

CSE 12 Week 5 Discussion

2-2-21

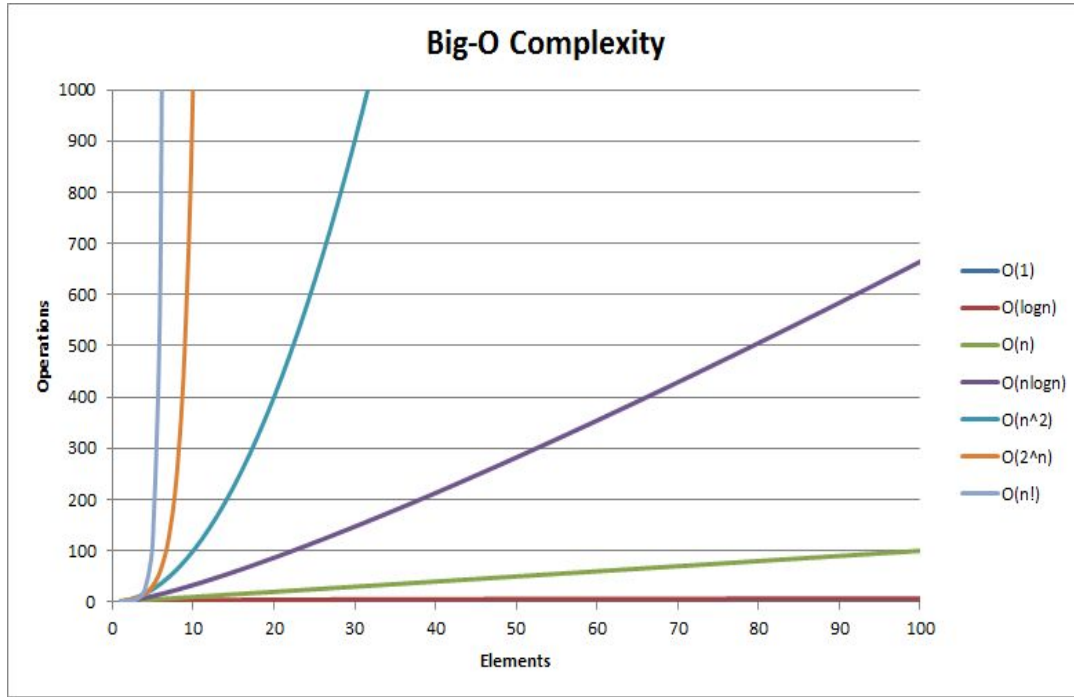
Focus: Sorting

Reminders

- PA5 is a **closed** assignment - no collaborating!
- PA2 Resubmission due Friday, February 5th 11:59 PM
- PA3 Resubmission due Friday, February 12th 11:59 PM

Sorting

Big-O review



- Relative to input n
- Constants do not matter
 - $O(3n) = O(n)$
- Higher order values dominate
 - $O(n^2 + n) = O(n^2)$

Algorithms review

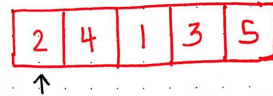
Which algorithm finds the minimum element in a list and moves it to the end of a sorted prefix in the list?

- a. Selection sort
- b. Insertion sort
- c. `insert()`
- d. `transform()`

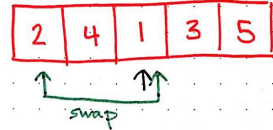
Answer - A (selection sort)

Simplified Selection Sort:

Our smallest number starts off as the first number - whatever it is.

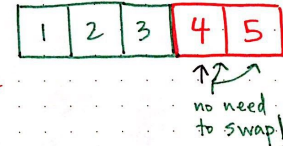
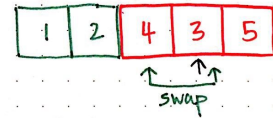
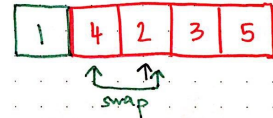


We'll iterate through the whole dataset until we find the actual smallest number. Then, we'll swap it to be in the 1st position.

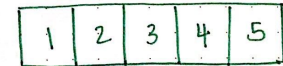


We'll continue this process:

- 1/ find smallest unsorted number,
- 2/ swap it to switch places with the unsorted number at the front of the list,
- 3/ do the same with the next number.



