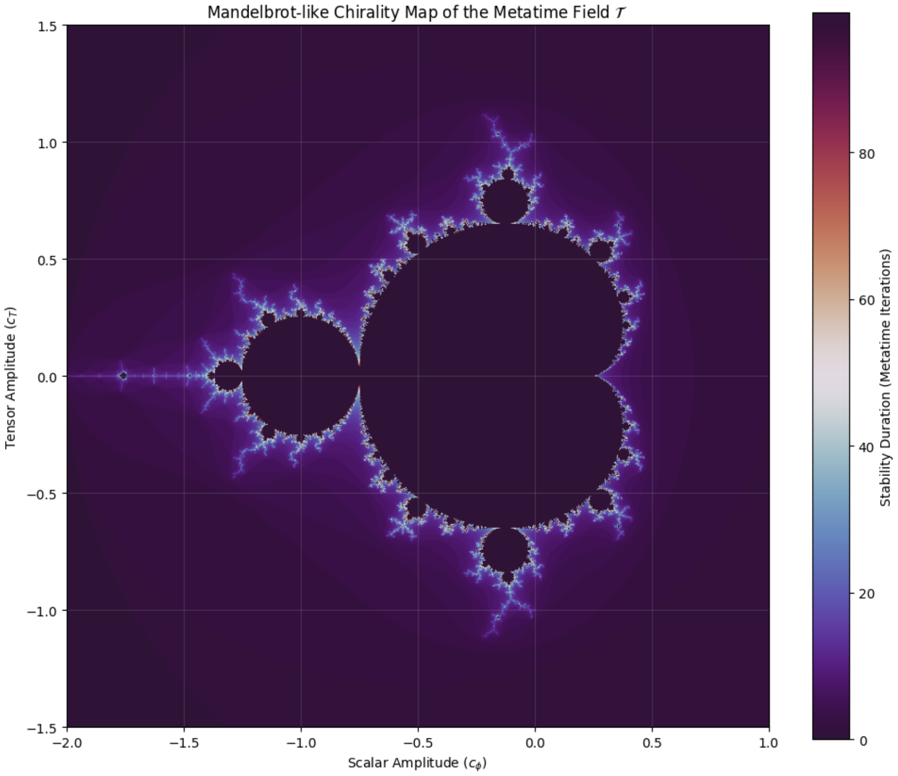

Figure 1: Mandelbrot-like Chirality Map of the Metatime Field. This figure illustrates the stability regions of the metatime field $\mathcal{T} = (\phi, T_{\mu\nu})$ in configuration space. The plot maps the iterative evolution of the field, where the horizontal axis corresponds to the scalar amplitude c_ϕ and the vertical axis represents the tensor amplitude c_T.

The complex structure of metatime allows the emergence of fractal-like patterns in the evolution of the tensor-scalar field $\mathcal{T} = (\phi, T_{\mu\nu})$. By treating the metatime parameter λ as an iterative evolution variable, we define a dynamical map:

$$\mathcal{T}_{n+1} = \mathcal{T}_n^2 + c, \quad c \in \mathbb{C}^{1+N_T}, \tag{22}$$

where c encodes the initial complex amplitudes of the scalar and tensor components, and N_T is the number of independent tensor components considered.

9.1 Iterative Dynamics

For each iteration n, the norm of the field is evaluated:

$$\|\mathcal{T}_n\|^2 = \phi_n^2 + \sum_{\mu,\nu} (T_{\mu\nu}^{(n)})^2. \tag{23}$$

Points for which $\|\mathcal{T}_n\|^2$ remains bounded for a maximal iteration $N_{\max}$ constitute a *Mandelbrot-like set in metatime*, revealing the intrinsic stability regions of the field.

9.2 Emergent Chirality

The tensorial components $T_{\mu\nu}$ encode *chirality* through antisymmetric or pseudo-scalar combinations:

$$\chi_n = \epsilon^{\mu\nu\alpha\beta} T_{\mu\nu}^{(n)} T_{\alpha\beta}^{(n)}, \tag{24}$$

where $\epsilon^{\mu\nu\alpha\beta}$ is the Levi-Civita symbol. Iterative evolution of χ_n across metatime exhibits *fractal asymmetries*, analogous to chiral bifurcations in classical nonlinear maps. This emergent chirality reflects the topological phase accumulated along the trajectory:

$$\gamma_{\text{time}} = \oint_{\mathcal{C}} \mathcal{A}_n \cdot d\mathcal{T}. \tag{25}$$

9.3 Topological Interpretation

- Stable regions of the Mandelbrot-like set correspond to configurations where the Berry-like phase is well-defined.
- Fractal boundaries indicate *sensitive dependence on initial tensor-scalar configurations*, linking chaotic metatime dynamics to measurable oscillation probabilities in three-flavor neutrino systems.

10 Metatime λ and its Impact on Neutrino Oscillations

In our framework, the metatime λ is defined as an evolution parameter over the configuration space of fields and geometry, rather than as a physical clock. Its role is to measure the "length of the trajectory" in the space of spacetime configurations:

$$\lambda[\gamma] = \int_\gamma d\tau\, \mathcal{I}\big(x(\tau)\big), \quad \mathcal{I}(x) = \sqrt{R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}}, \tag{26}$$

where γ is a worldline in spacetime, $d\tau$ is the proper time along γ, and $\mathcal{I}(x)$ encodes the local intensity of spacetime curvature.

