CSE 12 – Basic Data Structures and Object-Oriented Design
Lecture 13

Greg Miranda and Paul Cao, Winter 2021

This lecture is being recorded

Announcements

- Quiz 13 due Monday @ 8am
- Survey 5 due tonight @ 11:59pm
- PA5 due Wednesday @ 11:59pm

Topics

- Partition/Sort
- Questions on Lecture 13?

Quicksort: Another magical (recursive) algorithm

https://www.youtube.com/watch?v=ywWBy6J5gz8

14	4	9	12	15	8	19	2

Select a **pivot** element:

14	4	9	**12**	15	8	19	2

"Partition" the elements in the array (smaller or equal to pivot, larger or equal to pivot)

2	4	9	8	15	**12**	19	14

Magically sort the smaller elements and the larger elements (Quicksort)

pivot value → 12
pivot index → 4

2	4	8	9	12	15	19	21

< >

```
Quicksort(numbers, lowIndex, highIndex) {
   if (lowIndex >= highIndex) {
      return
   }

   lowEndIndex = Partition(numbers, lowIndex, highIndex)
   Quicksort(numbers, lowIndex, lowEndIndex)
   Quicksort(numbers, lowEndIndex + 1, highIndex)
}
```

There are many ways to partition!

```
Partition(numbers, lowIndex, highIndex) {
   // Pick middle element as pivot
   midpoint = lowIndex + (highIndex - lowIndex) / 2
   pivot = numbers[midpoint]

   done = false
   while (!done) {
      // Increment lowIndex while numbers[lowIndex] < pivot
      while (numbers[lowIndex] < pivot) {
         lowIndex += 1
      }

      // Decrement highIndex while pivot < numbers[highIndex]
      while (pivot < numbers[highIndex]) {
         highIndex -= 1
      }

      // If zero or one elements remain, then all numbers are
      // partitioned. Return highIndex.
      if (lowIndex >= highIndex) {
         done = true
      }
      else {
         // Swap numbers[lowIndex] and numbers[highIndex]
         temp = numbers[lowIndex]
         numbers[lowIndex] = numbers[highIndex]
         numbers[highIndex] = temp

         // Update lowIndex and highIndex
         lowIndex += 1
         highIndex -= 1
      }
   }

   return highIndex
}
```

Quick sort

inclusive

high

0 1 2 3 4 5 6 7

sort {12, 4, 9, 3, 15, 8, 19, 2}

pivot index = 3

pivot value = 3

lowIndex = 0 1 2 3 4

highIndex = 7

midpoint = 3

```
Partition(numbers, lowIndex, highIndex) {
   // Pick middle element as pivot
   midpoint = lowIndex + (highIndex - lowIndex) / 2
   pivot = numbers[midpoint]

   done = false
   while (!done) {
      // Increment lowIndex while numbers[lowIndex] < pivot
      while (numbers[lowIndex] < pivot) {
         lowIndex += 1
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      }

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      if (lowIndex >= highIndex) {
         done = true
      }
      else {
         // Swap numbers[lowIndex] and numbers[highIndex]
         temp = numbers[lowIndex]
         numbers[lowIndex] = numbers[highIndex]
         numbers[highIndex] = temp

         // Update lowIndex and highIndex
         lowIndex += 1
         highIndex -= 1
      }
   }

   return highIndex
}
```

Quick Sort Details

pivot = 5

1. We always pick the middle location as pivot
2. The data we sort is {2, 3, 1, 5, 4, 6, 7}

After the first split, what is the order of elements in the list that was <= pivot?