Eventually, we'll end up with a totally sorted dataset!!



Selection sort

```
public static void sSort(int[] arr) {  
    for(int i = 0; i < arr.length; i += 1) {  
        System.out.print(Arrays.toString(arr) + " -> ");  
        int minIndex = i;  
        for(int j = i; j < arr.length; j += 1) {  
            if(arr[minIndex] > arr[j]) { minIndex = j; }  
        }  
        int temp = arr[i];  
        arr[i] = arr[minIndex];  
        arr[minIndex] = temp;  
        System.out.println(Arrays.toString(arr));  
    }  
}
```

What is the worst case runtime for selection sort?

- a. $O(n^2)$
- b. $O(n)$
- c. $O(n \log n)$
- d. $o(n!)$
- e. None of the above

Answer - A

```
public static void sSort(int[] arr) {  
    for(int i = 0; i < arr.length; i += 1) {  
        System.out.print(Arrays.toString(arr) + " -> ");  
        int minIndex = i;  
        for(int j = i; j < arr.length; j += 1) {  
            if(arr[minIndex] > arr[j]) { minIndex = j; }  
        }  
        int temp = arr[i];  
        arr[i] = arr[minIndex];  
        arr[minIndex] = temp;  
        System.out.println(Arrays.toString(arr));  
    }  
}
```

In worst case, outer loop is order of $O(n)$ and inner loop is the order of $O(n)$.

$$O(n) * O(n) = O(n^2)$$

Selection sort

```
public static void sSort(int[] arr) {  
    for(int i = 0; i < arr.length; i += 1) {  
        System.out.print(Arrays.toString(arr) + " -> ");  
        int minIndex = i;  
        for(int j = i; j < arr.length; j += 1) {  
            if(arr[minIndex] > arr[j]) { minIndex = j; }  
        }  
        int temp = arr[i];  
        arr[i] = arr[minIndex];  
        arr[minIndex] = temp;  
        System.out.println(Arrays.toString(arr));  
    }  
}
```

What is the best case runtime for selection sort?

- a. $O(n^2)$
- b. $O(n)$
- c. $O(n \log n)$
- d. $O(n!)$
- e. None of the above

Answer - A

```
public static void sSort(int[] arr) {  
    for(int i = 0; i < arr.length; i += 1) {  
        System.out.print(Arrays.toString(arr) + " -> ");  
        int minIndex = i;  
        for(int j = i; j < arr.length; j += 1) {  
            if(arr[minIndex] > arr[j]) { minIndex = j; }  
        }  
        int temp = arr[i];  
        arr[i] = arr[minIndex];  
        arr[minIndex] = temp;  
        System.out.println(Arrays.toString(arr));  
    }  
}
```

Even in best case, outer loop runs through all iterations while going through entire inner loop each time

$$O(n) * O(n) = O(n^2)$$

Selection sort

```
public static void sSort(int[] arr) {  
    for(int i = 0; i < arr.length; i += 1) {  
        System.out.print(Arrays.toString(arr) + " -> ");  
        int minIndex = i;  
        for(int j = i; j < arr.length; j += 1) {  
            if(arr[minIndex] > arr[j]) { minIndex = j; }  
        }  
        int temp = arr[i];  
        arr[i] = arr[minIndex];  
        arr[minIndex] = temp;  
        System.out.println(Arrays.toString(arr));  
    }  
}
```

How can we optimize selection sort?

- a. Start inner loop at j=0 instead of j=i
- b. Swap elements inside the inner loop
- c. Make outer loop bound arr.length-1 instead of arr.length
- d. Start outer loop at i=1 instead of i=0
- e. Cannot be optimized any further for correct solution

Answer - C

```
public static void sSort(int[] arr) {  
    for(int i = 0; i < arr.length-1; i += 1) {  
        System.out.print(Arrays.toString(arr) + " -> ");  
        int minIndex = i;  
        for(int j = i; j < arr.length; j += 1) {  
            if(arr[minIndex] > arr[j]) { minIndex = j; }  
        }  
        int temp = arr[i];  
        arr[i] = arr[minIndex];  
        arr[minIndex] = temp;  
        System.out.println(Arrays.toString(arr));  
    }  
}
```