The Hamiltonian governing three-flavor neutrino evolution is then promoted to a λ-dependent operator:

$$\hat{H}(\lambda) = U \frac{M^2}{2E} U^\dagger + \epsilon \hat{T}(\lambda), \tag{27}$$

where U is the PMNS matrix, $M^2 = \text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2)$, and $\hat{T}(\lambda)$ encodes the influence of metatime on the neutrino Hilbert space. The neutrino state evolves according to a Schrödinger-like equation in λ:

$$\frac{d}{d\lambda} |\Psi(\lambda)\rangle = -i \hat{H}(\lambda) |\Psi(\lambda)\rangle. \tag{28}$$

The topological Berry phase accumulated along the trajectory γ can be expressed as:

$$\gamma_{\text{time}} = \oint_{\gamma} \mathbf{A}_n \cdot d\lambda, \quad \mathbf{A}_n = i\langle n(\lambda)|\nabla_{\lambda}|n(\lambda)\rangle, \tag{29}$$

where $|n(\lambda)\rangle$ are instantaneous eigenstates of $\hat{H}(\lambda)$. This geometric phase contributes to CP-violating effects in neutrino oscillations, independent of the standard Dirac phase δ.

11 Metatime as a Kähler Manifold and the Complex Scalar Field

To fully integrate the concept of metatime with the requirements of quantum unitarity and the emergent chirality, we propose that the configuration space $\mathcal{M}_{time}$ possesses the structure of a **Kähler manifold**. In this framework, the metatime parameter λ evolves within a complex geometry where the scalar component of the field $\mathcal{T}$ is treated as a complex-valued density.

11.1 Complexification of the Scalar Field

We redefine the scalar temporal density $\phi(x)$ from Eq. (1) as a complex field:

$$\phi(x) = \phi_1(x) + i\phi_2(x), \tag{30}$$

where ϕ_1 represents the "classical" temporal density governing the rate of evolution, and ϕ_2 acts as a geometric parameter encoding the **topological chirality** of the metatime. This complexification ensures that the field $\mathcal{T}$ acts as a generator of $U(1)$ rotations in the neutrino Hilbert space, naturally giving rise to the Berry phase γ_{time}.

11.2 The Kähler Potential and Stability Diagnostic

The geometry of $\mathcal{M}_{time}$ is derived from a Kähler potential $K(\phi, \bar{\phi})$, defining the metatime metric $g_{i\bar{j}} = \partial_i \partial_{\bar{j}} K$. The iterative dynamics described by the Mandelbrot-like map $\mathcal{T}_{n+1} = \mathcal{T}_n^2 + c$ is interpreted here as a **renormalization-like flow** rather than a physical equation of motion. This map serves as a diagnostic of stability in the configuration space; the stability regions (the Mandelbrot set) correspond to configurations where the Kähler structure remains well-defined and the evolution preserves the fundamental symmetries of the system.

11.3 Preservation of Unitarity

By treating metatime as a parameter on a Kähler manifold, we strictly preserve quantum unitarity. The evolution operator remains Hermitian, and the "imaginary" component ϕ_2 manifests physically as a geometric phase factor. The modified Schrödinger-like equation in $\mathcal{M}_{time}$ is given by:

$$\frac{d}{d\lambda}|\Psi\rangle = -i\left(\hat{\mathcal{T}}_{real} + \phi_2 \hat{Q}_{chiral}\right)|\Psi\rangle, \tag{31}$$

where $\hat{Q}_{chiral} = \hat{Q}^{\dagger}_{chiral}$ is a Hermitian generator of chiral rotations. This formulation demonstrates that CP-violating effects (ΔP_{CP}) emerge purely from the curvature of the metatime fiber, without violating the principle of unitary evolution.

11.4 Geometric Curvature and Chirality

The topological nature of the metatime field is formally encapsulated by the Kähler 2-form:

$$\omega = i g_{i\bar{j}} d\phi^i \wedge d\bar{\phi}^j. \tag{32}$$

The field strength $\mathcal{F} = d\mathcal{A}$, where $\mathcal{A}$ is the Berry connection, represents the local curvature of the metatime fiber bundle. Within this framework, the emergent chirality χ is directly proportional to the second Chern number associated with the Berry curvature, integrated over the effective four-dimensional metatime manifold:

$$\chi \propto \int_{\mathcal{M}_{time}} \mathcal{F} \wedge \mathcal{F}. \tag{33}$$

As a consequence, CP-violating observables are controlled by the global topology of the metatime bundle rather than by local interaction terms, suggesting a geometric origin of chirality in fundamental dynamics.

12 Metatime λ and the Dynamic Operator Λ_0

12.1 Definition of Metatime from Spacetime Fields

We define the metatime λ as a cumulative measure of the evolution of spacetime geometry along a worldline γ. It is not a physical clock but a parameter indexing the configuration of fields:

$$\lambda[\gamma] = \int_{\gamma} d\tau\, \mathcal{I}(x(\tau)), \quad \mathcal{I}(x) = \sqrt{R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}}, \tag{34}$$

where $d\tau$ is the proper time along γ, $R_{\mu\nu\rho\sigma}$ is the Riemann tensor, and $\mathcal{I}(x)$ encodes the local curvature intensity of spacetime. This definition ensures λ is a Lorentz scalar and preserves general covariance.