A. 1 2 3 4

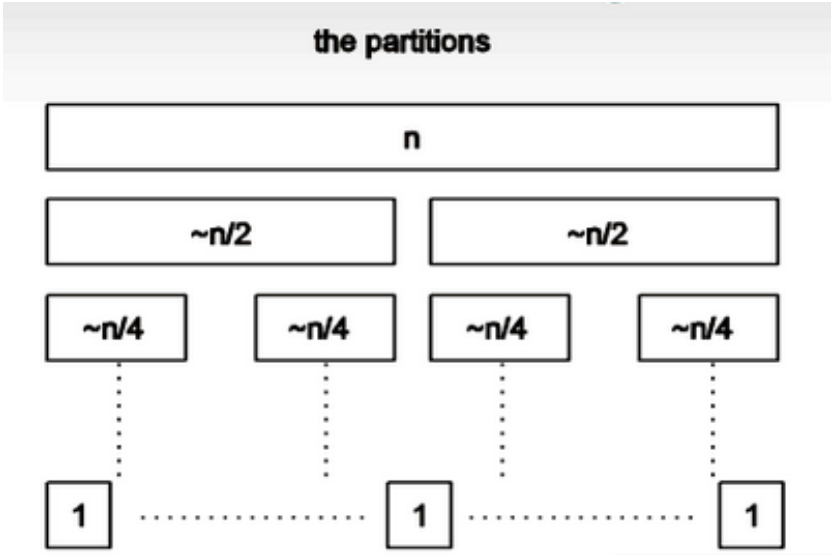
B. 2 3 1 4

C. 4 3 2 1

D. 3 4 1 2

E. None of the above

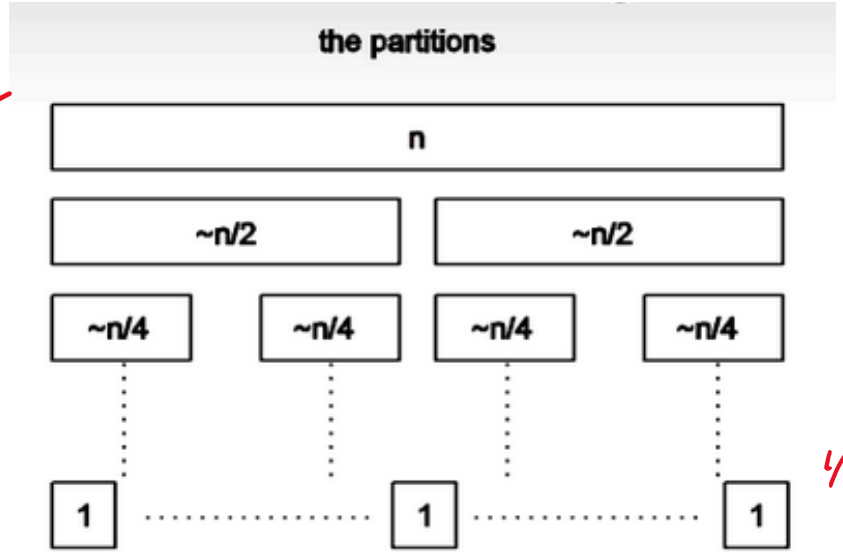
Quick Sort: Using a "good" pivot



How many levels will there be if you choose a pivot that divides the list in half?

A. 1
B. log(N)
C. N
D. N*log(N)
E. N^2

Quick Sort: Using a "good" pivot



If the time to partition on each level takes N comparisons, how long does Quicksort take with a good partition?

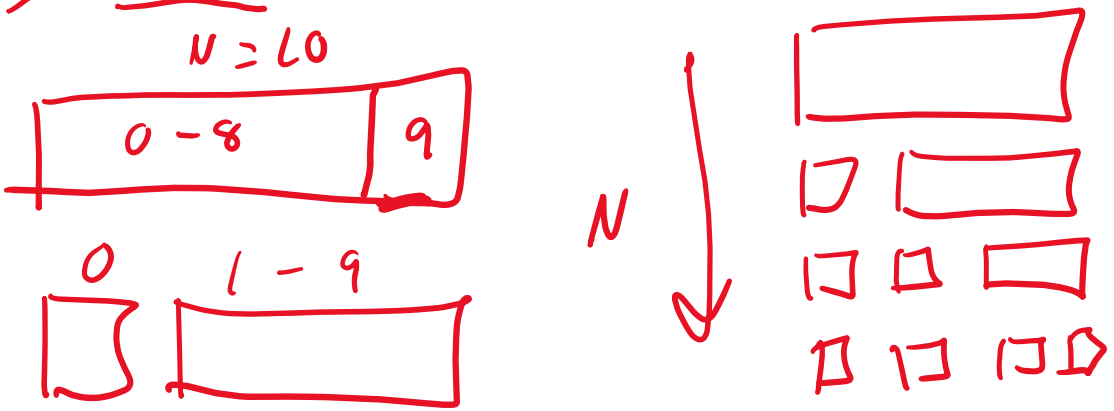
A. O(1)
B. O(log(N))
C. O(N)
D. O(N*log(N))
E. O(N^2)