Last element (arr[arr.length-1]) must already be in correct place, so cut down algorithm by 1 iteration

Cannot compare values with other indices because at end of list anyways

Selection sort

```
public static void sSort(int[] arr) {  
    for(int i = 0; i < arr.length-1; i += 1) {  
        System.out.print(Arrays.toString(arr) + " -> ");  
        int minIndex = i;  
        for(int j = i; j < arr.length; j += 1) {  
            if(arr[minIndex] > arr[j]) { minIndex = j; }  
        }  
        int temp = arr[i];  
        arr[i] = arr[minIndex];  
        arr[minIndex] = temp;  
        System.out.println(Arrays.toString(arr));  
    }  
}
```

By changing outer loop bound to `arr.length-1`, what is the new worst case runtime of selection sort?

- a. $O(n^2)$
- b. $O(n)$
- c. $O(n \log n)$
- d. $O(n!)$
- e. None of the above

Answer - A

```
public static void sSort(int[] arr) {  
    for(int i = 0; i < arr.length-1; i += 1) {  
        System.out.print(Arrays.toString(arr) + " -> ");  
        int minIndex = i;  
        for(int j = i; j < arr.length; j += 1) {  
            if(arr[minIndex] > arr[j]) { minIndex = j; }  
        }  
        int temp = arr[i];  
        arr[i] = arr[minIndex];  
        arr[minIndex] = temp;  
        System.out.println(Arrays.toString(arr));  
    }  
}
```

No change in runtime (still $O(n^2)$ for worst and best cases)

Reducing outer loop by 1 iteration results in $O(n-1)$ for outer loop, which is same as $O(n)$

Inner loop still runs $O(n)$

$O(n) * O(n) = O(n^2)$

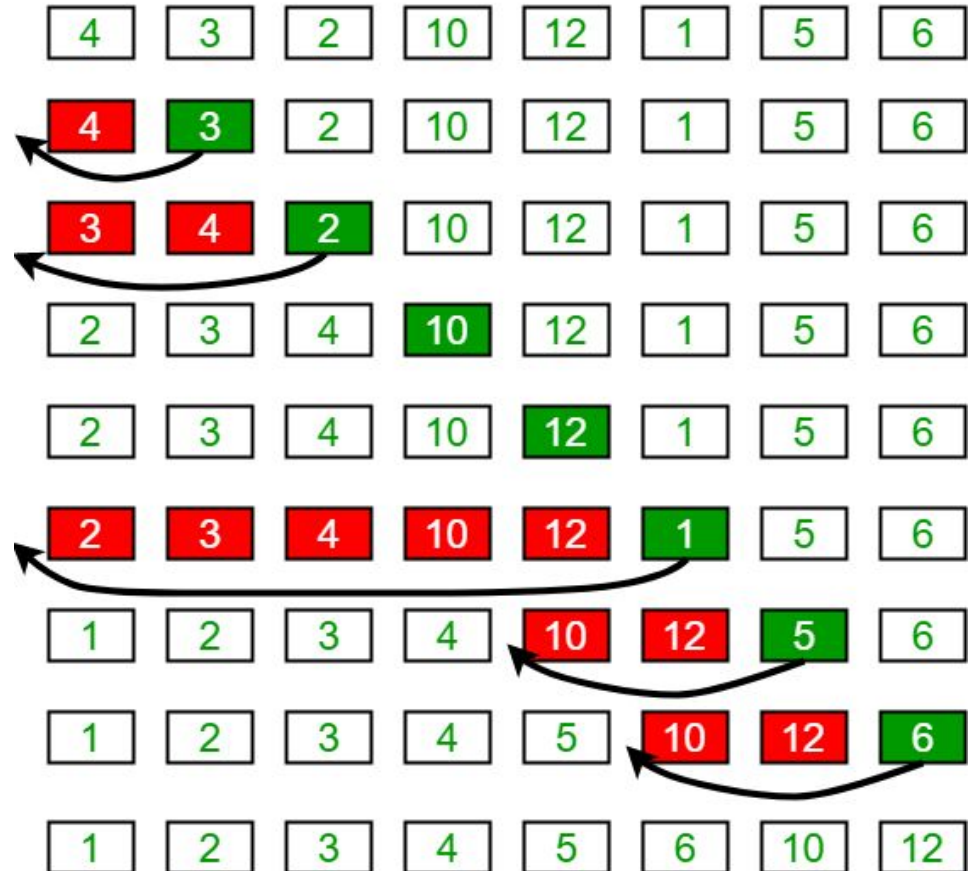
Algorithms Review

Which algorithm repeatedly takes the next element in a list inserts it into the correct ordered position within a sorted prefix of the list?