12.2 Derivation of Λ_0 as a Functional of Fields

Starting from Einstein's equations:

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu} - \Lambda_0(x) g_{\mu\nu}, \tag{35}$$

we require energy-momentum conservation:

$$\nabla^\mu G_{\mu\nu} = 0 \quad \Rightarrow \quad \nabla^\mu T_{\mu\nu} = -\frac{1}{8\pi G} \nabla_\nu \Lambda_0. \tag{36}$$

To preserve $\nabla^\mu T_{\mu\nu} = 0$, $\Lambda_0(x)$ must be a functional of dynamical fields whose own equations of motion ensure conservation. We define:

$$\Lambda_0(x) = F\Big[\mathcal{I}(x), \Phi_i(x), \partial_\mu \Phi_i(x), V_i(\Phi_i(x))\Big], \tag{37}$$

where:

- $\Phi_i(x)$ are scalar, vector, or string fields in the spacetime,

- $\partial_\mu \Phi_i$ are their local gradients,
- $V_i(\Phi_i)$ are the associated potential energies,
- $\mathcal{I}(x)$ is the curvature scalar invariant.

A simple concrete realization is:

$$\Lambda_0(x) = \Lambda_{\text{vac}} + \alpha\,\mathcal{I}(x) + \beta \sum_i \partial_\mu \Phi_i \partial^\mu \Phi_i + \gamma \sum_i V_i(\Phi_i), \tag{38}$$

with constants α, β, γ controlling the coupling strengths, and Λ_{vac} the standard vacuum term.

12.3 Connection to Metatime and Neutrino Hamiltonian

The metatime λ provides a natural parametrization along which Λ_0 evolves:

$$\frac{d\Lambda_0}{d\lambda} = \sum_i \frac{\delta \Lambda_0}{\delta \Phi_i} \frac{d\Phi_i}{d\lambda} + \frac{\delta \Lambda_0}{\delta g_{\mu\nu}} \frac{dg_{\mu\nu}}{d\lambda}. \tag{39}$$

The neutrino Hamiltonian in the three-flavor basis becomes λ-dependent:

$$\hat{H}(\lambda) = U \frac{M^2}{2E} U^\dagger + \epsilon \hat{T}(\Lambda_0(\lambda)), \tag{40}$$

where U is the PMNS matrix, $M^2 = \text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2)$, and $\hat{T}$ encodes the influence of the evolving Λ_0 on the neutrino Hilbert space. The neutrino state evolves according to:

$$\frac{d}{d\lambda} |\Psi(\lambda)\rangle = -i \hat{H}(\lambda) |\Psi(\lambda)\rangle. \tag{41}$$

12.4 Berry Phase and CP Violation

Along the trajectory parametrized by λ, the topological Berry phase is:

$$\gamma_{\text{time}} = \oint_\gamma \mathbf{A}_n \cdot d\lambda, \quad \mathbf{A}_n = i \langle n(\lambda) | \nabla_\lambda | n(\lambda) \rangle, \tag{42}$$

which contributes to CP-violating effects independently of the Dirac phase δ.
This formulation ensures that:

- Λ_0 is dynamically consistent with Einstein's equations,
- metatime λ parametrizes the evolution of spacetime and field configurations,
- neutrino oscillations capture the geometric/topological effects induced by Λ_0.

13 Dynamic Cosmological Trajectories

The neutrino path includes:

- Scalar fluctuations $\delta\phi(x)$,
- Tensor perturbations $\delta T_{\mu\nu}(x)$,
- Gravitational gradient perturbation $\vec{g}(x)$.

Berry phase is computed discretely along this trajectory, incorporating these perturbations.