$\log_2 N$

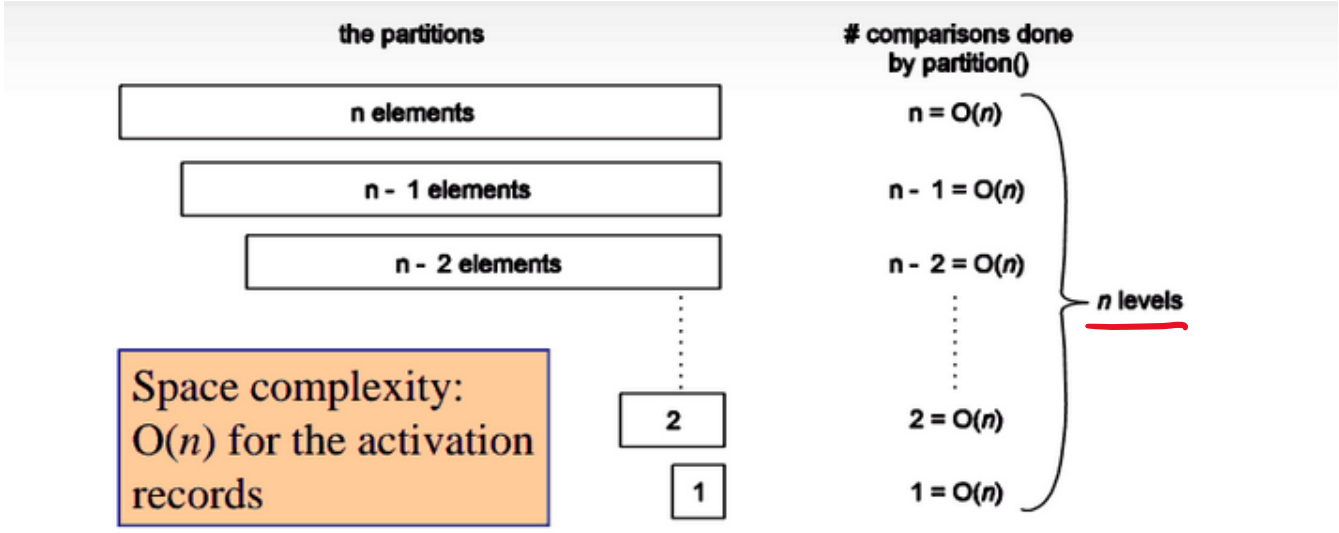
$N * \log_2(N)$

Which of these choices would be the *worst* choice for the pivot?

A. The minimum element in the list
B. The last element in the list
C. The first element in the list
D. A random element in the list



Quick sort with a bad pivot



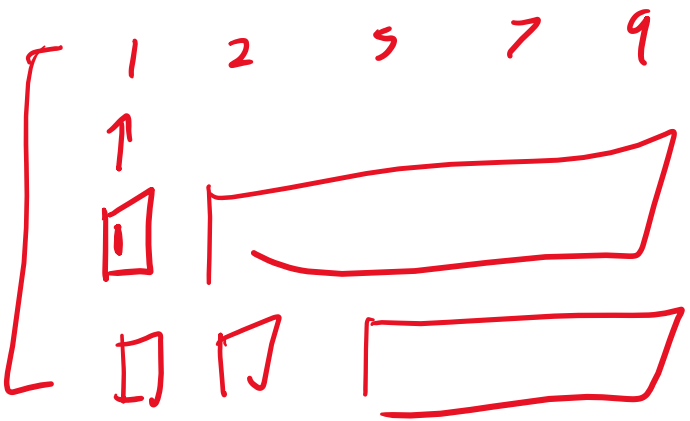
If the pivot always produces one empty partition and one with $n - 1$ elements, there will be n levels, each of which requires $O(n)$ comparisons: $O(n^2)$ time complexity

Which of these choices is a better choice for the pivot?

0 A. The first element in the list → bad for sorted array

10 B. A random element in the list

4/1 C. They are about the same



worst case N^2

$\log_2(100) = 8$

$N = 100$

$\frac{1}{100} + \frac{1}{99} + \frac{1}{98} + \frac{1}{97} \ldots$

QuickSort – Draw the picture of sort()

```
public class Sort {
  public static void swap(String[] array, int i1, int i2) {
    String temp = array[i1];
    array[i1] = array[i2];
    array[i2] = temp;
  }
  public static int partition(String[] array, int low, int high) {
    int pivotStartIndex = high - 1;
    String pivot = array[pivotStartIndex];
    int smallerBefore = low, largerAfter = high - 2;

    while (smallerBefore <= largerAfter) {
      if (array[smallerBefore].compareTo(pivot) < 0) {
        smallerBefore += 1;
      }
      else {
        swap(array, smallerBefore, largerAfter);
        largerAfter -= 1;
      }
    }

    swap(array, smallerBefore, pivotStartIndex);
    return smallerBefore;
  }
```

```
  public static void qsort(String[] array, int low, int high) {
    if (high - low <= 1) { return; }
    int splitAt = partition(array, low, high);
    qsort(array, low, splitAt);
    qsort(array, splitAt + 1, high);
  }

  public static void sort(String[] array) {
    qsort(array, 0, array.length);
  }

main() {
  String[] str = {"f", "b", "a", "e", "d", "c" };
  int[] result = Sort.sort(str);
  System.out.println(Arrays.deepToString(result));
}
```