- a. Selection sort
- b. Insertion sort
- c. insert()
- d. transform()

Answer - B
(insertion sort)

Insertion Sort Execution Example



Insertion sort

```
public static void iSort(int[] arr) {  
    for(int i = 0; i < arr.length; i += 1) {  
        System.out.print(Arrays.toString(arr) + " -> ");  
        for(int j = i; j > 0; j -= 1) {  
            if(arr[j] < arr[j-1]) {  
                int temp = arr[j-1];  
                arr[j-1] = arr[j];  
                arr[j] = temp;  
            }  
        }  
        System.out.println(Arrays.toString(arr));  
    }  
}
```

What is the worst case runtime for insertion sort?

- a. $O(n^2)$
- b. $O(n)$
- c. $O(n \log n)$
- d. $O(n!)$
- e. None of the above

Answer - A

```
public static void iSort(int[] arr) {  
    for(int i = 0; i < arr.length; i += 1) {  
        System.out.print(Arrays.toString(arr) + " -> ");  
        for(int j = i; j > 0; j -= 1) {  
            if(arr[j] < arr[j-1]) {  
                int temp = arr[j-1];  
                arr[j-1] = arr[j];  
                arr[j] = temp;  
            }  
        }  
        System.out.println(Arrays.toString(arr));  
    }  
}
```

In worst case, outer loop is order of $O(n)$ and inner loop is the order of $O(n)$.

$$O(n) * O(n) = O(n^2)$$

Insertion sort

```
public static void iSort(int[] arr) {  
    for(int i = 0; i < arr.length; i += 1) {  
        System.out.print(Arrays.toString(arr) + " -> ");  
        for(int j = i; j > 0; j -= 1) {  
            if(arr[j] < arr[j-1]) {  
                int temp = arr[j-1];  
                arr[j-1] = arr[j];  
                arr[j] = temp;  
            }  
        }  
        System.out.println(Arrays.toString(arr));  
    }  
}
```

What is the best case runtime for insertion sort?

- a. $O(n^2)$
- b. $O(n)$
- c. $O(n \log n)$
- d. $O(n!)$
- e. None of the above

Answer - A

```
public static void iSort(int[] arr) {  
    for(int i = 0; i < arr.length; i += 1) {  
        System.out.print(Arrays.toString(arr) + " -> ");  
        for(int j = i; j > 0; j -= 1) {  
            if(arr[j] < arr[j-1]) {  
                int temp = arr[j-1];  
                arr[j-1] = arr[j];  
                arr[j] = temp;  
            }  
        }  
        System.out.println(Arrays.toString(arr));  
    }  
}
```

Even in best case, outer loop runs through all iterations while going through entire inner loop each time

$$O(n) * O(n) = O(n^2)$$

Insertion sort

```
public static void iSort(int[] arr) {  
    for(int i = 0; i < arr.length; i += 1) {  
        System.out.print(Arrays.toString(arr) + " -> ");  
        for(int j = i; j > 0; j -= 1) {  
            if(arr[j] < arr[j-1]) {  
                int temp = arr[j-1];  
                arr[j-1] = arr[j];  
                arr[j] = temp;  
            }  
        }  
        System.out.println(Arrays.toString(arr));  
    }  
}
```

How can we optimize insertion sort?