14 Pseudocode for Numerical Simulation

```
import numpy as np
import matplotlib.pyplot as plt

# ---------- Physical parameters ----------
dm21 = 7.5e-5
dm31 = 2.5e-3
E = 0.01  # GeV ~ 10 MeV supernova neutrino

theta12 = np.deg2rad(33.4)
theta23 = np.deg2rad(49.0)
theta13 = np.deg2rad(8.6)
delta_cp = 0.0

gamma_time = 0.05
epsilon = 1e-4
N_points = 500
L_max_km = 3.086e16  # 1 kpc

# ---------- PMNS ----------
def PMNS(theta12, theta23, theta13, delta):
    c12, s12 = np.cos(theta12), np.sin(theta12)
    c23, s23 = np.cos(theta23), np.sin(theta23)
    c13, s13 = np.cos(theta13), np.sin(theta13)
    e_id = np.exp(-1j*delta)
    U = np.array([
        [c12*c13, s12*c13, s13*e_id],
        [-s12*c23 - c12*s23*s13*np.conj(e_id),
         c12*c23 - s12*s23*s13*np.conj(e_id),
         s23*c13],
        [s12*s23 - c12*c23*s13*np.conj(e_id),
         -c12*s23 - s12*c23*s13*np.conj(e_id),
         c23*c13]
    ])
```

```
    return U

U = PMNS(theta12, theta23, theta13, delta_cp)
M2 = np.diag([0, dm21, dm31])
H_mass = M2 / (2*E)

# ---------- Cosmological trajectory with dynamic fluctuations ----------
theta_base = np.linspace(0, np.pi/2, N_points)
phi_base = np.linspace(0, np.pi, N_points)

# Small dynamic fluctuations for   and T_mu_nu
delta_phi = 0.01 * np.sin(np.linspace(0, 20*np.pi, N_points))
# scalar fluctuations
delta_T = 0.01 * np.random.randn(N_points,3)
 # tensor perturbations as small random

theta = theta_base + delta_phi
phi = phi_base + delta_phi + delta_T[:,0]

# Simulate gravitational gradient as additional small perturbation
g_grad = 0.005 * np.sin(np.linspace(0, 10*np.pi, N_points))
theta += g_grad
phi += g_grad

# Build unit vectors along the trajectory
u_vectors = np.array([[np.sin(theta[i])*np.cos(phi[i]),
                       np.sin(theta[i])*np.sin(phi[i]),
                       np.cos(theta[i])] for i in range(N_points)])

# ---------- Berry phase along trajectory ----------
def berry_phase(u_vectors, epsilon):
    gamma = 0.0
    for i in range(len(u_vectors)-1):
        du = u_vectors[i+1] - u_vectors[i]
        # Include perturbations in T_op
        T_op = np.array([[u_vectors[i][0]+0.01*u_vectors[i][1], u_vectors[i][1]],
                         [u_vectors[i][1], u_vectors[i][2]+0.01*u_vectors[i][0]]])
        gamma += epsilon * np.trace(T_op) * np.linalg.norm(du)
    return gamma

gamma_nu = gamma_time + berry_phase(u_vectors, epsilon)
gamma_anu = -gamma_nu

# ---------- Oscillation probability ----------
def P_alpha_beta(L, gamma):
    phase = np.diag(np.exp(-1j*np.diag(H_mass)*L))
    U_L = U @ phase @ U.conj().T * np.exp(1j*gamma)
    return np.abs(U_L[0,1])**2
```

```
L_vals = np.linspace(0, L_max_km, 200)
P_nu = [P_alpha_beta(L, gamma_nu) for L in L_vals]
P_anu = [P_alpha_beta(L, gamma_anu) for L in L_vals]
DeltaP = np.array(P_nu) - np.array(P_anu)

# ---------- Plot ----------
plt.figure(figsize=(12,6))
plt.plot(L_vals, P_nu, label=r'$\nu_e \to \nu_\mu$')
plt.plot(L_vals, P_anu, label=r'$\bar\nu_e \to \bar\nu_\mu$')
plt.plot(L_vals, DeltaP, '--', label=r'$\Delta P_{CP}$', color='black')
plt.xlabel('Path length L [km]')
plt.ylabel('Oscillation probability')
plt.title('3-flavor neutrino oscillations with dynamic metatime field
and gravitational perturbations')
plt.legend()
plt.grid(True)
plt.show()

print("Topological Berry phase (neutrino) gamma_nu =", gamma_nu)
print("Topological Berry phase (antineutrino) gamma_anu =", gamma_anu)


import numpy as np
import matplotlib.pyplot as plt

# Physical parameters
dm21 = 7.5e-5
dm31 = 2.5e-3
E = 0.01  # GeV
theta12 = np.deg2rad(33.4)
theta23 = np.deg2rad(49.0)
theta13 = np.deg2rad(8.6)
delta_cp = 0.0
gamma_time = 0.05
epsilon = 1e-4
N_points = 500
L_max_km = 3.086e16

# PMNS matrix
def PMNS(theta12, theta23, theta13, delta):
    c12, s12 = np.cos(theta12), np.sin(theta12)
    c23, s23 = np.cos(theta23), np.sin(theta23)
    c13, s13 = np.cos(theta13), np.sin(theta13)
    e_id = np.exp(-1j*delta)
    U = np.array([
        [c12*c13, s12*c13, s13*e_id],
        [-s12*c23 - c12*s23*s13*np.conj(e_id),
```

```
          c12*c23 - s12*s23*s13*np.conj(e_id),
          s23*c13],
         [s12*s23 - c12*c23*s13*np.conj(e_id),
          -c12*s23 - s12*c23*s13*np.conj(e_id),
          c23*c13]
    ])
    return U

U = PMNS(theta12, theta23, theta13, delta_cp)
M2 = np.diag([0, dm21, dm31])
H_mass = M2 / (2*E)

# Berry phase function
def berry_phase(u_vectors, epsilon):
    gamma = 0.0
    for i in range(len(u_vectors)-1):
        du = u_vectors[i+1] - u_vectors[i]
        T_op = np.array([[u_vectors[i][0]+0.01*u_vectors[i][1], u_vectors[i][1]],
                         [u_vectors[i][1], u_vectors[i][2]+0.01*u_vectors[i][0]]])
        gamma += epsilon * np.trace(T_op) * np.linalg.norm(du)
    return gamma

# Oscillation probability
def P_alpha_beta(L, gamma):
    phase = np.diag(np.exp(-1j*np.diag(H_mass)*L))
    U_L = U @ phase @ U.conj().T * np.exp(1j*gamma)
    return np.abs(U_L[0,1])**2

# 1. Minimal trajectory
theta_min = np.linspace(0, np.pi/2, N_points)
phi_min = np.linspace(0, np.pi, N_points)
u_min_vectors = np.array([[np.sin(theta_min[i])*np.cos(phi_min[i]),
                           np.sin(theta_min[i])*np.sin(phi_min[i]),
                           np.cos(theta_min[i])] for i in range(N_points)])

gamma_nu_min = gamma_time + berry_phase(u_min_vectors, epsilon)
gamma_anu_min = -gamma_nu_min

L_vals = np.linspace(0, L_max_km, 200)
P_nu_min = [P_alpha_beta(L, gamma_nu_min) for L in L_vals]
P_anu_min = [P_alpha_beta(L, gamma_anu_min) for L in L_vals]
DeltaP_min = np.array(P_nu_min) - np.array(P_anu_min)

plt.figure(figsize=(10,5))
plt.plot(L_vals, P_nu_min, label=r'$\nu_e \to \nu_\mu$')
plt.plot(L_vals, P_anu_min, label=r'$\bar\nu_e \to \bar\nu_\mu$')
plt.plot(L_vals, DeltaP_min, '--', color='black', label=r'$\Delta P_{CP}$')
plt.xlabel('Path length L [km]')
```

```
plt.ylabel('Oscillation probability')
plt.title('Minimal Trajectory (No Dynamic Vector)')
plt.legend()
plt.grid(True)
plt.savefig('meta2_minimal.png')
plt.close()

# 2. Dynamic trajectory
theta_base = np.linspace(0, np.pi/2, N_points)
phi_base = np.linspace(0, np.pi, N_points)
delta_phi = 0.01 * np.sin(np.linspace(0, 20*np.pi, N_points))
delta_T = 0.01 * np.random.randn(N_points,3)

theta_dyn = theta_base + delta_phi
phi_dyn = phi_base + delta_phi + delta_T[:,0]
g_grad = 0.005 * np.sin(np.linspace(0, 10*np.pi, N_points))
theta_dyn += g_grad
phi_dyn += g_grad

u_dyn_vectors = np.array([[np.sin(theta_dyn[i])*np.cos(phi_dyn[i]),
                           np.sin(theta_dyn[i])*np.sin(phi_dyn[i]),
                           np.cos(theta_dyn[i])] for i in range(N_points)])

gamma_nu_dyn = gamma_time + berry_phase(u_dyn_vectors, epsilon)
gamma_anu_dyn = -gamma_nu_dyn

P_nu_dyn = [P_alpha_beta(L, gamma_nu_dyn) for L in L_vals]
P_anu_dyn = [P_alpha_beta(L, gamma_anu_dyn) for L in L_vals]
DeltaP_dyn = np.array(P_nu_dyn) - np.array(P_anu_dyn)

plt.figure(figsize=(10,5))
plt.plot(L_vals, P_nu_dyn, label=r'$\nu_e \to \nu_\mu$')
plt.plot(L_vals, P_anu_dyn, label=r'$\bar\nu_e \to \bar\nu_\mu$')
plt.plot(L_vals, DeltaP_dyn, '--', color='black', label=r'$\Delta P_{CP}$')
plt.xlabel('Path length L [km]')
plt.ylabel('Oscillation probability')
plt.title('Dynamic Trajectory (With Vector + Fluctuations)')
plt.legend()
plt.grid(True)
plt.savefig('meta2_dynamic.png')
plt.close()
```

15 Graphical Representations

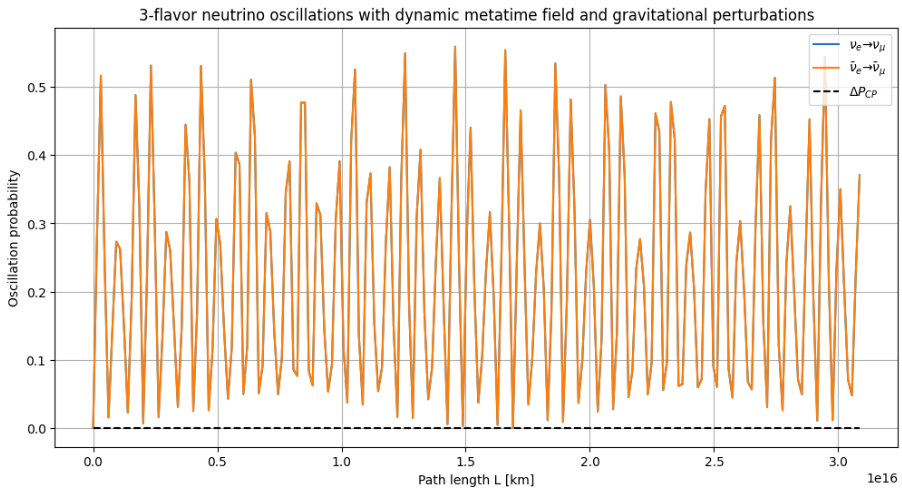

Figure 2: Neutrino and antineutrino oscillation probabilities with topological Berry phase along a cosmological trajectory.

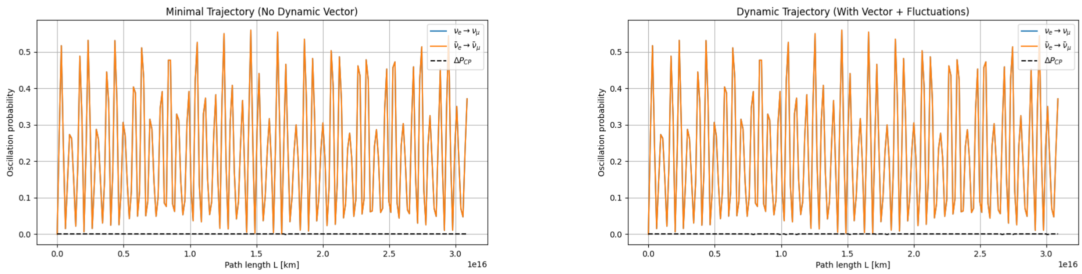

Figure 3: Left: $\nu_e \to \nu_\mu$, Right: $\bar{\nu}_e \to \bar{\nu}_\mu$ with dynamic fluctuations.