- Compare `arr[j]` to `arr[j-2]` also
- Break out of inner loop if `arr[j]` is `>= arr[j-1]`
- Make outer loop bound `arr.length-1` instead of `arr.length`
- Break out of outer loop if `arr[j]` is `>= arr[j-1]`
- Cannot be optimized any further for correct solution

Answer - B

```
public static void iSort(int[] arr) {  
    for(int i = 0; i < arr.length; i += 1) {  
        System.out.print(Arrays.toString(arr) + " -> ");  
        for(int j = i; j > 0; j -= 1) {  
            if(arr[j] < arr[j-1]) {  
                int temp = arr[j-1];  
                arr[j-1] = arr[j];  
                arr[j] = temp;  
            }  
            else {  
                break;  
            }  
        }  
        System.out.println(Arrays.toString(arr));  
    }  
}
```

Once we see that elements are in the right order, we do not need to look at earlier elements since they will already be ordered

Insertion sort

```
public static void iSort(int[] arr) {  
    for(int i = 0; i < arr.length; i += 1) {  
        System.out.print(Arrays.toString(arr) + " -> ");  
        for(int j = i; j > 0; j -= 1) {  
            if(arr[j] < arr[j-1]) {  
                int temp = arr[j-1];  
                arr[j-1] = arr[j];  
                arr[j] = temp;  
            }  
            else {  
                break;  
            }  
        }  
        System.out.println(Arrays.toString(arr));  
    }  
}
```

By adding the else statement with a break, what is the new worst case runtime of insertion sort?

- a. $O(n^2)$
- b. $O(n)$
- c. $O(n \log n)$
- d. $O(n!)$
- e. None of the above

Answer - A

```
public static void iSort(int[] arr) {  
    for(int i = 0; i < arr.length; i += 1) {  
        System.out.print(Arrays.toString(arr) + " -> ");  
        for(int j = i; j > 0; j -= 1) {  
            if(arr[j] < arr[j-1]) {  
                int temp = arr[j-1];  
                arr[j-1] = arr[j];  
                arr[j] = temp;  
            }  
            else {  
                break;  
            }  
        }  
        System.out.println(Arrays.toString(arr));  
    }  
}
```

No change in runtime (still $O(n^2)$ for worst case)

Will not call break statement in worst case because entire list is out of order

Inner and outer loops each run $O(n)$

$O(n) * O(n) = O(n^2)$

Insertion sort

```
public static void iSort(int[] arr) {  
    for(int i = 0; i < arr.length; i += 1) {  
        System.out.print(Arrays.toString(arr) + " -> ");  
        for(int j = i; j > 0; j -= 1) {  
            if(arr[j] < arr[j-1]) {  
                int temp = arr[j-1];  
                arr[j-1] = arr[j];  
                arr[j] = temp;  
            }  
            else {  
                break;  
            }  
        }  
        System.out.println(Arrays.toString(arr));  
    }  
}
```

By adding the else statement with a break, what is the new best case runtime of insertion sort?

- a. $O(n^2)$
- b. $O(n)$
- c. $O(n \log n)$
- d. $O(n!)$
- e. None of the above

Answer - B

```
public static void iSort(int[] arr) {  
    for(int i = 0; i < arr.length; i += 1) {  
        System.out.print(Arrays.toString(arr) + " -> ");  
        for(int j = i; j > 0; j -= 1) {  
            if(arr[j] < arr[j-1]) {  
                int temp = arr[j-1];  
                arr[j-1] = arr[j];  
                arr[j] = temp;  
            }  
            else {  
                break;  
            }  
        }  
        System.out.println(Arrays.toString(arr));  
    }  
}
```

Best case: sorted list

As a result, will call break statement each time inner loop is entered

Inner loop runs $O(1)$

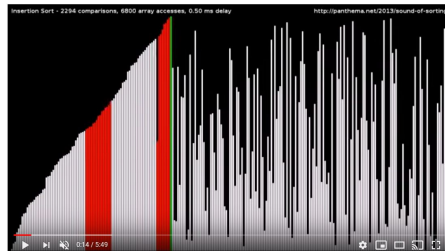
Outer loop still runs $O(n)$

$O(n) * O(1) = O(n)$

Sorting Algorithm Visualizations

WARNING: flashing lights

<https://www.youtube.com/watch?v=kPRA0W1kECg>



If you do not have a hearing sensitivity, we recommend having the volume on during these visualizations. Lower pitches correspond to operations on smaller bars/values, and higher pitches correspond to operations on larger bars/values. Check out the top left corner for the current sort method and number of certain operations.

The first 4 sort methods are the 4 you've learned so far: insertion, selection, quick, and merge sort.

Note on bogo sort: this is a bit of a joke in CS. Bogo sort works by randomly moving all of the values until it finds the sorted solution by chance; it is the last sort method in the video. It has run time upper bound: $O((n+1)!)$