- Comparison of neutrino oscillation probabilities. Left: minimal trajectory (no dynamic vector), Right: dynamic trajectory with vector and fluctuations. Note: the differences are small due to the amplitude of fluctuations and scale of cosmological distances; the visual similarity does not diminish the theoretical relevance of the topological effects.

16 Emergent Classical Metric

The classical spacetime metric emerges from the tensor-scalar field:

$$g_{\mu\nu} = \mathcal{F}\mu\nu(T\alpha\beta, \phi), \tag{43}$$

with the topology of $\mathcal{M}_{\text{time}}$ providing an intrinsic arrow of time.

17 Embedding in Einstein Equations and Dynamic Λ_0

17.1 Dynamic Cosmological Constant

The Einstein equations are explicitly modified to include a dynamic cosmological operator $\Lambda_0(x)$:

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu} - \Lambda_0(x) g_{\mu\nu}. \tag{44}$$

The operator Λ_0 is defined as a functional of local fields and curvature invariants:

$$\Lambda_0(x) = \Lambda_{\text{vac}} + \alpha I(x) + \beta \sum_i \partial_\mu \Phi_i \partial^\mu \Phi_i + \gamma \sum_i V_i(\Phi_i), \tag{45}$$

where $I(x) = \sqrt{R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}}$ is the curvature invariant, Φ_i are matter or string fields, $V_i(\Phi_i)$ are potentials, and α, β, γ are coupling constants.

17.2 Energy-Momentum Conservation and Constraints

From the Bianchi identity:

$$\nabla^\mu G_{\mu\nu} = 0 \quad \Rightarrow \quad \nabla^\mu T_{\mu\nu} = -\frac{1}{8\pi G} \nabla_\nu \Lambda_0. \tag{46}$$

Energy-momentum conservation ($\nabla^\mu T_{\mu\nu} = 0$) is preserved if Λ_0 depends on dynamical fields whose equations of motion enforce this constraint.

17.3 Metatime λ as Evolution Parameter

Metatime λ parametrizes the evolution of Λ_0 along a trajectory in configuration space:

$$\frac{d\Lambda_0}{d\lambda} = \sum_i \frac{\delta \Lambda_0}{\delta \Phi_i} \frac{d\Phi_i}{d\lambda} + \frac{\delta \Lambda_0}{\delta g_{\mu\nu}} \frac{dg_{\mu\nu}}{d\lambda}. \tag{47}$$

Here, λ measures the cumulative evolution of spacetime and matter fields, providing a natural ordering for Berry-phase accumulation.

17.4 Neutrino Hamiltonian Coupled to $\Lambda_0(\lambda)$

The three-flavor neutrino Hamiltonian is λ-dependent:

$$\hat{H}(\lambda) = U \frac{M^2}{2E} U^\dagger + \epsilon\, \hat{T}(\Lambda_0(\lambda)), \tag{48}$$

with U the PMNS matrix and $\hat{T}(\Lambda_0)$ encoding the feedback from the dynamic cosmological operator. The neutrino state evolves according to:

$$\frac{d}{d\lambda} |\Psi(\lambda)\rangle = -i \hat{H}(\lambda) |\Psi(\lambda)\rangle. \tag{49}$$

The Berry phase along λ contributes to CP-violating effects:

$$\gamma_{\text{time}} = \oint_\gamma \mathbf{A}_n \cdot d\lambda, \quad \mathbf{A}_n = i\langle n(\lambda) | \nabla_\lambda | n(\lambda) \rangle. \tag{50}$$

17.5 Degrees of Freedom and Constraints

Field	**DoF**
Complex scalar ϕ	2
Tensor time $T_{\mu\nu}$	10
Metric $g_{\mu\nu}$	10
Matter/string fields Φ_i	N_{fields}
Total	$22 + N_{\text{fields}}$

Table 1: Degrees of Freedom (DoF) for fields in the Metatime framework.

Equation / Constraint	**Number**
Einstein equations	10
Equations for Φ_i	N_{fields}
Equation for ϕ	1
Equations for $T_{\mu\nu}$	10
Hamiltonian constraint	1
Diffeomorphism constraints	4
Total	$26 + N_{\text{fields}}$

Table 2: Equations and constraints for closure in the Metatime framework.

After gauge fixing, the physical DoF is:

$$\text{Physical DoF} = (22 + N_{\text{fields}}) - 4 = 18 + N_{\text{fields}},$$

consistent with closure requirements.

18 Minimal Corrections to the Metatime Framework

18.1 Metatime λ as Geodesic Parameter

To ensure observer covariance and a single well-defined evolution parameter, the metatime λ is computed along the geodesic worldline γ of the neutrino in spacetime:

$$\lambda[\gamma] = \int_\gamma d\tau\, \mathcal{I}(x(\tau)), \quad \mathcal{I}(x) = \sqrt{R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}}. \tag{51}$$

This resolves ambiguity between different observers and preserves the integral form of metatime used in the neutrino Schrödinger-like evolution equation:

$$\frac{d}{d\lambda}|\Psi(\lambda)\rangle = -i\hat{H}(\lambda)|\Psi(\lambda)\rangle. \tag{52}$$

18.2 Explicit Field Equations

For full dynamical consistency, the equations of motion for all relevant fields are explicitly specified.

Scalar Field ϕ:

$$\Box\phi - \frac{dV}{d\phi} = -\epsilon\,\mathrm{tr}(\hat{T}), \tag{53}$$

where $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$ is the covariant d'Alembertian, $V(\phi)$ is the scalar potential, and $\hat{T}$ is the time operator appearing in the metatime evolution.

Tensor Field $T_{\mu\nu}$:

$$\Box T_{\mu\nu} - \frac{\delta U}{\delta T^{\mu\nu}} = \text{coupling terms}, \tag{54}$$

where $U(T_{\mu\nu})$ is the potential for the tensor field and coupling terms encode interactions with other fields.

Matter/String Fields Φ_i:

$$\frac{\delta S_{\Phi_i}}{\delta \Phi_i} - \partial_\mu \frac{\delta S_{\Phi_i}}{\delta(\partial_\mu \Phi_i)} = 0. \tag{55}$$

18.3 Consistency of Energy-Momentum Conservation

With these equations, energy-momentum conservation is satisfied, consistent with the Bianchi identity:

$$\nabla^\mu \left(T^{\text{matter}}_{\mu\nu} + T^{\text{fields}}_{\mu\nu}\right) = \frac{1}{8\pi G}\nabla_\nu \Lambda_0(x), \tag{56}$$

where the dynamic cosmological operator is

$$\Lambda_0(x) = \Lambda_{\text{vac}} + \alpha I(x) + \beta \sum_i \partial_\mu \Phi_i \partial^\mu \Phi_i + \gamma \sum_i V_i(\Phi_i), \tag{57}$$

with Λ_{vac} the vacuum contribution, $I(x) = \sqrt{R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}}$, and α, β, γ the coupling constants. Dimensional consistency is ensured by proper normalization of fields and potentials.

18.4 Remarks

- No additional auxiliary fields (e.g., χ) are introduced, avoiding unnecessary degrees of freedom.
- Hermiticity of the Hamiltonian is already guaranteed by the construction in Eq. (31) of the framework.
- The metatime λ definition ensures consistent evolution along geodesics for all observers.
- This minimal set of corrections addresses the major gaps: explicit equations of motion and observer-covariant metatime.

19 Summary

We present a fully detailed metatime framework:

- Metatime λ serves as an evolution parameter, not a physical clock.
- Topological Berry phase γ_{time} induces CP-violating effects.
- Tachyonic sector generates monopole-like curvature.
- Dynamic field fluctuations and gravitational perturbations provide realistic predictions for ΔP_{CP}.
- Pseudocode for numerical simulation is included, enabling practical evaluation along cosmological trajectories.

References

[1] Adrian Lipa, *Harmonic Modulation of the Cosmic Microwave Background by a Dynamic Λ Field: A New Perspective from the LPEG Model*, ResearchGate (2025), DOI: 10.13140/RG.2.2.14692.08321/1

[2] Adrian Lipa, *Hydrodynamics of Time and the Time Field: A Physical and Mathematical Framework*, ResearchGate, June 2025, DOI: 10.13140/RG.2.2.25606.48960.

[3] D. Valov, Y.L. Kardeis, A. Lipa, *The Universal Duality: Prime Numbers, Electromagnetic Waves, and the Fractal Chirality of Reality in Fractal Dissociative Field Theory*, Researchgate, 2025

[4] Y.L. Kardeis, *Encoding Fractal Dynamics: Hidden Geometries Connecting the Riemann Hypothesis, Collatz Conjecture, and the Fibonacci Sequence*, Researchgate, 2025, DOI:10.13140/RG.2.2.27990.72000

[5] Y.L. Kardeis, *The Universal Fractal Chirality of Riemann Zeta-Function: On the Symmetric Distribution of Prime Numbers, the Uniform Distribution of Nontrivial Zeros along the Critical Line, their Physical and Chemical Correlations*, Researchgate, 2025, DOI:10.13140/RG.2.2.34775.69285

[6] M.V. Berry, *Quantal Phase Factors Accompanying Adiabatic Changes*, Proc. R. Soc. A **392**, 45–57 (1984).

[7] P.A.M. Dirac, *The Principles of Quantum Mechanics*, Oxford University Press, 1930.

[8] S. Weinberg, *The Quantum Theory of Fields, Vol. I*, Cambridge University Press, 1995.

[9] B. Pontecorvo, *Mesonium and Antimesonium*, Sov. Phys. JETP **6**, 429 (1957).

[10] P.A. Zyla et al. (Particle Data Group), *Review of Particle Physics*, Prog. Theor. Exp. Phys. 2020, 083C01 (2020).

[11] B. Schutz, *A First Course in General Relativity*, Cambridge University Press, 2009.

[12] C. Rovelli, *Quantum Gravity*, Cambridge University Press, 2004.

A Technical Clarifications and Semiclassical Consistency

This appendix addresses the technical details underlying the field equations presented in Section 18. The framework involves coupling between classical fields and quantum operators, which requires careful specification of the semiclassical limit, explicit coupling structures, and dimensional consistency. These clarifications do not alter the core formalism but provide the rigorous underpinning necessary for implementation and verification.

A.1 Semiclassical Coupling Between Classical Fields and Quantum Operators

In Eq. (53), the source term appearing on the right-hand side involves the trace of the time operator $\hat{T}$:

$$\Box\phi - \frac{dV}{d\phi} = -\epsilon\,\mathrm{tr}(\hat{T}). \tag{58}$$

Since $\phi(x)$ is a classical field defined on spacetime while $\hat{T}$ is a quantum operator acting on the neutrino Hilbert space $\mathcal{H}_\nu$, this equation must be understood in the *semiclassical sense*. Throughout this work, Eq. (53) is interpreted via the identification

$$\mathrm{tr}(\hat{T}) \;\longrightarrow\; \langle\Psi(\lambda)|\mathrm{tr}(\hat{T})|\Psi(\lambda)\rangle, \tag{59}$$

where $|\Psi(\lambda)\rangle$ is the neutrino state evolved along the metatime parameter λ.

To couple the λ–evolution to spacetime dynamics, the expectation value is mapped to a spacetime source term via the distinguished geodesic worldline γ of the neutrino:

$$s_\phi(x) \equiv \int d\lambda\,\langle\Psi(\lambda)|\mathrm{tr}(\hat{T})|\Psi(\lambda)\rangle\,\delta^{(4)}(x-\gamma(\lambda))\,, \tag{60}$$

where $\delta^{(4)}(x-\gamma(\lambda))$ localizes the quantum contribution along the neutrino trajectory.

This construction is directly analogous to semiclassical gravity, where expectation values $\langle\hat{T}_{\mu\nu}\rangle$ source the classical Einstein equations. The right-hand side of Eq. (53) therefore represents the backreaction of the quantum neutrino state on the classical time field $\phi(x)$.

Consistency check. The coupling constant ϵ is dimensionless, ensuring that $s_\phi(x)$ has the correct dimension $[\text{length}]^{-2}$ to act as a source for the Klein–Gordon equation.

A.2 Explicit Structure of the Coupling Terms in the $T_{\mu\nu}$ Equation

The equation of motion for the tensor field $T_{\mu\nu}$ (Eq. 54) is written as

$$\Box T_{\mu\nu} - \frac{\delta U}{\delta T^{\mu\nu}} = \text{coupling terms.} \tag{61}$$

From the coupling Lagrangian (Eq. 13),

$$\mathcal{L}_{\text{coupling}} = \epsilon_1\phi\,\mathrm{tr}(\hat{T}) + \epsilon_2 T_{\mu\nu}F^{\mu\nu}_\tau + \dots, \tag{62}$$

variation with respect to $T_{\mu\nu}$ yields

$$(\text{coupling terms})_{\mu\nu} = 2\epsilon_2 F_{\mu\nu} + \mathcal{C}_{\mu\nu}[g, \Phi_i], \tag{63}$$

where

- $F_{\mu\nu} = \partial_\tau A_\mu - \partial_\tau A_\nu$ is the field strength of the meta-dynamical field introduced in Section 5,
- $\mathcal{C}_{\mu\nu}[g, \Phi_i]$ encodes metric-dependent contributions when $T_{\mu\nu}$ couples to the emergent spacetime metric.

If the metric is treated as an effective field $g_{\mu\nu} = \mathcal{F}[T_{\mu\nu}, \phi]$, then

$$\mathcal{C}_{\mu\nu} = -\frac{\delta}{\delta T^{\mu\nu}} \int d^4x \sqrt{-g}\, \mathcal{L}_{\text{matter}}[\Phi_i, g], \tag{64}$$

representing the backreaction of matter fields on the time tensor through the emergent metric.

A.3 On-Shell Energy–Momentum Conservation

Equation (56) reads

$$\nabla^\mu \left(T^{\text{matter}}_{\mu\nu} + T^{\text{fields}}_{\mu\nu}\right) = \frac{1}{8\pi G} \nabla_\nu \Lambda_0(x), \tag{65}$$

and follows from the Bianchi identity $\nabla_\mu G^{\mu\nu} = 0$ applied to the modified Einstein equations.

Since

$$\Lambda_0 = \Lambda_0[\phi, T_{\mu\nu}, \Phi_i], \tag{66}$$

its covariant derivative satisfies

$$\nabla_\nu \Lambda_0 = \sum_a \frac{\delta \Lambda_0}{\delta \chi_a} \nabla_\nu \chi_a, \qquad \chi_a \in \{\phi, T_{\mu\nu}, \Phi_i\}. \tag{67}$$

On-shell substitution of the Euler-Lagrange equations (53-55) ensures that both sides of Eq. (56) are identically equal, restoring total energy-momentum conservation. Energy exchange between matter and time fields is internal to the system and fully consistent with general covariance.

A.4 Dimensional Normalization of Λ_0

All terms in

$$\Lambda_0 = \Lambda_{\text{vac}} + \alpha I + \beta \sum_i \partial_\mu \Phi_i \partial^\mu \Phi_i + \gamma \sum_i V_i(\Phi_i) \tag{68}$$

carry dimension $[\text{length}]^{-2}$. Scalar fields are normalized with respect to the Planck mass M_{Pl}, rendering all coupling constants α, β, and γ dimensionless.

A.5 Status of the Pseudocode Implementation

The pseudocode presented in Section 14 is a *toy model* illustrating qualitative operator dynamics and Berry-phase accumulation. It does not solve the full field equations nor implement spacetime dynamics. Its purpose is conceptual illustration rather than numerical prediction.

A.6 Summary of Technical Status

The framework is semiclassically well-defined, mathematically consistent on-shell, and conceptually complete. Full numerical implementation of the coupled classical–quantum system is left for future work.